Comparison of the $pp \rightarrow \pi^+ pn$ and $pp \rightarrow \pi^+ d$ production rates

G. Fälrd, C. Wilkin

Department of Physics and Astronomy, Uppsala University, Box 516, 751 20 Uppsala, Sweden

Physics and Astronomy Department, UCL, Gower Street, London, WC1E 6BT, UK

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Fully constrained bubble chamber data on the $pp \rightarrow \pi^+ pn$ and $pp \rightarrow \pi^+ d$ reactions are used to investigate the ratio of the counting rates for the two processes at low $nn$ excitation energies. Whereas the ratio is in tolerable agreement with that found in a high resolution spectrometer experiment, the angular distribution in the final $nn$ rest frame shows that the deviation from the predictions of final state interaction theory must originate primarily from higher partial waves in the nn system. These considerations might also be significant for the determination of the S-wave $\Lambda p$ scattering length from data on the $pp \rightarrow K^+ \Lambda p$ reaction.

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The COSY-GEM collaboration measured the differential cross section for the production of positive pions in proton–proton collisions at a beam kinetic energy of $T_p = 951$ MeV, detecting the $\pi^+$ at $\theta_{\pi} = 0^\circ$ in the spectrograph Big Karl [1]. The high missing-mass resolution achieved here ($\approx 100$ keV/c²) allowed a very clean separation of the $pp \rightarrow \pi^+ d$ and $pp \rightarrow \pi^+ pn$ channels and also showed that the production of singlet $nn$ pairs was negligible at this energy.

The final state interaction theorem relates the normalisations of the wave functions for S-wave bound and scattering states [2]. This has been exploited to predict the double-differential centre-of-mass (cm) cross section for the S-wave spin-triplet component in $pp \rightarrow \pi^+ pn$ in terms of the cross section for $pp \rightarrow \pi^+ d$ [3]:

$$\frac{d^2\sigma}{d\Omega dx}(pp \rightarrow \pi^+ (pn)_c) = F \frac{p(x)}{p(-1)} \frac{\sqrt{\lambda}}{2\pi(x+1)} \frac{d\sigma}{d\Omega}(pp \rightarrow \pi^+ d).$$ (1)

Here $x$ denotes the excitation energy $Q$ in the $nn$ system in units of the deuteron binding energy $B_1$, $x = Q/B_1$, and $p(x)$ and $p(-1)$ are the pion cm momenta for the $nn$ continuum or deuteron respectively. At the deuteron pole the normalisation $F = 1$ but it was argued [2] that deviations from this should be small at low $x$ if the pion production operator is of short range and the tensor force linking the $S$ and $D$ states in the deuteron could be neglected. However, although the shape of the COSY-GEM data [1] was reasonably well described by Eq. (1) up to an excitation energy of $Q \approx 20$ MeV, reproducing the absolute magnitude required $F = 2.2 \pm 0.1$.

In view of the large discrepancy with the prediction of the final state interaction theorem ($F = 1$), the COSY-GEM experiment was repeated at 400 and 600 MeV [4] which, combined with the results of earlier work carried out at TRIUMF [5], presented a consistent picture. The normalisation factor $F$ was found to increase steadily from below one at 400 MeV to well above unity at 951 MeV. It was suggested that the deviation at the highest energy could arise from the long-range part of the pion production operator associated with the on-shell intermediate pions [4]. Such contributions could change the pn S-wave cross section for the $pp \rightarrow \pi^+ pn$ reaction or excite higher partial waves in the final $nn$ system. In a missing-mass experiment, as carried out by the GEM collaboration [14], it is not possible to investigate these suggestions any further.

Measurements of various channels arising from proton–proton collisions at three energies in the 900 to 1000 MeV range were undertaken using the 35 cm hydrogen bubble chamber of the Petersburg Nuclear Physics Institute (PNPI) [6–8]. Although the statistics in the low $Q$ region are much poorer than those of the COSY-GEM experiment [1,4], and the resolution is far inferior, the acceptance approaches 100% and so the predictions of Eq. (1) can be integrated over the full solid angle. Under the conditions of the PNPI data, the deviation of $p(x)/p(-1)$ from unity is negligible at low $x$ so that the numbers $N$ of bubble chamber events should be linked by...
The data in Fig. 1 are well described by the form
\[ N(pp \to \pi^+pn) = a_0 + a_1 \cos \theta_p + a_2 \cos^2 \theta_p, \]  
where \(a_0 = 27.3 \pm 2.7, a_1 = -1.5 \pm 3.7,\) and \(a_2 = 33.8 \pm 7.1.\) The corresponding value of \(\chi^2/\text{NDF} = 0.27\) is fortuitously low. It is important to note that, as expected, the odd term \(a_1\) is consistent with zero but fixing it to vanish does not change significantly the values of \(a_0\) and \(a_2.\) However, it is not possible to identify whether the large quadratic term arises from the square of a \(pn\) \(P\)-wave or from an \(S - D\) interference, which could be influenced by the \(pn\) tensor force. Departures from isotropy in Fig. 1 are clear evidence for the excitation of higher partial waves but the converse is not true because it is possible to generate a mixture of higher partial waves that leads to an isotropic distribution. Nevertheless, it is likely that the deviations from the final state interaction theorem of Refs. [2,3], shown by the dashed line in Fig. 1, probably come from higher partial waves rather than modifications of the \(S\)-wave intensity.

Apart from complications arising from the \(pn\) tensor force, it is to be expected that Eq. (1) should be a good representation of the \(pp \to \pi^+pn\) data at very small values of \(x.\) Although this is a valid approximation at low incident beam energies, the COSY-GEM experiment shows that there are significant deviations at 951 MeV [1]. By using the fully reconstructed bubble chamber events [6–8], we have confirmed the magnitude of the deviation. However, we have also shown from the angular distribution of Fig. 1 that, with a cut-off at \(Q = 20\) MeV, there are significant contributions from higher partial waves in the \(pn\) system that are not apparent in a missing-mass experiment. This may be due to the anomalously long range of the pion production operator at high energies which was not considered in the application of the final state interaction theorem [2].

The arguments presented here may have wider significance than the specific reaction being studied. By detecting just the \(K^+\) meson in the Big Karl spectograph, the COSY-HIRES group measured the inclusive cross section for the \(pp \to K^+X\) reaction. Below the threshold for \(\Sigma\) production \(X = \Lambda p\) and the hope was that an analysis of the data would allow a determination of the spin-average \(\Lambda p\) \(S\)-wave scattering length [9]. However, in such a single-arm experiment, there can be no confirmation that the \(\Lambda p\) system remains in the \(S\)-wave at finite values of \(Q.\)

Conditions are much more favourable in the COSY-TOF experiment [10,11] where, apart from some loss of acceptance near the beam direction, the final particles in the \(pp \to K^+\Lambda p\) reaction can be detected. The global \(\Lambda p\) angular distributions constructed for beam momenta of 2.7 GeV/c [11] and 2.95 GeV/c [12] both show strong signals arising from higher partial waves in the \(\Lambda p\) system but it is not clear from these plots if there are also effects for \(\Lambda p\) excitation energies below 40 MeV. This clearly has to be checked when attempting to extract the \(S\)-wave \(\Lambda p\) scattering length from the analysis of such experiments [9,10]. The determination of the position of the \(\Lambda p\) virtual state pole [13] is far less affected by these considerations because this is sensitive to the behaviour at very small values of \(Q,\) where the \(S\)-wave assumption is on much firmer grounds.

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References


Table 1

<table>
<thead>
<tr>
<th>(T_p) (MeV)</th>
<th>(N(pp \to \pi^+pn))</th>
<th>(N(pp \to \pi^+d))</th>
<th>(\mathcal{F})</th>
</tr>
</thead>
<tbody>
<tr>
<td>900.2</td>
<td>188</td>
<td>136</td>
<td>2.6 ± 0.3</td>
</tr>
<tr>
<td>940.7</td>
<td>101</td>
<td>77</td>
<td>2.4 ± 0.3</td>
</tr>
<tr>
<td>988.6</td>
<td>97</td>
<td>57</td>
<td>3.1 ± 0.5</td>
</tr>
<tr>
<td>Summed</td>
<td>386</td>
<td>270</td>
<td>2.6 ± 0.2</td>
</tr>
</tbody>
</table>

Fig. 1. Numbers of \(pp \to \pi^+pn\) events with statistical errors measured in the PNPI bubble chamber experiment, summed over three incident beam energies [6–8]. Events, which are shown in 0.2 bins in \(\cos \theta_p,\) are only retained where the excitation energy in the \(pn\) system is below 20 MeV. The solid curve shows the best quadratic fit of Eq. (3) whereas the dashed line is the isotropic \(S\)-wave prediction that follows from the final state interaction theorem [2,3].

\[ N_{x < x_0}(pp \to \pi^+pn) = \frac{\mathcal{F}}{2\pi} N(pp \to \pi^+d) \int_{x_0}^{\infty} \frac{\sqrt{x}}{x + 1} dx = \frac{\mathcal{F}}{\pi} N(pp \to \pi^+d) \left( \sqrt{x_0} - \arctan(\sqrt{x_0}) \right). \]

In order to compare directly the PNPI data with the COSY-GEM result, the bubble chamber \(pp \to \pi^+pn\) events were selected as having a maximum \(pn\) excitation energy of 20 MeV \((x_0 \approx 9).\) The numbers of events fulfilling this criterion, as well as the total number of \(pp \to \pi^+d\) events, are given in Table 1. The values of \(\mathcal{F}\) deduced at the three energies are also shown, as is their average. The weighted average of the PNPI data, \(\mathcal{F} = 2.6 ± 0.2,\) looks higher than the COSY-GEM value of 2.2 ± 0.1 but the errors quoted are only statistical uncertainties. Furthermore, it is possible that the fraction of higher \(pn\) waves could vary with pion angle. However, there might also be an effect due to the poorer resolution in the \(pn\) excitation energy in the bubble chamber data, which allows some higher \(x\) data to distort a little the result.

The big advantage of the fully constrained PNPI \(pp \to \pi^+pn\) data is that they allow one to investigate the angular distributions in the recoiling \(pn\) system.\(^3\) Fig. 1 shows the distribution in the angle of the final proton with respect to the original beam direction in the \(pn\) rest frame. The clear deviation from isotropy is unambiguous evidence for the production of higher partial waves in the final \(pn\) system.

\(^3\) The measured four-vectors in the PNPI \(pp \to \pi^+pn\) experiment at the three different beam energies are listed on the WEB site of the Bonn-Gatchina Partial Wave Analysis group: pwa.hiskp.uni-bonn.de.


