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CONCEPTUALISING TRANSLATIONS BETWEEN REPRESENTATIONS
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Representations and translations between them are central in mathematics education. For example, in the NCTM standards it is emphasized students need to be able to “select, apply, and translate among mathematical representations to solve problems” (NCTM 2000, p.67). A variety of research studies have contributed to the knowledge about translations the last decades. This variety is both an asset and an obstacle when this research is used to implement new strategies in the school practice or as a base to plan new research studies. To enable an accumulation of the emerging knowledge there is a need to categorize studies that focus on similar questions and that conceptualizes translation similarly. The current paper suggests some classifications that such a categorization can be based on in an emerging framework.

INTRODUCTION

In school mathematics students can be engaged in translations between representations, between modes, or between semiotic resources. The translation can be part of a solution process or the task given in a test. In short, translation can refer to different processes in different contexts and this comprehensive use of the term calls a need to clarify. The purpose of the current paper is to present an emerging framework for the study of a concept used with slightly different meanings: translations. Such a framework is useful both as a structure to categorize previous research and when planning studies about translations. It is suggested future studies are more explicit about how, where, and why the translation of interest is conducted. The paper as a whole contributes to the research field on translations by highlighting some important concepts, by incorporating those concepts in structure useful as a backbone to a framework on translations, and by relating the structure to Pierce’s theory on signs.

TERMINOLOGY

Representations

The term representation is used in a variety of ways and the conceptualisation of the term contributes to what is meant by translations between representations. Von Glasersfeld (1987) argues the term representation is fuzzy. He gives four distinct different meanings of the term. Representation can refer to i) depicting something like a flower, ii) to mentally represent something, iii) to denote an unknown quantity, and iv) for a person to replace someone. The first three are representations that can be translated and the distinction made is therefore important for how a particular translation is conceptualized. A difference between i) and ii) is that in the first case there is something
existing that are represented but in the second case there is no prior object that is re-presented. In iii) the representation is pure mental. It applies to the use of a representation such as $x$ as a substitute for something, a common usage in mathematics texts. It is obvious the translations are of different types depending on if a representation is a depiction of a visual object (i) or an abstract substitution for something (iii). Besides this difference the representation can be depicted in different ways something that also affect how a particular translation can be understood. The next section addresses this issue of kinds of representations.

Semiotic resources

In mathematics education research one way to distinguish between types of semiotic resources used in printed text is the divide between natural language, images, and mathematical notation (see e.g., O’Halloran, 2008; Dyrvold, 2016). The different semiotic resources are often interwoven in printed text, for example in images with words and mathematical notation. Therefore referring to these different parts of a printed text as representations may obscure the intricacy in how they are composed, and thus, the distinction between different semiotic resources is useful. Natural language consists of letters that constitute the words and sentences. For images the function of the ‘marks’ are of a much bigger variety. A line in an image can represent a similar line that exists in reality, such as the horizon, but the line can also represent a continuous pattern of dots representing $y=x$. For mathematical notation there is also a large variety in how the ‘marks’ shall be read. Pimm (1987) distinguishes four types of symbols used as mathematical notation: logograms that represent whole concepts (e.g., $\pi$ and $\div$), pictograms (e.g., $||$ and $\angle$), letters (e.g., $\mu$ and $\beta$), and punctuation marks (e.g., ! and ]). Letters, punctuation marks and logograms, are all truly symbolic in that they are arbitrarily chosen and there is a necessary agreement that they shall stand for something. Pictograms on the other hand are iconic since they refer to another item by means of sensory motor similarity (see von Glasersfeld, 1987) something that is not the case with non-iconic signs.

The differences between representations in the group images as well as mathematical notation is worth to consider in research about translations in mathematics since the variety in for example how an image is represented means the translations from/to images are of different types. A translation between a mathematical expression with an iconic sign such as $\angle$ and a pictorial image of the angle of a soffit is a translation between representations that partly uses the same means to represent. Therefore the translation can be perceived as less pervading than if the representations differ more.

Multimodality and multisemiotics

Two concepts of importance for a particular use of the term translations are multimodality and multisemiotics. Selander and Kress (2010) explain the communication is multimodal when different modes are used in communication. The variety of resources that make the communication multimodal can for example be objects, gestures, words
and symbols. That is, different types of media and signs. O’Halloran (2008) addresses the problem of terminology in studies of multimodality. She emphasizes that the use of the terms mode and semiotic causes confusion, especially in the terms multisemiotic and multimodal. O’Halloran suggests the term mode is used for the channel used to represent, for example auditory, visual or tactile. Medium on the other hand may be used to refer to the material resources for the channel, for example a radio or a newspaper. Semiotic is used to refer to the semiotic resources that have unique grammatical systems through which they are organized, for example visual images, natural language, and mathematical notation (O’Halloran, 2008). With these distinctions multisemiotic is used for texts with more than one semiotic resource and multimodal for discourses with more than one mode of semiosis. A website can be both multimodal (visual and auditory) and multisemiotic (natural language and visual images) but the medium is one, the screen.

In research about translations the conceptualisation of a particular communicative act or representation as multisemiotic or multimodal can have big impact on what translation means in that context. Since the terms multimodal and multisemiotic is used in such a variety of ways, knowing that for example a classroom study focus on multimodal translation does not tell exactly what is studied. It is important when a study is reported and for readers to be aware of the need to communicate and to carefully read what the translation actually concerns.

**Summing up about terminology**

The request to translate can be communicated through different media in different modes and a particular representation can consist of several different semiotic resources. Therefore the term translation can refer to a plethora of different communicative acts. If essential terms are clearly defined more is revealed about the translation but as part of an emerging framework on translation is valuable also to focus on where something is translated, how the translation emerges and why something is translated. These issues are addressed in next section.

**TRANSLATIONS**

In mathematics a concept, a mathematical object, or a process can often be represented in different ways. Janvier, Girardon, and Morand (1993) suggest it is in such cases translations between representations are possible. The avoidance to define translation in an exhaustive way in the current paper is deliberate since some of the issues raised here correspond to different definitions of translation. A broad definition of translation is Kress’ (2010) description of translation as a shift in meaning that can occur for example between modes, genres or cultures. In many occasions however, it is of necessity to distinguish exactly what kind of translation that is of interest and where the potential boundaries between different types of translations are.

**Where**

One issue that distinguishes translations is where the translation takes place. This
question depends on how translations are defined. Both Kress (2010) and Duval (2006) do from distinct different theoretical perspectives separate between different types of translations. Kress suggests the term *transformation* for translations without change in mode. One mode can for example be words and another images. A translation between different images is with Kress terminology a transformation, but if the translation occur between an image and words Kress would categorize it as transduction. A similar division is made by Duval (2006). This divide is relevant since images, mathematical notation, and natural language differ in grammar and syntax and representations such as gestures have other means that in many ways differ from those that occur in print. In addition there is representations that is visualised on computer screens, which have the means of movement and 3D.

The grammar and syntax of natural language and mathematical notation are similar in some issues; for example the decoding from left to right, and the use of parentheses to signal order. A 3D image on screen and an image in print also share parts of the grammar for how it shall be interpreted. Taking that into account, it seems reasonable to assume that a translation between natural language in print and a 3D object on screen entails a larger change than for example a translation between natural language and mathematical notation, both in print. Thus, referring to both as just translations obscure how different the two are.

**How**

Another issue in relation to translations is the question whether any active reformulation take place, the question of *how* the translation is present. The question posed is whether the translation/s addressed is something requested by the representations in the act of interpretation or whether an active agent perform the translation when a new representation is created. This variety is also, implicitly, evident from Kress’ description of translation, in the use of both ‘shift in meaning’ and ‘meaning is moved’. The former indicate existence, and the latter some action (Kress, 2010). ‘Meaning is moved’ indicate that meaning is actively moved between representations whereas ‘shift in meaning’ may refer to what the text offer. This distinction between active/passive translations is not posed by Kress, presumably since in his view the text is partly created by the reader in the reading process.

The point here is what this issue may mean for the phenomenon that is studied. It is possible to distinguish two different foci when the issue of *how* is considered: translations can be i) performed by the reader when offered by the representations, or ii) performed by the reader when a representation that translates meaning posed in another representation is created. The difference is noteworthy because there are different things that are studied with these different conceptualisations of translations. The first type of translations is focused in many studies on the relation between a function and its graph (e.g., Zazkis, Liljedahl, & Gadovsky, 2003). In such studies the phenomena studied is the translation process as part of the reading. In the other type of studies students actively construct translated representations. A difference between these two
types is also the object of study. In studies on translations as part of the reading process the translation is done mentally, compared to when a representation is constructed and therefore possible to visually evaluate.

**Why**

At last, there is also a question of *why* a translation is conducted. This issue is related to the question of how; that is, which kind of activity does the student engage in. The question of why regards an agent who does the translation but this issue regard *why* the translation is needed. This issue is significant in relation to what is actually studied since being asked to translate between representations evokes a reading behaviour that is potentially different than if the translation is needed in the reading of a text but that is not explicitly said. Several studies on translations analyses success rate for particular groups of participants who are *asked to* translate between representations (e.g., Capraro & Joffrin, 2006). If the tasks used are of that type the issue studied is whether students are able to correctly translate when asked.

If not evoked by a request to translate, translations may be conducted as part of the solution process. Translations can for example be needed in the interpretation of the mathematics. The answer to *why* a translation is conducted may also be that the solution of a problem demands a translation. Consider the task *Leo and Ben started reading the same book on Monday. Leo read four pages a day and Ben read nine pages a day. What page was Ben on when Leo was on page one hundred?*. In the solution of such a task, the presentation of the solution would reasonably utilize mathematical notation and therefore the solver translates the problem. In addition, the more complex a task is the more likely it is that several different translations are needed. Since mathematical problems often are presented using several representations and several semiotic resources, the interpretation as well as the solution of a task may result in several translations done by the solver. Part of the competence in problem solving is actually the ability to choose the most apt representation in the solution.

**The backbone to a framework on translations: a diagram**

The concepts and issues in relation to translations in mathematics mentioned previously are related in a complex pattern. A reminder of the main points and how they are related is given in a diagram (Figure 1). The arrows represent a path of choices. The first choice regard representations and after that each issue concern types of translations (TR). One important point is that it is impossible to study only internal representations, the possible first choice is whether to include also mental representations as part of the object of study, something that will be discussed further in relation to theory on signs. Every new step in the diagram represent two options or more, which means that if every step is taken into account the number of different types of translations are huge. In some instances the choice can also be to include several of the options. In addition, there are other perspectives that are of importance for studies on translations; for example the choice of theoretical perspective.
In the remaining part of the paper Pierce’s theory of signs will be discussed to illustrate the interrelation between theoretical issues and methodical choices. In two instances in Figure 1 a focus on mental representations and processes is emphasized. These options are moved to the rightmost part of the figure to illustrate the relation to a choice of theoretical emphasis; the rightmost vertex of the angle in Figure 2.

**Influence from theory on the object of study in research on translations**

One essential ingredient in translations is the sign, in its most broad definition, and therefore this theoretical discussion takes its starting point in Pierce’s definition of sign. In this paper the italicized ‘sign’ is used to refer to Pierce’s comprehensive notion of sign, which includes three sign components. The first component, the representamen, is what we usually mean by the word “sign” for a symbol as π or an image. An important issue within this theory of signs is the relation between the three components. The representamen exist in relation with the second component, the semiotic object (or semiotically real object). This object is called the semiotic object since our perception of it is never absolute; it is what the sign stands for. The third component of the sign is the interpretant. The interpretant can be understood as the meaning of the sign and it is related to the other two components. Merrell (2001) argues the interpretant “relates to and mediates between the representamen and the semiotic object in such a way as to bring about an interrelation between them at the same time and in the same way that it brings itself into interrelation with them” (p.28). Every component acts as an intermediary between the other two, and thus there is no ‘sign’ if any of the three components is missing since they all are dependent of the other two. This interdependence is visualized in Figure 2. For example, an object such as a *protractor* ex-
ists, but the semiotic object *protractor* is dependent of the interpretant (the agreed meaning) and the representamen (any representation) of a protractor. The complex action or process where the relations between the components are established is called semiosis, also visualised in Figure 2. If we consider that in translations several processes of semiosis are, at least partially, simultaneously present the complexity of translations is very apparent.

![Figure 2: Pierce’s model of the ‘sign’ and the process of semiosis.](image)

In the diagram (Figure 1) choices leading to an emphasis on mental issues are deliberately made visible. We cannot study thoughts, but in research (e.g. about translations) we may concentrate on visual data or on processes of comprehension; for example by analyses of utterances made by the solver. It is useful to reflect over the model of the ‘sign’ (Figure 2) in relation to this difference. Essential is of course that all three sign components are needed because of the interdependence between them, but there is a difference in how pronounced the different sign components are in studies on translations. For example in a study on student’s constructions of solutions requested in a particular semiotic resource (translated), an emphasis on the accuracy of the representation corresponds to a pronounced interest in the representamen. If the emphasis in the study instead is on the student’s construction of understanding in relation to their translated representation this indicates a more pronounced interest in the interpretant. All sign components are in play in both studies but it is valuable, both when reading research and when studies are designed, to evaluate the research interest in relation to the sign components.

The perspective of context is also important in relation to studies on translations, in particular in relation to the sign component *interpretant*. Representations in mathematics may mistakenly be seen as definite and exact but as in all semiotic processes ‘signs’ are construed and re-constured. This aspect is taken into account by Steinbring (2006) whose epistemological triangle can contribute with the perspective of context to a framework on translations.
In summary, representations and relations between them are important to focus on in teaching. Engaging in relations between representations in mathematics means engagement in translations that can differ substantially in what they demand from the student. Therefore it is necessary to implement knowledge gained from research on translations in schools, and also to further study different aspects of translations. A framework on translations would contribute to such a development since it provides a structure that facilitates an accumulative development of the collected results. Hopefully the emerging framework and the issues stressed in the current paper can contribute to a nuanced discussion about the concept translations, and to enlighten some aspects of importance in relation to the developing field.

References


