Optimal Link Scheduling for Age Minimization in Wireless Systems

Qing He, Di Yuan and Anthony Ephremides

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***Abstract*—Information age is a recently introduced metric to represent the freshness of information in communication systems. We investigate age minimization in a wireless network and propose a novel approach of optimizing the scheduling strategy to deliver all messages as fresh as possible. Specifically, we consider a set of links that share a common channel. The transmitter at each link contains a given number of packets with time stamps from an information source that generated them. We address the link transmission scheduling problem with the objective of minimizing the overall age. This minimum age scheduling problem (MASP) is different from minimizing the time or the delay for delivering the packets in question. We model the MASP mathematically and prove it is NP-hard in general. We also identify tractable cases as well as optimality conditions. An integer linear programming formulation is provided for performance benchmarking. Moreover, a steepest age descent algorithm with better scalability is developed. Numerical study shows that, by employing the optimal schedule, the overall age is significantly reduced in comparison to other scheduling strategies.**

***Index Terms*—information age, link scheduling, optimization, wireless networks.**

I. Introduction

For a wireless system with applications that require availability of fresh information, such as a monitoring system, which obtains information from environmental sensors, or a cellular system where channel information needs to be periodically acquired, freshness of the received information is important. The information generated by the source reflects the most recent status value. However, the reception of the information is delayed and hence aged due to the time spent in queueing and transmission. To measure the freshness of information, the concept of age of information at a given time t has been defined as the difference between the current time t and the time when the latest received information sample was generated [1].

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The studies of age of information have been motivated by a host of applications including sensor-based networks [2], intelligent vehicles [3], Internet of Things (IoT) [4], mobile communication [5], and social media [1], etc. The authors of [6] further extend the relevant applications to cloud computing and route caches in ad hoc networks. In Figure 1, we show a general end-to-end scenario, which is an abstraction of the above applications.

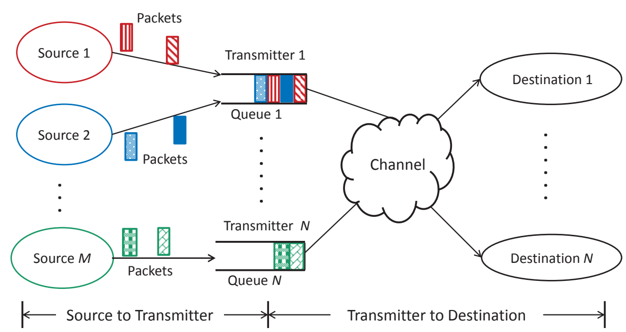

Fig. 1. An end-to-end scenario of age of information.

Firstly, information is generated by the sources and transmitted as packets that carry time stamps indicating the generation time of the information samples. Usually, a source sends the packets immediately to a server node, which we refer to as a "transmitter" in Figure 1. This is mainly because the sources, e.g., sensors, are not capable of buffering or processing the packets due to the restrictions of hardware or energy consumption. The transmitter here can be a real server or a logical node where the packets are queued and managed before being delivered to the destination. A transmitter is either associated with single source (like Transmitter N) or shared by multiple independent sources (e.g., Transmitter 1) [6], [7]. The packets are delivered by the transmitters to their respective destinations through a shared wireless channel.

As each packet represents a status update for a source, the age at the receiver with respect to information from a source in the above scenario is defined as the elapsed time since the most recently received update was generated. That is, assuming that the latest packet received by the destination carries a time stamp of τ, the age for the corresponding source status at time t is calculated as $t-\tau$. In Figure 2, we illustrate the age evolution for a source [1].

In previous works, queueing models are used to explore the problems of age. The information generation process is usually assumed to be Poisson. The serving time of packets, which typically consists of the waiting time in queue and the delivering time from the transmitter to the destination, is

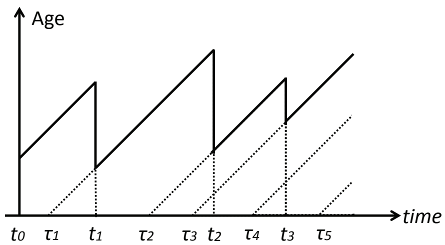

Fig. 2. Age evolution for a source. The ith packet with time stamp τ_i is received by the destination at t_i. The age at t_i equals $t_i - \tau_i$. The age increases linearly if no packet is received.

described in terms of a simple process. In a wireless system, the time taken by a packet to be delivered to its destination depends on channel conditions as well as the scheduling strategy used in the network. The scheduling aspect has not been considered in the previously studied models. That is, there is a lack of studies dealing with scheduling multiple users to minimize age.

In this paper, we propose a novel approach of improving the freshness of information by optimizing the transmission scheduling with respect to age. Link scheduling is a key aspect of access coordination in a wireless system with a shared channel, where the links that are simultaneously activated cause interference to each other. The scheduling problem consists of the fundamental question of which of the mutually interfering links should transmit at each time so that some criteria, such as throughput, energy, time, or their combinations, are optimized. In our case, it will be the age, that is, develop a schedule to ensure that on the whole the information received by the destinations is as fresh as possible.

The concept of age is relevant to a host of applications requiring fresh information. We consider a general setup that is independent of any physical-layer system specifications so that the insights derived in this paper are valid for a wide range of wireless systems. Specifically, we consider a set of transmitter-receiver pairs, or links, that share a wireless medium. Each link has a given set of packets to be delivered. We address the problem that aims to find an optimal schedule, such that the overall age is minimized. In what follows, we refer to the problem as the minimum age scheduling problem (MASP). Although the link packet sets are given in MASP, we do not assume that all packets have to be buffered before scheduling takes place. Rather, the optimization problem appears naturally in networks that run scheduling in cycles; packets that arrive during a scheduling cycle are scheduled for transmission in the next cycle. Hence, for each scheduling instance, the MASP deals with the packets that have been queued in the order of first-come-first-served (FCFS) at the respective transmitters in the previous cycle.

A. The MASP and the MTSP

It is worth noting that minimizing freshness of information is in general different from minimizing the completion time or delay for all packets. The latter, corresponds to the so called, minimum time link scheduling problem (MTSP). Firstly, the order of activation of the selected link sets is immaterial in the MTSP, while it plays an important role for the purpose of minimizing the overall age. What's more, we show in the following example that even if we consider the solutions that minimize time, none of them yield minimum age.

Consider a network consisting of four sources, each of them being associated with one transmitter. Each transmitter queue has one packet to deliver. Let the initial age be $9, 9, 1, 2$ for sources $1, 2, 3, 4$, respectively. Time is slotted and each transmitter delivers one packet per slot if it is activated. Assume that the sets of links that can be activated together are $\{1\}$, $\{2\}$, $\{3\}$, $\{4\}$, $\{1, 2\}$, $\{1, 3\}$ and $\{2, 4\}$. It can be easily verified that, for this case, an optimal solution of the MTSP contains two link sets $\{1, 3\}$ and $\{2, 4\}$, each occupying one slot. Due to the different order, there are two minimum-time schedules: (i) $\{1, 3\}$, $\{2, 4\}$ and (ii) $\{2, 4\}$, $\{1, 3\}$. We calculate the age of these two in Figure 3. From the result, we observe that the order of link sets does impact the result of age. The overall age of the two schedules are 34 and 33, respectively. Then we calculate the age for another schedule, that is, $\{1, 2\}$, $\{4\}$, $\{3\}$, which consists of three link sets, each for one time slot. The overall age is 29 (see the details in Figure 3). In fact, one can verify that this is actually the optimal solution for the MASP. Therefore, the schedule of minimum age differs from that of minimum time. This of course is not surprising given that the MTSP is oblivious of the ages of the packets in the queues.

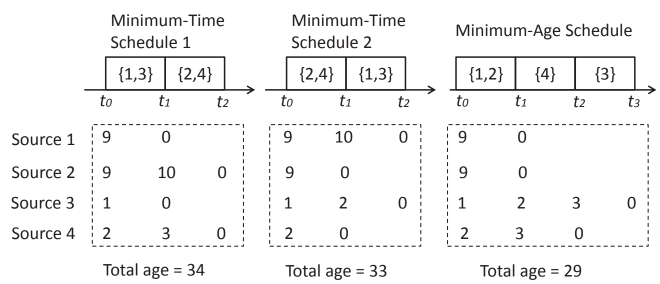

Fig. 3. An example of age evolution in different schedules.

B. Contributions

In this study, our main contributions consist of investigating the age from a network perspective and proposing a new approach to improve the freshness of information by optimizing the scheduling strategy in the network. We formulate the MASP mathematically and prove it is NP-hard in general. Several insights are presented including identifying tractable cases and deriving optimality conditions. An integer linear programming (ILP) is provided to enable the computation of a global optimal solution using off-the-shelf optimization methods. We also develop a sub-optimal, but fast, algorithm for the problem, with better scalability than ILP. Numerical results are provided to validate the approach and to confirm the effectiveness of the algorithm.

In Section II we review and discuss related work. In Section III, we define the system model and formalize the problem, followed by the complexity analysis in Section IV. Then we derive structural results in Section V. In Section VI, an ILP for the MASP is developed. In Section VII, we propose a scalable algorithm. Numerical study is presented in Section VIII. Extension to rolling horizon is discussed in Section IX. In Section X, we provide concluding remarks.

II. Related Work

Link scheduling is one of the classic problems in wireless systems with multiple access. The investigation of scheduling has a long history that has ranged from simple transmission models to fully cross-layered ones that combine rate and power control with overall network resource allocation (see the surveys [8], [9]). In [10], the authors propose a spatial time division multiple access (STDMA) scheme in which feasible compatible set of links are activated in each time slot. The scheduling problem can be represented using a set-covering formulation with the objective of optimizing a given cost criterion [11], [12]. For problem complexity of the MTSP, the general hardness under the protocol model is provided in [13], [14]. Under the physical model, the MTSP with an arbitrary or a geometric gain matrix, is proved to be NP-hard in [12], [15] and [16], respectively. The MTSP with continuous link rates is proved to be hard in general in [17].

A variety of algorithm design and problem approximations have been proposed and studied for the scheduling problem, e.g., [18], [19], etc. Under the protocol model, graph-based scheduling algorithms employing implicit or explicit coloring strategies are widely used. For scheduling problems under the physical model, a column-generation based solution method is used in [20], which can approach an optimal solution, with the advantage of a potentially reduced complexity. In [21]–[23], the investigations indicate that it is possible to integrate the problems of routing, scheduling, and physical layer effects in a very abstract fashion that provided general structural results like the back-pressure algorithm. Although there is a rich amount of literature available, none of those has considered scheduling with minimum age.

The study of age of information is yet at an early stage. In [1], the importance of real-time status updates in networks is recognized. The authors employ a time average age metric for the performance evaluation and study the problem of keeping the status updates generated by a source as fresh as possible at the appropriate receivers. A single source and server system is investigated with queuing models, under the queue discipline of FCFS. The average age of multiple sources is characterized in [6]. In [24], the authors investigate the age of information for a status updating system through a network cloud, considering random transmission and service processes, no waiting time and the possibility of packets arriving out of order. As the randomness of service time may render some packets obsolete, a new queue management technique at transmitters have been proposed in [7] and [25] for multiple sources and single source, respectively. The new policy of packet management is to maintain a queue with only the latest update of each source, overwriting and discarding any previous queued packets with older status from that source. To analyze the age of information, an alternative metric, called peak age, has been defined in [25] to provide information about the maximum value of age achieved immediately prior to the reception of an update. In [26], the authors study the optimization of the weighted sum of age of information (which is similar to ours), in a wireless network with a single base station and multiple clients. Unlike our work, however, the focus of [26] is on scheduling policy, and each time slot can accommodate at most one packet by the system scenario.

III. System Model

A. Minimum Age Scheduling in Wireless Systems

We proceed now to describe our model in details. We consider a system with N sources, denoted by S_1, S_2, …, S_N, which generate information samples that are carried by packets. A packet containing the time stamp of the information sample is sent to its intended transmitter without delay. Assuming that each transmitter is associated with a source, we define the N transmitters as TX_1, TX_2, …, TX_N. The packets wait in a queue at the transmitter before they are delivered to the respective destination through a shared wireless channel based on the transmission schedule to be optimized. A destination might be common for multiple transmitters or not. For the general case, we define the destinations, i.e., receivers, as RX_1, RX_2, …, RX_N, where RX_n is the receiver of TX_n, $n = 1, \dots, N$. Due to the one-to-one mapping, we use the same index for the source, transmitter, queue at transmitter and receiver.

For source S_n, $n = 1, \dots, N$, the number of packets to be delivered is denoted by K_n. These packets are to be scheduled following the discipline of FCFS. We denote the ith packet of S_n by U_{ni}, and the time stamp it carries by τ_{ni}. For each source S_n, $\tau_{n,i-1} < \tau_{ni}, i = 2, \dots, K_n$. Time is slotted. We use t_j to denote the time by the end of time slot j, thus time slot j is defined by $[t_{j-1}, t_j]$, with the convention that t_0 is the starting time of the scheduling horizon. We use a_{nj} to denote the instantaneous age for source S_n of time slot j. Note that the value of a_{nj} is set by the end of the time slot, and the specific value depends on whether or not a packet of source S_n is scheduled in time slot j. We use a_{n0} to denote the initial age, i.e., the age at t_0, of S_n. For an arbitrary source S_n, the instantaneous age a_{nj}, is calculated in Equation (1) below. In the subsequent, we will simply use the term age to refer to this entity.

$$a_{nj} = \begin{cases} a_{n,j-1} + 1 & \text{if no packet is received by } \text{RX}_n \text{ in} \\ & \text{time slot } j, \\ t_j - \tau_{ni} & \text{if } U_{ni} \text{ is received by } \text{RX}_n \text{ in time} \\ & \text{slot } j. \end{cases} \tag{1}$$

In this study, we focus on the single hop scenario, that is, the packets are delivered from the transmitter to the receiver without passing through any relay nodes. Denote by link n the pair of TX_n and RX_n. We use the term *group* to refer to a subset of the N links that can simultaneously transmit, i.e., a "compatible" link set. The criteria of compatible link sets depend on the interference models. The concept will be discussed further in Section III-B. Denote by c a group and $\mathcal{C}$ the set of the groups for the N links under a given interference model, that is, $c \in \mathcal{C}$. The transmitting rate of a link is one packet per time slot. That is, if a group is activated, each link in this group delivers one packet in a time slot.

For MASP, a schedule, i.e., a sequence of link groups, is feasible if and only if

- all packets can reach their destinations by the end of the schedule;
- for an arbitrary S_n, packet $U_{n\iota}$, $\forall \iota < i$, is delivered before U_{ni}.

Hence we define the scheduling problem in the following

Definition 1. *Given a_{n0} and τ_{ni}, $n = 1, \ldots, N, i = 1, \ldots, K_n$, the MASP is to find a feasible schedule, such that $\sum_{n=1,\ldots,N,\ j=0,\ldots,T_n} a_{nj}$ is minimal, where a_{nj} is defined in* (1) *and T_n is the time slot in which the last packet of S_n is delivered with the schedule.*

To understand MASP better, we illustrate in Figure 4 an example of the age for a source with four packets being delivered in the first, fourth, fifth, and last time slots, respectively. Here, each time slot is assumed to have a time unit of one. Hence the overall age is represented by the size of the shaded area.

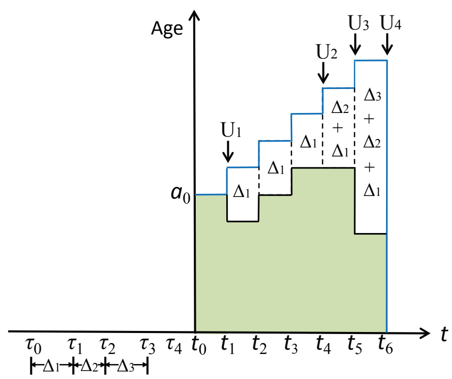

Fig. 4. Transformation of the sum of age of a source.

The illustration enables us to compute the overall age in another way, which leads to a new mathematical formulation of MASP that facilitates the optimality characterization and algorithm design in later sections. For a source S_n, if no packet is delivered, then its age increases by one in each time slot, as depicted by the staircase curve in Figure 4. The overall area under this curve represents the total age, or data staleness, if no update is received. Assuming that the schedule length is T_n, this area's size is given by the expression below.

$$a_{n0} + (a_{n0}+1) + \cdots + (a_{n0}+T_n-1) = \frac{T_n(2a_{n0}+T_n-1)}{2} \tag{2}$$

We use Δ_{ni} to denote the difference between the time stamp of source n's update i and that of update $i-1$, i.e., $\Delta_{ni} = \tau_{ni} - \tau_{n,i-1}$, with the convention that $\tau_{n0} = t_0 - a_{n0}$. An interpretation of this definition is that the age of source n equals zero at time point τ_{n0}. Once the packet for update i is delivered at time slot j, by (1), the age at t_j decreases from $a_{n,j-1}+1$ to $t_j - \tau_{ni}$. Since $a_{n,j-1} = t_{j-1} - \tau_{n,i-1} = t_j - 1 - \tau_{n,i-1}$, the age reduction caused by the packet delivery equals Δ_{ni}, i.e., the difference between the two consecutive time stamps. Denote by $T_{ni} = t_{ni} - t_0$, where t_{ni} represents the time when packet U_{ni} is delivered. The quantity T_{n,K_n}, which is the time point when S_n is emptied, is simply written as T_n. As shown in Figure 4, the sum of the age reduction, which equals the area between the staircase curve and the age curve, is given by

TABLE I
NOTATION

Notation	Description
S_n	Source n
K_n	The number of packets from S_n
U_{ni}	The ith update packet from S_n
τ_{ni}	The time stamp carried by U_{ni}
t_j	The time corresponds to the end of the jth time slot
a_{n0}	The initial age of S_n
a_{nj}	The age of S_n at t_j
Δ_{ni}	$\tau_{ni} - \tau_{n,i-1}$, which is the difference between the time stamp of source n's update i and that of update $i-1$
TX_n	Transmitter n
RX_n	Receiver n
c	A link group, a member of $\mathcal{C}$
$\mathcal{C}$	The set of candidate groups
T_{ni}	$t_{ni} - t_0$, where t_{ni} is the time when U_{ni} is delivered
T_n	$t_{nK_n} - t_0$, the schedule length of S_n

$$\sum_{i=1}^{K_n-1} \Delta_{ni}(T_n - T_{ni}). \tag{3}$$

Therefore, the overall age of S_n, given by subtracting (3) from (2), reads

$$\frac{T_n^2}{2} + (a_{n0} - \frac{1}{2} - \sum_{i=1}^{K_n-1}\Delta_{ni})T_n + \sum_{i=1}^{K_n-1}\Delta_{ni}T_{ni}. \tag{4}$$

Hence the MASP is formulated as follows.

$$\underset{\{T_{ni},T_n\in Z^+\}}{\text{minimize}} \sum_{n=1}^{N} \frac{T_n^2}{2} + (a_{n0} - \frac{1}{2} - \sum_{i=1}^{K_n-1}\Delta_{ni})T_n + \sum_{i=1}^{K_n-1}\Delta_{ni}T_{ni} \tag{5a}$$

subject to

$$1 \le T_{n1} < T_{n2} < \cdots < T_n \quad \forall n = 1, \ldots, N, \tag{5b}$$

$$\{n \in \{1,2,\ldots,N\} : T_{ni} = j,\ i = 1,\ldots,K_n\} \in \mathcal{C}$$
$$\forall j = 1, 2, \ldots, \sum_{n=1}^{N} K_n. \tag{5c}$$

We summarize the key notation in Table I.

In the definition of MASP, the age value of a source is considered zero, after all packets of the source are delivered in the schedule. There are a couple of underlying reasons. First, in a broader application context, information sources may dynamically enter and exit the system. When a source is no longer part of the system, its age becomes irrelevant and reasonably zero. The task of MASP is to make a schedule of minimum age for all the given packets, without assuming there will always be additional arrivals. Consequently the age is set to zero after a source is emptied (for the given packets). Second, the modeling approach makes the individual completion times relevant in the optimization. One effect of this can be illustrated by considering the simple scenario of time division multiple access (TDMA), i.e., one packet per slot, for N sources, each having one packet to be scheduled. The sources differ significantly in their initial age values $a_{n0}, n = 1, \ldots, N$, and for the given packet time stamps $\tau_{n1}, n = 1, \ldots, N$, the age reduction due to packet scheduling, i.e., Δ_{n1}, is identical for the sources. If we consider the age of a source increasing linearly after its packet is delivered, all $N!$ sequences are equally optimal. With our model, the optimum is to schedule the sources in the descending order of initial age, which is more intuitive.

For a given source, setting the age to zero after the last packet has been delivered is equivalent to assuming that, irrespective of how many more slots are used for the other sources' packets, the given source pays no penalty in age in the current schedule cycle. This assumption is however relaxed by cycle-by-cycle scheduling and its extension to rolling horizon. Namely, even if the age is treated as zero after after a source is emptied, the cycle-by-cycle scheduling process shall still keep track of the age increase, which, in fact, will define the initial age for each cycle. This also applies to the more general approach of rolling horizon scheme discussed later in Section IX.

B. Interference Models

For a wireless network with a shared channel, two interference models are widely used [27]. Under the protocol model, any two links can be active simultaneously if and only if they are sufficiently spatially separated from each other. For the physical model, aka the signal-to-interference-and-noise ratio (SINR) model, a transmission is successful requires that the SINR value at the intended receiver exceeds a threshold. Specifically, if a channel matrix G of dimension $N \times N$ is provided, where its element G_{ln} is the gain between the transmitter of link l and the receiver of link n, and σ_n^2 is the noise variance, then in group $\mathcal{c}$, the SINR of link n transmitting with power P_n is given by

$$\gamma_{n\mathcal{c}} = \frac{P_n G_{nn}}{\sum_{l \in \mathcal{c}, l \neq n} P_l G_{ln} + \sigma_n^2} \ . \tag{6}$$

Denoting the SINR threshold by γ, a group $\mathcal{c}$ is feasible if and only if $\gamma_{n\mathcal{c}} \geq \gamma, \forall n \in \mathcal{c}$.

The problem of constructing compatible link sets, or feasible groups, is the so called *Link Activation (LA)*, which has been studied extensively in the literature, under the two interference models [8]. Hence in developing solution approaches for the MASP, we will focus on how to minimize age with given candidate groups. However, it is worth noting that the MASP contains the LA as a building block.

We end this section by an example, which is simple, yet serves the purpose of motivating the theoretical investigation of complexity and structure results that follow. The example illustrates that intuition may fail in deriving the optimal schedule. Consider a system with two sources. Sources S_1 and S_2 have three and two packets, respectively. Assume TDMA is the only possible link activation. Let the initial age and time stamps be $a_{10} = a_{20} = 12$, and $\tau_{11} = 6, \tau_{12} = 7, \tau_{13} = 8, \tau_{21} = 5$, and $\tau_{22} = 10$. The schedule starts at $t_0 = 15$. Intuitively, the solution is to choose the one with the largest age reduction in each slot. Then we obtain solution 1, for which the overall age is 94, as shown in Figure 5. However, for this case, solution 2 leads to the optimum.

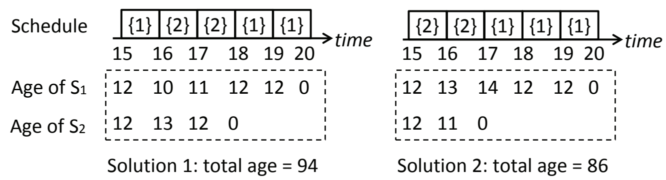

Fig. 5. Comparison of age in two schedules.

IV. Complexity Considerations

From Section III, it is clear that the MASP is essentially to determine which group should transmit in each time slot. The optimization decisions are of discrete-choice nature. Hence the MASP is inherently a combinatorial optimization problem, for which complexity is a fundamental aspect. Note that, the previous complexity analysis of the scheduling problem with other metrics, e.g., minimum time, cannot apply for the MASP, since the objective function and the constraints have been changed in the latter. In the following, we formally settle the NP-hardness of the MASP under the physical model.

Theorem 1. *The MASP under the physical model is NP-hard.*

Proof: The recognition version of MASP is to determine whether or not there is a scheduling solution such that the overall age of the sources is less than or equal to a given number. We establish the proof by constructing a polynomial-time reduction from the 3-satisfiability (3-SAT) problem, which is NP-complete [28] (see the Appendix for the definition and terminology of 3-SAT).

For any 3-SAT instance with M Boolean variables and D clauses, we define an MASP instance as follows. First, we define two sets of information sources, $\mathcal{M}$ and $\mathcal{D}$. Set $\mathcal{M}$ has $2M$ sources corresponding to the $2M$ literals of the 3-SAT instance. For the variable represented by source m, we denote by m' the source corresponding to the negation. Hence $\mathcal{M} \triangleq \{1, \ldots, M, 1', \ldots, M'\}$. Set $\mathcal{D} \triangleq \{1, \ldots, D\}$ is mapped to the D clauses in the 3-SAT instance. The initial ages of the literal sources and clause sources are uniformly set to positive integers a_0 and d_0 (e.g., one). Each source has one packet whose time stamp has no significance to the proof.

We consider $2M + D$ wireless links, each being associated with a source. Due to the one-to-one mapping between the sources and links, we use the same index for both. Let the noise variance as well as the transmitting power for all the links be equal to one. The SINR threshold γ is set to $0.5 + \epsilon$, where ϵ is a small positive number satisfying $\epsilon \leq \frac{1}{2(2M-1)}$. The values of channel gain are defined as follows: $G_{mm} =$

$G_{m'm'} = G_{dd} = 1,\ \forall m, m' \in \mathcal{M},\ \forall d \in \mathcal{D}$, and for links l and n, $\forall l, n \in \mathcal{M},\ l \neq n$,

$$G_{ln} = G_{nl} = \begin{cases} 1 & \text{if } n = l' \text{ or } n' = l \\ 0 & \text{otherwise} \end{cases}$$

For links of clause sources, $G_{ij} = 0,\ \forall i, j \in \mathcal{D},\ i \neq j$. Moreover, for links of sources in different sets, $G_{dm} = G_{dm'} = 0,\ \forall d \in \mathcal{D},\ \forall m, m' \in \mathcal{M}$, i.e., the links of clause sources do not generate any interference to the links of literal sources. If m is one of the three literals in clause d, then $G_{md} = 0$, otherwise $G_{md} = \frac{1}{M}$.

By construction, a feasible schedule must use at least two time slots, as the SINR of the two links of sources m and m' cannot meet the threshold if they both transmit, $\forall m = 1, \ldots, M$. Suppose there is a solution of two time slots. Then, each time slot must contain exactly M links, and the two links of m and m' must be in different time slots. Moreover, in the most optimistic case, the overall age equals $2Ma_0 + Dd_0 + M(a_0 + 1)$, assuming all D links of the clause sources are in time slot one. Note that, due to the SINR requirement, for any clause source d, its link can be in time slot one, if and only if this time slot contains at least one of the three links of the literal sources that correspond to the three literals of the clause in the 3-SAT instance, as otherwise the SINR of the link of d is $\frac{1}{M \times \frac{1}{M} + 1} = 0.5 < \gamma$.

From the above observations, we can now establish a solution mapping. Namely, a Boolean variable of the 3-SAT instance has value true, if and only if the link of the corresponding literal source is in the first time slot. Moreover, the answer to the 3-SAT instance is yes, if and only if the overall age of $2Ma_0 + Dd_0 + M(a_0 + 1)$ can be achieved (i.e., all D links of clause sources are in time slot one). Because 3-SAT is NP-complete and the reduction is clearly polynomial, the recognition version of the MASP is NP-complete, and the MASP is NP-hard. ■

For the MASP under the protocol model, following a similar proof, the complexity result is given below.

Theorem 2. *The MASP under the protocol model is NP-hard.*

We omit the proof since it resembles the arguments in the proof of Theorem 1, except that the feasibility criterion of a group is given by the protocol model instead of an SINR threshold in constructing the polynomial reduction from the 3-SAT problem.

Given the fact that both the MASP under the protocol model and the MASP under the physical model contain LA as a building block, and LA itself is a hard problem [9], it arises a question about the underlying rationale of the NP-hardness, i.e., is it merely a consequence of the hardness of LA? In the following theorem, we provide a negative answer, stating that the MASP is hard even if the candidate groups are known.

Theorem 3. *The MASP with given candidate groups is NP-hard.*

Proof: We construct a polynomial-time reduction from the 3-SAT problem to the MASP for which the candidate group set $\mathcal{C}$ is known. For any 3-SAT instance with M variables and D clauses, denote by v_m the mth variable and $\bar{v}_m$ its negation. Reusing the notation in the proof of Theorem 1, we define two sets of information sources, $\mathcal{M} \triangleq \{1, 2, \ldots, M\}$ and $\mathcal{D} \triangleq \{1, 2, \ldots, D\}$, corresponding to the M variables and the D clauses, respectively. The initial ages are uniformly set to a_0 for all literal sources and d_0 for all clause sources. In addition, we define a set of auxiliary sources $\mathcal{B} \triangleq \{1, 2, \ldots, B\}$, of which the initial age is b_0, satisfying $B > \sqrt{DM(M+1)(M+d_0)}$ and $a_0 > b_0 > D(d_0 + M + B)$. The time stamp of the packets can be any meaningful value since each source has only one packet.

We consider a wireless network with $M + D + B$ links. Each of them is associated with a source and indexed by that. The candidate group set $\mathcal{C}$ consists of the following three subsets:

1) the $D + B$ single-link groups;
2) the M groups, of which the mth group is formed by link m and the links in $\mathcal{D}$ that correspond to the clauses containing v_m as one of the three literals;
3) another M groups, of which the mth group is formed by link m and the links in $\mathcal{D}$ that correspond to the clauses containing $\bar{v}_m$ as one of the three literals.

Note that the M single-link groups corresponding to the 3-SAT variables are also feasible for use. However, using such a single-link group is not optimal, because there are multi-link groups, namely, the second and third subsets, that contain these M links, and each of these groups can be scheduled to deliver multiple packets. For this reason, we do not explicitly define the M single-link groups. Through the construction, we observe that:

1) a feasible schedule of this MASP instance occupies at least $M + B$ time slots, because a group in $\mathcal{C}$ contains at most one link in $\mathcal{M} \cup \mathcal{B}$ and each link in a group transmits one packet per time slot;
2) in an optimal solution, the groups are sorted in the descending order of their initial age, which is defined as the sum of the initial ages of the elements; otherwise, swapping any two groups that are not in this order will improve the objective.

Suppose there exists an optimal solution using $M + B$ time slots, then the links $d \in \mathcal{D}$ must transmit together with the links $m \in \mathcal{M}$. Putting together the condition $a_0 > b_0$, the optimal schedule consists of two segments: M groups from the second and the third subsets of $\mathcal{C}$, occupying the first M slots, and the B single-link groups for the links $b \in \mathcal{B}$, using the following B slots. By (4), it is easy to calculate that the total age contributed by the second segment is $\sum_{t=M+1}^{M+B} \frac{t^2}{2} + (b_0 - \frac{1}{2})t$. For the first segment, considering the observation that the groups have to be arranged in the descending order of their initial ages, the resulting age may range from $\sum_{t=1}^{M} \frac{t^2}{2} + (a_0 - \frac{1}{2})t + Dd_0$ to $\sum_{t=1}^{M} \frac{t^2}{2} + (a_0 - \frac{1}{2})t + \lceil \frac{D}{M} \rceil [\sum_{t=1}^{M} \frac{t^2}{2} + (d_0 - \frac{1}{2})t]$. The former corresponds to the case that all the links in $\mathcal{D}$ can be transmitted in one group (and hence being scheduled in the first time slot); while the latter is the upper bound that is attained when the links $\mathcal{D}$ are evenly distributed in the M groups. Therefore, the total age of an optimal schedule using

$M+B$ slots is no more than

$$A_{M+B} = \sum_{t=1}^{M} \frac{t^2}{2} + (a_0 - \frac{1}{2})t + \lceil \frac{D}{M} \rceil [\sum_{t=1}^{M} \frac{t^2}{2} + (d_0 - \frac{1}{2})t] + \sum_{t=M+1}^{M+B} \frac{t^2}{2} + (b_0 - \frac{1}{2})t. \quad (7)$$

Next, we consider the case for which the optimal schedule has at least $M+B+1$ slots. Since $a_0 > b_0 > D(d_0+M+B)$, in an optimal solution, the first M slots are assigned to the groups containing links representing literal sources, followed by the B single-link groups, and the single-link groups with links from $\mathcal{D}$ are scheduled lastly. Therefore, the minimum objective value is achieved by the case in which $D-1$ links of $\mathcal{D}$ can be scheduled in the first slot and the remaining one occupies the last slot. Hence the lower bound of the total age for all the solutions with more than $M+B$ slots is

$$(D-1)d_0 + \sum_{t=1}^{M} \frac{t^2}{2} + (a_0 - \frac{1}{2})t + \sum_{t=M+1}^{M+B} \frac{t^2}{2} + (b_0 - \frac{1}{2})t + \frac{(M+B+1)^2}{2} + (d_0 - \frac{1}{2})(M+B+1). \quad (8)$$

Recall the condition $B > \sqrt{DM(M+1)(M+d_0)}$. It can be verified that the total age of any solution with more than $M+B$ slots is strictly greater than A_{M+B}. We now check the recognition version of the defined MASP with the objective value A_{M+B}. If it is feasible, a solution mapping from the MASP to the 3-SAT problem can be established by setting the binary variable to be true, if its corresponding link has the packet delivered in a group of the second subset; otherwise, the variable is set to be false. Since in this case, all the packets of the clause sources are delivered together with the packets representing the binary variables, the 3-SAT problem is feasible. On the other hand, if the answer to the 3-SAT problem is true, the recognition version of the defined MASP with the objective value A_{M+B} must be feasible. Since the 3-SAT problem is NP-complete and the reduction is clearly polynomial, the recognition of MASP is NP-complete. Hence the MASP with given candidate groups is NP-hard. ■

V. Structural Results

Since computing the optimal schedule is hard in general, it is of interest to identify tractable cases and investigate optimality conditions, which may point to practical and realistic algorithms for approximating, or precisely determining, an optimal schedule. We first consider the MASP in which only single-link groups are allowed to transmit, i.e., the classic access scheme of TDMA. Earlier in Figure 5, we have demonstrated that even with TDMA, deriving the optimal schedule is not intuitive. Hence we start from a simple case where each source has one packet to be delivered.

Theorem 4. *The MASP with single packet per source and TDMA is tractable.*

Proof: We provide an analytic solution and prove it is globally optimal. For the case considered here, the objective function (5a) is simplified to $\sum_{n=1}^{N} \frac{T_n^2}{2} + (a_{n0} - \frac{1}{2})T_n$. Since each source has only one packet, for any TDMA-based schedule, packet delivering reaches completion for exactly one link at each time slot. Hence $T_n,\ n = 1, \ldots, N$, take the N different integer values in $[1, N]$. The overall age of a feasible schedule therefore equals $\sum_{n=1}^{N} \frac{1}{2}(n^2 - n) + \sum_{n=1}^{N} a_{n0}T_n$. To achieve the minimum age, at the optimum, T_n must be in descending order of a_{n0}. Otherwise the objective function value can be improved, or kept the same (in the case with multiple optimal solutions), by swapping the values of T_n. Therefore, the optimal solution is to schedule the N links one by one in this order. To compute this optimal solution, the bottleneck is to sort a_{n0} in descending order which has complexity $O(N \log N)$. Hence the conclusion follows. ■

We now consider the general TDMA case, where the sources may have multiple packets to be delivered. We study the case where the sources are identical in their frequency of information sampling, that is, information generation is periodic and the period is the same for all sources. An example scenario consists of sensor networks for environmental detection and control.

Lemma 5. *For MASP with TDMA, if the time intervals of any two consecutive packets of any source, i.e., Δ_{ni}, are identical, then in the optimal solution, the packet delivery at each link is contiguous, i.e., without being interrupted by other links.*

Proof: Assuming that Δ is the common time interval, the objective function in (5a) reads

$$\sum_{n=1}^{N} \frac{T_n^2}{2} + (a_{n0} - \frac{1}{2} - (K_n - 1)\Delta)T_n + \sum_{i=1}^{K_n - 1} \Delta T_{ni} = \sum_{n=1}^{N} \frac{T_n^2}{2} + (a_{n0} - \frac{1}{2} - K_n\Delta)T_n + \sum_{i=1}^{K_n} \Delta T_{ni}. \quad (9)$$

With TDMA, it is clear that the optimal schedule occupies $\sum_{n=1}^{N} K_n$ slots, and consequently, $T_{ni},\ n = 1, \ldots, N,\ i = 1, \ldots, K_n$ take the different integer values in $[1, \sum_{n=1}^{N} K_n]$. Then the last term in (9), i.e., $\sum_{n=1}^{N}\sum_{i=1}^{K_n} \Delta T_{ni}$, equals $\frac{K(K+1)}{2}\Delta$, where $K = \sum_{n=1}^{N} K_n$, and hence is a constant. Therefore the optimality of a solution is completely determined by the values of $T_n,\ n = 1, \ldots, N$. Evidently, for any feasible schedule, we have $T_n \geq K_n$. The objective function in (9) is monotonically increasing in T_n in its feasible region, because $a_{n0} > K_n\Delta$ holds as a_{n0} represents the elapsed time since the packet before U_{nK_n} was generated.

Suppose Ω is an optimal solution in which the links are not emptied one by one; then there exists at least one link whose packets delivering is interrupted by others. Without loss of generality, we re-index the links by the delivering order of their last packets and assume link l is the first one for which the packets are not delivered consecutively, that is, there are packets of link m, $m > l$, being delivered before T_l, and hence $T_l > \sum_{n=1}^{l} K_n$. We then move all those interrupted packets to the slots right after U_{l,K_l} being delivered and preserve the delivering order of them. In the new solution Ω', T_l decreases to $\sum_{n=1}^{l} K_n$, and T_n, $\forall n \neq l$, remains the same as that in Ω. Hence the total age of Ω' is strictly less than Ω. This, however, contradicts the assumption that Ω is optimal, and the theorem follows. ∎

Starting from this optimality condition, we identify two scenarios that can be solved in polynomial time.

Theorem 6. *The MASP with TDMA and identical Δ_{ni} is solved in polynomial time, if the initial ages of all sources, i.e., a_{n0}, $\forall n \in \mathcal{N}$, are of the same value.*

Proof: By Lemma 5 and its proof, the optimal scheduling solution consists of N segments, one for each link, and the objective value is determined by $\sum_{n=1}^{N} \frac{T_n^2}{2} + (a_0 - \frac{1}{2} - K_n\Delta)T_n$, wherein a_0 is the common initial age for all sources. For the first part of the total age, i.e., $\sum_{n=1}^{N} \frac{T_n^2}{2}$, the optimum is achieved when $T_n,\ n = 1, \ldots, N$, take the minimum possible values. The second part, $\sum_{n=1}^{N}(a_0 - \frac{1}{2} - K_n\Delta)T_n$, reaches the lower bound when $T_n,\ n = 1, \ldots, N$, take the minimum values and are in the ascending order of K_n. In view of that, we provide an analytic solution where both parts achieve the minimum values and hence lead to the optimal solution. Assuming link $N(u)$ is the uth link that completes packet delivering, it follows that $T_{N(u)} = \sum_{v=1}^{u} K_{N(v)}$ holds. To minimize $\sum_{v=1}^{u} K_{N(v)}$, $u = 1, \ldots, N$, the indices $N(u)$ have to be consistent with the ascending order of the number of packets of each link. That is, $N(1) = \text{argmin}_n\{K_n,\ n \in \{1, \ldots, N\}\}$, $N(2) = \text{argmin}_n\{K_n,\ n \in \{1, \ldots, N\} \setminus \{N(1)\}\}$, and so on. Clearly, in this solution, $T_{N(u)}$ are minimum and in the ascending order of K_n as well. Therefore, the optimal schedule is to activate the N links in the ascending order of K_n and for each link, use the same amount of consecutive time slots as the number of its packets. The time complexity on computing this optimal solution therefore depends on sorting K_n, which is $O(N \log N)$, and hence the theorem follows. ∎

We now consider a more general case in which the information update is uniformly distributed in time, and hence the initial age a_{n0} is determined by the last received update before the packets under consideration. Denote by U_{n,K_n+1} this packet for S_n, then $a_{n0} = t_0 - \tau_{n,K_n+1}$. Since at t_0, there are K_n packets in the queue of S_n, we have $K_n\Delta < a_{n0} < (K_n + 1)\Delta$.

Theorem 7. *The MASP with TDMA and identical Δ_{ni} is solved in polynomial time, if the condition $K_n\Delta < a_{n0} < (K_n + 1)\Delta$ holds.*

Proof: By Lemma 5, the optimal solution is to activate each link one by one. We show in the following that the optimal activation order of the links follows the ascending order of K_n, $\forall n \in \mathcal{N}$. Suppose the opposite; then in the optimal solution Ω, there exist two neighboring links, e.g., links l and $l+1$, being activated in reverse order, that is, $K_l > K_{l+1}$ and $T_l < T_{l+1}$. By swapping the activation order of these two links, we obtain a new solution Ω', in which links l and $l+1$ are emptied at T'_l and T'_{l+1}, respectively. Clearly, the total age resulted by the links other than l and $l+1$ remains the same in Ω'. Hence the difference between the objective values of Ω and Ω' is given by

$$\begin{aligned}&\frac{T_l^2}{2} + (a_{l0} - \frac{1}{2} - K_l\Delta)T_l + \frac{T_{l+1}^2}{2} + \\ &(a_{l+1,0} - \frac{1}{2} - K_{l+1}\Delta)T_{l+1} - \frac{(T'_{l+1})^2}{2} - \frac{(T'_l)^2}{2} - \\ &(a_{l+1,0} - \frac{1}{2} - K_{l+1}\Delta)T'_{l+1} - (a_{l0} - \frac{1}{2} - K_l\Delta)T'_l \end{aligned} \tag{10}$$

Putting together the assumptions $K_l > K_{l+1}$ and $T_l < T_{l+1}$, as well as the facts that $T_{l+1} = T'_l$ and $T_l > T'_{l+1}$, one can verify that (10) is greater than zero as long as $K_l\Delta < a_{l0} < (K_l + 1)\Delta$ and $K_{l+1}\Delta < a_{l+1,0} < (K_{l+1} + 1)\Delta$. Hence the objective value improves in Ω', contradicting that Ω is optimal. Therefore, the solution that activates the N links one by one, following the ascending order of K_n, is optimal. Since the time complexity on computing this solution is $O(N \log N)$, the theorem follows. ∎

Next, we consider another case, where groups up to a certain cardinality are all feasible. That is, whether a link set is compatible or not solely depends on its cardinality instead of the individual elements of the set. Specifically, for any group, replacing any link of it with another link that is not in this group, the new group remains feasible. An example scenario, which has been studied in [17] for minimum time scheduling, is that the transmitters in an isotropic medium have same distances (and hence identical geometric gains) to their common, or co-located, receivers. If the transmitters use identical power, then the interference is a function of the number of concurrently active links only. We show in the following a necessary condition of the optimal schedule for this case.

Theorem 8. *For the MASP with cardinality-based groups, in the optimal solution, the active groups follow the descending order of group cardinality.*

Proof: Suppose there exists an optimal solution Ω consisting of a group sequence in which group $\mathcal{C}_1$ is active ahead of $\mathcal{C}_2$ and $|\mathcal{C}_1| < |\mathcal{C}_2|$. Then there must be at least one link l, such that $l \notin \mathcal{C}_1$ and $l \in \mathcal{C}_2$. We construct a new schedule Ω' by moving the occurrence of link l from $\mathcal{C}_2$ to $\mathcal{C}_1$. By the property of the cardinality-based groups, the new two groups remain feasible. The other groups as well as the order of these groups (and hence including the other occurrences of l), are kept the same as Ω. It is easy to verify that Ω' is a feasible schedule and the packets of the links except l are delivered in the same pattern as in Ω. Hence, except l, the age of any other source remains the same. By moving link l from $\mathcal{C}_2$ to $\mathcal{C}_1$, one packet of link l is delivered earlier than in Ω. Without loss of generality, assuming that this is the vth packet, it follows that T_{lv} is strictly decreased. For $T_{li},\ i = 1, \ldots, K_l,\ i \neq v$, the values remain the same. Therefore, by the objection function (5a), the overall age of Ω' is strictly less than that of Ω. This, however, contradicts the assumption that Ω is optimal. Hence the theorem follows. ∎

VI. Integer Linear Programming (ILP) Formulation

In this section, we propose an ILP formulation of the MASP, to allow for the computation of the global optimal solution for problem instances of small and moderate sizes, using off-the-shelf optimization tools [29], [30]. We firstly present an ILP formulation for the MASP with given candidate group set $\mathcal{C}$, that is, in each slot, a group $c \in \mathcal{C}$ is selected to construct a schedule. After presenting the formulation, we will show that, by updating some sets of linear constraints, the ILP formulation applies also to the MASP under the physical model.

We define four sets of binary variables.

$$x_{nij} = \begin{cases} 1 & \text{if packet } U_{ni} \text{ is delivered at } t_j, \\ 0 & \text{otherwise.} \end{cases}$$

$$y_{nj} = \begin{cases} 1 & \text{if link } n \text{ is active in the } j\text{th time slot,} \\ 0 & \text{otherwise.} \end{cases}$$

$$z_{cj} = \begin{cases} 1 & \text{if group } c \text{ is active in the } j\text{th time slot,} \\ 0 & \text{otherwise.} \end{cases}$$

$$u_{nj} = \begin{cases} 1 & \text{if all packets of link } n \text{ have been delivered before/at } t_j, \\ 0 & \text{otherwise.} \end{cases}$$

Following the definition and notation in Section III-A, each link in an active group delivers one packet per time slot. It is clear that at least one packet can be delivered in each slot during a schedule. Hence for the links with $K_n,\ n = 1, \ldots, N$ packets, the schedule length is at most $\sum_{n=1}^{N} K_n$. Letting $T = \sum_{n=1}^{N} K_n$, we define $\mathcal{J} \triangleq \{1, 2, \ldots, T\}$, and recall that, t_j, $j \in \mathcal{J}$ represents the time corresponding to the end of the jth time slot. Define $\mathcal{N} \triangleq \{1, 2, \ldots, N\}$, $\mathcal{K}_n \triangleq \{1, 2, \ldots, K_n\}$, and $\mathcal{K}'_n \triangleq \{1, 2, \ldots, K_n - 1\}$, $\forall n \in \mathcal{N}$. In addition, we use $\mathcal{C}_n$ to denote the subset of groups containing link n. The ILP formulation is given by (11).

The objective function defined in (11a) is the overall age of all sources. The age of source n at time t_j, i.e., a_{nj}, is calculated by (11b) and (11c), which are in fact linear variations of (1). Suppose there is no packet from S_n being delivered at t_j and there is a packet in queue n; then, by definition, $y_{nj} = u_{nj} = 0$, and we have that the right-hand sides of (11b) and (11c) are $a_{n,j-1}+1$ and $-\sum_{i\in\mathcal{K}'_n} \tau_{ni} x_{nij}$, respectively. As $a_{nj} \geq 0$, in this case the inequality (11c) is always satisfied, or equivalently, void, irrespective of the specific values of τ_{ni} and x_{nij}. Therefore, only (11b) with $a_{nj} \geq a_{n,j-1} + 1$ takes effect in this case, and the constraint is tight at an optimal solution since the objective is to minimize the sum of a_{nj} and there is no more constraints containing a_{nj} in (11d)–(11i). If any packet other than the last one of S_n is delivered at t_j, that is, $y_{nj} = 1$ and $u_{nj} = 0$, then the right-hand side of (11b) becomes negative as the age of S_n in any time slot cannot be more than $a_{n0} + T - 1$. Hence (11b) is always satisfied and a_{nj} is constrained by (11c), which implies $a_{nj} \geq t_j - \sum_{i\in\mathcal{K}'_n} \tau_{ni} x_{nij}$. Following the same reasoning as above, the constraint is tight at the optimum. Therefore, the inequalities in (11b) and (11c) together represent the age a_{nj} as in (1). Once all packets of S_n are delivered, i.e., $u_{nj} = 1$, it is easy to verify that both (11b) and (11c) are void, resulting in zero age for S_n.

$$\underset{\{x_{nij},\ y_{nj},\ z_{cj},\ u_{nj}\in\{0,1\},\ a_{nj}\in Z^*\}}{\text{minimize}} \quad \sum_{n\in\mathcal{N}, j\in\mathcal{J}} a_{nj} \tag{11a}$$

subject to

$$a_{nj} \geq a_{n,j-1} + 1 - (y_{nj} + u_{nj})(a_{n0} + T) \quad \forall n \in \mathcal{N},\ \forall j \in \mathcal{J}, \tag{11b}$$

$$a_{nj} \geq t_j - \sum_{i\in\mathcal{K}'_n} \tau_{ni} x_{nij} - (1 - y_{nj} + u_{nj}) t_j \quad \forall n \in \mathcal{N},\ \forall j \in \mathcal{J}, \tag{11c}$$

$$x_{n,i+1,j} \leq \sum_{b=1}^{j} x_{nib} \quad \forall n \in \mathcal{N},\ \forall i \in \mathcal{K}'_n,\ \forall j \in \mathcal{J}, \tag{11d}$$

$$y_{nj} = \sum_{i\in\mathcal{K}_n} x_{nij} \quad \forall n \in \mathcal{N},\ \forall j \in \mathcal{J}, \tag{11e}$$

$$u_{nj} = \sum_{b=1}^{j} x_{n,K_n,b} \quad \forall n \in \mathcal{N}, \quad \forall j \in \mathcal{J}, \tag{11f}$$

$$u_{nT} = 1 \quad \forall n \in \mathcal{N}, \tag{11g}$$

$$y_{nj} \leq \sum_{c\in\mathcal{C}_n} z_{cj} \quad \forall n \in \mathcal{N},\ \forall j \in \mathcal{J}, \tag{11h}$$

$$\sum_{c\in\mathcal{C}} z_{cj} \leq 1 \quad \forall n \in \mathcal{N},\ \forall j \in \mathcal{J}. \tag{11i}$$

The inequalities in (11d) ensure that the transmitting order of packets for any source fulfills the FCFS discipline. That is, packet $U_{n,i+1}$ can be delivered at slot j only if its previous packet $U_{n,i}$ has been delivered before or at this time slot. Together with (11e), which implies that only one packet of an active link can be delivered in each slot, the transmission of $U_{n,i}$ is strictly before that of $U_{n,i+1}$. The equalities in (11e) imply that link n is active at time slot j if and only if one of the packets of S_n is delivered at t_j. Since y_{nj} is binary, the equalities also imply that a link can transmit up to one packet per slot. The definition of u_{nj} is given in (11f), in which the right-hand side denotes the delivery status of the last packet of S_n at t_j. By (11g), it is guaranteed that the last packet of each source has been delivered within a schedule. Therefore, the constraint sets (11d)–(11g) together impose the two feasibility conditions of a schedule defined in Section III-A. The activation constraint relating to link and group is provided in (11h). By (11h), link n can be activated only if it is included in the active group in the time slot. The inequalities in (11i) state the fact that at most one group is active in each time slot.

For the MASP under the physical model, an ILP formulation can be obtained by simply replacing (11h) and (11i) with the following SINR constraints

$$\frac{P_n G_{nn} y_{nj} + Q_n(1 - y_{nj})}{\sum\limits_{l\in\mathcal{N}, l\neq n} P_l G_{ln} y_{lj} + \sigma_n^2} \geq \gamma \quad \forall n \in \mathcal{N},\ \forall j \in \mathcal{J}. \tag{12}$$

Here, the value of parameter Q_n is set to $Q_n = \gamma(\sum_{l\in\mathcal{N}, l\neq n} P_l G_{ln} + \sigma_n^2)$. This parameter is introduced for linearization. The specific value is chosen to be large enough to ensure the constraint becomes void if link n is not active. If link n is active in slot j, i.e., $y_{nj} = 1$, then in the left-hand side of (12), the numerator is the signal power while the denominator is the sum of the interference generated by the other active links plus noise. The transmission of link n is successful if and only if the SINR value is greater than or equal to the threshold γ. On the other hand, if $y_{nj} = 0$, no SINR requirement should be in place. For the chosen value of Q_n, one can easily verify that (12) is always satisfied if $y_{nj} = 0$. The inequalities in (12) are easily converted to be linear by multiplying both sides with the positive denominator. Hence by substituting (12) for (11h) and (11i), we obtain an ILP for the MASP under the physical model.

VII. Steepest Age Descent Algorithm

Since the MASP is hard in general, we develop an optimization algorithm that is suboptimal, but fast, with better scalability than ILP. Throughout the scheduling horizon, the age for a source is reduced if and only if a packet of the corresponding link is scheduled, and the amount of reduction varies by link and the packet number. By the formulation in (5) as well as the illustration in Figure 4, it is intuitive to schedule packets yielding large age reduction as early as possible. To this end, our scheduling algorithm selects, for each time slot, the link group of which the packets in question, if scheduled, lead to the largest sum age reduction. As the algorithm aims to reduce the age as much as possible in each step, it is a steepest age descent algorithm. Moreover, the algorithm considers schedule construction in both ascending and descending orders of time slot indices; these are referred to as forward and backward constructions, respectively, and detailed below.

A. Basic Design

For each time slot, the algorithm calculates the age reduction enabled by each of the candidate link groups. Consider link group $c \in \mathcal{C}$, and, for link $n \in c$, denote by $\rho(n)$ ($1 \leq \rho(n) \leq K_n$) the index of the packet that will be transmitted if the link is scheduled. Throughout this section, we use δ to denote age reduction, instead of Δ as in Section III-A, because for the last packet of each link, the amount of age reduction in the algorithm is set differently from that in Section III-A. Recall that, for link n and its $\rho(n)$th packet ($1 \leq \rho(n) \leq K_n - 1$), scheduling the packet results in an age reduction of exactly $\delta_{n,\rho(n)} = \tau_{n,\rho(n)} - \tau_{n,\rho(n)-1}$, i.e, the difference between the time stamps of packets $\rho(n)$ and $\rho(n) - 1$. The age reduction for group c, denoted by δ_c, equals the sum of the age reduction due to the individual links in the group. That is, $\delta_c = \sum_{n \in c} \delta_{n,\rho(n)} = \sum_{n \in c} (\tau_{n,\rho(n)} - \tau_{n,\rho(n)-1})$. The algorithm then selects the group maximizing the age reduction, i.e., $\text{argmax}_{c \in \mathcal{C}} \delta_c$. Note that, in any step, if packet delivery is complete for a link, then the candidate group set $\mathcal{C}$ is also updated to remove the link from the groups that contain it.

For any link n, the above discussion applies to packets except for $\rho(n) = K_n$, for which the age drops to zero if n is scheduled. Denoting by j the time slot under consideration, the amount of reduction equals $a_{n,j-1} + 1$. In addition, unlike scheduling packets up to K_{n-1}, the age will not increase further because n is now emptied. To take this effect into account, we add the term $\frac{(T-j)(T-j+1)}{2}$ to the age reduction, where T is the (yet unknown) schedule length, to reflect that scheduling the packet will prevent the age from increasing during the rest of the schedule. Note that the term encourages the use of link groups that will empty many sources. That is, $\delta_{n,K_n} = a_{n,j-1} + 1 + \frac{(T-j)(T-j+1)}{2}$. As T is unknown until the scheduling is complete, the algorithm uses two phases. First, we set T to the upper bound of $T = \sum_{n \in N} K_n$ to obtain a schedule, of which the length is used to construct a second schedule. The algorithm then selects the one with lower overall age.

B. Backward Construction

The basic algorithm design, as in Section VII-A, is to start from time t_0 and to perform scheduling slot by slot. We enhance the algorithm by backward construction that constructs a solution backwards in the timeline. For each link, the backward construction processes packets in the reverse order, namely $K_n, K_{n-1}, \ldots, 1$. This order of processing is possible due to the observation that, for any link n and its packet $i < K_n$, the age reduction of scheduling the packet equals $\delta_{n,i} = \tau_{ni} - \tau_{n,i-1}$ that is not dependent on which time slot the packet (or its predecessor) is scheduled.

Denote again by $\rho(n)$ the index of the packet in question if link n is scheduled. Recall that the algorithm strives to schedule link groups with large age reduction early in the schedule. Since now the schedule is constructed backwards in the time line, link group selection uses minimization (instead of maximization as in forward construction), that is, the selection is based on $\text{argmin}_{c \in \mathcal{C}} \delta_c$, where $\delta_c = \sum_{n \in c} (\tau_{n,\rho(n)} - \tau_{n,\rho(n)-1})$ if $1 \leq \rho(n) \leq K_n - 1$.

Backward construction starts by setting a schedule length T (for which more details are given later), and performs scheduling in the descending order of time slot indices. Suppose all packets are delivered by time slot j ($1 \leq j \leq T$). If $j > 1$, the entire schedule is shifted from time slots $j, j+1, \ldots, T$ to time slots $1, \ldots, T - j + 1$, which then form the output, and the corresponding overall age is computed.

Consider the last packet of link n; this packet is processed first for the link in backward construction. In forward construction, the age reduction of scheduling the packet in time slot j is $\delta_{n,K_n} = a_{n,j-1} + 1 + \frac{(T-j)(T-j+1)}{2}$. However, in backward construction, $a_{n,j-1}$ is not known. In our implementation, we set $a_{n,j-1} = a_{n0} + j - 1$, which corresponds to assuming constant age increase since the first time slot. Note that this setting encourages to first schedule the last packet of all links.

As for forward construction, there are two phases in backward construction. First, the algorithm sets T to the upper bound $\sum_{n \in N} K_n$ to obtain a schedule. The length of this schedule is then used as T to construct another solution backwards. The schedule with lower overall age provides the output.

C. Algorithm Flow

The flow of the algorithm is presented in Algorithm 1. Note that in all the phases of the algorithm, once a link is emptied (either in forward construction or backward construction), it is discarded from all groups containing the link.

Algorithm 1 Steepest age descent algorithm

Input: $\mathcal{N},\ \mathcal{K}_n,\ \tau_{ni},\ a_{n0},\ t_0,\ \mathcal{C}$
Output: $\Omega^*,\ a^*$
1: $complete \leftarrow$ **false**, $empty(n) \leftarrow$ **false**, $\mathcal{C}' \leftarrow \mathcal{C},\ \tau_{n0} \leftarrow t_0 - a_{n0},\ n \in \mathcal{N}$
2: $T \leftarrow \sum_{n\in\mathcal{N}} K_n,\ j \leftarrow 0,\ \rho(n) \leftarrow 1, \Omega \leftarrow \emptyset$ // Forward construction: Phase I
3: **while** $complete =$ **false do**
4: $\quad j \leftarrow j+1,\ t \leftarrow t_0 + j$
5: $\quad$ **for** $n \in \mathcal{N}$ **do**
6: $\quad\quad$ Compute $\delta_{n,\rho(n)}$
7: $\quad$ **for** $c \in \mathcal{C}$ **do**
8: $\quad\quad \delta_c \leftarrow \sum_{n\in c} \delta_{n,\rho(n)}$
9: $\quad c_j \leftarrow$ argmax$(\delta_c),\ \Omega(j) \leftarrow c_j$
10: $\quad$ **for** $n \in c_j$ **do**
11: $\quad\quad$ **if** $\rho(n) < K_n$ **then**
12: $\quad\quad\quad \rho(n) \leftarrow \rho(n)+1$
13: $\quad\quad$ **else**
14: $\quad\quad\quad empty(n) \leftarrow$ **true**, $c_j \leftarrow c_j \setminus n$, update $\mathcal{C}$
15: $\quad$ **if** $empty(n) =$ **true** $\forall n \in \mathcal{N}$ **then**
16: $\quad\quad complete \leftarrow$ **true**
17: $T \leftarrow j,\ \hat{a} \leftarrow$ the overall age of Ω computed by (1)
18: $a^* \leftarrow \hat{a},\ \Omega^* \leftarrow \Omega$
19: $complete \leftarrow$ **false**, $\mathcal{C} \leftarrow \mathcal{C}',\ j \leftarrow 0,\ \rho(n) \leftarrow 1,\ \Omega \leftarrow \emptyset$, repeat lines 3 - 17 // Forward construction: Phase II
20: **if** $\hat{a} < a^*$ **then**
21: $\quad a^* \leftarrow \hat{a},\ \Omega^* \leftarrow \Omega$
22: $complete \leftarrow$ **false**, $\mathcal{C} \leftarrow \mathcal{C}',\ T \leftarrow \sum_{n\in\mathcal{N}} K_n,\ j \leftarrow T,\ \rho(n) \leftarrow K_n$, $\Omega \leftarrow \emptyset$ // Backward construction: Phase I
23: **while** $complete =$ **false do**
24: $\quad t \leftarrow t_0 + j,\ j \leftarrow j-1$
25: $\quad$ **for** $n \in \mathcal{N}$ **do**
26: $\quad\quad$ Compute $\delta_{n,\rho(n)}$
27: $\quad$ **for** $c \in \mathcal{C}$ **do**
28: $\quad\quad \delta_c \leftarrow \sum_{n\in c} \delta_{n,\rho(n)}$
29: $\quad c_j \leftarrow$ argmin$(\delta_c),\ \Omega(j) \leftarrow c_j$
30: $\quad$ **for** $n \in c_j$ **do**
31: $\quad\quad$ **if** $\rho(n) > 1$ **then**
32: $\quad\quad\quad \rho(n) \leftarrow \rho(n)-1$
33: $\quad\quad$ **else**
34: $\quad\quad\quad empty(n) \leftarrow$ **true**, $c_j \leftarrow c_j \setminus n$, update $\mathcal{C}$
35: $\quad$ **if** $empty(n) =$ **true** $\forall n \in \mathcal{N}$ **then**
36: $\quad\quad complete \leftarrow$ **true**
37: Shift Ω from $j, \ldots T$ to $1, \ldots, T-j+1$ $T \leftarrow T-j+1,\ \hat{a} \leftarrow$ the overall age of Ω computed by (1)
38: **if** $\hat{a} < a^*$ **then**
39: $\quad a^* \leftarrow \hat{a},\ \Omega^* \leftarrow \Omega$
40: $complete \leftarrow$ **false**, $\mathcal{C} \leftarrow \mathcal{C}',\ j \leftarrow T,\ \rho(n) \leftarrow K_n,\ \Omega \leftarrow \emptyset$, repeat lines 23 - 39 // Backward construction: Phase II
41: **return** $(a^*,\ \Omega^*)$

Remark 1. *The reasoning of the algorithm can be verified by applying it to the examples discussed in previous sections. The algorithm achieves the global optimum for both cases described in Figures 3 and 5. The optimal solution in the first case is given by the forward construction. For the second case, which is a counter example of the optimality of the forward construction as we have shown earlier, the backward construction provides the optimal solution.*

Remark 2. *We remark that our steepest age descent algorithm leads directly to the optimum for the case discussed in Theorem 4. This is because each source has one packet and hence the age reduction of scheduling a link equals the initial age and the number of time slots elapsed; the latter is the same for all links. By the steepest descent principle and due to TDMA, the algorithm will schedule links in descending order of initial ages, which is optimal by the theorem. For the scenario in Lemma 5, we remark that for any time slot, the potential age reduction is identical for all links as the time stamps of any two consecutive packets are the same. Recall that exactly one link is scheduled in each time slot due to TDMA. Hence, if the tie is broken by link index, either in ascending or descending order, the scheduling solution fulfills the optimality condition in the lemma.*

VIII. Numerical Study

In this section, we provide numerical results to assess the benefit in reducing the age by employing the optimal schedule of the MASP in wireless networks, and evaluate the performance of the proposed optimization algorithm. We perform the simulation for two sets of networks of different sizes.

We consider two baseline scheduling solutions for comparison. For TDMA, i.e., one link can be scheduled in each time slot, the baseline solution is the round robin strategy. That is, all links having packets remaining are scheduled one by one in the order of their indices; this is repeated until all packets are delivered. For the more general case where there may be multiple links per time slot, it is not fair to compare to round-robin scheduling. Hence, we consider maximum-cardinality scheduling as the baseline solution. Specifically, for each time slot, the baseline schedule chooses the group that enables the largest number of packets that can be delivered. If there are multiple choices, then these groups will be chosen sequentially as long as they remain maximal in cardinality. This solution process represents a greedy method for minimum-time scheduling. Subsequently, the term improvement refers to the amount of reduction in age achieved by our algorithm in comparison to the baseline solutions. Moreover, for small networks (Section VIII-A), the performance of our algorithm will also be compared to the global optimum computed via the ILP in Section VI. We use the term optimality gap to refer to the relative difference between the age value from our algorithm and that of the global optimum.

A. The MASP for Small Networks

We consider networks with $N = 5$ sources, each having up to 4 packets to be delivered. The starting time is $t_0 = 30$. The initial age a_{n0} of the sources and the time stamps τ_{ni} of the packets are uniformly distributed random integers in their respective (defined or feasible) intervals. That is, $a_{n0} \in [10, 25]$ and $\tau_{ni} \in (t_0 - a_{n0}, t_0)$. In total 50 instances are generated and tested, to gain insights on performance evaluation.

We firstly consider the MASP with TDMA, where the candidate group set consists of singletons only. For performance comparison, we consider round-robin scheduling and the global optimum. The latter is computed by the ILP shown in (11), using CPLEX 12.6 [29], for accurate performance assessment. The results of the proposed algorithm are also computed for performance evaluation. For each instance, the age of the optimum and the solution of the algorithm are normalized by the baseline solution of round-robin. Figure 6 shows the empirical cumulative distribution functions (CDFs) of the results.

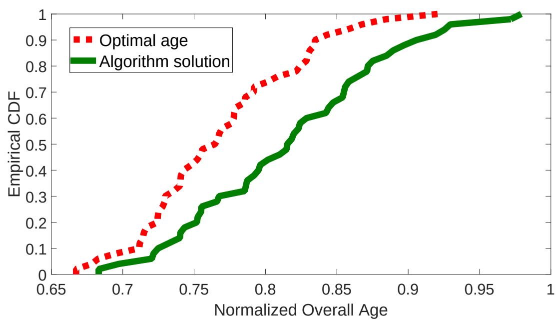

Fig. 6. The MASP with TDMA.

The result demonstrates a noticeable improvement in decreasing the overall age by employing the optimal scheduling solution. For the 50 instances, the optimal age ranges from 0.66 to 0.92, with an average value of 0.76, with respect to the baseline solution that uses round-robin. The proposed algorithm performs well for this network setting. The average optimality gap is 6.4% in comparison to the global optimum. The solution derived from the algorithm yields approximately 20% improvement on average over the baseline solution.

Next, we consider the general case of MASP, in which multi-link groups are allowed. We generate the candidate group set $\mathcal{C}$ under the SINR model. Specifically, the $N = 5$ links are randomly distributed in an area of 500×500 meters. The transmitting power and noise variance are uniformly set to 30 dBm and -100 dBm for all links, respectively. The channel gain follows a distance-based propagation model with a path loss exponent of 4. The SINR threshold is set to be 0 dB. The candidate group set $\mathcal{C}$ consists of all SINR-feasible groups under this setting. For this set of instances, we use the aforementioned maximum-cardinality scheduling as the baseline solution for comparison. The overall age of the optimized scheduling derived from ILP and the proposed algorithm are normalized by this baseline solution. In Figure 7, we present the results of 50 instances, in form of empirical CDFs.

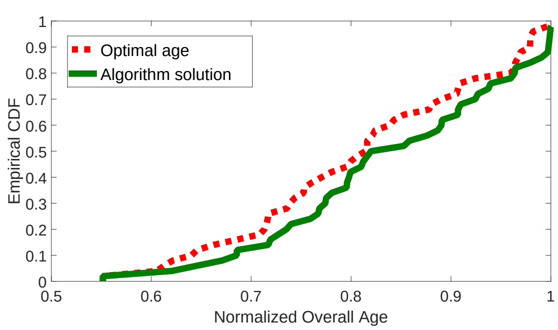

Fig. 7. The MASP with multiple-link sets for small networks.

For this dataset, optimizing the scheduling strategy leads to an average improvement of 19% over the baseline. The results also demonstrate the good performance of the algorithm, for which the optimality gap is less than 3% on average.

B. The MASP for Larger Networks

We also consider larger networks with $N = 20$ links. The number of packets to be delivered is up to 10 for each link. The initial ages a_{n0}, $n \in \mathcal{N}$, are random integers in $[10, 250]$. The starting time is $t_0 = 300$. The time stamps are uniformly randomly distributed in $(t_0 - a_{n0}, t_0)$. For this set of scenarios of the MASP, the size of the dataset prohibits the use of ILP, hence we compute the scheduling solutions by the algorithm and the optimized solutions are normalized by the baseline. In addition, defining the candidate link groups by checking the SINR value is not feasible for this dataset. Therefore, we construct the candidate group set $\mathcal{C}$ as follows. The group set $\mathcal{C}$ consists of 10 groups with the cardinalities randomly ranging from 2 to a predefined C, as well as the single-link groups. We set $C = 5, 10, 15$ and generate the datasets respectively. In addition, the MASP with TDMA corresponding to setting $C = 1$ is tested. For each setup, 100 instances are generated and tested. The results are presented in Figure 8.

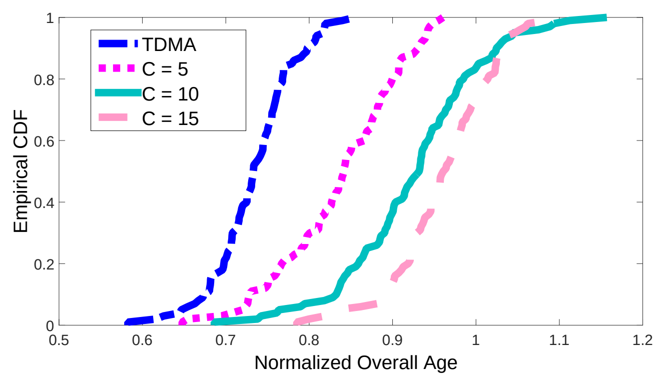

Fig. 8. The MASP with multiple-link sets.

In comparison to the baseline solution, i.e., the minimum time scheduling, the solution of our algorithm yields better performance in age. For the maximal group cardinalities $C = 15, 10, 5$ and TDMA, the average improvement over the baseline solution are 4%, 8%, 16% and 27%, respectively. For all the instances with smaller C, the objective of age has been improved. For larger C, the algorithm outperforms the minimum time scheduling for the majority of instances (more than 80%). Hence the results further demonstrate that minimizing age is different from optimizing the traditional criterion for link scheduling. From the curves, one can observe that, the improvement of age is more prominent for the instances with small group cardinalities, implying that the contribution of employing optimized scheduling strategy is more significant in severely interference-limited networks, e.g., networks with densely located links. This observation actually agrees with intuition since the sequencing that resulted from scheduling affects the age and the smaller the group cardinality is the more the packets that have their age affected.

IX. Extension to Rolling Horizon

Recall that MASP corresponds to solving one scheduling instance in scenarios where scheduling takes place in cycles;

a schedule is computed by the end of each cycle, for packets that arrived during the cycle. Instead of a static view of scheduling cycle, a natural extension is the use of the rolling horizon strategy. With rolling horizon, the scheduling process optimizes the schedule based on the knowledge of packets currently present at the transmitters as in MASP, but the optimized schedule is only executed partially, i.e., for some number of time slots, before a new scheduling optimization is performed to account for the remaining packets as well as the new packets that have arrived.

For rolling horizon, an important parameter is how frequent (re-)scheduling should take place, i.e., how much of the current schedule shall be executed before running the next optimization instance. Updating the schedule frequently enables to better account for new arrivals but leads to more computational effort. On the other hand, running a schedule almost to completion will not perform well from the age standpoint. This is because towards the end of the current schedule, packet delivery is seemingly complete for many sources (for the previous input used for computing the schedule); the optimum of the remaining time of the schedule would be significantly different if new arrivals are taken into consideration. Consider TDMA, for which the schedule length equals $\sum_{n=1}^{N} K_n$. The schedule should then be re-optimized at a time point that is considerably less than $\sum_{n=1}^{N} K_n$ time slots. In general, the scheduling frequency should also make use of any knowledge available on packet generation at the sources. Detailed analysis of the relation between scheduling frequency, packet generation, and performance, as well as extensive numerical study for rolling horizon together form a line of our future investigation.

X. Concluding Remarks

We have considered wireless link scheduling in the context of minimum age in wireless networks. Fundamental insights including problem complexity and structural results have been obtained. An ILP formulation and a steepest age descent algorithm have been developed to solve the MASP with optimality and scalability, respectively. Simulation results have also been obtained to show the benefit of employing the optimal scheduling solution in terms of the overall age, and to confirm the effectiveness of the proposed algorithm.

It is worth remarking that the complexity and structural analysis, as well as the solution approaches remain applicable, if the age of a source is not assumed to drop to zero, but continues to increase after the last packet. For example, the ILP formulation is easily adapted by removing the variables indicating when the last packet of each source is delivered.

There are many extensions in addition to rolling horizon. For example, we may extend the study to the scenarios with multiple sources being associated with each transmitter, or consider multi-hop networks. Another extension is the fundamental solution characterization under a multi-objective setting that incorporates both the freshness of information and the fairness among the links.

Appendix
3-SATISFIABILITY

3-SATISFIABILITY (in short, 3-SAT) is a decision problem from Boolean logic. Let $V = \{v_1, v_2, \dots, v_m, \dots, v_M\}$ be a set of Boolean variables. A literal is either a variable v_m or its negation $\bar{v}_m$. The literals v_m and $\bar{v}_m$ are true if and only if the variable v_m is assigned to be true and false, respectively. A clause is a disjunction of those literals, e.g., $v_1 \vee \bar{v}_2 \vee v_3$. A clause is satisfied if and only if at least one of its literals is true. A Boolean formula F is in conjunctive normal form (CNF) of clauses. The SATISFIABILITY (in short, SAT) problem is to determine if there is an assignment of Boolean values to the variables in set V, such that F is true, i.e., all the clauses are simultaneously satisfied. 3-SAT is a restricted version of SAT in which all instances have exactly three literals per clause.

A 3-SAT instance is reducible if its formula F can be simplified in the case of

- the value(s) of one or more literals is able to be determined trivially, e.g., only one of v_m and $\bar{v}_m$ appears in F;
- a clause is always satisfied and hence can be eliminated from F, e.g., a clause contains both v_m and $\bar{v}_m$.

In this paper, we assume the 3-SAT instances used in the proofs are irreducible, that is, it is non-trivial to eliminate any literal or clause.

SAT is the first known NP-complete problem (Cook's Theorem). 3-SAT is as hard as SAT since any CNF formula F can be reduced to some 3-CNF formula that is satisfiable if and only if F is. 3-SAT is one of the most widely used problems for proving other NP-completeness results.

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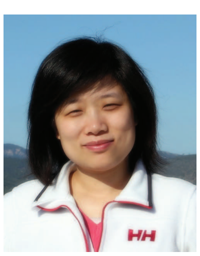

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