Correlation between Track Irregularities and
Vehicle Dynamic Response
Based on Measurements and Simulations

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Preface

This licentiate thesis is the summary of my research work at the Department of Aeronautical and Vehicle Engineering at KTH Royal Institute of Technology, Stockholm, Sweden, starting in November 2014.

The financial support from the KTH Railway Group and its partners is greatly acknowledged.

I want to thank my supervisors professors Mats Berg and Sebastian Stichel. Your guidance helped defining the direction of this research and provided valuable pieces of advice and remarks on ideas, work in progress, paper drafts and more. It is a pleasure to work at KTH in your research group.

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Finally, to our son Taisei. Your growth is inspiring me to grow as well.

Tomas Karis
Stockholm, May 2018
Abstract

Deviations from the design track geometry are called track irregularities, which are a main excitation mechanism in the track–vehicle dynamic system, and very important to monitor and maintain to have traffic on a line run safely and comfortably. Especially during vehicle acceptance testing, it is important that a new vehicle behaves close to design predictions and within limit values, thus it is important to be able to describe track irregularities in a consistent way.

There are several methods which quantify the effects track irregularities have on a vehicle while running along the track. Most common is analyse standard deviations and percentiles and maximum values over sections with pre-defined length. However, these quantities do not correlate well with the vehicle dynamic response, e.g. two track sections with similar maxima and standard deviations can result in very different response of the vehicle.

To improve the correlation between track irregularities and vehicle response measures, it is recommended by past research to use multiple regression analysis to take e.g. vehicle speed and track curvature into account as well. Other methods range from derivatives of track irregularities, via transfer functions and vehicle filters to neural networks. Common for all these methods are that there is either still slight inconsistencies in the results or that they are tailored for certain vehicle types on specific lines. As a result, the preferred method to evaluate track irregularities is still to use standard deviations.

In this thesis, data from three vehicles in two measurement campaigns is evaluated using a single degree of freedom model as inspiration to break down the path from track to vehicle into several steps. A weak link in these steps is identified, which shows significantly lower correlation coefficients than the other steps. The weak link is the step from vertical track irregularity second spatial derivative to vertical axle box acceleration divided by the squared vehicle speed. A variable wavelength range $D_x$ is introduced, as an alternative to the common $D_1$ (3–25 m), $D_2$ (25–70 m) and $D_3$ (70–150 m) wavelength ranges. Its wavelength range corresponds to the vehicle response band-pass filter frequencies and is thus speed dependent.

Simulations are also carried out to investigate the weak link and for the possibility to vary parameters that cannot be changed during on-track measurements. A multi-body system model of the passenger coach Bim 547.5 is used, together with recorded track data and vehicle speed from the on-track measurements. The varied parameters have rather low sensitivity and affect results to a small extent. Most impact has the randomly varying vertical track stiffness which affects the vertical wheel–rail forces and axle box accelerations.

In future work, it should be explored if it is possible – and in such case how – to separate the effects of varying track stiffness from the track irregularities. This to better understand when a vehicle response is linked more to the track irregularities or to the track flexibility. The weak link identified in the steps from track to vehicle should also be further explored, perhaps by extending the underlying model or evaluate a different set of measurements.
**Keywords:** track irregularities, vehicle response, wheel–rail force, axle box acceleration, Dynotrain, Gröna Tåget, multi-body system, simulations
Sammanfattning

Spårlägesfel är avvikelsor från den nominella spårgeometrin. De är en viktig excitationsmekanism i det dynamiska system som bana och fordon utgör och är viktiga att övervaka och åtgärda för att trafiken ska kunna flyta säkert och komfortabelt. Eftersom det vid mätningar för typgodkännande av fordon är viktigt att fordonet beter sig som förväntat och inom gränsvärden, är det viktigt att kunna beskriva spårlägesfel på ett sätt som är konsekvent och motsvarar hur fordonet ”känner av” dem.


I den här avhandlingen används en enfrihetsgradsmodell som inspiration för att bryta ner excitationsvägen från spår till fordon i ett antal steg, som sedan undersöks genom att utvärdera mätdata från tre fordon i två forskningsprojekt. En svag länk bland stegen identiferas, vilken visar signifikant lägre korrelationsvärde än övriga steg. Den svaga länken är steget från spatial andraderivata av spårlägesfel till axelboxacceleration dividerad med fordonshastigheten i kvadrat. Ett variabelt våglängdsområde $Dx$ introduceras som ett alternativ till de vanligt förekommande $D1$ (3–25 m), $D2$ (25–70 m) och $D3$ (70–150 m). Det variabla våglängdsområdet motsvarar de frekvenser som används för utvärdering av fordonssrespons och är därmed hastighetsberoende.

Nyckelord: spårlägesfel, fordonssrespons, hjul-räl-krafter, axelboxacceleration, Dynotrain, Gröna Täget, flerkroppssystem system, simuleringar
Thesis outline

This thesis explores how track–vehicle interaction can be broken down into steps from track to vehicle in order to find key parameters to relate track irregularities to vehicle responses in a consistent manner.

The thesis consists of two parts: **Overview** and **Appended Papers**. The overview introduces topics relevant to this thesis, then continues with a general section on track–vehicle interaction, a review of previous work done within the field of track–vehicle interaction and track geometry assessment and ends with a summary of the present work and suggestions for future work. The second part consists of the following two papers as scientific contributions of the thesis work:

**Paper A**


**Paper B**


Division of work between authors

**Paper A**

Berg and Karis outlined the study, with Stichel, Li, Thomas and Dirks reviewing results and paper drafts. Karis wrote the paper drafts and the Matlab code used for evaluation of measurement results and Berg aided in interpreting the preliminary results.

**Paper B**

Berg and Karis outlined the study, whereas Stichel contributed with ideas and feedback on preliminary results. The Matlab code used for Paper A was re-used and slightly modified by Karis. Karis wrote the paper draft and Berg and Stichel reviewed it before submission.
Publications not included in this thesis

Conference papers


Thesis contributions

This thesis contributes to the understanding of track–vehicle interaction as follows:

- A variable wavelength range $Dx$ for track irregularities, which corresponds to the filter frequencies of vehicle responses, is introduced

- It is shown how track–vehicle interaction can be broken down into smaller steps, inspired by a single degree of freedom model

- A weak link in the above steps is observed, which has significantly lower correlation coefficients in all evaluated cases for measured data than the other steps

- A sensitivity analysis of the track-vehicle interaction steps is presented

- Randomly varying track stiffness along the track is shown to have a significant impact on vertical wheel–rail forces and axle box accelerations
Contents

I OVERVIEW 1

1 Introduction 3
   1.1 Track irregularities ........................................ 3
   1.2 Vehicle responses ........................................... 6
   1.3 Research motivation ....................................... 7

2 Track–vehicle interaction 9
   2.1 Excitation sources .......................................... 10
   2.2 Vehicle responses .......................................... 12

3 Previous work 15
   3.1 Standard deviations ......................................... 17
   3.2 Isolated defects .............................................. 18
   3.3 Combination of parameters .................................. 19
   3.4 Methods based on vehicle response ......................... 19
   3.5 Concluding remarks ....................................... 24

4 The present work 27
   4.1 Summary of Paper A ......................................... 27
   4.2 Summary of Paper B ......................................... 29

5 Conclusions and future work 31
   5.1 Conclusions .................................................. 31
   5.2 Future work .................................................. 32

Bibliography 33

II APPENDED PAPERS 37
Part I

OVERVIEW
Chapter 1

Introduction

For all tracks, there is a design geometry with horizontal and vertical curve radii, track cant and gauge. As vehicles run on the track, the alignment with the design geometry deteriorates and causes track irregularities to form, which contribute to the dynamic input of the track–vehicle system. The interaction between a track and a vehicle running along it is of interest for several reasons. It might be to run the vehicle at high speed, with high payload, quietly and comfortably. Especially during vehicle acceptance testing, it is important that a new vehicle behaves close to design predictions and within limit values. Without proper knowledge about the interaction between the track and vehicle, this is not possible.

There are several methods which quantify the effects track irregularities have on a vehicle while running along the track. Most common is to analyse standard deviations, percentiles and maximum values over sections with pre-defined length [1, 2, 3]. The statistical values for each section can then be compared between track and vehicle to analyse limit values and vehicle behaviour. This is further discussed in Chapter 3.

This chapter continues by introducing some basics within the field of track–vehicle interaction that are further discussed and explored within this thesis in later chapters. Section 1.1 introduces track irregularities, which are discussed in Chapters 2 and 3. Vehicle responses are introduced in Section 1.2 and further detailed in Chapter 2.

1.1 Track irregularities

Deviations from the design track geometry are called track irregularities. Track irregularities are a main excitation mechanism in the track–vehicle dynamic system and very important to monitor and maintain to have traffic on a line run safely and comfortably. Thus, a track that very closely follows the chosen track geometry, i.e. small track irregularities, is considered to have a high quality. Track irregularities are generally interpreted as the deviations from design geometry within 1–200 m.
wavelength range. During evaluation, track irregularity data is filtered into different wavelength ranges, the most common being 3–25 m (denoted $D1$), 25–70 m ($D2$) and 70–150 m vertically or 70–200 m laterally (both $D3$). Shorter waves, at least shorter than 1 m, are defined as roughness or corrugation of the rails and longer wavelengths about 200 m and above represent the vertical and horizontal curves of the track, although the exact limit is not consistently defined for the latter and ranges between 70 m to 200 m depending on measurement accuracy and maintenance policies.

The four track irregularities defined in EN 13848-1 [4] are shown in Figure 1.1 and explained below.

- Longitudinal level irregularities are the track irregularities in the vertical direction and defined as the mean value of both rails’ vertical deviations from their nominal vertical position to their running tables. The term vertical (track) irregularities will be used throughout this thesis.

- Alignment irregularities are the track irregularities in the lateral direction, defined as the mean value of both rails’ lateral deviations from the nominal track centre. The term lateral (track) irregularities will be used throughout this thesis.

- Cant irregularities (called cross level irregularities in EN 13848) are the unintentional rotation of the track around its longitudinal axis, defined as the difference in height between both rails. In curves, the cant irregularities are the deviations from the nominal cant.

- Track gauge is the smallest distance between the inside of the two rails, measured within 14 mm below the running surface. The deviation from the nominal track gauge is the track gauge irregularity.

Figure 1.1: The four types of track irregularities defined in EN 13848-1 [4]. Figure source: [5].
1.1. TRACK IRREGULARITIES

These four track-related track irregularities are equivalent to the four rail-related track irregularities: lateral and vertical deviation of each rail. Since they are interchangeable, one or the other can be used depending on the situation and data of interest.

Track measurements

For measuring the track irregularities, different types of machines and vehicles are used, generally called track-recording vehicles (TRV) or cars (TRC). Most common on main lines are measurement coaches equipped with inertia-based measurement systems, which use accelerometers, gyroscopes and lasers to record the track geometry and irregularities. Figure 1.2 shows the principle of an inertia-based measurement system. Three examples of inertia-based track recording cars are Railab [7], operated by DB in Germany, IMV 200 [8] operated by Infranord in Sweden and its predecessor STRIX [6], which was operated by Banverket and later Infranord.

Another monitoring technique that is becoming increasingly common is track condition monitoring using revenue trains. By mounting accelerometers on the axle boxes, bogie frames and in the car bodies, track irregularities can be estimated through numerical methods. Although the derived track irregularities are quite close to the true irregularities, this method is generally used to faster identify where track maintenance is needed in order to carry out preventive maintenance and prevent traffic disruptions.

Figure 1.2: Principle of inertia-based track measurement system. A gyroscope, lateral and vertical accelerometers, linear variable differential transformers (LVDT) and an optical system with lasers and cameras record information about the track geometry. Figure source: [6].
CHAPTER 1. INTRODUCTION

1.2 Vehicle responses

As a vehicle run on a track, it will react to various imperfections, e.g. track irregularities, joints, variation in track stiffness. These reactions, or responses, can be either measured directly (acceleration levels from accelerometers) or indirectly (wheel-rail forces from strain-gauges on a wheelset) or be simulated using the imperfections as part of the input data. There are various parameters that can be measured on a vehicle, but commonly measured and evaluated are wheel-rail forces, accelerations on bogie frames and in the carbody. Other parameters of interest can be the acceleration on axle boxes and movement between suspension steps. Wheel-rail forces, axle box accelerations and to some extent bogie frame accelerations are related to safety. Accelerations in the carbody and partially bogie frame are rather ride comfort parameters. Movements between suspension steps can be a safety risk, if the allowed movements are restricted by the design of the suspension system, although this is normally not the case. A schematic view of some vehicle responses is shown in Figure 1.3.

Vehicle response signals are often filtered in the frequency domain, depending on the intended use. Safety-related signals are often filtered to disregard frequencies higher than 10 Hz (e.g. accelerations) or 20 Hz (e.g. wheel-rail forces) [3], whereas ride comfort related signals (i.e. carbody accelerations) are filtered to emphasise certain frequency ranges in which the human body is more sensitive, which is about 1–10 Hz depending on direction [9]. Corrugation and wheel defects such as wheel-out-of-roundness generally cause vibrations at higher frequencies (up to 2000 Hz) [10]. Both are considered out of scope for the work in this thesis, since neither is counted as a track defect as defined in EN 13848-1 [4].

![Figure 1.3: Cross section showing examples of vehicle responses. Part of figure from [5].](image)
1.3 Research motivation

Track irregularities have a large impact on the dynamic response of a rail vehicle, as stated earlier. However, the connection between what is occurring at track level and how the vehicle reacts is not always clear. In terms of correlation there is often a large variation between different vehicles and situations, which has been seen in many research projects.

One issue is the discrepancy between what wavelengths infrastructure owners use when evaluating their recorded track irregularities and what frequencies that are taken into account during evaluation of vehicle responses. For example, at a speed of 160 km/h, the common 3–25 m range corresponds to 1.78–14.8 Hz, but the filtering for vehicle responses is usually neglecting anything above 10 Hz (e.g. accelerations in safety context) or 20 Hz (e.g. wheel-rail forces).

Another issue to take into account is that infrastructure owners and maintenance organisations often want a measure of track quality that is close to how the track is, e.g. amplitude of track irregularities. Thus, trying to introduce a very sophisticated method of classifying track irregularities, but which obscures the actual track irregularities, might make maintenance work harder. In many ways, the currently used methods are not perfect, but are deemed good enough for the time being.

This research project was started as there are still many open questions regarding track–vehicle interaction, as indicated above and further detailed in Chapter 3. The aim is to resolve questions around how any given rail vehicle reacts to track irregularities and how to quantify that behaviour so maintenance can be carried out more efficiently and vehicle acceptance testing can be carried out more reliably.
Chapter 2

Track–vehicle interaction

This chapter covers the basics in track–vehicle interaction including important sources of excitation and examples of vehicle response as well as some key international standards. As an example, Figure 2.1 shows a two degree of freedom (2DOF) model running along a rigid track with track irregularities $z_t$ present. In this model, the excitation alone comes from the track irregularities, but in reality several excitation sources exist.

![Two-dimensional model with two DOF ($z, \chi$). Masses $m$, $m_{w1}$ and $m_{w2}$. Mass inertia moment $J$. Damping $c_1$ and $c_2$; stiffnesses $k_1$ and $k_2$. Longitudinal distance from centre of gravity to wheelset $L_1$ och $L_2$. Speed $v$. Displacements $z(t)$, $z_{w1}(t)$ och $z_{w2}(t)$ and track irregularities $z_t(s)$. Rotation, or pitch, $\chi(t)$. Source: [5].](image)
2.1 Excitation sources

Several sources can excite the track–vehicle system. Track irregularities cause continuous and stochastic excitation, whereas the vehicle response from turnouts and joints is transient in behaviour. Track flexibility also provides additional excitation, as it can vary along the track, even from sleeper to sleeper [5]. Wheel out-of-roundness can cause strong, high-frequency, vibrations and can be considered a self-induced excitation as it does not come from the track.

Track irregularities

The degradation of track nominal geometry and imperfections at construction and renewal of tracks gives a stochastic input excitation to the track–vehicle mechanical system. This is the most important parameter for the dynamic behaviour of a vehicle and thus very important to monitor, as large track irregularities will cause poor ride comfort and in the worst case derail a vehicle. Traditionally, isolated defects (maximum values measured as mean to peak values) and standard deviations have been used to classify a track’s quality level. In recent years, this quality measure have been questioned and many new methods evaluated [11]. See also the subsection ”EN 13848” below.

Rail corrugation

Corrugation of rails are very short-wave track irregularities, with a wavelength range about 0.03–1.0 m. Corrugation in the 30–300 mm range is the result of resonances in the wheel-rail interaction, while 300 mm up to 1 m is usually due to the rail manufacturing process [5]. Since the wavelength range is short compared to track irregularities, rail corrugation generally results in an audible vibration. Although not causing any large discomfort for passengers or goods, its high-frequency vibration can cause large dynamic forces, mostly on wheels, rails and track [12]. Rail corrugation can be removed by grinding the rails, which is done with special grinding trains that can essentially restore the nominal rail profile.

Turnouts and joints

Turnouts, crossings and joints have parts where a wheel will travel from one rail to the other as it changes track. At these places, large, transient forces can be found, especially if the components are worn. As this is a safety issue – the vehicle might derail if the components are too worn – it is very important to monitor and carry out proper maintenance. The maintenance is more manual in nature, compared to e.g. maintaining track irregularities on the line, as there are more special and delicate components in especially turnouts, e.g. hydraulic or electro-mechanical actuators to move the switch rails. The switch nose is often worn heavily and needs to be restored by welding new pieces into place or replacing it together with the
2.1. EXCITATION SOURCES

wing and nose rails. Due to the transient nature of the vehicle response at these points, they will be disregarded in the present work.

**Track flexibility**

Track flexibility is the overall stiffness of the track from the rail to the ground, both laterally and vertically. It varies along the track, with the largest variations being caused by turnouts and crossings as well as bridges. As the present work focus on vertical track–vehicle interaction, any mention of track flexibility refers to the vertical direction. Switches and turnouts tend to have increased stiffness (lower flexibility), due to the stronger material used at the crossing, but also caused by the longer sleepers and overall larger mass concentrated in the turnout. Another factor is the varying stiffness caused by differences in the subgrade and substructure along the track. The former cause transient, high-frequency vibrations and the latter continuous, low-frequency vibrations [12]. Track flexibility has been identified to have an impact on the behaviour of running vehicles, similar to track irregularities and is thus important to assess in order to optimise the life cycle and maintenance of track components [5]. It is possible to measure the track flexibility, similar to track irregularities, to find points of interest along the track. For example, the stiffness variation in a turnout or just before a bridge can be adjusted by changing pad stiffness and sleeper material [13].

**Wheel out-of-roundness**

Wheel-out-of-roundness corresponds to rail corrugation (cf. Rail Corrugation above), but for wheels. It can also be called wheel polygonisation when the wheel shape is no longer a simple oval, triangle or rectangle, but a higher order polygon. For example, triangular and quadratic wheels can be an issue after turning the wheelsets in a lathe with three or four cutting heads. Higher order shapes have a different origin and is more common on vehicles with block brakes [12, 14].

**EN 13848**

The most important European standard on track irregularities is the series of EN 13848, which has six parts:

- Part 1: Characterisation of track geometry
- Part 2: Measuring systems – Track recording vehicles
- Part 3: Measuring systems – Track construction and maintenance machines
- Part 4: Measuring systems – Manual and light weight devices
- Part 5: Geometric quality assessment
- Part 6: Characterisation of geometric quality
Part 1 [4] defines the track parameters that need to be measured, while Parts 2–4 cover measurement systems to measure those parameters. Parts 5 [1] and 6 [2] describe how to assess the measured track data and different methods to characterise the track geometry quality. More specifically, in Part 5, limit values for maximum allowed amplitudes of isolated defects are defined for three quality levels, for three wavelength ranges. In Part 6 limit values for standard deviations of track irregularities are defined. Different methods to assess track irregularities are also discussed, since they differ between infrastructure owners within Europe. For the work in this thesis, Parts 1, 5 and 6 are of interest and will be discussed more in Chapter 3.

2.2 Vehicle responses

There are several parameters that can be defined as vehicle response and some example are given in Figure 1.3. In this thesis, focus is on wheel-rail forces and accelerations measured on axle boxes. Other responses can be measured as well, e.g. movements between bogie and carbody or accelerations of bogie frames.

Wheel–rail forces

The contact forces between wheels and rails are usually measured with strain gauges mounted on wheelsets. Special measurement wheelsets with thinner web are typically needed, since the strain gauges need a certain movement to measure properly. As measurement techniques have improved regular wheelsets can also be used, saving cost and labour for test preparation.

Typically, the dynamic part of the wheel–rail forces is of most interest, as it can be several times larger than the static or quasi-static part. Dynamic wheel-rail forces in the lateral and vertical directions are generally caused by track irregularities, track stiffness variation and wheel and rail corrugations [5]. Traction- and braking-caused longitudinal forces and will not be discussed in the scope of this thesis.

Accelerations

Accelerations measured on the vehicle are used for various evaluation criteria. Carbody accelerations are used to objectively measure ride comfort for passengers or transported goods. Bogie accelerations can be used to monitor ride stability of the vehicle during tests above normal operating speed. Axlebox accelerations are used for condition monitoring of roller bearings, wheel defects or track irregularities. Figure 2.2 shows an accelerometer mounted on an axle box of a Swedish X2 power head, for vehicle condition monitoring purposes.
2.2. VEHICLE RESPONSES

Figure 2.2: Accelerometer (yellow with black cable) mounted on an axle box of a SJ X2 motor bogie. Photo by author.

EN 14363

EN 14363 [3] defines testing methods and evaluation criteria for vehicle behaviour assessment. It covers several aspects of the vehicle acceptance procedure and provides criteria on vehicles, infrastructure, wheel-rail interface and operating conditions. There are limit values on isolated defects (mean to peak values) and standard deviations of lateral and vertical track irregularities. Standard deviations must be within a certain range, which changes depending on the test speed, for the assessment to be acceptable. The standard also states which wavelength ranges should be used for the assessment.

UIC Code 518

The UIC 518 [15] leaflet is an international standard by the International Union of Railways. It is very similar to EN 14363, as the latter was based on the former. One difference is that UIC 518 focus purely on running dynamics, whereas EN 14363 also includes static tests. Another is that UIC 518 defines its own limit values on track irregularities and the allowed distribution. The limit values are QN1 (best), QN2 and QN3 (worst), and are chosen to be representative for the quality of the European railway lines. Most of the other evaluation criteria and measured quantities are very similar to EN 14363.
Chapter 3

Previous work

This chapter covers various methods used to evaluate track–vehicle interaction and the correlation between track irregularities and vehicle responses. These range from proven, conventional, methods like percentiles or standard deviations to newer, often more advanced, methods like transfer functions, neural networks and multiple regression analysis, all of which aim to connect track geometry quality characteristics to certain vehicle responses. The sections in this chapter roughly follow the state-of-the-art overview of track quality assessment methods found in EN 13848-6, Chapter 5 [2]. Much of the work presented in EN 13848-6 has been carried out within the research project Dynotrain, of which an overview can be found below.

Dynotrain

The Dynotrain research project was an EU-funded project that was carried out 2009–2013 [16]. It aimed to close open points in the then official Technical Specification for Interoperability (TSI) [17, 18]. The TSIs shall be adhered to when constructing new railway lines or new trains which are to be used within the European Union, to ensure common basic safety levels. As described in Section 1.2, it is not uncommon to find poor agreement between the severity of track irregularities and the resulting vehicle responses. To address this, the first work package (WP1) in Dynotrain collected data from railway lines in several countries, using a train consisting of one locomotive (DB Class 120, see Figure 3.1), the Railab track recording car and several passenger and freight cars. In some countries the locomotive was inactive and the train was pulled by one or two local locomotives due to differences in electrical and signalling systems. The recorded track and vehicle data were used in WP2, which focused on finding a general method to classify track geometry quality in such a way that it is closely related to vehicle responses.
There are several reports and papers describing the work progress within Dynotrain that go into detail regarding findings within the project. An overview of track geometry measurement and assessment for both rail and road, together with many of the findings in Dynotrain, are presented by Haigermoser et al. (2015) [19]. In another paper, which summarise work regarding track geometry quality assessments, Haigermoser et al. (2014) [11] evaluate and compare methods to assess track–vehicle interaction. Measured track data is evaluated using one of several methods included in the comparison. Different methods that are investigated are:

- Track irregularities filtered with various wavelength ranges
- First and second spatial derivatives of track irregularities
- Track data resampled to chord geometry
- Parametrise track geometry using triangles
- Wavelet transforms of track data
- Point mass acceleration
- Vehicle response analysis
- Transfer functions and vehicle filters, derived from measured and simulated data or by proprietary software (e.g. Pupil [20])
Then both track and vehicle data are processed using statistical methods (standard deviations, percentiles, maximum values etc.) over track sections with varying length depending on speed. Last, the statistical values are used in a multiple regression analysis, to quantify how well the different track analysis methods perform. The multiple regression takes track irregularities and speed into account on tangent track and also cant deficiency, curvature and track irregularities in curves. Two measures are used to quantify the regression analysis: the coefficient of determination $R^2$ and the root mean square error (RMSE) of the regression. $R^2$ tends to increase (i.e. indicate better results) as the number of input parameters increase, so Haigermoser et al. use the compensated $\bar{R}^2$, which only increase if the new input parameter actually do contribute to the overall regression. Track data are then evaluated against vehicle reactions for a large number of signals. The benchmarking is always against the recommended method in EN 14363:2005 [21], which is to filter track irregularities with a 3–25 m band pass filter and then evaluate maximum values and standard deviations in all track sections. Results show that some methods can increase the regression value between track and vehicle data, but only slightly. One recommendation is to include track curvature in the EN 14363 assessment, to increase sample size and decrease unexpected variation in the results. This is now also included in EN 14363:2016 [3]. Further work is considered to be needed in regards to alternative ways of describing track irregularities and the measurement reliability for both track data and vehicle reactions.

The following sections describe the methods mentioned above in more detail and examples of usage. The sections are divided into Standard deviations (Section 3.1), Isolated defects (Section 3.2), Combinations of parameters (Section 3.3), Methods based on vehicle response (Section 3.4) and last some concluding remarks in Section 3.5.

### 3.1 Standard deviations

Standard deviation of track irregularities over a distance of 200 m is commonly used as a maintenance measure on many railway networks. Longer sections can also be used and sometimes a single standard deviation value (per track parameter) is used to represent a specific line or a whole network. Among the European railway networks, it is common to calculate the standard deviation for lateral and vertical track irregularities, and track gauge, twist and cant irregularities can also be included. For lateral and vertical track irregularities, the 3–25 m wavelength range is generally used but the 25–70 m range can be evaluated as well.

**EN 13848**

The European standard for track irregularities (or track geometry quality) EN 13848, part 5 [1], defines limit values on track gauge, lateral (in the standard called "alignment"), vertical ("longitudinal level") and cant ("cross level") irregularities and
There are three quality levels defined: immediate action limit (IAL), intervention limit (IL) and alert limit (AL). The IAL limits are normative and must be followed, but the IL and AL limits are rather reflecting the average maintenance practice among the European railway networks and thus only recommendations. Part 6 of EN 13848 [2] describes different methods to characterise track geometry quality. It also defines quality classes with regards to standard deviations of vertical and lateral track irregularities, filtered in the 3–25 m wavelength range, divided into five speed ranges. EN 13848-6 recommends to calculate the standard deviation of lateral and vertical track irregularities for each rail separately, but also mentions that other ways are possible e.g. the outer rail in curves, worst/best rail or from a mean value of both rails.

The quality classes are based on the results of a survey on track geometry quality among European rail networks. A cumulative distribution of the total weighted average of standard deviations (per speed range) is calculated. Class A is the range of standard deviations corresponding to the lowest 10 % in this frequency distribution, Class B the range corresponding to 10–30 %, Class C 30–70 %, Class D 70–90% and Class E 90–100 %. Class D shall be used as the AL reference range for standard deviations of vertical and lateral track irregularities in EN 13848-5.

3.2 Isolated defects

Isolated defects are often evaluated per occurrence due to the derailment risk, but also used as a quality measure in which the number of peaks exceeding a defined limit value are counted. The percentage of a line exceeding the allowed number of isolated defects may also be calculated, e.g. for maintenance purposes. Among the European railway networks, it is common to calculate the number of defects per track length for lateral, vertical, track gauge, twist and cant irregularities. For lateral and vertical track irregularities, the 3–25 m wavelength range is generally used but the 25–70 m range can be evaluated as well. Track section lengths from 100 m up to several kilometres (depending on speed and curve radius) are common when counting the number of isolated defects exceeding the allowed limit value.

An example of work that have used isolated defects extensively is Karis [22], in which evaluation of track irregularities and their effect on wheel-rail forces and ride comfort values was investigated. Karis evaluated track irregularities and various vehicle responses from measurements carried out in the summer of 2008 within the Green Train (Gröna Tåget) research programme [23]. The method was to use the Pearson correlation coefficient [24] as a measure of how closely related track irregularities were to e.g. wheel-rail forces or ride comfort index values. In [22], standard deviations and percentiles are calculated for track irregularities, wheel-rail forces and ride comfort values, over track sections defined according to UIC 518 (3rd edition) [25]. Results vary from excellent correlation to very poor, depending on the track section, speed, wavelength range, among other things. Conclusions are that the, at the time, Swedish limit values [26] on lateral track irregularities
are strict enough for higher speed (about 300 km/h), whereas the limit values for vertical track irregularities needs to be stricter. It is also proposed that a minimum limit value for track gauge over 100 metres should be part of a new track standard, which covers higher speeds than 200 km/h.

3.3 Combination of parameters

Combined standard deviations may be used to get a single parameter representation of a railway network’s track quality level. For each track section, the square root of the sum of all four track irregularity’s squared standard deviations is calculated. Each irregularity have a weighting factor that is to be determined by the infrastructure manager to tailor the total measure for different purposes and network characteristics. For example, setting the track gauge weighting factor to zero to evaluate the need for tamping [2].

Standard deviations of parameter combinations, not to be confused with the above combined standard deviations, is a method where usually two of the measured track irregularities are combined to better represent vehicle behaviour. An example is when a lateral irregularity of the right rail happens at the same position as a cant irregularity where the right rail is lower than the left, which could lead to a potentially dangerous situation but would not be seen in the lateral or cant irregularity signals separately. The combined signals are calculated using sign conventions in such a way that situations as above are emphasized [26].

Point-mass acceleration method (PMA) simplifies the vehicle to a point mass without inertia, i.e. the vehicle’s centre of gravity, a certain distance above the track. Formulae are used to calculate the corresponding lateral and vertical accelerations, based on line speed, track irregularities and centre of gravity height. The assessment value is the root of the squared sum of the two calculated accelerations. This value gives better correlation with wheel-rail forces than using the track irregularities directly. Evaluation a floating average value over e.g. 100 m is recommended [2].

3.4 Methods based on vehicle response

Vehicle Response Analysis (VRA) uses assessment functions to calculate corresponding vehicle responses. The assessment functions can be calculated by dynamic simulations, vehicle filters, neural networks and other methods and should be done for each representative vehicle type. The output from an assessment function is the utilised limit value according to EN 14363 for important vehicle response parameters.
Vehicle filters, transfer functions, power spectra

Yet another method to assess track geometry quality is to analyse the vehicle behaviour for given track irregularities. Since, ultimately, it is the rolling stock using the track that needs to run safely, it is in a way more logical to analyse how the vehicles behave rather than only studying the track. Usually this is done by using a vehicle filter in the time domain or a transfer function in the frequency domain, both of which are applied to the measured track data to estimate the vehicle responses. A filter or transfer function is normally vehicle and track-specific, so if the vehicle or track change too much, a new filter or transfer function needs to be calculated.

A well-known example is Pupil [20], developed by Lloyd’s Register and used by Prorail in the Netherlands. The software uses various assessment filters, applied in real-time to the measured track irregularities, which shows the utilisation of limit values. If a limit value is exceeded, track maintenance is planned accordingly. A vehicle filter is produced by simulating the vehicle responses for a set of track irregularities, i.e. the filter is unique for each vehicle type.

Power spectral density (PSD) can be used on either track data or vehicle data. It shows the level of energy at each frequency (spatial, in the case of track irregularities), which can indicate what part of a system is causing or amplifying behaviour linked to that frequency. EN 13848-6 recommends that PSDs are calculated over relatively long track sections, e.g. 5 km, with as wide wavelength range as possible, preferably at least 3–70 m. It is also stated that some advantages are that recurring defects like welds are easily seen and that vehicle manufacturers and infrastructure owners get a deeper understanding of the track quality.

Measurements using in-service vehicles

Measured vehicle responses are often used to assess the track geometry quality on high-speed lines, but sometimes also on conventional lines. Since the measurement vehicle should represent the in-service rolling stock used on the line, isolated systems with similar traffic and vehicles benefit the most from this method. Accelerations on bogie frames and carbody are measured, but other parameters like wheel-rail forces can also be recorded if needed. By also recording the train’s position, it is possible to connect possible defects to a position in the track itself [2].

Similar to vehicle filters and transfer functions, vehicle responses can be directly analysed, but in this case completely omitting the track data in the analysis. This is useful if the vehicle characteristics are well known, such that resonance frequencies can be taken into account during the analysis. By measuring on vehicles in revenue service, frequent updates on the track and vehicle conditions are possible. Even if the vehicle characteristics are not known in detail, the continuous monitoring of the track makes it possible to quickly see the relative change in e.g. ride comfort or axle box accelerations over time and plan maintenance efforts accordingly. The
3.4. METHODS BASED ON VEHICLE RESPONSE

above in combination with an increased digitalisation of the railways is becoming a very powerful tool to improve maintenance efficiency.

Naganuma et al. [27] measure vertical acceleration on axle boxes and test three different filtering techniques to calculate vertical track irregularities. All three methods could produce a 10 m versine vertical track irregularity signal with high enough accuracy compared to a track recording vehicle to be usable for track irregularity assessments. To implement the method on in-service vehicles, cf. Figure 3.2, new hardware is developed to minimise computational load. The final device has high reliability and good measurement repeatability and is installed on six series N700 Shinkansen trains for continuous monitoring of vertical track irregularities. As the axle boxes constitute a harsh environment for sensors, Tsunashima et al. [28] improve on the Naganuma et al. work by suggesting to mount sensors in the carbody. To test the approach, they create a vehicle model and use it to make a Kalman filter. The filter can reproduce vertical track irregularities with a maximum error of 1.0–1.5 mm depending on method. Further development is done by Odashima et al. [29]. The Kalman filter approach is now developed into a small box with triaxial accelerometers, gyroscope and GPS. A small computer inside collects data and stores it locally, with possibilities to offload data via mobile internet or memory card. As the basis of this example is a Kalman filter, which requires a vehicle model to work, it needs to be tailored for each vehicle type on a railway line.

Figure 3.2: Example of a track condition monitoring system on the Tokaido Shinkansen. Axle box and carbody accelerations are processed onboard the train to estimate vertical track irregularities. Source: [27].
Terashima and Sato [30] developed and mounted track monitoring equipment on an in-service train on the Keihin-Tohoku line in the Tokyo area. The system measures track irregularities and monitors track components and is part of an effort to move towards condition-based maintenance. From the results, it is possible to quickly see the growth of track irregularities as well as missing or loose components e.g. fastening bolts. It is not mentioned how well the track irregularities correspond to the ones measured with a track recording vehicle.

Schenkendorf et al. [31] present a hybrid approach where recorded vehicle data is first used to estimate vertical track irregularities, then continuous wavelet functions are used to classify what type of defect was recorded. The method is tested and can designate between long-wave defects, short-wave defects and no defects. Future work includes using measured instead of simulated data and extend the method’s internal vehicle model from a quarter-car model to a full-car model.

Track irregularity derivatives

Li et al. [32] examine the use of second-order spatial derivatives of track irregularities and propose this as a measure of track geometry quality. A 1DOF model, pictured in Figure 3.3 and a 3DOF model are used in the investigation, both simulated in Gensys [33] for the purpose of evaluating the use of derivatives. It is found that the second-order derivatives in general have higher correlation with the vertical wheel-rail force than the vertical track irregularities themselves. PSDs are also used to assess the models and results in the frequency domain. For the second derivatives it is shown that the PSDs are easier to work with, as the amplitudes differ less than in a PSD of the track irregularity. The use of derivatives in daily maintenance work is also discussed and it is mentioned that it can be easily implemented within current infrastructure management systems due to the similarities with the actual vertical track irregularities.

Lönnbark [34] continued the analysis of derivatives as an alternative track quality measure. This work uses measured data from Dynotrain [16] to investigate the use of spatial derivatives of track irregularities as a track quality measure. Vertical track irregularities and their first and second order derivatives are compared with vertical wheel-rail forces for several vehicle types in a similar manner to that of Karis [22]. Also here the correlation coefficient is used as a measure of how closely related track and vehicle data are. Results are rather inconclusive. For some wavelength ranges and vehicles, derivatives are better, but in other cases the amplitude is better. It is concluded that other parameters not included in the study also affects the track–vehicle interaction.
3.4. METHODS BASED ON VEHICLE RESPONSE

Neural networks

The use of neural networks to map track geometry quality and vehicle responses is not wide spread, but can be powerful. Of the various methods mentioned in EN 13848-6 [2], using neural networks is a type of vehicle response analysis. A neural network consists of a number of pre-defined input and output parameters, of which the values of the input parameters are known. The relationship between input and output parameters, however, is not known and it is not necessary to know either, except that some kind of relationship exists. Each input and output parameter are modelled as input and output nodes in a network, connected to each other through one or several intermediate layers of nodes. In Figure 3.4, a small neural network with three input nodes, four hidden intermediate nodes and two output notes is depicted. Mathematically, the connections are a set of equations with weighting factors for each parameter and a base constant (called \textit{bias}). The network is then “trained” by using one or more sets of known input and output parameters, in order to calibrate all weighting factors and biases (constants). First, the input values are applied and the calculated output is compared to the known output. Then, if the calculated output differs too much from the known output, an error correcting process is started by passing backwards through the network and each node’s weighting factors are modified to minimise the error. When the neural network is trained properly, variations in the input will then produce new output values which should closely reflect the system behaviour in reality [35].

One drawback with neural networks in the present context is that they are tailored for certain vehicles on certain parts of a railway line, and if a new vehicle or new line is going to be evaluated, the network needs to be re-trained. Thus, it is easiest to implement on isolated systems or parts of a railway with homogeneous traffic, e.g. metros, freight lines or high-speed lines.
Li et al. [36] present a setup in which a neural network is trained to calculate vehicle responses, based on given track data. The network is the core of a performance-based track geometry assessment system, designed to complement the conventional track geometry quality assessment methods. Much effort is put on establishing a database for the input and output data, which are obtained from measurements, as they will affect the quality level of the neural network. Results show good agreement between measured and predicted vehicle response.

Guler [37] uses neural networks to predict track geometry deterioration on 180 km of the Turkish railways. Not only track geometry data was collected, but also track components, layout, traffic, environment and maintenance data. Much effort is put into modelling track and ballast deterioration and analysing maintenance efforts to include these effects into the neural network. While the results are good and show rather high $R^2$ (about 0.73–0.83), there is still quite a bit of dispersion in the scatter plots. The resulting neural network could be used to plan maintenance activities, as it can predict future track geometry deterioration with good accuracy.

### 3.5 Concluding remarks

The use of track irregularities and their isolated defects and standard deviations over different track sections is well explored and also still the preferred method in many applications to quantify track geometry quality. Vehicle responses are evaluated in a similar fashion during e.g. vehicle acceptance testing. Track and vehicle data can also be compared through correlation or regression analysis of
3.5. CONCLUDING REMARKS

the above quantities. Using derivatives of track irregularities, however, is not as thoroughly investigated as the irregularities themselves, but shows some potential and should be further explored.

The different methods which directly use vehicle responses are interesting. In this way, the need for continuous and sometimes frequent measurements by special track recording vehicles is reduced and in-service trains – which are going to run on the track anyway – can be used collect the data needed for evaluation of the track irregularities. Maintenance resources can be used optimally and preventive maintenance can be carried out to larger extent. Using the above methods during vehicle acceptance testing can also aid in quickly evaluating if the track used for testing fulfils the track geometry quality criteria. This area can also benefit from digitalisation and big data assessment to further direct resources where they are needed, and in a just-in-time-manner to avoid prolonged closing of tracks.

Neural networks have great potential, but are often only tailored for one combination of vehicle and railway line. It is thus quite specialised on certain conditions and is of most use on lines with homogeneous vehicle fleet and traffic, e.g. metro, tram or high-speed lines. It could be interesting to study how general a neural network can be made, so it can be used on other vehicles or tracks to replace simulations in some cases. This would be beneficial for assessing track irregularities based on vehicle responses in real time, which is now mostly done by applying vehicle filters on measured track data (in real time).

Some of the identified research gaps show potential for further research. This includes the use of derivatives, measuring vehicle responses directly and the rather ambitious task of making a neural network for any combination of track and vehicle. Especially using spatial derivatives of track irregularities need to be explored more to further understand – or rule out – their role in track–vehicle interaction.
Chapter 4

The present work

The scientific work carried out in the present project is summarised in this chapter. Two papers are appended, both of which investigate the correlation between vertical track irregularities and vertical wheel–rail forces and accelerations.

In the first paper, Paper A, data from two measurement campaigns are investigated and evaluated with focus on breaking down track–vehicle interaction into three input–response pairs, which make up a path from track irregularity to vehicle response. Measurements recorded on three different vehicles are analysed in this paper.

In Paper B, one of the vehicles from Paper A is used also in simulations to replicate measurement results and vary track and vehicle parameters. Simulation results are compared with measurement results to aid the understanding of what parameters are important for the track-to-vehicle pairs introduced in Paper A.

Both Paper A and B use the same 1DOF model as inspiration for the track-to-vehicle path (cf. Relation 4.1). Summaries of the two papers can be found below, whereas the full papers are appended in Part II of this thesis.

4.1 Summary of Paper A

Paper A [38] concentrates on evaluation of two measurement campaigns. The measurements come from the EU project Dynotrain and the Swedish high-speed EMU project Green Train (Gröna Tåget). Building on the 1DOF model in Figure 3.3 used by Li et al. [32] as inspiration, the track–vehicle interaction is broken down into several steps or ”interaction pairs”:

- Wheel-rail force to inertia force from unsprung mass,
- axle box acceleration divided by squared speed to track irregularity second derivative and
- track irregularity second derivative to track irregularity.
The "path" can be expressed mathematically as in Relation 4.1

\[
Q_{\text{dyn}} \text{ vs } z_t \Leftrightarrow \begin{cases} 
Q_{\text{dyn}} \text{ vs } m_w \ddot{z}_w \\
\ddot{z}_w \text{ vs } z''_t \\
z''_t \text{ vs } z_t
\end{cases}
\]  

(4.1)

where \(Q_{\text{dyn}}\) is the dynamic vertical wheel–rail force, \(m_w\) the unsprung mass, \(\ddot{z}_w\) the vertical acceleration of the unsprung mass, \(v\) the vehicle speed, \(z_t\) the vertical track irregularity (here per rail) and \(z''_t\) the spatial second derivative of \(z_t\).

Standard deviations and the 99.85th percentiles are calculated over track sections as in EN 14363 [3] for each pair, then the correlation coefficient is used as an indication of how well-connected each pair is. Only track sections with straight track and a vehicle speed in the range of 160–170 km/h is taken into account during the correlation analysis. One of the pairs, \(\ddot{z}_w \text{ vs } z''_t\), is found to generally have much lower correlation value than the other two pairs, and is pointed to as the weak link and the aim for further studies.

A new wavelength range \(D_x\) is also introduced and used throughout the analysis alongside the standardised \(D1\) and \(D2\). \(D_x\) represents the wavelength range that corresponds to 0.4–20 Hz, i.e. it is speed dependent. Each part is then mainly analysed using standard deviations and cross-correlation coefficients.

Generally, the wheel–rail force to unsprung mass and track irregularity second derivative to track irregularity show the highest correlation coefficients, as shown in Figures 4.1 and 4.2 for wavelength range \(D1\). The other pair shows various degrees of poor correlation. The main finding from this paper is that the step from axle box acceleration divided by squared speed to track irregularity second derivative is the weak link. It is suggested to investigate this part further, preferably using dynamic simulations for the possibility to alter some of the input parameters.

![Figure 4.1: Correlation coefficients for each "interaction pair" for locomotive BR 120. \(Q\) and \(\ddot{z}_w\) filtered 0.4–20 Hz, \(z_t\) and \(z''_t\) 3–25 m.](image-url)
4.2. SUMMARY OF PAPER B

Figure 4.2: Correlation coefficients for each "interaction pair" for passenger coach Bim 547.5. \( Q \) and \( \ddot{z}_w \) filtered 0.4–20 Hz, \( z_t \) and \( z_t'' \) 3–25 m.

4.2 Summary of Paper B

Paper B [39] is a continuation of the findings in Paper A. One of the vehicles from the analysis in Paper A, the passenger coach Bim 547.5, is further used in time domain simulations. Measured straight track geometry, track irregularities and vehicle speed profile are used to more accurately reproduce the on-track tests.

Although the weak link in the measurement evaluation was the step from axle box acceleration divided by speed squared to track irregularity second derivative, this step now shows high correlation coefficients when using simulated data, as shown in Figure 4.3.

Parameter variations are carried out, in which vertical track stiffness, unsprung mass and vertical primary suspension are varied as well as different combinations of track irregularities. It is shown that the weak link is mostly affected by track stiffness. Changing the nominal vertical track stiffness has minor impact on the correlation coefficients, whereas adding a random stiffness variation on top of the nominal stiffness has a larger impact. Varying the unsprung mass also affects the correlation coefficients for track–vehicle evaluation pairs containing the vertical wheel–rail force or axle box acceleration.

It is suggested to investigate the effect varying stiffness might have on track irregularities, in an attempt to improve track–vehicle interaction assessments. Another suggestion is to expand the 1DOF model to include both rails and a full wheelset, as the effects from cross-level and wheelset roll movement are not currently taken into account. Finally, evaluation of further measurements from isolated networks like metro systems might be suggested, as it might give more consistent results since factors like different axle loads, operating speed and vehicle suspension will remain fairly constant.
Figure 4.3: Baseline results. $Q$ and $\ddot{z}_w$ filtered 0.4–20 Hz, $z_t$ and $z_t''$ 3–25 m. Standard deviations. Note the very different results for measurements and simulations in the bottom left scatter plot, which represents the step from track irregularity second derivative to axle box acceleration divided by speed squared.
Chapter 5

Conclusions and future work

The research motivation behind this thesis is to explore why, in some cases, the vehicle response does not correspond well to the expected behaviour based on the given track irregularities. While there are still open questions, this thesis gives some new contributions that will help future research in the area.

5.1 Conclusions

The varying wavelength range $Dx$ introduced in Paper A has little effect on the overall results and is not improving the correlation values much. This is somewhat surprising, as matching the wavelength range of the track data with the frequency range of the vehicle responses intuitively should give better results than two more or less independent ranges.

The track–vehicle interaction is broken down into several steps, inspired by a single degree of freedom model. The three steps from vertical track irregularity to vertical wheel–rail force are analysed using correlation coefficients and scatter plots. A weak link is identified to be the step from second spatial derivative of track irregularities to axle box accelerations divided by speed squared.

It is also identified that using the vertical track irregularities’ second spatial derivative gives about the same level of correlation value as using the vertical track irregularities. Further, it is shown that the dynamic vertical wheel–rail force is very closely related to the unsprung mass’ inertia force.

In Paper B, simulation results show that the weak link from Paper A instead has high correlation value when using simulated data. There is very low sensitivity for the varied parameters, as the high correlation values are not significantly lowered in any case. Since randomly varying track stiffness affect the vertical wheel-rail force and axle box acceleration, it is important to take the stiffness variation along the track into account to ease further understanding of how track irregularities affect vehicle responses.
5.2 Future work

There are a few directions to continue the work in this thesis.

Evaluate additional measurements

First, it should be of interest to study measured data from a more isolated railway network or line with a more homogeneous vehicle fleet, e.g. a high speed line, metro system or commuter network, than in papers A and B. In that way, influences from varying types of traffic, vehicle types, and running speed can be minimised, as they affect degradation of the track geometry quality.

Track stiffness

Further, looking into analysis methods to reduce the influence from track stiffness on the track irregularities could be of further interest. Since the track irregularities are measured when the track is loaded, they will contain some information about the track stiffness as well. Paper B shows a strong connection between the axle box acceleration and the varying track stiffness, which could be used to estimate e.g. the relative difference in track stiffness between parts of a railway line. The possibility to use axle box accelerations measured on in-service vehicles for this purpose should also be explored.

A more advanced model

Another direction could be to extend the 1DOF model used as inspiration for Relation 4.1 to include e.g. the whole wheelset and re-work Relation 4.1 to include these changes. This should improve the understanding of the different pairs, although with the downside of making the model more complicated.
Bibliography


Part II

APPENDED PAPERS