Abstract: When adding a new train to an existing timetable, this often has to be done in a way that gives an efficient train path for the added train, while the existing trains in the timetable only can be shifted in restricted ways. A first step in the addition procedure is to find all the available track capacity that can be used for the train to be added, given its earliest possible departure time and latest possible arrival time. This available track capacity will form one or more capacity corridor from the origin to the destination. In this paper we propose a method for finding all available capacity corridors that can be used when adding the train. The basis of the method is an optimization model, which includes all scheduling restrictions, securing that the capacity corridors are both feasible from a security viewpoint and also feasible with respect to the already planned trains.
1 Introduction

The Swedish Transport Administration is about to implement new methods for the train timetabling process. The goal is a train timetabling process that is adaptable to the market need and more simple for both the operators and the infrastructure planner. The fundamental idea is to only specify details of a train path when these details needs to be known. Thus, the train timetable will gradually become more and more detailed. Today when an operator applies for track access they apply for a train path, i.e. arrival times to and departure times from the departure and arrival station as well as all intermediate stations. All these arrival and departure times are illustrated with the crosses in Figure 1a. From an operator point of view, most of these times are unimportant at the time they apply for a train path. The important times for the operator are the departure times from and arrival times to stations where passengers can start or end their journeys for passenger trains or goods will be loaded or unloaded for freight trains. In the future, these important times for the operator, are enough to apply for in the train timetabling process. The Swedish Transport Administration will then commit to fulfill these important times when "delivering" the train timetable. Thus, these important times are the delivery commitments.

Figure 1b displays the delivery commitment marked with crosses and three possible train paths fulfilling the delivery commitment. Since the departure and arrival times at all intermediate stations are not fixed, there are some options for the timetable planner to plan and make operational adjustments to the train path. As a result the infrastructure manager can plan crossings, overtakings and other details of a train path later than in the Annual train timetabling process and thus the train timetable becomes more adaptable to future applications for track access. Imagine that an operator applies for a train path in the Short term-process. With delivery commitments, there are some flexibility in the train timetable such that the already accepted trains can be moved and fit the new train path. Figure 2a illustrates a case when the train timetable is planned using train paths. Figure 2b illustrates how the introduction of delivery commitments in the train timetabling process eases the problem and makes the train timetable more adaptable. In conclusion, the train timetable can be made more efficient and include more train paths using delivery commitments.
Figure 1: The difference between a train path and a delivery commitment for a train running from A to D. (a) The operator applies for a train path defined by the crosses. The infrastructure manager has only one option for planning the train path (the red solid line). (b) The operator applies for a delivery commitment defined by the crosses. The infrastructure manager has a number of options for planning the train path. Three suggestions are shown (red dashed line).

2 Problem description

When an application for track access arrives and the train timetable consists of train paths, it is clear what the available track capacity for that application is. Since a train path has all arrival and departure times from all stations fixed, all the available track capacity is the empty space in a graphic train timetable where the train path applied for can fit. This is not the case with delivery commitments. The arrival and departure times are not fixed and which trains that wait for the other in interactions is not decided. In Figure 3, the empty space in a graphic train timetable where a delivery commitment application can be planned is illustrated for three different feasible train timetables for the same delivery commitments. The empty space varies in size depending on the train timetable. The question is what the available track capacity is for the delivery commitment application.

We define the available track capacity to be the maximum empty space in a train timetable where the train path for a delivery commitment application can be planned. To find the available track capacity, the empty track capacity is given a mathematical expression. This expression is then maximized using mathematical optimization. The remainder of this report
Figure 2: Difference between a train timetable consisting of train paths and delivery commitments. The track is single track. (a) The blue solid line represents already planned train paths, i.e. all station arrival, departure and passing times are fixed, and the black dashed line represents a train path application. The train path application is impossible to include in the train timetable. (b) The smaller crosses are already planned delivery commitments corresponding to the train path in the same color. Using delivery commitments, only the times marked with a cross are fixed. The train paths are used to ensure that the delivery commitments are possible to plan in a train timetable. The operator applies for delivery commitments marked by the large crosses. The infrastructure planner investigates if the train path would violate the requested delivery commitments given the train paths from the already planned delivery commitments. The application can be included in the train timetable without violating any other delivery commitment.
Figure 3: Three different train paths for the same delivery commitments yield different empty spaces in the train timetable.

describes the mathematical expression for empty track capacity and the optimization model that finds the available track capacity for a delivery commitment application.

3 Finding the available track capacity

The available track capacity for a delivery commitment request, given the already planned delivery commitments, is the largest empty space in a train timetable where a delivery commitment can be scheduled. To make a mathematical model of the available track capacity, we introduce the concept of track capacity corridors. A track capacity corridor is defined as some
track capacity in time and space that is not occupied by any train path from an already planned delivery commitment and on which the train that should operate a delivery commitment request can be planned. Figure 4 illustrates one such track capacity corridor. The red area is the size of the track capacity corridors. We want to find as large timetable space, i.e. union of track capacity corridors, as possible. This is done by formulating an optimization problem. The idea is that the delivery commitment application can be scheduled on the union of all track capacity corridors, which is the available track capacity for the delivery commitment request.

Let $C$ be the set of all track capacity corridors and let $L^d$ be the set of all segments that is used by the train that should operate the delivery commitment request. The track capacity corridor $i \in C$, is defined by a time interval $[h_{i,g}^{\min}, h_{i,g}^{\max}]$, for every station or track section $g \in G_l$ for track segment $l \in L^d$, where $h_{i,g}^{\min}$ and $h_{i,g}^{\max}$ are continuous variables to be determined by the optimization model. As Figure 4 describes, there cannot be any train paths for other delivery commitments in the time interval $[h_{i,g}^{\min}, h_{i,g}^{\max}]$ for all geographic locations $g$, since the track capacity corridors should be free of interactions with other trains. Thus, the union of the track capacity corridors is the available track capacity for the delivery commitment application.

The number of track capacity corridors, i.e. the size of the set $C$ should be high enough to ensure that all the available track capacity is found. This is ensured by adding a track capacity corridor from $C$ and noting that the outcome of the optimization is not changed. The following algorithm is used:

1. Let $n_{cap}$ be any number and let $o^*$ equal to 0.
2. Set the number of track capacity corridors to $n_{cap}$.
3. Solve the optimization model.
4. If the optimal objective value equals $o^*$, all available track capacity has been found and the algorithm terminates. Otherwise, set the number of capacity corridors to $n_{cap} + 1$, set $o^*$ to the optimal objective value and go to step 3.

Figure 5 illustrates this, where the track capacity corridors are the red parallelograms. Starting from only one track capacity corridor in Figure 5a. The optimization model is solved for one track capacity corridor and two track capacity corridors, yielding the result in Figure 5b. The optimal solution has changed when adding one more track capacity corridor, since
Figure 4: A track capacity corridor $i$ between stations A and B, defined by the time intervals $[h_{i,A}^{\text{min}}, h_{i,A}^{\text{max}}]$ and $[h_{i,B}^{\text{min}}, h_{i,B}^{\text{max}}]$ given a requested delivery commitment (red crosses) and two already planned delivery commitments (blue crosses and lines). The track capacity corridor spans some track capacity in space and time in the train timetable that is not occupied by other train paths. The crosses are the delivery commitments and the track capacity corridor must consider the red delivery commitments.

more available track capacity can be found. By adding one more track capacity corridor and solving the optimization model again yields the result in Figure 5c. Even more available track capacity has been found. If one further track capacity corridor is added and the optimization model is solved again in Figure 5d, the found available track capacity does not increase. The number of track capacity corridors does not have to increase even further since the available track capacity can not increase more.

4 Capacity corridor optimization problem

A track capacity corridor is some available track capacity where a feasible train path that fulfills a delivery commitment can be planned. This section describes the constraints added to the optimization problem for finding the track capacity corridors. The constraints for the infrastructure and the already planned delivery commitments is described in [Gestrelius et al., 2015]. These constraints make sure that the trains
Figure 5: The number of track capacity corridors used in the optimization must be high enough to saturate the available track capacity. This means that when adding another track capacity corridor there should not be more track capacity found in the optimization where the train path for the delivery commitment request, marked with red crosses, can be planned. The red parallelograms are the track capacity corridors and the blue crosses and lines are the already planned delivery commitments. (a) One track capacity corridor is used. (b) Two track capacity corridors are used, and the available track capacity has increased from (a). (c) Three track capacity corridors are used and the available track capacity have increased even further than in (b). (d) Four track capacity corridors are used. The available track capacity found are the same as in (c). Thus, the available track capacity is saturated.
do not travel faster than their maximal speed (including acceleration and deceleration), that the delivery commitments are enforced and that the safety regulations regarding single and double track are followed. The rest of this section describes the tack track capacity corridor constraints.

Let $C$ be the set of track capacity corridors. The union of all track capacity corridors represents the track capacity on which a delivery commitment request can be scheduled. The route of the delivery commitment request is split into track segments $\mathcal{L}$. Let $\mathcal{G}_l$ denote the geographic locations in each track segment $l \in \mathcal{L}$. The delivery commitment request constrains the arrival and departure time from some of the geographic locations in $\mathcal{G}_l$. Thus, there are a latest arrival time $l_{i,g}^{\text{max}}$ and an earliest arrival time $l_{i,g}^{\text{min}}$ from the geographic location $g \in \mathcal{G}_l$. Introduce the continuous variables $l_{i,g}^{\text{min}}$ and $l_{i,g}^{\text{max}}$. These variables denote a possible earliest and latest time a train can arrive to the geographic location $g$. Thus, each track capacity corridor $i$ consists of a time interval $[l_{i,g}^{\text{min}}, l_{i,g}^{\text{max}}]$ at every geographic location $g \in \mathcal{G}_l$.

The union of the track capacity corridors should be the available track capacity for a delivery commitment. Thus, the corridors should consider the train that should operate the delivery commitment request. Constraints for continuity, interactions and safety against other trains should thus also be implemented on the track capacity corridors. Therefore, the constraints on the track capacity corridors are a modification to the constraints for the trains introduced in [Gestrelius et al., 2015].

4.1 Continuity constraints

The track capacity corridors should be able to contain the train that should operate the delivery commitments. The variables $h_{i,g}^{\text{min}}$ and $h_{i,g}^{\text{max}}$ must be a time interval $[h_{i,g}^{\text{min}}, h_{i,g}^{\text{max}}]$. Tho achieve this introduce the constraint

$$h_{i,g}^{\text{min}} \leq h_{i,g}^{\text{max}} \quad \forall g \in \mathcal{G}_l, \; l \in \mathcal{L}, \; i \in C. \quad (1)$$

The track capacity corridor must also fulfill the movements of the train that will operate the delivery commitments. Let the continuous variable $\omega_{i,g}$ be the dwell time for this train on station or track segment $g$. If the train would arrive to the geographic location $g \in \mathcal{G}_l$ at the time $h_{i,g}^{\text{min}}$, then the earliest time for the variable $h_{i,g}^{\text{min}}$ is $h_{i,g}^{\text{min}} + \omega_{i,g}$. The same hold for $h_{i,g}^{\text{max}}$. The continuity constraints are
The minimum dwell time on a track section depend on whether the train stops or not on the stations adjacent to the track segment. To include this in the optimization problem, introduce the binary variables $\gamma_{i,s}$ and $\gamma_{i,l}^{\text{both}}$. These variables are defined as

$$
\gamma_{i,s} = \begin{cases} 
1, & \text{if the train for capacity corridor } i \text{ stops at station } s, \\
0, & \text{otherwise.} 
\end{cases} \quad (4)
$$

and

$$
\gamma_{i,l}^{\text{both}} = \begin{cases} 
1, & \text{if the train for capacity corridor } i \text{ stops at both ends of track section } l, \\
0, & \text{otherwise.} 
\end{cases} \quad (5)
$$

Further, let the set $L_i^{SF}$ denote all track sections where the train in the track capacity corridors can stop at the first station and pass the second station. Likewise, $L_i^{FS}$, $L_i^{SS}$, $L_i^{FF}$ denote the sets where the train for the track capacity corridor can pass the first station and stop at the second, stop at both stations and pass both stations, respectively. Let $\omega_{i,l}^{FF}$ be the minimum travel time on a track section when the train does not stop at either ends, $\omega_{i,l}^{SF}$ be the minimum travel time on a track section where the train accelerates from a stop, $\omega_{i,l}^{FS}$ be the minimum travel time on a track section where the train decelerates to a stop and $\omega_{i,l}^{SS}$ be the minimum travel time on a track section where both acceleration and deceleration are included. Let $\omega_{i,s}^{\text{min}}$ be the minimum dwell time as a station and $\omega_{i,s}^{\text{max}}$ be the maximum dwell time at a station. If the minimum dwell time at a station $\omega_{i,s}^{\text{min}}$ is prolonged by the time $\delta_{i,s}$, then the train is assumed to have stopped at the station. The constraints enforcing restrictions on the dwell time is
\( \omega_{i,s} \leq \omega_{i,s} \quad i \in \mathcal{C}, s \in \mathcal{S}, l \in \mathcal{L} \)  
(6)

\( \omega_{i,s} \leq \omega_{i,s}^{\text{min}} + \delta_{i,s} + M\gamma_{i,s} \quad i \in \mathcal{C}, s \in \mathcal{S}, l \in \mathcal{L} \)  
(7)

\( \omega_{i,l}^{\text{FF}} \leq \omega_{i,l} \quad i \in \mathcal{C}, l \in \mathcal{L}_{l}^{\text{FF}}, l \in \mathcal{L} \)  
(8)

\( \omega_{i,l}^{\text{SF}} \gamma_{i,l-1} \leq \omega_{i,l} \quad i \in \mathcal{C}, l \in \mathcal{L}_{l}^{\text{SF}}, l \in \mathcal{L} \)  
(9)

\( \omega_{i,l}^{\text{FS}} \gamma_{i,l+1} \leq \omega_{i,l} \quad i \in \mathcal{C}, l \in \mathcal{L}_{l}^{\text{FS}}, l \in \mathcal{L} \)  
(10)

\( \gamma_{i,l-1} + \gamma_{i,l+1} \leq 1 + \gamma_{i,l}^{\text{both}} \quad i \in \mathcal{C}, l \in \mathcal{L}_{l}^{\text{SS}}, l \in \mathcal{L} \)  
(11)

\( \omega_{i,l}^{\text{SS}} \gamma_{i,l}^{\text{both}} \leq \omega_{i,l} \quad i \in \mathcal{C}, l \in \mathcal{L}_{l}^{\text{SS}}, l \in \mathcal{L} \)  
(12)

\( \omega_{i,g} \leq \omega_{i,g}^{\text{max}} \quad i \in \mathcal{C}, g \in \mathcal{G}, l \in \mathcal{L}. \)  
(13)

Note that \( l - 1 \) and \( l + 1 \) are stations.

The track capacity corridor is also constrained by the delivery commitment request. The constraints that enforce this are

\[
\begin{align*}
    h_{i,g}^{\text{max}} &\leq l_{i,g}^{\text{max}} \quad i \in \mathcal{C}, g \in \mathcal{G}, l \in \mathcal{L} \quad (14) \\
    h_{i,g}^{\text{min}} &\geq l_{i,g}^{\text{min}} \quad i \in \mathcal{C}, g \in \mathcal{G}, l \in \mathcal{L}, \quad (15)
\end{align*}
\]

where \( l_{i,g}^{\text{max}} \) is the latest arrival time according to the delivery commitment to a station or track section \( g \) and \( l_{i,g}^{\text{min}} \) is the earliest arrival time according to the delivery commitment to a station or track section \( g \).

### 4.2 Interaction constraints

Inside the track capacity corridors, there should be no conflicts with other trains. To enforce this, the interaction constraints from [Gestrelius et al., 2015] are slightly modified. There are also interaction constraints for when corridors interact with each other. In this section, we will describe these constraints.

#### 4.2.1 Capacity corridor interaction

The track capacity corridors are not allowed to overlap each other. To enforce this, the track capacity corridors are ordered. Let \( i_0 \) denote the first track capacity corridor. The interaction between the track capacity corridors is then constrained with the following constraint,

\[
\begin{align*}
    h_{i-1,g}^{\text{max}} &\leq h_{i,g}^{\text{min}} \quad \forall g \in \mathcal{G}, l \in \mathcal{L}, \ i \in \mathcal{C} \setminus \{i_0\}. \quad (16)
\end{align*}
\]

This constraint makes sure that the track capacity corridors do not overlap.
4.2.2 Capacity corridor and train interaction

There are two types of interactions between a train and a track capacity corridor; crossings and overtaking. A crossing occurs when a train is driving in the opposite direction than the train for the track capacity corridor. An overtaking is when a train is driving in the same direction as the train for the track capacity corridor. This section describes the constraints both interaction types.

To model the interaction between track capacity corridors and trains, the binary variables $y^{i,r}_{g}$ and $y^{r,i}_{g}$ are introduced, such that

$$y^{i,r}_{g} = \begin{cases} 
1, & \text{if corridor } i \text{ arrives before } r \text{ at geographic location } g, \\
0, & \text{otherwise.} 
\end{cases}$$

and

$$y^{r,i}_{g} = \begin{cases} 
1, & \text{if train } r \text{ arrives before corridor } i \text{ at geographic location } g, \\
0, & \text{otherwise.} 
\end{cases}$$

Further, let $S^S(i,r)$ be the set of stations on a single track where corridor $i$ and train $r$ can interact. Similarly, let $S^D(i,r)$ and $L^D(i,r)$ be possible interaction locations for corridor $i$ and train $r$ on stations on a double track and double track sections, respectively. The corridors and trains that can interact is given in the set $K_C$. The set of corridors and trains that can interact for which the arrival order matter is given by the set $K^A_C$, and for which the arrival order does not matter is given by the set $K^N_C$. The set of stations where overlaps between trains may occur is denoted as $S^K_O(r,r')$ and the set of stations where overtakings may occur is denoted as $K_O$. The rest of this section describes the constraints for crossings at stations and links and overtakings.

Crossings at single track stations

Let $t_{r,s}$ be the time train $r$ arrives to station $s$ and let $\Delta^{i,r}_s$ be the safety buffer time between a track capacity corridor $i$ and a train $r$ at station $s$. This safety buffer time is the same as for interactions between trains since the track capacity corridor should span track capacity where a train can be planned. The constraints for track capacity corridor and train crossing at a single track station are
\[ h_{i,s}^{\text{max}} + \Delta_{s}^{i,r} - t_{r,s} \leq M(1 - y_{s}^{i,r}) \quad s \in S(i, r), (i, r) \in K_{C}^{A} \quad (19) \]
\[ t_{r,s} - h_{i,s+1}^{\text{min}} \leq M(1 - y_{s}^{i,r}) \quad s \in S(i, r), (i, r) \in K_{C}^{A} \quad (20) \]
\[ y_{s}^{i,r} \leq \gamma_{i,s} \quad s \in S(i, r), (i, r) \in K_{C}^{A} \quad (21) \]
\[ t_{r,s} + \Delta_{s}^{i,r} - h_{i,s}^{\text{min}} \leq M(1 - y_{s}^{r,i}) \quad s \in S(r, i), (r, i) \in K_{C}^{A} \quad (22) \]
\[ h_{i,s}^{\text{max}} - t_{r,s+1} \leq M(1 - y_{s}^{r,i}) \quad s \in S(r, i), (r, i) \in K_{C}^{A} \quad (23) \]
\[ y_{s}^{r,i} \leq \gamma_{r,s} \quad s \in S(r, i), (r, i) \in K_{C}^{A}. \quad (24) \]

Equation (19)-(21) impose a correct interaction with safety precautions if the track capacity corridors ends before the train arrives. Equation (22)-(24) impose constraints if the train arrives to the interaction location before the track capacity corridor starts.

**Crossings at double track stations**

To enforce a safe crossing for a track capacity corridor and a train on a double track section, the constraints

\[ h_{i,s}^{\text{max}} - t_{r,s} \leq M(1 - y_{s}^{i,r}) \quad s \in S^{D}(i, r), (i, r) \in K_{C}^{A} \quad (25) \]
\[ t_{r,s} - h_{i,s+1}^{\text{min}} \leq M(1 - y_{s}^{i,r}) \quad s \in S^{D}(i, r), (i, r) \in K_{C}^{A} \quad (26) \]
\[ t_{r,s} - h_{i,s}^{\text{min}} \leq M(1 - y_{s}^{r,i}) \quad s \in S^{D}(i, r), (r, i) \in K_{C}^{A} \quad (27) \]
\[ h_{i,s}^{\text{max}} - t_{r,s+1} \leq M(1 - y_{s}^{r,i}) \quad s \in S^{D}(i, r), (r, i) \in K_{C}^{A} \quad (28) \]

are introduced. Equation (25) and (26) impose constraints for interactions between train and track capacity corridor when the track capacity corridor starts before the arrival of the train. Equation (27) and (28) impose constraints for interactions if the train arrives to the interaction before the start of the track capacity corridor.

**Crossings at double track sections**

The constraints enforcing a safe crossing on a double track section are

\[ h_{i,l}^{\text{max}} - t_{r,l} \leq M(1 - y_{l}^{i,r}) \quad l \in L^{D}(i, r), (i, r) \in K_{C}^{A} \quad (29) \]
\[ t_{r,l} - h_{i,l+1}^{\text{min}} \leq M(1 - y_{l}^{i,r}) \quad l \in L^{D}(i, r), (i, r) \in K_{C}^{A} \quad (30) \]
\[ t_{r,l} - h_{i,l}^{\text{min}} \leq M(1 - y_{l}^{r,i}) \quad l \in L^{D}(i, r), (r, i) \in K_{C}^{A} \quad (31) \]
\[ h_{i,l}^{\text{max}} - t_{r,l+1} \leq M(1 - y_{l}^{r,i}) \quad l \in L^{D}(i, r), (r, i) \in K_{C}^{A}. \quad (32) \]
Equation (29) and (30) enforce safety precautions when the track capacity corridors ends before the arrival of the train. Equation (31) and (32) enforce the same safety precautions if the train arrives to the interaction location before the start of the track capacity corridor.

No crossing
The crossing between the train and the track capacity corridor need not occur on the railway network in the optimization problem. It can also occur outside the railway network. The constraints enforcing this crossing are

\[
\begin{align*}
    h_{r,g_i}^{\text{max}} - t_{r,g_i}^l & \geq M(y_{\text{out}}^{i,r} - 1) \quad (r, i) \in \mathcal{K}_C \quad (33) \\
    t_{r,g_i}^f - h_{r,g_i}^{\text{min}} & \geq M(y_{\text{out}}^{r,i} - 1) \quad (i, r) \in \mathcal{K}_C. \quad (34)
\end{align*}
\]

Equation (33) enforce a constraint if the train arrives before the track capacity corridor starts. Equation (34) enforces a constraint if the track capacity corridor ends before the train arrives.

Choosing interaction location
A crossing between a train and a track capacity corridor can only occur once in the railway network. Thus, there are only one interaction location. The constraint for choosing an interaction location is

\[
\sum_{s \in \mathcal{S}^i(r)} (y_{s}^{i,r} + y_{s}^{r,i}) + \sum_{s \in \mathcal{S}^o(i,r)} (y_{s}^{i,r} + y_{s}^{r,i}) + \sum_{l \in \mathcal{L}^o(i,r)} (y_{l}^{i,r} + y_{l}^{r,i})
+ y_{\text{out}}^{i,r} + y_{\text{out}}^{r,i} = 1, \quad (i, r) \in \mathcal{K}_C. \quad (35)
\]

Overtakings
For a train to overtake a track capacity corridor on a station, or vice versa, both the train and the track capacity corridor need to occupy the station at the same time. This is an overlap. To set constraints on overtakings between corridor and trains, introduce the binary variables \(x_{s}^{r,i}, p_{s}^{r,i}\) and \(p_{s}^{i,r}\) such that

\[
x_{s}^{r,i} = \begin{cases} 
1, & \text{train } r \text{ and corridor } i \text{ overlap at station } s, \\
0, & \text{otherwise.}
\end{cases} \quad (36)
\]

\[
p_{s}^{r,i} = \begin{cases} 
1, & \text{corridor } i \text{ arrives to a station } s \text{ before train } r \text{ departs,} \\
0, & \text{otherwise.}
\end{cases} \quad (37)
\]
and

\[ p_{i,r}^s = \begin{cases} 
1, & \text{train } r \text{ arrives to a station } s \text{ before corridor } i \text{ departs,} \\
0, & \text{otherwise.} 
\end{cases} \] \quad (38)

To ensure secure overtakings between track capacity corridors and trains, the constraints

\[
t_{r,s+1} - h_{i,g}^{\text{max}} \leq Mp_{i,r}^s \quad s \in S^O(r, i), (r, i) \in K_O \quad (39)
\]

\[
h_{i,g} - t_{r,s+1} \leq M(1 - p_{i,r}^s) \quad s \in S^O(r, i), (r, i) \in K_O \quad (40)
\]

\[
h_{i,g} - t_{r',s} \leq Mp_{i,r}^s \quad s \in S^O(r, i), (r, i) \in K_O \quad (41)
\]

\[
t_{r',s} - h_{i,g}^{\text{max}} \leq M(1 - p_{i,r}^s) \quad s \in S^O(r, i), (r, i) \in K_O \quad (42)
\]

\[
p_{i,r}^s + p_{i,r}^s \leq x_{r,r'}^s + 1 \quad s \in S^O(r, i), (r, i) \in K_O \quad (43)
\]

\[
x_{s,i}^r \leq p_{i,r}^s \quad s \in S^O(r, i), (r, i) \in K_O \quad (44)
\]

\[
x_{s,i}^r \leq p_{i,r}^s \quad s \in S^O(r, i), (r, i) \in K_O \quad (45)
\]

\[
y_{i,r}^g \leq x_{s,i}^r \quad s \in S^O(r, i), (r, i) \in K_O \quad (46)
\]

are enforced. The constraints in Equation (39)-(42) enforce the right values on \( p_{i,r}^s \) and \( p_{i,r}^s \). The constraints in Equation (43)-(46) enforce the right values on the binary variables \( x_{s,i}^r \) and \( y_{i,r}^g \).

**Ordering variables**

When an overtaking occurs, the arrival order to the subsequent stations is affected. This order should be kept track of when track capacity corridors and trains overtake each other in the optimization problem. To do this, introduce the binary variable \( v_{s,i}^r \), such that

\[
v_{s,i}^r = \begin{cases} 
1, & \text{if corridor } i \text{ arrives before train } r \text{ to geographic location } g, \\
0, & \text{otherwise.} 
\end{cases} \] \quad (47)

The constraints ensuring the appropriate train and track capacity corridor order are

\[
v_{s,i}^r - v_{s+1,i}^r \leq y_{i,r}^g \quad s \in S(r, i), (r, i) \in K_O \quad (48)
\]

\[
v_{s+1,i}^r - v_{s,i}^r \leq y_{i,r}^g \quad s \in S(r, i), (r, i) \in K_O. \] \quad (49)
If an interaction occurs at a station, then there should be some added stopping time. The constraints ensuring this added stopping time at a station are
\[ v_s^{r,i} - y_s^{j,r} \leq 1 + \gamma \quad s \in S(r,i), (r,i) \in K \quad (50) \]
\[ 1 - v_s^{r,i} - y_s^{j,r} \leq 1 - \gamma \quad s \in S(r,i), (r,i) \in K \quad (51) \]

4.3 Safety regulation constraints

When trains interact, extra buffer time is added by the infrastructure manager to ensure the safety of the trains. Similar safety measurements are taken when trains interact with a track capacity corridor. In the optimization model, there are the following cases:

1. Station requiring 3 minutes between train \( r \) and \( r' \).
2. Station requiring 2 minutes between train \( r \) and \( r' \).
3. Station requiring 1 minutes between train \( r \) and \( r' \) or both trains must stop.

These are the same cases as when trains interact. In this section, the reformulated constraints from [Gestrelius et al., 2015] enforcing the safety buffer times are stated.

Safety buffer times at stations

The idea of how to enforce a safety buffer time between the track capacity corridors and trains at stations is similar to how it is done for the safety buffer time between trains. The binary variable \( w_s^{r,i} \) is introduced and defined as
\[ w_s^{r,i} = \begin{cases} 1, & \text{safety buffer time is 1 minute,} \\ 0, & \text{no safety buffer time.} \end{cases} \quad (52) \]

Let \( \Delta_s^{r,r'} \) be the safety buffer time. The constraints enforcing a this
safety buffer time between train and track capacity corridor are

\[ h_{i,g}^{\min} - t_{r,s} - \Delta_s^{i,r}(1 - \gamma_{i,s}) \geq M(v_s^{r,i} - 1), \]
\[ \forall s \in S^M \cap S^O(r, i), (r, i) \in K_O \] (53)

\[ h_{i,g}^{\min} - t_{r,s} - \Delta_s^{i,r}(1 - \gamma_{r,s}) \geq M(v_s^{r,i} - 1), \]
\[ \forall s \in S^M \cap S^O(r, i), (r, i) \in K_O \] (54)

\[ t_{r,s} - h_{i,g}^{\max} - \Delta_s^{r,i}(1 - \gamma_{r,s}) \geq - M v_s^{r,i}, \]
\[ \forall s \in S^M \cap S^O(r, i), (r, i) \in K_O \] (55)

\[ t_{r,s} - h_{i,g}^{\max} - \Delta_s^{r,i}(1 - \gamma_{i,s}) \geq - M v_s^{r,i}, \]
\[ \forall s \in S^F \cap S^O(r, i), (r, i) \in K_O \] (56)

\[ h_{i,g}^{\min} - t_{r,s} - w_s^{r,i} \geq M(v_s^{r,i} - 1), \]
\[ \forall s \in S^F \cap S^O(r, i), (r, i) \in K_O \] (57)

\[ t_{r,s} - h_{i,g}^{\max} - w_s^{r,i} \geq - M v_s^{r,i}, \]
\[ \forall s \in S^F \cap S^O(r, i), (r, i) \in K_O \] (58)

**Safety buffer times at single track sections**

The safety buffer time at single track sections between track capacity corridors and trains are enforced using the constraints

\[ h_{i,l}^{\min} - t_{r,l} - \Delta_l^{j,r}(1 - \gamma_{i,l-1}) \geq M(v_s^{r,i} - 1), \]
\[ \forall l \in L^S(r) \cap L^S(i), (r i) \in K_O \] (59)

\[ t_{r,l} - h_{i,l}^{\max} - \Delta_l^{r,i}(1 - \gamma_{r,l-1}) \geq - M v_s^{r,i}, \]
\[ \forall l \in L^S(r) \cap L^S(i), (r i) \in K_O \] (60)

**Safety buffer times at double track sections**

The safety buffer time must also be implemented on double track sections. The constraints enforcing this on interactions between trains and track capacity corridors on double track sections are

\[ h_{i,l}^{\min} - t_{r,l} - \Delta_l^{j,r} \geq M(v_s^{r,i} - 1), \]
\[ l \in L^D(r) \cap L^D(i), (r i) \in K_O \] (61)

\[ t_{r,l} - h_{i,l}^{\max} - \Delta_l^{r,i} \geq - M v_s^{r,i}, \]
\[ l \in L^D(r) \cap L^D(i), (r i) \in K_O \] (62)

### 4.4 Objective function

The objective of the optimization model and the purpose of the track capacity corridors is to find the available track capacity for a delivery
commitment request given a number of existing delivery commitments. For each corridor, the total time each station and track section $g \in G_l$ is not occupied by other train paths and where a train path for the delivery commitment application can be scheduled can be computed by $h_{i,g}^{\text{max}} - h_{i,g}^{\text{min}}$. To find the available track capacity for a delivery commitment application, we first need to find the minimum time window over all stations and track sections $g$ must be maximized on all track segments that the delivery commitments request traverses, i.e. all segments in $L^d$. The objective in the optimization is

$$\max \sum_{l \in L^d} \sum_{i \in C} \min_{g \in G_l} \left\{ \left( h_{i,g}^{\text{max}} - h_{i,g}^{\text{min}} \right) \right\}$$

The objective sum up all time intervals on each geographic location and takes the smallest time interval. This is done to maximize the train paths that can fit on the available track capacity. This is explained in Figure 6. The broadest time interval is at station C, but no more train paths, then the number of train paths that can run through the bottleneck in station A, can be planned on the available track capacity.

This objective function is not linear. In order to linearize it, we introduce the continuous variable $o_{i,l}$ for all $i \in C$ and $l \in L^d$. Further, we introduce the constraints
\[ o_{i,l} \leq h_{i,g}^{\text{max}} - h_{i,g}^{\text{min}} \quad \forall i \in \mathcal{C}, l \in \mathcal{L}^d, g \in \mathcal{G}_l. \] (64)

The objective can then be expressed as

\[ \max \sum_{l \in \mathcal{L}^d} \sum_{i \in \mathcal{C}} o_{i,l} \] (65)

5 Conclusion

The optimization model introduced in this paper is used to find the available track capacity for some delivery commitment request. The optimization model has been tested on parts of the Swedish railway network; the railway network between the stations Skymsossen, Norrköping and Boxholm and the railway network between the stations Sellnäs, Kumla and Jädersbruk. Result for the former optimization is displayed in Figure 7. The optimization model for this case is solved within minutes.

References

Figure 7: The results for the optimization model on the railway network between Skymosse, Norrköping, Boxholm. The red crosses are the delivery commitment application and the red area is the available track capacity.