Fibre Bragg Gratings
Realization, Characterization and Simulation

Ingemar Petermann

Royal Institute of Technology
Department of Microelectronics and Applied Physics
Stockholm, Spring 2007

Doctoral Thesis in Physics
The main topic of this thesis is realization and characterization of fibre Bragg gratings.

A novel versatile grating fabrication technique is developed and a number of gratings are realized, showing the potential of the system. Arbitrarily-shaped gratings are sequentially imprinted in the fibre by a moving interference pattern created with a continuous-wave ultraviolet (UV) source. This scheme allows for a very good control and stability of the grating shape, which is also shown experimentally. As opposed to most other present fabrication techniques, the proposed method offers a total control over the grating parameters by software, enabling simple implementation of new designs.

Different kinds of error sources when stitching long gratings are identified and investigated regarding impact on the final grating result.

Another important question within this field is how to characterize gratings. We propose a new characterization method based on optical low-coherence reflectometry (OLCR). A new interferometer design allows for simple simultaneous detection of the reflection response from two different points in the interrogated grating, so that differential measurements can be performed. The advantage of this is that the sensitivity to noise caused by e.g. thermal fluctuations in the system is substantially reduced. Several test gratings have been investigated and a very good agreement to the expected results is noted.

A second characterization technique using interferometric detection of the side diffraction from the grating under test is investigated both theoretically and experimentally. With aid of two-dimensional theory for wave propagation, it is shown that there is a linear relation between the detected phase and modulation depth and the corresponding grating properties. The technique is evaluated with a novel scheme of implementation where the UV source provided in a fabrication setup is used as
source for the side probe. This approach results in a very simple implementation and opens for an integration of the characterization and fabrication systems.

Finally, a tuning method for transmission filters based on local heating of linearly chirped fibre Bragg gratings is analysed and further developed to allow for fully software-controlled operation. The potential of this technique is illustrated by some promising initial experimental results.
Even though there is only one name on the cover of this thesis, it is by no means the result of one single person’s work. I have many to thank for contributions, direct or indirect, some of which have earned a special remark.

I am very grateful to professor Ari T. Friberg for accepting me as a student, for his assistance in theoretical considerations and for being a big help in refining and improving my articles.

Many thanks to my supervisors Raoul Stubbe and Sten Helmfrid, Raoul for introducing me into the wonderful field of fibre optics and Sten for having patience with all my questions whenever the equations decided to work against me. Thanks also to Bengt Sahlgren for numerous fruitful discussions in the laboratory and, especially, for always having time to share his outstanding physical intuition. Leif Nyholm also earns special thanks for all his help in the laboratory.

Most of the work with the OLCR has been done in collaboration with Johannes Skaar from the Norwegian University of Science and Technology in Trondheim, Norway. I have gained a lot of insight into the world of grating synthetization during our discussions. Many thanks also for sharing the very elegant treatment of the theory behind OLCR measurements.

Finally, I want to thank all my colleagues at Acreo AB (former Institute of Optical Research, IOF) and Proximion Fiber Systems AB, for many fruitful discussions and good friendship. Among these, I have to specifically mention Ola Gunnarsson, who not only bravely coped with the hard task of sharing cubicle with me for several years, but also gave me the idea for the final publication, and Åsa Claesson who gave me very valuable support when my professional duties diverged from the scope of this thesis in 2003.

The main part of this work (up to mid 2003) was carried out within Acreo’s Industrial Research School in Optics, partially funded by the Knowledge Competence (KK) Foundation in Sweden.

Ingemar Petermann, April 2007
Publications included in the thesis


Publications not included in the thesis

Conference contributions


Patents


Reports


# Contents

Abstract i

Preface iii

List of Publications iv

Contents vii

## 1 Introduction 1

1.1 Background ................................. 1
1.2 Motivation ................................ 1
1.3 Outline .................................. 2

## 2 Fibre Bragg Grating Theory 3

2.1 Wave propagation in linear media ..................... 3
2.2 Transverse modes in standard optical fibre .............. 6
2.3 Mode coupling ................................ 9
2.4 The fibre Bragg grating .......................... 12
   2.4.1 Fundamental properties ....................... 12
   2.4.2 Grating characteristics ....................... 13
   2.4.3 Coupled-mode equations ....................... 14
   2.4.4 Weak gratings ............................... 16
   2.4.5 Uniform gratings ............................. 16

## 3 Bragg Grating Simulation 19

3.1 Introduction ................................ 19
3.2 Thin layer approach ............................. 19
3.3 Fundamental transfer matrix approach ................. 22
3.4 Optimized transfer matrix implementation .......... 26
4 Grating Realization 29
4.1 Background ................................................. 29
4.2 Fabrication methods ........................................ 30
  4.2.1 Basic principles ......................................... 30
  4.2.2 Multiple printing in fibre ............................... 31
  4.2.3 Sequential exposure with a continuous-wave source 33
4.3 Common sources of error .................................. 36
  4.3.1 Vibrations .................................................. 36
  4.3.2 Fibre issues ............................................. 37
  4.3.3 Stitching .................................................. 38

5 Grating Characterization 39
5.1 Background .................................................. 39
5.2 Low-coherence reflectometry .............................. 40
5.3 Interferometric side diffraction ........................... 44
5.4 Method comparison ......................................... 46

6 Grating Applications 47
6.1 Introduction .................................................. 47
6.2 Grating tuning ................................................ 49

7 Conclusions and Future Work 51

8 Summary of Publications 54

Bibliography 57
1.1 Background

The ever-increasing need for communication capacity has brought the field of optics into focus for the past decades. Fibre optics is the only technique known today to have the power to meet the strong demands for flexibility and high bandwidth posed by the rapidly growing communication networks.

One of the key elements in many components for optical communication — active as well as passive — is the fibre Bragg grating. These gratings interact with the light in the fibre by means of mode coupling, and by tailoring grating parameters such as modulation depth and period, advanced functionalities can be achieved.

Since the first report on photo-induced fibre Bragg gratings was published in 1978 [1], a whole new research field has evolved and today, several text books dealing with both theoretical and experimental aspects of this technique can be found [2, 3].

The field is vast and not merely restricted to telecommunication components. Fibre-based Bragg gratings also play an important role in e.g. sensor applications, where small size, low weight and high resistance to harsh environments are central issues [4].

1.2 Motivation

While the framework for most of today’s existing techniques for realization [5], characterization [6-8] and simulation [9] of fibre Bragg gratings was formulated as long as 10 – 20 years ago, these basic concepts need further development and refinement to keep pace with the high demands posed by modern applications.

When the work with this thesis started, photo-induced Bragg grating realization was usually based on exposure by pulsed light. This both reduced the control of the dose distribution along the fibre and limited the maximum average light
power that could be used, since very high peak powers may damage the fibre. As a result, it was very hard — if not impossible — to obtain enough accuracy to realize advanced and long grating structures. Furthermore, the theoretical background of many characterization techniques was not fully formulated and the implementations were in an early stage and sometimes rather cumbersome to use, e.g. with respect to stability and repeatability.

The main purpose of the present thesis is to introduce and describe new methods for realization and characterization of advanced fibre Bragg gratings that overcome these limitations, as well as provide further understanding of theoretical and experimental problems encountered in the quest for the “perfect” grating.

1.3 Outline

It is the author’s intention that this thesis be readable for an audience that has a general knowledge in the field of physics, optics and mathematics. The theory behind modes and mode coupling in optical single-mode fibres is therefore reviewed along with a short introduction to the basic properties of Bragg gratings in chapter 2. This material is well-known and can be found in different embodiments in numerous textbooks and articles.

Chapter 3 gives an overview of some common grating simulation techniques. Access to a flexible simulation tool is important for developing and understanding the physical basis of new grating-based components and the simulation tools outlined in this section have been used extensively in the work towards this thesis.

Chapter 4 deals with the realization of Bragg gratings in optical fibres using the UV sensitivity of optical glass. First, the general principles of the technique are reviewed with a closer look at the multiple printing in fibre (MPF) method. Secondly, I describe a setup based on a development of the MPF method. Finally, a few words about common error sources and how to avoid them are given. Gratings can also be induced in optical fibres by chemical, thermal or mechanical means, but this is outside the scope of this thesis.

In chapter 5, the focus is turned to different grating characterization methods. After a summary of general considerations and methods, I present the principle of a new method based on optical low-coherence reflectometry. An introduction to the interferometric side diffraction technique follows and finally, I briefly discuss the differences between these two methods.

Chapter 6 gives a short overview of applications for fibre Bragg gratings with a focus on grating tuning.

Conclusions of the thesis are presented in chapter 7 followed in chapter 8 by short summaries of the six included publications.
Fibre Bragg Grating Theory

This chapter presents some of the theory behind Bragg gratings and wave propagation in optical fibres. I concentrate on the theory that is necessary to understand the basic properties of fibre Bragg gratings as treated in this thesis, with no intention to cover every aspect of the field. For a more complete treatment, the reader is referred to the textbooks on Bragg gratings [2, 3] and further literature dealing with modes and mode coupling in optical waveguides, see e.g. refs. 10–13.

2.1 Wave propagation in linear media

In order to understand how a Bragg grating interacts with light, it is necessary to have an idea about wave propagation in optical fibres in general. Let us therefore start with a closer look at how a lightwave propagates in a linear medium such as an optical fibre.

The starting point for most electromagnetic calculations is the Maxwell equations (see e.g. ref. 14), given by

\[ \nabla \times \mathbf{E}(r, t) = -\frac{\partial \mathbf{B}(r, t)}{\partial t}, \]  
\[ \nabla \times \mathbf{H}(r, t) = \mathbf{J}(r, t) + \frac{\partial \mathbf{D}(r, t)}{\partial t}, \]  
\[ \nabla \cdot \mathbf{D}(r, t) = \rho(r, t), \]  
\[ \nabla \cdot \mathbf{B}(r, t) = 0, \]

where \( \mathbf{E} \) and \( \mathbf{H} \) are the electric and magnetic field vectors, respectively, and \( \mathbf{D} \) and \( \mathbf{B} \) the corresponding electric and magnetic flux densities. \( \mathbf{J} \) is the current density that, along with the charge density \( \rho \), vanishes in the absence of free charges, as is
the case in optical fibres. All entities are (in general) functions of space \( r \) and time \( t \).

The flux densities contain the material response of the propagating fields and are given by the constitutive relations

\[
\mathbf{D}(r,t) = \varepsilon_0 \mathbf{E}(r,t) + \mathbf{P}(r,t),
\]

\[
\mathbf{B}(r,t) = \mu_0 \mathbf{H}(r,t) + \mu_0 \mathbf{M}(r,t),
\]

where \( \varepsilon_0 \) is the vacuum permittivity, \( \mu_0 \) the vacuum permeability and \( \mathbf{P} \) and \( \mathbf{M} \) the induced electric and magnetic polarizations. For a non-magnetic medium, \( \mathbf{M} = 0 \).

From now on, we assume that the medium is an optical fibre, i.e. \( \mathbf{J}, \rho \) and \( \mathbf{M} \) are all zero.

Taking the curl of eq. (2.1) and using eq. (2.2) and the constitutive relations from above results in

\[
\nabla \times \nabla \times \mathbf{E}(r,t) = -\varepsilon_0 \mu_0 \frac{\partial^2 \mathbf{E}(r,t)}{\partial t^2} - \mu_0 \frac{\partial^2 \mathbf{P}(r,t)}{\partial t^2}.
\]

Applying the well-known identity

\[
\nabla \times \nabla \times \mathbf{E} = \nabla (\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E}
\]

then yields

\[

\nabla^2 \mathbf{E}(r,t) - \varepsilon_0 \mu_0 \frac{\partial^2 \mathbf{E}(r,t)}{\partial t^2} - \mu_0 \frac{\partial^2 \mathbf{P}(r,t)}{\partial t^2} = \nabla [\nabla \cdot \mathbf{E}(r,t)],
\]

which is the most general form of the wave equation for the electric field in an optical fibre. A similar equation is easily derived for the magnetic field, resulting in:

\[

\nabla^2 \mathbf{H}(r,t) - \varepsilon_0 \mu_0 \frac{\partial^2 \mathbf{H}(r,t)}{\partial t^2} = -\frac{\partial}{\partial t} \nabla \times \mathbf{P}(r,t).
\]

As a next step, one has to find an expression for the polarization \( \mathbf{P} \). The polarization reflects the response of the material to the field and it is reasonable that this response is highly dependent of the field frequency. Therefore, the polarization is more easily described in the Fourier domain. For most applications, the polarization can be defined by the power series

\[
\tilde{\mathbf{P}}(r, \omega) = \varepsilon_0 \left( \chi^{(1)} \tilde{\mathbf{E}} + \chi^{(2)} \tilde{\mathbf{E}} \tilde{\mathbf{E}} + \chi^{(3)} \tilde{\mathbf{E}} \tilde{\mathbf{E}} \tilde{\mathbf{E}} + \ldots \right),
\]

where \( \chi^{(i)} \) is the susceptibility of (tensor) order \( i \) and a tilde (\( \sim \)) denotes the Fourier transform. Note that all entities are now functions of space \( r \) and frequency \( \omega \). In the time domain, the polarization takes the more complex form

\[
\mathbf{P}(r,t) = \varepsilon_0 \left( \chi^{(1)} * \mathbf{E} + \chi^{(2)} * (\mathbf{E} \mathbf{E}) + \chi^{(3)} * (\mathbf{E} \mathbf{E} \mathbf{E}) + \ldots \right),
\]
where the asterisk (*) denotes convolution and all entities are again functions of space and time.

In the present context, we limit the calculations to the linear case and omit all nonlinear parts of the polarization, which results in

\[
\tilde{P}(\mathbf{r}, \omega) \approx \epsilon_0 \chi^{(1)}(\mathbf{r}, \omega) \tilde{E}(\mathbf{r}, \omega) .
\]  

(2.13)

In the linear case, we can furthermore use the Fourier domain for all calculations, since different frequencies do not “mix”. The first constitutive relation (eq. (2.5)) then is restated in the Fourier domain as

\[
\tilde{D}(\mathbf{r}, \omega) = \epsilon_0 (1 + \chi^{(1)}(\mathbf{r}, \omega)) \tilde{E}(\mathbf{r}, \omega) = \epsilon_0 \epsilon_r(\mathbf{r}, \omega) \tilde{E}(\mathbf{r}, \omega)
\]  

(2.14)

where \( \epsilon_r = 1 + \chi^{(1)} \) is the (linear) relative permittivity. Since, according to eq. (2.3),

\[
\nabla \cdot \tilde{D}(\mathbf{r}, \omega) = \epsilon_0 \nabla \epsilon_r(\mathbf{r}, \omega) \cdot \tilde{E}(\mathbf{r}, \omega) + \epsilon_0 \epsilon_r(\mathbf{r}, \omega) \nabla \cdot \tilde{E}(\mathbf{r}, \omega) = 0
\]

\[\Leftrightarrow \nabla \cdot \tilde{E}(\mathbf{r}, \omega) = -\nabla \ln \epsilon_r(\mathbf{r}, \omega) \cdot \tilde{E}(\mathbf{r}, \omega) ,
\]  

(2.15)

the Fourier transform of eq. (2.9) now takes the form

\[
\nabla^2 \tilde{E}(\mathbf{r}, \omega) + \omega^2 \mu_0 \epsilon_0 \epsilon_r(\mathbf{r}, \omega) \tilde{E}(\mathbf{r}, \omega) = -\nabla \left[ \nabla \ln \epsilon_r(\mathbf{r}, \omega) \cdot \tilde{E}(\mathbf{r}, \omega) \right] .
\]  

(2.16)

For a medium with constant permittivity \( \epsilon_r \), the right hand side of the above equation is zero. Comparing the resulting homogeneous equation to the general wave equation reveals that the term \( \mu_0 \epsilon_0 \epsilon_r \) must equal the inverse square \( 1/v^2 \) of the phase velocity. From this it is straightforward to show that the index of refraction \( n \) is given by

\[
n^2 = \left( \frac{c}{v} \right)^2 = \epsilon_r ,
\]  

(2.17)

where \( c \) is the phase velocity in free space (see e.g. ref. 15). This also holds for non-constant permittivities. Finally introducing the wave number of free space \( k_0 = \omega/c \), eq. (2.16) can now be written as

\[
\nabla^2 \tilde{E}(\mathbf{r}, \omega) + n^2(\mathbf{r}, \omega) k_0^2(\omega) \tilde{E}(\mathbf{r}, \omega) = -\nabla \left[ \nabla \ln n^2(\mathbf{r}, \omega) \cdot \tilde{E}(\mathbf{r}, \omega) \right] .
\]  

(2.18)

This equation fully describes the propagation of the electric field in a linear medium. A similar equation is obtained for the magnetic field in eq. (2.10):

\[
\nabla^2 \tilde{H}(\mathbf{r}, \omega) + n^2(\mathbf{r}, \omega) k_0^2(\omega) \tilde{H}(\mathbf{r}, \omega) = -i\omega \mu_0 \left( \nabla n^2 \right) \times \tilde{E}(\mathbf{r}, \omega) ,
\]  

(2.19)

where “i” denotes the imaginary part.
2.2 Transverse modes in standard optical fibre

Consider a standard fibre structure consisting of a cylindrical core of radius $a$ with index of refraction $n_{co}$, surrounded by a cladding with an index of refraction $n_{cl} < n_{co}$ (see fig. 2.1). Since we have cylindrical symmetry, it is natural to use cylindrical coordinates $r = (r, \phi, z)$ with the fibre centered around the $z$ axis and the coordinates defined as usual. Looking at the regions with radius $r \leq a$ and $r > a$ separately, the right hand sides of eq. (2.18) and eq. (2.19) vanish for each region.

Starting with the $z$ component $E_z$ of the electric field, eq. (2.18) then takes the form

$$\nabla^2 \tilde{E}_z(r, \omega) + n^2(r, \omega)k_0^2(\omega) \tilde{E}_z(r, \omega) = 0,$$

with

$$n(r, \omega) = n(r) = \begin{cases} n_{co}, & r \leq a \\ n_{cl}, & r > a \end{cases}, \quad (2.21)$$

where the frequency dependence of $n(r)$ has been omitted in order to improve readability.

A ansatz to solve eq. (2.20), we choose a separable solution on the form

$$\tilde{E}_z(r, \phi, z, \omega) = A_z(\omega)R_z(r)e^{im\phi}e^{i\beta z}, \quad (2.22)$$

where $m$ and $\beta$ are constants to be determined later from the boundary values. Note that complex representations are used for all fields and that the corresponding physical fields are given by the real parts of these entities. It is reasonable to assume that the wave is propagating along the fibre in $z$ direction, so $\beta$ can be thought of as a propagation constant. The term $A_z(\omega)$ is the field spectrum which is constant for a given frequency $\omega$. Remembering that

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2}, \quad (2.23)$$
eq. (2.22) substituted in eq. (2.20) yields
\[ \frac{\partial^2}{\partial r^2} R_z(r) + \frac{1}{r} \frac{\partial}{\partial r} R_z(r) + \left( \zeta^2 - \frac{m^2}{r^2} \right) R_z(r) = 0 , \] (2.24)

where \( \zeta^2 = n^2(r)k_0^2 - \beta^2 \). This is a form of Bessel’s differential equation with a general solution for positive \( \zeta^2 \) according to
\[ R_{z,m}(r) = AJ_m(\zeta r) + BY_m(\zeta r) , \] (2.25)

where \( A \) and \( B \) are constants and \( J_m \) and \( Y_m \) are Bessel functions of first and second kind, respectively. The subscript \( m \) refers to the mode number corresponding to the value of the constant \( m \). For positive \( \gamma^2 = -\zeta^2 \) eq. (2.24) takes the form of the modified Bessel equation,
\[ \frac{\partial^2}{\partial r^2} R_{z,m}(r) + \frac{1}{r} \frac{\partial}{\partial r} R_{z,m}(r) - \left( \gamma^2 + \frac{m^2}{r^2} \right) R_{z,m}(r) = 0 . \] (2.26)

Here, the solutions are given by
\[ R_{z,m}(r) = CI_m(\gamma r) + DK_m(\gamma r) , \] (2.27)

where \( C \) and \( D \) are constants and \( I_m \) and \( K_m \) are modified Bessel functions of first and second kind, respectively. Obviously, the set of solutions that is to be used is determined by the sign of the difference \( n^2k_0^2 - \beta^2 \).

If \( \beta^2 \geq n_{co}^2k_0^2 \), the solutions would be given by eq. (2.27) both in the core and cladding. However, solutions in the core containing \( K_m \) are not physically realizable, since \( K_m \to \infty \) for \( r \to 0 \). \( I_m \) on the other hand tends towards infinity for large \( r \), so this function cannot be a part of a physical solution in the cladding. Boundary conditions state that both the field and its derivative have to be matched in the core/cladding interface (see e.g. ref. 14 for a derivation), which is not possible in this case and we can conclude that no physical solution exist for \( \beta^2 \geq n_{co}^2k_0^2 \).

In the other extreme case we have that \( \beta^2 \leq n_{cl}^2k_0^2 \) and the solution is given by eq. (2.25) everywhere. Both \( J_m \) and \( Y_m \) are sinusoidal for \( r \to \infty \), which means that energy would be transferred out of the fibre and no guided mode is possible. These solutions are called radiation modes and will not be further treated here.

We can thus conclude that solutions for guided modes only exist in the regime
\[ n_{cl}^2k_0^2 < \beta^2 < n_{co}^2k_0^2 . \] (2.28)

It is often convenient to define an effective index \( n_{eff} = \beta/k_0 \), which corresponds to the refractive index that a mode with propagation constant \( \beta \) will “see” in the fibre.

From eq. (2.28) it is clear that \( \zeta^2 \) is positive in the core (eq. (2.25) holds) and negative in the cladding (eq. (2.27) holds). We can dismiss core solutions involving
Ym and cladding solutions involving Im, since these functions tend towards infinity for \( r \to 0 \) and \( r \to \infty \), respectively. Thus, the physical general solution for the radial part of the z component of the electric field is given by

\[
R_{z,m}(r) = \begin{cases} 
AJ_m(\zeta r), & r \leq a \\
DK_m(\gamma r), & r > a 
\end{cases},
\]  

(2.29)

where we have redefined \( \zeta^2 = n_0^2 \kappa_0^2 - \beta^2 \) and \( \gamma^2 = \beta^2 - n_0^2 \kappa_0^2 \). As a next step, the constants \( m, \beta, A \) and \( D \) have to be determined from the boundary conditions.

According to our ansatz, the field varies sinusoidally as a function of angle \( \phi \). Then we must have that

\[
e^{im(\phi + 2\pi)} = e^{im\phi}
\]

\[
\Leftrightarrow \quad m(\phi + 2\pi) = m\phi + \alpha 2\pi, \quad \alpha \in \mathbb{N}
\]

\[
\Leftrightarrow \quad m = \alpha.
\]  

(2.30)

Thus \( m \) can be any natural number and it is therefore convenient to use it as counter of different transverse modes as has been done above.

In general, the tangential electromagnetic field components at a boundary must obey

\[
\begin{cases} 
E_{t,+} = E_{t,-} \\
\mathbf{n} \times (H_{t,+} - H_{t,-}) = J_{\text{surf}}
\end{cases},
\]  

(2.31)

where “+” and “−” denote the field just before and after the boundary, respectively, “t” denotes the transverse components, \( \mathbf{n} \) is the boundary normal and \( J_{\text{surf}} \) the surface current density (see ref. 14). The latter is only nonzero for boundaries between medias that include a perfect conductor. In the present case we can therefore write these conditions on the form

\[
\begin{cases} 
E_{t,+} = E_{t,-} \\
H_{t,+} = H_{t,-}
\end{cases}.
\]  

(2.32)

With cylindrical geometry, the tangential component consists of the \( z \) and \( \phi \) components and, considering the former, we have

\[
AJ_m(\zeta a) = DK_m(\gamma a)
\]

\[
\Leftrightarrow \quad D/A = \frac{J_m(\zeta a)}{K_m(\gamma a)}.
\]  

(2.33)

Knowing the quotient \( D/A \), we can choose e.g. \( A \) freely to comply with the overall field strength. Now we can write eq. (2.22) as

\[
\tilde{E}_{z,m}(r, \phi, z, \omega) \propto \begin{cases} 
J_m(\zeta r)e^{im\phi}e^{i\beta z}, & r \leq a \\
J_m(\zeta a)K_m(\gamma r)e^{im\phi}e^{i\beta z}, & r > a
\end{cases}.
\]  

(2.34)
leaving $\beta$ to be determined by the boundary condition for the $\phi$ component. The solution for the $z$ component of the magnetic field takes exactly the same form.

All other components of the electromagnetic field can be expressed in terms of these $z$ components. Combining eq. (2.1) and eq. (2.2) leads to the following expressions for the $\phi$ components:

\[
\tilde{E}_\phi(r, \omega) = \frac{i}{\zeta^2(r, \omega)} \left[ \beta \frac{\partial \tilde{E}_z(r, \omega)}{\partial \phi} - \sqrt{\mu_0 \epsilon_0 k_0(\omega)} \frac{\partial \tilde{H}_z(r, \omega)}{\partial r} \right],
\]

\[
\tilde{H}_\phi(r, \omega) = \frac{i}{\zeta^2(r, \omega)} \left[ \beta \frac{\partial \tilde{H}_z(r, \omega)}{\partial \phi} + \sqrt{\epsilon_0 \mu_0 k_0(\omega)} n^2(r, \omega) \frac{\partial \tilde{E}_z(r, \omega)}{\partial r} \right],
\]

where the same $z$ dependence as in eq. (2.22) has been assumed. Similar equations for the radial components are also found:

\[
\tilde{E}_r(r, \omega) = \frac{i}{\zeta^2(r, \omega)} \left[ \beta \frac{\partial \tilde{E}_z(r, \omega)}{\partial r} + \sqrt{\mu_0 \epsilon_0 k_0(\omega)} \frac{\partial \tilde{H}_z(r, \omega)}{\partial \phi} \right],
\]

\[
\tilde{H}_r(r, \omega) = \frac{i}{\zeta^2(r, \omega)} \left[ \beta \frac{\partial \tilde{H}_z(r, \omega)}{\partial r} - \sqrt{\epsilon_0 \mu_0 k_0(\omega)} n^2(r, \omega) \frac{\partial \tilde{E}_z(r, \omega)}{\partial \phi} \right].
\]

Applying the boundary conditions of eq. (2.32) to eq. (2.35) results in the transcendental equation

\[
\left( \frac{\gamma_m(\gamma)}{\zeta_m(\gamma)} + \frac{K'_m(\gamma)}{\gamma K_m(\gamma)} \right) \left( \frac{\gamma_m(\gamma)}{\zeta_m(\gamma)} + \frac{n^2}{n_0^2} \frac{K'_m(\gamma)}{\gamma K_m(\gamma)} \right) = \left( \frac{m \beta k_0(n^2 - n_0^2)}{\alpha \gamma^2 n_{co}} \right)^2,
\]

where the prime denotes derivation. This equation has to be solved numerically in order to obtain a value for $\beta$. In general, several solutions may be found within the guide limits $n^2_{cl} k_0^2 < \beta^2 < n^2_{co} k_0^2$.

### 2.3 Mode coupling

I will in this thesis restrict the calculations to optical single-mode fibres, where only one transverse mode can propagate, i.e. eq. (2.37) has only two solutions $+|\beta_0|$ and $-|\beta_0|$. The propagation is in either forward or backward direction, and in a plain piece of fibre there will be no cross-talk between these two propagation modes. Introducing a variation of the refractive index in the fibre can cause the modes to couple to each other, i.e. incident light is reflected throughout the structure. The corresponding field equations can be solved in different ways in order to determine the reflected field, in the following a perturbation approach is used (see also e.g. refs. 2, 10 and 11). In this context the variation of the refractive index is assumed small enough to be viewed as a perturbation of the initial refractive index $n_0$, i.e. we have

\[
n = n_0 + \delta n.
\]
As the refractive index is caused by the material polarization, let us now express the latter as a combination of an unperturbed part $P_0$ and a perturbation $P_{\text{pert}}$:

$$P = P_0 + P_{\text{pert}}. \quad (2.39)$$

Equation (2.9) then takes the form

$$\nabla^2 \mathbf{E}(\mathbf{r}, t) - \epsilon_0 \mu_0 \frac{\partial^2 \mathbf{E}(\mathbf{r}, t)}{\partial t^2} = \nabla (\nabla \cdot \mathbf{E}(\mathbf{r}, t)) + \mu_0 \frac{\partial^2 P_{\text{pert}}(\mathbf{r}, t)}{\partial t^2}, \quad (2.40)$$

which, in the linear approximation for the polarization, leads to

$$\nabla^2 \tilde{\mathbf{E}}(\mathbf{r}, \omega) + n_0^2(\mathbf{r}, \omega) k_0^2(\omega) \tilde{\mathbf{E}}(\mathbf{r}, \omega) = -\omega^2 \mu_0 \tilde{P}_{\text{pert}}(\mathbf{r}, \omega), \quad (2.41)$$

following the discussion leading to eq. (2.20). The total field consists of a superposition of the two guided mode fields, i.e. for the $z$–component

$$\tilde{E}_z(\mathbf{r}, \omega) = A(z, \omega) \mathcal{E}_z(\mathbf{r}, \phi) e^{-i\beta z} + B(z, \omega) \mathcal{E}_z(\mathbf{r}, \phi) e^{+i\beta z}, \quad (2.42)$$

where $A(z, \omega)$ and $B(z, \omega)$ contain the slow longitudinal dependency of the positively and negatively propagating field, respectively, and $\mathcal{E}_z(\mathbf{r}, \phi)$ consists of the $z$ component of the transverse dependencies of these fields. The calculations can be carried out with any of the three field components and from now on we will drop the $z$ subscript. Since the two modes in this case are the same apart from their direction of propagation, the transverse dependencies will be the same for both fields. Bear in mind that, according to last section, $\mathcal{E}_z(\mathbf{r}, \phi)$ satisfies the unperturbed wave equation

$$\nabla^2 (\mathcal{E}_z(\mathbf{r}, \phi) e^{i\beta z}) + n_0^2(\mathbf{r}, \omega) k_0^2(\omega) (\mathcal{E}_z(\mathbf{r}, \phi) e^{i\beta z}) = 0 \quad \Leftrightarrow \quad \nabla^2 \mathcal{E}_z(\mathbf{r}, \phi) + (n_0^2 k_0^2 - \beta^2) \mathcal{E}_z(\mathbf{r}, \phi) = 0. \quad (2.43)$$

From now on we will omit the $\omega$ dependence. Substituting eq. (2.42) into eq. (2.41) gives (with functional dependencies left out)

$$e^{-i\beta z} [A (\nabla^2 \mathcal{E}_z + (n_0^2 k_0^2 - \beta^2)) \mathcal{E}_z + \mathcal{E}_z (A'' - 2i \beta A')] + e^{+i\beta z} [B (\nabla^2 \mathcal{E}_z + (n_0^2 k_0^2 - \beta^2)) \mathcal{E}_z + \mathcal{E}_z (B'' + 2i \beta B')] = -\omega^2 \mu_0 \tilde{P}_{\text{pert},z}, \quad (2.44)$$

where the primes denote derivation with respect to $z$ and $\tilde{P}_{\text{pert},z}$ is the perturbed polarization in $z$ direction. According to eq. (2.43), the first term within each parenthesis in the above equation equals zero. A consequence of the weak coupling (perturbation) assumption is that the envelopes $A$ and $B$ are slowly varying, i.e.

$$A'' \ll \beta A' \quad (2.45)$$

and correspondingly for $B$. This is known as the slowly varying envelope approximation (SVEA). Equation (2.44) then simplifies to

$$-2i \beta \mathcal{E}_z e^{-i\beta z} A' + 2i \beta \mathcal{E}_z e^{+i\beta z} B' = -\omega^2 \mu_0 \tilde{P}_{\text{pert},z}. \quad (2.46)$$
If we now multiply each side with the mode field $E^*$ and then integrate over the transverse coordinates we have that
\begin{equation}
(-2i\beta e^{i\beta z}A' + 2i\beta e^{i\beta z}B') \int \int E_x E^*_x r dr d\phi = -\omega^2 \mu_0 \int \int \tilde{P}_{pert,x} E^*_x r dr d\phi .
\end{equation}
(2.47)

It can be shown (see e.g. ref. 12), that the mode fields obey the orthogonality relation
\begin{equation}
\frac{\beta_m}{2\omega \mu_0} \int \int E_n E^*_m r dr d\phi = \delta_{mn} ,
\end{equation}
(2.48)
where $\delta_{mn}$ is Kronecker’s delta and $n$ and $m$ the numbers of the modes. This relation ensures that the total power in each mode is given by $|A(z)|^2$ and $|B(z)|^2$, respectively. Applying (2.48) to eq. (2.47) with $n = m$ yields
\begin{equation}
A' e^{-i\beta z} - B' e^{+i\beta z} = -\frac{i\omega}{4} \int \int \tilde{P}_{pert,x} E^*_x r dr d\phi ,
\end{equation}
(2.49)
The perturbed polarization is caused by a perturbed permittivity,
\begin{equation}
\epsilon_r = \epsilon_{r,0} + \delta \epsilon_r ,
\end{equation}
(2.50)
which is related to the refractive index by
\begin{align}
\epsilon_{r,0} + \delta \epsilon_r &= (n_0 + \delta n)^2 \\
&= n_0^2 + 2n_0 \delta n + \delta n^2 \\
&\approx n_0^2 + 2n_0 \delta n \\
\Rightarrow \delta \epsilon_r &\approx 2n_0 \delta n .
\end{align}
(2.51)
We then have
\begin{align}
\tilde{P}_{pert} &= \tilde{P} - \tilde{P}_0 \\
&= \epsilon_0 (\epsilon_r - 1) \tilde{E} - \epsilon_0 (\epsilon_{r,0} - 1) \tilde{E} \\
&= \epsilon_0 \delta \epsilon_r \tilde{E} \\
&= 2\epsilon_0 n_0 \delta n \tilde{E} .
\end{align}
(2.52)
Using (2.42) and (2.52), eq. (2.49) now takes the form
\begin{equation}
A' e^{-i\beta z} - B' e^{+i\beta z} = -\frac{i\omega}{2} \epsilon_0 n_0 \left( A e^{-i\beta z} + B e^{+i\beta z} \right) \int \int \delta n \ E_x E^*_x r dr d\phi ,
\end{equation}
(2.53)
where the last term on the right side is known as the overlap integral. The modes may in general extend beyond the fibre core with the refractive index perturbation and this integral accounts for the resulting changes in the coupling characteristics between the modes. We will in the following assume that the field strength is negligible outside the fibre core, which means that the refractive index can be considered...
constant over the whole effective integration area, i.e. \( \delta n(r, \phi, z) = \delta n(z) \). The equation then simplifies to

\[
A' e^{-i\beta z} - B' e^{+i\beta z} = \frac{i\omega^2 n_0}{c^2 \beta} \delta n (A e^{-i\beta z} + B e^{+i\beta z}) ,
\]

where the same orthogonality relation as before and the identity \( c^2 = 1/\mu_0 \epsilon_0 \) have been used.

The right hand side of the above equation corresponds to the material response due to the perturbed refractive index. In order for a term on this side to make a contribution to a specific rate of change term on the left hand side, the corresponding exponents must be equal. If the phase-velocity dependencies are not at least approximately the same, the quickly oscillating differential phase term will make sure that the contribution is averaged to zero in any physical application and thus no coupling occurs. It is therefore evident that if the index perturbation \( \delta n \) is constant along the fibre, i.e. there is no grating and only a constant index change, no coupling between counter-propagating modes can take place. In this case, the rate of change in either \( A \) or \( B \) will only be affected by the corresponding field itself. If, on the other hand, \( \delta n \) varies along the fibre, the resulting phase component of this variation will add to the exponents, thus enabling cross coupling for wavelengths matching the variation period.

2.4 The fibre Bragg grating

2.4.1 Fundamental properties

Fibre gratings consist of a spatial quasi-periodic variation of the refractive index in an optical fibre. If the variation is described by a cosine, the total refractive index \( n(z) \) at position \( z \) in the fibre takes the form

\[
n(z) = n_{av} + \Delta n(z) \cos(K z + \phi(z)) ,
\]

where \( n_{av} \) is the average refractive index, \( \Delta n(z) \) the modulation depth, \( K = 2\pi/\Lambda \) the spatial frequency of the grating having a central period or pitch \( \Lambda \), and \( \phi(z) \) a variable phase factor. Note that \( n_{av} \) is not necessarily the same as \( n_0 \) in eq. (2.38), since there is in general an additional constant increase of the initial refractive index due to the grating fabrication process.

The modulation depth may in general change slowly along the length of the grating, thus forming an index modulation profile, an apodization. Gratings with a variable spatial frequency \( K(z) = K + \Delta K(z) \) (or pitch \( \Lambda(z) \)) are called chirped gratings. By differentiating \( K(z)z \) we conclude that this variation can be expressed
Figure 2.2: A grating of length $L$, period $\Lambda$ and average refractive index $n_{av}$ containing $N + 1$ fringes. Light is incident from the left and is partially reflected back by an amount determined by the reflection coefficient $r(\omega)$.

in terms of a variable phase factor according to

$$\frac{d\phi(z)}{dz} = (1 + z)\Delta K(z) ,$$

(2.56)

which means that eq. (2.55) is able to describe both apodization and chirp of a grating.

Light incident on the grating according to figure 2.2 with a (vacuum) wavelength fulfilling the Bragg condition for normally incident light,

$$\lambda_B = 2n_{av}\Lambda ,$$

(2.57)

will always have the same phase at the front face of the grating after reflection, independent of which fringe $m = 0, \ldots, N$ served as reflector. Since all reflections interfere constructively in this case, light with this Bragg wavelength is reflected better than light with any other wavelength not fulfilling the condition.

In general, a Bragg grating can be said to couple different light propagation modes to each other. Here, I will restrict the calculations to gratings in single-mode fibres, where only one transverse mode is allowed. Thus the coupling can only occur between different propagation directions of the same transverse mode.

It is also possible to realize slanted Bragg gratings that couple propagation modes in the fibre core to cladding and/or radiation modes and vice versa. However, the theory behind these kinds of gratings is beyond the scope of the present thesis.

2.4.2 Grating characteristics

One of the most important characteristics for a Bragg grating based fibre component is its (complex) reflection and/or transmission coefficients. These are the transfer functions for the component in the frequency domain and fully describe how it affects
the light. The same information is also contained in the time domain through the component’s *impulse response*, which is related to the reflection coefficient by a Fourier transform.

The reflection amplitude spectrum is easily measured with an optical spectrum analyser, but the phase distribution is often much more important for the grating performance. This phase has to be measured by interferometric means (see chapter 5).

The *group delay* and its *ripple* (GDR) are parameters that are commonly used as figures of merit for chirped gratings. The group delay is defined as the derivative of the reflection coefficient phase according to

$$\tau(\omega) = \frac{d}{d\omega} \arg(r(\omega)),$$

where $\omega = k_0c = 2\pi c/\lambda$ is the angular velocity of the light as before. Furthermore, the *dispersion* is defined as

$$D(\lambda) = \frac{d}{d\lambda} \tau(\omega(\lambda)).$$

Intuitively, the group delay can be described as the time delay that a light pulse of each frequency experiences when passing the component in question and the dispersion is the rate of change of this delay per unit wavelength. One important application area for chirped fibre gratings is as compensators for the dispersion resulting from the wavelength dependence of the effective index in optical fibres. It is then important to minimize the error-induced GDR for the grating to avoid performance deterioration. Even though this parameter is still the most commonly used in the literature, it should be noted that recent studies show that the phase itself is more directly connected to the system performance [16].

The next few sections deal with the determination of the reflection coefficient for a Bragg grating with given apodization and phase profiles. The reverse problem, i.e. determining the grating profiles giving a certain reflection response, has been a hot topic for the past decade, but will not be dealt with in this thesis. Several solutions for the reverse problem can be found in e.g. ref. 17.

**2.4.3 Coupled-mode equations**

Let us assume that a Bragg grating in a single mode fibre consists of a refractive index that is varied with the period $\Lambda$ and a modulation amplitude that is added to the initial refractive index in the fibre. The perturbation term in eq. (2.38) is then given by

$$\delta n = \Delta n + \nu(z) \Delta n \cos(Kz + \phi(z))$$

$$= \Delta n + \frac{\nu(z) \Delta n}{2} \left[ e^{i(Kz + \phi(z))} + e^{-i(Kz + \phi(z))} \right],$$

(2.60)
where $\Delta n = n_{av} - n_0$ is the average index change over one period, $0 \leq \nu(z) \leq 1$ is the grating visibility and $K = 2\pi/\Lambda$.

Inserting the last equation in eq. (2.54) and matching exponents yields the following two coupled differential equations (with $z$ dependencies omitted):

$$\begin{cases}
\frac{\partial A}{\partial z} = -iMA - i\kappa e^{i(2\beta - K)z + \phi}B \\
\frac{\partial B}{\partial z} = iMB + i\kappa e^{-i(2\beta - K)z + \phi}A
\end{cases} \quad (2.61)$$

The variables $M$ and $\kappa$ are given by

$$M = \frac{k_0^2 n_0}{\beta} \Delta n, \quad \kappa = \frac{k_0^2 n_0}{\beta} \nu \Delta n \quad (2.62)$$

with $k_0 = \omega/c$ as before. The coupling coefficient $\kappa$ determines how “visible” the modes are to each other, i.e. how large the coupling between the modes is. The term $M$ represents a small correction to the propagation constant due to the presence of the disturbance. Let us now redefine $A$ and $B$ as

$$\begin{align*}
A(z) &= R(z)e^{-iMz}, \\
B(z) &= S(z)e^{iMz}.
\end{align*} \quad (2.63)$$

Equations (2.61) then take the form

$$\begin{cases}
\frac{\partial R}{\partial z} = -i\kappa e^{i(2\delta z + \phi)}S \\
\frac{\partial S}{\partial z} = i\kappa e^{-i(2\delta z + \phi)}R
\end{cases} \quad (2.64)$$

where $\delta = \beta - K/2 + M$. This is a form of the so-called coupled-mode equations, which govern the interaction between two counter-propagating modes due to a perturbation in the refractive index.

There are two special cases in which the coupled-mode equations have analytical solutions: (1) in the case of very weak perturbations, where the transmitted light can be regarded as unaffected by the perturbations, and (2) when the coupling coefficient $\kappa$ is constant as a function of $z$.

For the reflective response of a more complex grating including apodization and/or chirp it is necessary to turn to numerical methods, which is the topic of chapter 3.
2.4.4 Weak gratings

For weak gratings, one can assume that no multiple reflections occur and that the amount of light reaching each fringe is approximately constant. Since in this case each spatial frequency of the grating causes one certain light frequency according to the Bragg condition (2.57) to be reflected, it is reasonable to believe that the Fourier transform of the refractive index variation is directly related to the reflection spectrum.

If the forward travelling wave is unaffected by the grating, this means that \( R = R_0 \) can be taken as a constant in the second equation in (2.64), resulting in a first order differential equation. Let us furthermore choose the boundary conditions

\[
\begin{aligned}
S(0) &= rR_0 \\
S(\infty) &= 0,
\end{aligned}
\]

(2.65)

i.e. the value of the backward propagating wave \( S(z) \) at the beginning of the grating (where we define \( z \equiv 0 \)) is the product of the reflection coefficient \( r \) and the forward propagating wave, and no backward propagating wave exists infinitely far away. Integrating the equation using these boundary conditions yields

\[
0 - rR_0 = iR_0 \int_{-\infty}^{\infty} \kappa(z) e^{-i(2\delta z + \phi(z))} dz.
\]

(2.66)

In this equation, we have further extended the integration limits to range from minus infinity. This can be done since no grating exists for \( z < 0 \) and thus \( \kappa = 0 \). This equation is easily recognized as a Fourier transform and we have

\[
\begin{aligned}
r(2\delta) &= -i \int_{-\infty}^{\infty} \kappa(z) e^{-i\phi(z)} e^{-i2\delta z} dz = -iF \{ \kappa(z) \},
\end{aligned}
\]

(2.67)

where \( F \) denotes the Fourier transform and \( \kappa(z) = \kappa(z) e^{-i\phi(z)} \) is the complex coupling coefficient. We thus conclude that, for weak gratings, the reflection coefficient and the grating properties are related by a Fourier transform. For stronger gratings this is no longer the case, but much of the intuition from Fourier analysis is still valid [17]. Consequently, it is often possible to get at least a crude idea of the reflective response of a given grating by considering this method.

Note that the fact that the Fourier transform of the grating reflection coefficient is also the impulse response implies that the complex coupling coefficient is proportional to the impulse response for weak gratings (see also paper I).

2.4.5 Uniform gratings

A uniform coupling coefficient \( \kappa \) means that \( \partial \kappa / \partial z = 0 \). Differentiating the first of the coupled mode equations (2.64) with respect to \( z \) once more then gives

\[
R'' = (2\delta + \phi') e^{i(2\delta z + \phi)} S - i\kappa e^{i(2\delta z + \phi)} S'.
\]

(2.68)
Inserting both equations into the last one results in
\[ R'' - i(2\delta + \phi')R' - \kappa^2 R = 0 \]  
(2.69)

In order to solve the above equation we assume that the phase derivative \( \phi'(z) = \phi' \), i.e. the phase varies linearly with position \( z \). Next, inserting the ansatz \( R(z) \propto e^{\beta z} \) into this equation gives
\[
\tau^2 + i(2\delta + \phi')\tau - \kappa^2 = 0
\]
(2.70)

Thus the general solution takes the form
\[
R(z) = (\alpha_1 e^{\beta z} + \alpha_2 e^{-\beta z}) e^{i(\delta + \phi'/2)z}
\]
(2.71)

where \( \beta = \sqrt{\kappa^2 - (\delta + \phi'/2)^2} \) and \( \alpha_1 \) and \( \alpha_2 \) are constants to be determined by the boundary conditions. Given this solution, the expression for the wave travelling in the opposite direction is directly found from eq. (2.64) as
\[
S(z) = \frac{i}{\kappa} e^{-i(\delta - \phi'/2)z-i\phi}
\]
(2.72)

Let us now further limit the calculations to the case where the phase shift is constant, i.e. \( \phi(z) = \phi \) and consequently \( \phi' = 0 \). This results in
\[
\begin{aligned}
R(z) &= (\alpha_1 e^{\beta z} + \alpha_2 e^{-\beta z}) e^{i\delta z} \\
S(z) &= \frac{1}{\kappa} [\alpha_1 (i\delta + \sigma)e^{\beta z} + \alpha_2 (i\delta - \sigma)e^{-\beta z}] e^{-i\delta z - i\phi}
\end{aligned}
\]
(2.73)

The above equations describe the propagating waves at any point in the grating with coefficients \( \alpha_1 \) and \( \alpha_2 \) determined from the boundary conditions.

It is often of interest to determine the fields at one side of the grating given the fields at the other side. In order to find an expression for the coefficients in this case, we take as boundary condition that
\[
\begin{aligned}
R(0) &= R_0 \\
S(0) &= S_0
\end{aligned}
\]
(2.74)

Note that \( R_0 \) and \( S_0 \) are actually functions of frequency \( \omega \), which determine the spectrum of the corresponding light waves. For readability reasons, this dependency is omitted here, though. Put into the last two equations the above conditions yield
\[
\begin{aligned}
\alpha_1 &= R_0 - \alpha_2 \\
\alpha_2 &= \frac{i\delta + \sigma}{i\delta - \sigma} \alpha_1 - \frac{ik}{i\delta - \sigma} S_0 e^{i\phi}
\end{aligned}
\]
(2.75)
Inserting these coefficients into eq. (2.73) gives the equations
\[
\begin{align*}
R(z) &= \left[ (\cosh \sigma z - i \frac{\delta}{\sigma} \sinh \sigma z) \right] R_0 - \left[ \frac{i \kappa}{\sigma} \right] e^{i \phi} \sinh \sigma z S_0 \right] e^{i \delta z} \\
S(z) &= \left[ \frac{i \kappa}{\sigma} e^{-i \phi} \sinh \sigma z R_0 + \left( \cosh \sigma z + i \frac{\delta}{\sigma} \sinh \sigma z \right) S_0 \right] e^{-i \delta z},
\end{align*}
\]
where the Euler identities for the hyperbolic sine and cosine have been used. In terms of the propagating fields we then have
\[
\begin{align*}
A(z) e^{-i \beta z} &= \left[ (\cosh \sigma z - i \frac{\delta}{\sigma} \sinh \sigma z) A_0 - \left[ \frac{i \kappa}{\sigma} \right] e^{i \phi} \sinh \sigma z B_0 \right] e^{-i K z/2} \\
B(z) e^{i \beta z} &= \left[ \frac{i \kappa}{\sigma} e^{-i \phi} \sinh \sigma z A_0 + \left( \cosh \sigma z + i \frac{\delta}{\sigma} \sinh \sigma z \right) B_0 \right] e^{i K z/2},
\end{align*}
\]
where \( A(0) = A_0 = R_0 \) and \( B(0) = B_0 = S_0 \). These equations give the propagating field anywhere in the perturbed structure as function of the field values at \( z = 0 \).

The reflection coefficient at the front face of a grating with length \( L \) is given by the quotient between the inserted and backward travelling waves,
\[
r(\lambda) = \frac{S_0}{R_0}. \tag{2.78}
\]
Assuming that no light of any wavelength is inserted beyond the grating end gives the additional boundary value
\[
B(L) = S(L) = 0. \tag{2.79}
\]
The second line in eq. (2.77) then gives
\[
r(\lambda) = \frac{-i \kappa \sinh (\sigma L)}{\sigma \cosh (\sigma L) + i \delta \sinh (\sigma L)} e^{-i \phi}, \tag{2.80}
\]
which constitutes the reflective spectral response of a uniform Bragg grating. This solution is also very convenient to use as basis for numerical simulation methods, as presented in the next chapter.
3.1 Introduction

There are several methods for simulating fibre Bragg gratings, but I will here concentrate on the most commonly used one, which is based on discretizing the grating structure in a cascade of subsections and then stepwise calculating the forward and backward propagating waves at each intersection. This kind of method is generally referred to as a transfer matrix approach, since each subsection is mathematically described as a matrix transforming the fields on one side to the fields on its other side.

There are several variants of the transfer matrix method in use [18–22]. These mainly differ in how each grating subsection is defined and modelled and are optimized for slightly different application areas. Other associated schemes where the matrix relates the total transverse tangential electric and magnetic fields at each side of the subsection have also been reported [23].

I will in this chapter discuss two of the most general transfer matrix methods, the thin layer [18] and the fundamental transfer matrix approach [9]. Other methods include e.g. the discretized grating model [19, 20], which was especially tailored for an inverse algorithm, but which is also useful in the standard formulation of the problem.

3.2 Thin layer approach

The conceptually most straightforward subsection model is to keep each section in the simulation so thin that its index of refraction is virtually constant from one side to the other. The effective index for a propagating light mode in section $p$ is then
given by

\[ n_p = n(pl) , \]  

(3.1)

where \( l \) is the length of the section and \( n(z) \) is the effective index as function of grating position \( z \). Each section consists of an interface to the preceding section and the propagation length \( l \) leading to the next section [18]. In order to accurately model a grating, each section cannot be larger than a small fraction of the grating pitch (see fig. 3.1).

The electromagnetic field in the interface between adjacent sections is determined by the Fresnel equations for reflection \( r \) and transmission \( t \) at an interface [14],

\[
\begin{cases}
r_p = \frac{n_{p-1} - n_p}{n_{p-1} + n_p} \\
t_p = \frac{2n_p}{n_{p-1} + n_p},
\end{cases}
\]  

(3.2)

where \( n_p \) and \( n_{p+1} \) are the refractive index values of two consecutive grating sections. Note that we can use the equations for perpendicularly incident light, since e.g. polarization effects are taken into account by the propagation modes of the structure; modes of different polarizations will have different effective indices.

The relation between the reflection and transmission coefficients in the interface from section \( p \) to \( p+1 \) and the corresponding (primed) coefficients in the “backward” direction is given by

\[
\begin{cases}
r'_p = -r_p \\
t_p t'_p = 1 - r_p^2,
\end{cases}
\]  

(3.3)

Denoting the forward and backward travelling waves just before an intersection by \( A, B \) and the corresponding fields just after the intersection by \( A', B' \) we have that

\[
\begin{cases}
A' = tA + r'B' \\
B = rA + t'B'
\end{cases}
\]
\[ A = \frac{1}{\ell} A' + \frac{\ell}{\ell} B' \]
\[ B = \frac{\ell}{\ell} A' + \frac{1}{\ell} B' \]  \hspace{1cm} (3.4)

where eq. (3.3) has been used. Rewriting these expressions in matrix form, adding subscripts for the \( p \)-th element and substituting according to eq. (3.2) results in

\[
\begin{pmatrix}
A_{p-1} \\
B_{p-1}
\end{pmatrix} = \frac{1}{2n_p} \begin{pmatrix}
n_{p-1} + n_p & n_{p-1} - n_p \\
n_{p-1} - n_p & n_{p-1} + n_p
\end{pmatrix} \begin{pmatrix}
A'_{p} \\
B'_{p}
\end{pmatrix},
\]  \hspace{1cm} (3.5)

which relates the forward and backward travelling waves just before and after an intersection. In order to model the whole section we have to introduce a matrix taking the propagation distance through the section into account. The distance between consecutive intersections is assumed to consist of a homogeneous material with a constant effective index equal to the local index at that position. The additional phase \( \delta \phi_p \) that light propagating through section \( p \) gains is therefore given by the simple expression

\[ \delta \phi_p = \frac{2\pi}{\lambda} n_pl \]  \hspace{1cm} (3.6)

where \( \lambda \) is the (vacuum) wavelength of the light and \( l \) the geometrical section length as indicated in fig. 3.1. This means that the fields at each side of a grating section are related by

\[
\begin{pmatrix}
A_{p-1} \\
B_{p-1}
\end{pmatrix} = \frac{1}{2n_p} \begin{pmatrix}
n_{p-1} + n_p & n_{p-1} - n_p \\
n_{p-1} - n_p & n_{p-1} + n_p
\end{pmatrix} \begin{pmatrix}
e^{-i\frac{2\pi}{\lambda} n_pl} & 0 \\
e^{i\frac{2\pi}{\lambda} n_pl} & 0
\end{pmatrix} \begin{pmatrix}
A_{p} \\
B_{p}
\end{pmatrix}
\]  \hspace{1cm} (3.7)

\[
\begin{pmatrix}
A_{p-1} \\
B_{p-1}
\end{pmatrix} = \frac{1}{2n_p} M_p T_p \begin{pmatrix}
A_{p} \\
B_{p}
\end{pmatrix},
\]

where \( M_p \) and \( T_p \) are the intersection and propagation matrices, respectively. When the sections are put together, the field at the right side of section \( p \) will be the same as the field at the left side of section \( p+1 \). Therefore the transfer matrix of the total grating consisting of \( N \) sections is just the product of the different section transfer matrices, i.e.

\[
\begin{pmatrix}
A_0 \\
B_0
\end{pmatrix} = \prod_{p=1}^{N} \frac{1}{2n_p} M_p T_p \begin{pmatrix}
A_N \\
0
\end{pmatrix}. \]  \hspace{1cm} (3.8)

Note that we have assumed that no light is incident beyond the end face of the grating, hence \( B_N = 0 \).

The reflection coefficient \( r \) at the front face of the grating is given by the quotient of the inserted and backward travelling waves. Defining

\[
\begin{pmatrix}
I_{11} & I_{12} \\
I_{21} & I_{22}
\end{pmatrix} = \prod_{p=1}^{N} \frac{1}{2n_p} M_p T_p
\]  \hspace{1cm} (3.9)
we then have that
\[ r(\lambda) = \frac{B_0}{A_0} = \frac{I_{21}}{I_{11}}. \]  
(3.10)
The transmission coefficient \( t \) is calculated similarly as the quotient of the inserted and transmitted waves,
\[ t(\lambda) = \frac{A_N}{A_0} = \frac{1}{I_{11}}. \]  
(3.11)
The thin layer method works with all kinds of refractive index perturbations and is the most general of the transfer matrix methods. However, since it is necessary to have many layers for each grating period, it is necessary to include a very large number of sections even for moderate grating lengths, rendering the method quite slow in most realistic cases. It should therefore only be used when other methods fail. An example of this is in the rare case when there are fast refractive index changes that cause the SVEA approximation in eq. (2.45) to break down.

### 3.3 Fundamental transfer matrix approach

A way to speed up the calculations is to use the fact that there is an analytical solution to the special case of uniform gratings. The grating structure can therefore be discretized as a cascade of \( N \) uniform subgratings, each having a specific period \( \Lambda \), phase offset \( \phi \) and modulation depth \( \Delta n \) (see fig. 3.2). Since each section now contains many grating periods, the total number of sections and corresponding calculation steps is greatly reduced. The propagating fields at the end faces of each subgrating obey eq. (2.76) and the boundary conditions are in each case determined by the adjacent subgratings and, at the end faces, by the conditions in eq. (2.74) and eq. (2.79). By changing subgrating parameters from section to section, any grating apodization and phase profile can be approximated. This solution constitutes the most commonly used method for fibre Bragg grating simulation. It was first presented in 1987 [9] and is both simple to implement [24] and gives good results for a wide variety of gratings.

In order to benefit from fewer iteration steps in the simulations, each section should typically be at least 10–100 periods long. It is of course not possible to model refractive index variations on a scale smaller than a section length, but this is usually not a problem for most kinds of Bragg gratings. For instance, any phase shift or other sudden variation inscribed into a grating will be averaged over the width of the interfering UV beams, which is often in the order of at least 50–100 grating periods for common telecommunication applications.

The starting point of this method is the equations (2.77). As for the thin layer approach, it is convenient to restate the equations in matrix form for a subgrating
Figure 3.2: A grating may be approximated by a number of subgratings (a), each having a specific period $\Lambda$, phase $\phi$ and modulation depth $\Delta n$. In (b) a single subgrating of length $l$ is shown.

of length $l$:

$$\begin{pmatrix} A(l)e^{-i\beta l} \\ B(l)e^{i\beta l} \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \begin{pmatrix} A_0 \\ B_0 \end{pmatrix}. \quad (3.12)$$

The elements of the transfer matrix at the right hand side are given according to eq. (2.77) by

$$T_{11}^* = T_{22} = \left( \cosh \sigma l + \frac{i\delta}{\sigma} \sinh \sigma l \right) e^{iKl/2},$$

$$T_{12}^* = T_{21} = \frac{i\kappa}{\sigma} \sinh \sigma l e^{iKl/2-i\phi}, \quad (3.13)$$

where the star (*) denotes complex conjugation. A closer look at this matrix reveals that it is unitary, so we can easily invert it and thus solve the equation for the field at the front face, giving

$$\begin{pmatrix} A_0 \\ B_0 \end{pmatrix} = \begin{pmatrix} T_{22} & -T_{12} \\ -T_{21} & T_{11} \end{pmatrix} \begin{pmatrix} A(l) e^{-i\beta l} \\ B(l) e^{i\beta l} \end{pmatrix}. \quad (3.14)$$

The transfer matrix of the total grating of length $L$ is as for the thin layer method given by the product of the different subgrating transfer matrices, i.e.

$$\begin{pmatrix} A_0 \\ B_0 \end{pmatrix} = \begin{pmatrix} I_{11} & I_{12} \\ I_{21} & I_{22} \end{pmatrix} \begin{pmatrix} A(L) e^{-i\beta L} \\ 0 \end{pmatrix}, \quad (3.15)$$

where

$$\begin{pmatrix} I_{11} & I_{12} \\ I_{21} & I_{22} \end{pmatrix} = \prod_{p=1}^{N} \begin{pmatrix} T_{22} & -T_{12} \\ -T_{21} & T_{11} \end{pmatrix}. \quad (3.16)$$

The reflection and transmission coefficients are then calculated in the same way as before,

$$r(\lambda) = \frac{B_0}{A_0} = \frac{I_{21}}{I_{11}}, \quad t(\lambda) = \frac{A(L) e^{-i\beta L}}{A_0} = \frac{1}{I_{11}}. \quad (3.17)$$

By looking at the values of $A$ and $B$ between each pair of subgratings, it is also simple to extract the field distribution within the grating.
It is very intuitive to look at a few simulations of basic structure types in order to visualize the theory and gain a better understanding of experimental results. A few examples are shown here, all simulated using the fundamental matrix method.

Figure 3.3 shows the impulse responses for an unapodized 2 cm long Bragg grating for some different grating strengths. The impulse response is calculated by Fourier transforming the complex reflection coefficient obtained in the simulation. It is evident that the impulse response resembles the refractive index profile in the weak limit, but not at all in the strong limit where the approximations leading to eq. (2.67) do not hold. This is the reason why the characterization method based on low-coherence reflectometry described in chapter 5 is less effective for stronger gratings.

Any sharp changes in the modulation depth give rise to oscillations in the spectral response. Introducing an apodization that smooths the sharpness greatly reduces or removes this oscillation. This is illustrated in fig. 3.4, where the reflective response from two 1 cm long gratings are shown where one is uniform (a) and one is apodized (b). The prominent side lobes caused by the uniform grating are completely removed by the apodized grating. The circles in the graphs are measured data points from a realization by the MPF method presented in chapter 4.

Finally, fig. 3.5 shows the field strength inside two gratings obtained by plotting the fields between each pair of grating sections. The field is plotted versus grating position (horizontal scale) and wavelength (vertical scale) for a 1 cm long uniform
Figure 3.4: Reflective response of a 1-cm uniform grating that is (a) unapodized and (b) apodized. The circles are measured data points from a realization of the gratings.

Figure 3.5: Field distribution inside a 1-cm unapodized and unchirped grating that (a) is uniform and (b) contains a 180° phase shift in the middle. The field strength is increasing from black to white shades.
grating in fig. 3.5a and the same structure with a 180° phase shift in the middle in fig. 3.5b. The phase shift causes negative interference for reflected light (left in the figures) at the central wavelength and thus creates a narrow band pass filter in transmission. This is clearly seen in fig. 3.5b, where the field strength for light at the central wavelength is increased at the phase shift and a narrow transmission line appears to the right. The same phenomenon for chirped gratings can be used to realize a tunable band pass filter as described in chapter 6.

The cover image of this thesis shows another field distribution example for an apodized and linearly chirped grating. In this case, a false spectrum is chosen as colour map.

### 3.4 Optimized transfer matrix implementation

The first step in any implementation of the fundamental transfer matrix method is to determine the subsection lengths, modulation depths and periods from the structure of the grating to be simulated. Secondly, the matrix elements in eq. (3.13) have to be calculated for each section and the product in eq. (3.16) evaluated. Finally, the complex reflection coefficient is calculated from eq. (3.17).

The most straightforward algorithm consists of dividing the grating in equally long subsections with a length according to the wanted spatial resolution and then performing element calculations and matrix multiplications stepwise in a simple loop, working through the grating from the end to the front face matrix by matrix. The process is then repeated for each response wavelength. This is an intuitive but rather inefficient algorithm, since many time-consuming calculations are unnecessarily repeated a large number of times. In this section I present a new and flexible algorithm that automatically optimizes the subsectioning and reduces the simulation time for any grating structure.

The first obvious step in any code optimization is to remove as many operations as possible from the main loops that are executed a large number of times. If we define the subgrating lengths in terms of a constant number \( p \) of local periods, i.e. \( l = p n_{av} \Lambda \), instead of a constant geometrical length, we additionally save all operations needed for keeping track of a changing start phase that would appear for each new section. Furthermore, the phase term \( e^{iKl/2} \) in the matrix elements reduces to the simple factor \( \pm 1 \). The sections in a chirped grating will not have the exact same length throughout the whole structure if this scheme is followed, but the differences are very small for all conceivable gratings so this is of no importance for the simulation.

In order to execute the main loop as few times as possible, we want to minimize
Figure 3.6: Flow chart for an optimized fundamental transfer matrix implementation.

The number of grating sections without compromising the spatial accuracy. Many classes of Bragg gratings are partially uniform and can be modelled by only a few sections, whereas others change continuously in some parameters and have to be modelled by many sections in order to obtain realistic results. A convenient way to automate the subsectioning process is to define a smallest section length corresponding to the desired spatial resolution and then let an initial loop step through the grating and increase the length for each section until any parameter (modulation depth, period or phase) is changed. The result is a dynamic sectioning with a minimized number of sections for the given structure and spatial resolution (see fig. 3.6).

The dynamic sectioning is only done once when the response for the first wavelength is calculated. The lengths of all sections are then saved into a vector indexed by section number. To further decrease the number of redundant calculations, it is advisable to save all major wavelength independent calculations into vectors as
well, each again indexed by section number. This will increase the memory usage somewhat, but since the typical number of intermediate vectors needed is around 10 (depending on implementation), this will not be a problem for a modern computer.

The response for all but the first wavelength is calculated by simply stepping through the intermediate vectors and executing all remaining wavelength dependent calculations for each section as shown in the right part of fig. 3.6. The effect of this algorithm is especially prominent for partially uniform gratings, where the number of sections and calculation steps is considerably reduced. Structures with continuously changing parameters such as linearly chirped gratings do not benefit from the dynamic sectioning, but still have simulation times considerably smaller than for a straightforward implementation due to the use of intermediate result vectors.

Different implementations of the above simulation method have been used throughout all the work in this thesis and had a central role in papers V and VI.
4.1 Background

The refractive index of an optical fibre may be increased by exposure to ultraviolet (UV) light. The UV sensitivity in standard optical fibre is very low, but it can be raised considerably by adding different dopants such as germanium and boron during the fabrication process [25] and by post-processing techniques such as hydrogen loading [26]. In this way, refractive index modulation depths of the order of $\Delta n \approx 10^{-2}$ are achievable.

Even though there has been extensive research in the field during the past decades, the physical mechanisms behind photosensitivity in glasses are still not fully understood. This is a research topic of its own [27, 28] and will not be discussed in further detail here.

The history of permanent UV-induced Bragg gratings in optical fibre began in 1978, when Hill et al. [1] first demonstrated the phenomenon. In this experiment, the grating was formed by the standing-wave pattern from two counter-propagating laser beams launched by end-fire coupling. While demonstrating the principle of photosensitivity in optical fibres, this technique is rather limited with small possibilities to realize customized complex grating structures.

In 1989, Meltz et al. [5] introduced the transverse holographic method that constitutes the basis of most fabrication techniques used today. The grating is now formed by exposing the fibre to a UV interference pattern focused upon the core from the side through the cladding (see fig. 4.1).

Usually, source wavelengths around 244 nm are used since the fibre sensitivity is comparatively high in this region, but there are also reports on fabrication setups
The period $\Lambda$ of the pattern is easily changed by increasing or decreasing the angle $\alpha$ between the two interfering beams. For a given UV wavelength $\lambda$ we have that

$$\Lambda = \frac{\lambda}{2 \sin(\alpha/2)},$$

which is easily derived by some geometrical considerations. As an example, a UV light source with $\lambda = 244$ nm needs an angle $\alpha \approx 27^\circ$ for a fringe pattern that exposes gratings for the 1550 nm regime.

It is also possible to form fibre Bragg gratings by other means such as direct point-to-point writing [32] or chemical composition [33], but these techniques are beyond the scope of this thesis.

## 4.2 Fabrication methods

### 4.2.1 Basic principles

There are two widely used ways to form the diffraction pattern for the grating exposure: either by means of a free space interferometer with a beamsplitter to divide the light into two paths (see e.g. ref. 5), or with aid of an etched phase mask comprising the modulation that is to be imprinted into the fibre. In the latter case, the phase mask works as a beamsplitter (see e.g. ref. 34). Figure 4.2 shows a schematic of each of these methods.

In the most straightforward implementation of these basic techniques, the interferometer is fixed with regard to the fibre. In this case, it is only possible to fabricate
Figure 4.2: Principles for creating the UV interference pattern for the transverse holographic fabrication method: (a) beamsplitter based interferometer, (b) example of a setup using a phase mask.

In order to realize longer and more advanced structures, it is necessary to include a possibility to displace the interferometer pattern and fibre relative to each other during the fabrication process. Several methods for realizing gratings with complex structures using phase mask based techniques have been developed [35–40]. While having the benefit of a potentially high reproducibility, the disadvantage of all these methods is that they rely on phase masks that have to be of very high quality with a low amount of fabrication errors (and therefore rather expensive) to ensure high quality gratings. Furthermore, new grating designs will in many cases demand new phase masks, which substantially lowers the flexibility and raises costs.

I will in the following sections focus on fabrication systems where no phase masks are necessary to form the grating.

4.2.2 Multiple printing in fibre

In 1995, Stubbe et al. [41] first proposed the multiple printing in fibre (MPF) technique, where long gratings are formed by consecutively exposing short subgratings consisting of a few hundred periods with a pulsed UV source while at the same time moving the fibre. With a high precision regulation of the fibre’s position relative
the interference pattern, the phase and position of each subgrating is precisely controlled. This technique does not have to include any phase mask — the interference pattern can just as well be generated by an interferometer with an ordinary beamsplitter. The control over the imprinted grating fringes can therefore be made direct without involving the possible source of errors that a phase mask constitutes.

Different grating designs are realized by changing the relative phase between the subgratings [42]. For uniform gratings, the fibre is translated an integral number of interference periods between each subgrating exposure. The subgratings then add to form a long continuous grating.

A single phase shift is introduced by translating the fibre a non-integral number of periods at one point. Chirps are realized in a similar way by continuously adding phase shifts following eq. (2.55). With a fixed subgrating period, the subgrating length will determine the strength of the largest chirp that can be realized without causing overlapping subgratings to cancel each other and thus decrease the grating visibility. If larger chirps are needed, the angle of the interfering beams have to be changed during the fabrication process [43], such that the subgrating period always corresponds to the required local grating period (see fig. 4.3). In order to maintain a well-defined phase of each subgrating, it is important that the beam angles are changed symmetrically, otherwise the interference pattern will move within the focus spot.

A straightforward way to change the modulation depth along the grating (i.e. introducing an apodization profile) would be to modify the UV power for each subgrating. This is not advisable, though, since firstly the power of a pulsed laser beam might be hard to control to high precision and secondly, this would also change the average refractive index $n_{av}$ throughout the fibre. This is due to the fact that the UV light is only capable of inducing an increase in the refractive index, not a
decrease. A fluctuating average refractive index translates to a fluctuating optical period of the subgratings, i.e. an additional chirp. Instead, a better method is to introduce a phase shift $\pm \Delta \varphi$ with different signs for consecutive subgratings. As a result, two gratings with a relative phase difference $2\Delta \varphi$ will be superposed along the fibre, so that the total refractive index is given by

$$\Delta n(z) \propto \cos(Kz - \Delta \varphi) + \cos(Kz + \Delta \varphi) \propto \cos \Delta \varphi \cos Kz,$$

(4.2)

with variables given as before. The phase dither $\pm \Delta \varphi$ thus determines the visibility of the grating through the cosine whereas the average refractive index, which is only determined by the UV dose used to expose each subgrating, can be kept constant.

Another advantage of the MPF technique is now evident: since all parameters of the grating are determined by the relative phase shifts between the subgratings, new grating designs are easily implemented by mere programmatic changes in the software synchronizing the fibre movement and UV pulse generation. The MPF technique also has a few drawbacks. To avoid vibrations and strains in the fibre during the grating fabrication, the fibre translation speed should be as constant as possible and thus the fibre must move continuously during the exposure of each subgrating. If a good visibility is to be maintained, this in turn sets a limit on the UV pulse length. Since the top effect of a short pulse is very high even for moderate average intensities, the irradiance at the fibre has to be kept low to avoid inducing optical damage. This means a longer exposure time, resulting in a higher probability for errors due to temperature fluctuations and material movement during the fabrication process.

Moreover, it is hard to keep the pulse energy constant, which results in phase errors due to a fluctuating average refractive index. Solutions with a mechanically chopped continuous-wave laser beam may reduce this effect, but this means at the same time that only a very small amount of the optical energy is actually used for the exposures.

### 4.2.3 Sequential exposure with a continuous-wave source

A fabrication scheme comprising a continuous-wave source, where all of the light is used for the grating exposure is certainly beneficial. Apart from lower power consumption and shorter fabrication times, always working with continuous entities makes it easier to fulfil the demands for a constant distribution of the UV dose in the fibre and a static environment without sudden bursts of heat and corresponding material fluctuations. In paper II, we propose and demonstrate a Bragg grating fabrication method that has these properties. This method has also been patented [44].
Figure 4.4 shows a schematic of the setup. The fibre is mounted on an air bearing born carriage, which is translated by a feed-back controlled linear drive in the same way as in ref. 41. The position relative the UV interference pattern is determined with a heterodyne interference detection system. Since the fibre is moving, the fringes of the continuous-wave interference pattern must follow this movement in order to maintain the visibility of the grating. This is accomplished by changing the path difference between the two interfering beams with aid of a pair of piezo translators mounted on mirrors c in the UV interferometer as depicted in fig. 4.5. The additional phase difference results in a shift of the fringes within the focal spot. Driving the piezo translators with a sawtooth signal controlled by the fibre movement causes the pattern to move with the fibre some distance (typically one period) and then quickly jump back to expose next part of the grating. This principle was also reported in ref. 45. A computer controlled shutter in the UV beam path additionally prevents unwanted exposure of the fibre before and after the grating writing process.

Complex structures such as phase shifts, apodizations and chirps are realized in very much the same way as in the MPF scheme, with the difference that all phase shifts are now controlled through the shape of the sawtooth signal driving the piezo translators as described in paper II. A step motor is used to move the mirror pair d (see fig. 4.5), which symmetrically translates the interfering beams in the plane of incidence. This translation is transformed to an angular change $\Delta \alpha$ by the cylindrical lenses e and thus the period of the interference pattern is changed. Due to
Figure 4.5: The UV interferometer for producing the interference pattern for the exposures. Mirrors $a$ and $b$ consist of sections with different transmission coefficients and divide the light into two paths. The interference pattern is translated with aid of piezo translators mounted on the mirrors $c$ and the fringe period is changed by moving the mirror pair $d$ as depicted. The pattern is focused into the fibre core with a cylindrical lens $f$.

The speed with which the fibre can be translated during the grating fabrication is limited by the response time of the piezo crystals — for large velocities, mechanical
ringing will substantially reduce the visibility of the imprinted grating. In the present setup, writing velocities of the order of centimeters per second have been achieved. Further optimization of size and shape of the piezo crystals may increase this figure.

As with the MPF scheme, all grating parameters are controlled by the computer software and new grating designs are easily implemented. In this case, we have the additional benefit of working with a continuous-wave laser source, where all of the light is used for the grating fabrication. The result is a very well-defined UV dose distribution along the fibre.

4.3 Common sources of error

There are a number of noise sources that any constructor of a fibre grating fabrication system will meet and that have to be dealt with in order to yield the highest possible grating quality. In this section I list a few of the most common ones and point at necessary precautions in order to minimize their impact on the end result.

4.3.1 Vibrations

It is important that the fibre and exposing system remain in a well defined position relative to each other during the fabrication process. With grating periods of typically $5 \times 10^{-7}$m, it does not take much to move the fibre a substantial part of this period, which may cause both phase errors and dose fluctuations in the resulting grating. Different kinds of vibrations are therefore one of the largest error sources and have to be dealt with.

Naturally, as a start, the fabrication system has to be rigidly mounted on a vibra-
tion absorbing foundation but this is by far not enough. Note that the wavelength of the UV light usually employed for the interference pattern is rather small and therefore extremely sensitive to temperature fluctuations and convection movements in the air. It is important to shield all light paths as thoroughly as possible to reduce this error.

In systems where fibre and interferometer are translated relative each other during fabrication, the movement has to be as smooth as possible. The smoothness can usually be improved somewhat by increasing the speed, taking benefit of the linear momentum of the mechanical parts. Keeping the fabrication time down also reduces the error induced by air fluctuations.

There are nowadays very high precision translators commercially available that have a relative precision and repeatability of the order of nanometres or even less. Using such a motion system certainly improves the grating quality. However, it is not necessarily enough to meet the extreme demands of e.g. dispersion compensation components for the telecommunication market. Even if the translator is perfectly controlled, this is no guarantee that there are no residual relative vibrations between fibre and exposing system from other origins. In order to achieve the highest possible grating quality it is therefore important to continuously monitor the actual fibre position and adjust the exposure position accordingly.

4.3.2 Fibre issues

The material properties of optical fibres are usually very homogeneous, but there may still be small inhomogeneities that have a negative effect on gratings exposed in the fibre. For highly sensitized fibres, there may be inhomogeneous distributions of different dopants, which causes a variation in the UV sensitivity. This, in turn, affects both the apodization and Bragg wavelength of an inscribed grating. One way to deal with this problem is to post process the fibre after the exposure [46]. This includes using some method for determining the position and amplitude of the inhomogeneities (see chapter 5) and adjustment of the refractive index modulation and/or its average value accordingly.

Another thing that has to be considered is the linear positioning of the fibre. A curved fibre during the exposure will cause a variation in the local period of the grating resulting in a spectral broadening of the response. The best way to minimize this effect is to induce a small longitudinal tension in the fibre during the exposure. The grating period will decrease somewhat when relaxing the fibre, but this will be the same for the whole grating and is easily accounted for.
4.3.3 Stitching

In applications such as chromatic dispersion compensation components for optical networks, there is a need for very long gratings. As an example, 80 km standard fibre over the entire C-band (wavelength range 1525 nm–1565 nm) requires a grating length of the order of 6 m for full compensation. In order to realize this kind of grating it is usually necessary to stitch several different consecutive exposures and each of these stitches is a potential source of errors due to phase, period or modulation depth mismatch between the adjacent structures.

Paper V identifies and investigates by simulation different errors that occur in stitches and discusses their impact on the resulting grating response. It is shown for the simulated structure that phase errors have to be less than 10° (corresponding to a positioning error of the order of ten nanometres) and modulation errors less than a few parts per million in order for the grating to be an acceptable component for a 10 Gbit/s transmission system.

It is evident that relative positioning of the different stitched subgratings has to be very precise. Apart from a state-of-the-art positioning system, it is also necessary to measure the position of the last exposed grating as a reference for the positioning of the next. This can be implemented by variations of the side diffraction characterization method described in chapter 5.

Differences in average refractive index between different subgratings cause a difference in optical period. However, this can be corrected by exposing grating parts with smaller averages to constant UV light (without any interference pattern) after the grating exposures. Of course, this also requires that some characterization method is implemented in order to determine where and how much to adjust the UV dose.
5.1 Background

Fabrication of high quality fibre Bragg gratings is not a simple task and there are many factors that can go wrong during the process, resulting in phase errors and unwanted refractive index fluctuations. Even a perfect setup will not produce error-free gratings, since inhomogeneities in the fibre material, both in absorption efficiency and density, will deteriorate the result. It is therefore very important to be able to characterize the gratings in order to determine their quality. Monitoring the properties of a grating during the fabrication process may furthermore open for real-time compensation for errors in the fibre and fabrication equipment.

In the case of weak gratings where the Fourier approximation (2.67) is applicable, it is always theoretically possible to calculate the refractive index profile and chirp along the fibre from the corresponding complex reflection spectrum. The problem is that even for moderate requirements on the end result, a rather high wavelength resolution is needed to accurately make these calculations. Moreover, in order to achieve a complete evaluation of all grating parameters, the phase spectrum, which is not obtained by ordinary spectrum analysers, must be measured as well. The demand for expensive high-resolution spectrometers along with the restriction to weak gratings makes this method unsuitable for most applications. Instead, more direct methods to measure the grating parameters are needed. A number of such methods have been demonstrated during the past decade that are suitable for a wide range of applications.

One possibility is to scan the grating with a probe applying heat [7, 47, 48] or pressure [49] and at the same time detect the change in the reflection spectrum of
the grating. From these changes, the distribution of phase and index modulation along the fibre can be determined.

Another interesting and simple evaluation technique is presented in ref. 50. With aid of the interference between light reflected from the grating and the light from a reference reflector such as the end face of the fibre, the group delay and dispersion are determined.

Characterization techniques for determining the polarization along the fibre have also been demonstrated [51].

I will here focus on two different techniques for evaluation of both apodization and phase distribution that are based on optical low-coherence reflectometry (OLCR) and side diffraction, respectively.

Regardless of evaluation method, it is helpful to relate the analysis to the fabrication parameters of the grating in question. It is in most cases unnecessary to use a higher spatial resolution than the resolution with which the grating was created as given by the width of the inscribing beam. This is the limit of a physical low-pass filtering during the fabrication process and virtually no exposure-inflicted errors will appear below this scale.

On the other hand, using a resolution much lower than the fabrication resolution will hide errors that may be of importance for grating performance. At this point, it is also valuable to consider the context for which the evaluation results are intended. For example, it has been shown that group delay ripple in chirped gratings is caused by defects on the millimetre scale and above only [52,53]. If this is the only parameter of interest it is therefore advisable to lower the resolution in order to get rid of irrelevant data.

The topic of relating the characterization analysis to fabrication parameters is further discussed in paper IV.

5.2 Low-coherence reflectometry

Measurement techniques based on low-coherent light are widely used in different metrology applications. During the past decade, low-coherent light has also found its way to the field of grating characterization (see e.g. refs. 6,54,55). These methods have all in common that they use the short coherence length of a broad-band light source to determine the spatial origin of detected interference. The technique is therefore sometimes also referred to as white light interferometry in the literature.

Figure 5.1 shows the principle in a simple Michelson setup for grating characterization. The only difference from an “ordinary” interferometer setup as presented in introductory optics textbooks is the spectral characteristics of the source, which is chosen to emit light over a wide range of wavelengths and thus have a very short
Grating Characterization

Figure 5.1: A white light interferometer. Only light reflected from a small part of the device under test around the balance position $d$ corresponding to the coherence length $\Delta l$ contributes to the interference pattern.

cohere length $\Delta l$. The light contributing to the interference originates from a small region around the balanced position $d$ corresponding to this coherence length. Reflections from outside this region will only add incoherently to the overall intensity at the detector since these have no stable phase relation to each other. Scanning the reference mirror moves the balanced position throughout the grating, thus allowing for a spatially resolved interferometrical measurement of the reflection properties of the grating under test. The resolution is in general given by the coherence length.

As shown in paper I, low-coherence measurements according to fig. 5.1 yield the (complex) impulse response of the grating under test which, in turn, is the Fourier transform of the complex reflection spectrum. This is understood intuitively by noting that continuous broad-band light can be seen as a superposition of a vast amount of uncorrelated overlapping short pulses that appear randomly in time. These pulses are all inserted into the grating and then the interferometric detection scheme isolates the signal for the time delay corresponding to the current position of the scanner mirror. A full scan from zero to whatever time value necessary consequently gives the response of the grating from an impulse as function of time delay. Measuring the impulse response in the time domain [56] yields exactly the same data. As an alternative, the impulse response can also be obtained in the frequency domain [57–59].

Every aspect of the grating can be deduced from the complex reflection spectrum and thus low-coherence reflectometry gives a complete characterization of the grating under test.

In the special case of weak gratings, the measurement result directly corresponds to the coupling coefficient $\kappa$ and phase factor $\exp(-i\phi)$ according to eq. (2.67).
Figure 5.2: Schematic 2-dimensional equivalent of a double-balanced interferometer for characterization of both phase and index modulation distributions in fibre Bragg gratings. The interrogating beam is split into two channels by the beamsplitter $a$. The grating is scanned by moving retroreflector $b$, whereas retroreflector $c$ changes the distance between the balanced points.

Stronger gratings may be evaluated by combining low-coherence measurements with reconstruction algorithms [60, 61].

We have proposed a novel interferometer design for low-coherence characterization of fibre Bragg gratings [55], which is closely described in paper I. Figure 5.2 shows a schematic 2-dimensional equivalent of the interferometer in our setup. This interferometer has two different balance positions, “channels”, that are scanned simultaneously through the grating under test. In this way, differential measurements are made, significantly reducing the sensitivity to noise due to thermal fluctuations in the system. During the measurement, data from both channels are recorded to a computer, which then calculates the modulation depth and phase distribution as a function of (optical) position in the fibre. The actual interferometer was built with bulk optics consisting mainly of a cube beamsplitter with some further optics attached (see fig. 5.3). The concept was further developed in ref. 62 by a different interferometer layout and including temporal switching between the two channels to further increase the common path and thus reduce the sensitivity to thermal fluctuations to an even greater extent.

One of the advantages of the low-coherence reflectometry technique as described here is that refractive-index fluctuations are taken into account, i.e. the measurement result corresponds to the actual grating structure that light inserted into the fibre will experience and interact with. Many other characterization solutions only give the grating parameters as function of geometrical position, and thus no information about optical inhomogeneities are obtained. Another important property of the
Figure 5.3: Schematic view of the interferometer. The laser beam travels through a beamsplitter in three levels (a)-(c). Beam paths $a_1$ and $a_2$ are the two reference paths and $b_1$ and $b_2$ the corresponding test paths. In (c) the paths are mixed before arriving at the detectors via multi-mode fibres. Retroreflector 1 is used for scanning through the device under test. The second retroreflector is used for adjusting the difference between the two reference paths. For each pair of paths, two detectors detecting complementary outputs are used (c).
technique is its simplicity: the fibre grating does not have to be prepared in any way prior to the measurement — it is simply attached to one arm of the interferometer. This fact minimizes the risk for accidental destruction of the fairly fragile fibre grating.

Even though the method in theory is applicable to all kinds of gratings, it is not necessarily practically suitable for very large modulation depths [63]. In this case, most of the light is reflected in the first encountered part of the grating and the signal from the rest of its length is very weak. A large part of the signal that carries information about the grating therefore runs the risk of ending up below the noise level, which may deteriorate the result. Using a technique where the grating is interrogated from two sides [64] increases the number of gratings that are characterizable, but does not remove the problem completely.

For weak and fairly strong gratings, the OLCR method has proven to be valuable, giving good results and being quite simple to use.

5.3 Interferometric side diffraction

Another very useful characterization technique was first proposed by Krug et al. [8] in 1995. Several papers on this technique have appeared in the literature since then (see e.g. ref. 47). The fibre grating is illuminated from the side with a probe beam from some coherent light source. The light is then scattered (diffracted) by the grating with an efficiency that is proportional to the square of the refractive index modulation. Detecting the first diffracted order while scanning the grating thus gives the refractive index profile.

The method was extended in ref. 65 to include interferometric detection, which enables retrieval of the phase distribution as well. A derivation of the theory behind this extended scheme is presented in paper III.

The main principle is that the first diffracted order is brought to interfere with the zeroth (transmitted) order. The resulting interference pattern is then detected as the fibre is scanned past the probe beam (see fig. 5.4). As shown in paper III, the detected irradiation is directly proportional to the modulation depth of the refractive index and the change of phase equals the change of phase of the grating relative the fixed probe beam during the scan. It also turns out that the signal-to-noise ratio is practically independent of probe beam power, a fact that was verified experimentally in paper IV.

The side diffraction technique can be applied to fairly weak gratings, but is best used for stronger structures, where the signal to noise ratio is larger. In this sense, it is a good complement to the OLCR method discussed earlier, which works best for weaker gratings. If weak signals are to be detected with the side diffraction method,
it is very important to keep the fibre clean during the measurement, since the main noise term can be expected to originate from the surface roughness of the fibre. Other noise sources such as Rayleigh scattering within the fibre material itself, are for ordinary optical fibres orders of magnitude lower.

Some care has to be taken when investigating very strong gratings with a modulation depth of the order of percents, since the response in the amplitude measurements then becomes nonlinear. The theory indicates that gratings with modulation depths ranging from at least $\Delta n \approx 10^{-5}$ to $\Delta n \approx 10^{-3}$ should be possible to characterize without larger problems, though. The lower limit may further be extended by use of different lock-in modulation schemes [66, 67]. We report successful evaluation of gratings with an approximate refractive index modulation depth of $4.3 \cdot 10^{-4}$ in paper IV.

All parameters are obtained as functions of geometrical lengths and hence, no information about the fluctuation of the average refractive index is retrievable.

The interferometric side diffraction method can be implemented in a very simple way by taking advantage of the existing hardware in a grating fabrication setup (see fig. 5.5). The grating to be characterized is mounted as usual and the UV light source can be used as probe as long as the power is kept low and the interference pattern does not move with the fibre during scanning. The only things that have to be added to the fabrication setup are a UV detector and some acquisition hardware and software. Beam alignment and scanning equipment are automatically provided by the realization system [68].
Both evaluation methods described in the previous sections are comparable regarding quality of the results. The choice of technique is rather a matter of using the most convenient setup for the application at hand.

Since the OLCR method actually gives the impulse response of the grating under investigation, this method may be less accurate for stronger gratings in applications where the spatial grating characteristics are to be determined. The stronger the grating, the harder it is to solve the inverse problem and reconstruct the grating parameters from the measured data.

The OLCR method is convenient from a practical point of view, since it does not involve any delicate optical alignment once the interferometer is mounted. Also, the grating to be characterized is simply connected to the interferometer without having to consider temperature fluctuations or reference path lengths due to the differential measurement scheme.

If the objective is to determine the optical response of a grating-based component, the OLCR method has a further advantage in that it includes the refractive index of the fibre. The result will therefore correspond to the structure that is “seen” by the light in the actual application regardless of any fluctuating average index of refraction. This advantage is instead turned into a disadvantage if the aim is to determine geometrical characteristics such as the position of the grating fringes. This information can be used to monitor the performance of a grating fabrication system or to accurately stitch several shorter gratings together to one long compound structure [69, 70] as described in chapter 4. The interferometric side diffraction method constitutes the natural choice for this kind of applications.
6.1 Introduction

The largest market for fibre Bragg gratings is found in telecommunications. Bragg gratings are highly wavelength selective and customizable devices and as such perfect for filter components in telecommunication networks with high demands on accuracy, rejection ratios and in-band profiles. Bragg gratings are today an essential constituent of most optical network systems, especially those based on wavelength division multiplexing (WDM) techniques where the data channels are divided by optical carrier wavelength. Another important application area within the telecom market concerns long transmission lines, where the tailored response from gratings with complex chirps and apodizations are very suitable for spectral dispersion compensation.

Grating-based filters in optical fibre are also relatively easy to tune, i.e. adjust their characteristic properties, which increases the flexibility and usability of these devices. Filters are easily realizable as well for classical telecommunication wavelengths in the infrared regime as for micro- and radiowave applications.

Since a fibre grating reflects light in a way that is determined by its period, chirp and apodization, this naturally leads to the idea of using gratings as sensors by letting the entity that is to be measured affect these properties in one way or another. Such sensors can easily be made very small and incorporated in delicate systems, virtually without adding any mass. When the gratings are packaged, they can endure nasty environments where classical sensor solutions are very short-lived or even impossible, e.g. in oil wells.

The simplest way to affect the properties of fibre gratings is to let temperature
and tension or geometrical deformation change the grating period, thus changing the location of the peak in the reflectance spectrum. By using intricate combinations of detection methods and different gratings, uniform as well as chirped, it is even possible to differentiate between temperature and other influences (see e.g. ref 71). Other techniques may involve slanted gratings that couple out parts of the light, which then interacts with the environment.

One of the greatest advantages with fibre grating sensors is the simplicity, with which large multiplexed sensor arrays can be constructed. One single fibre may contain a large number of gratings and, embedding it e.g. in a bridge foundation or an aeroplane wing, it can continuously monitor vibrations and/or strains over large areas with high precision [72–75].

There are several techniques for extracting the information from one specific grating in an array of units [76]. One possibility is to use so-called time resolved multiplexing [77] by sending short light pulses into the fibre and measure the time it takes for the reflections to return. Each grating will then be represented by a specific time delay (fig. 6.1a). It is also possible to let the gratings reflect different frequencies, thus being distinguishable in the frequency (or wavelength) domain of the reflected light [78]. This is an example of frequency resolved multiplexing (fig. 6.1b). The latter method works well for a smaller amount of gratings, whereas the former may be used for virtually any number, having the disadvantage of a rather low spatial resolution.

While there are comparatively few competing technologies to the fibre Bragg grating in many applications within telecommunications, this is not true in the world of sensing. The field of sensing in general is old and there are numerous kinds of e.g. electrical gauges on the market that are accurate and reliable as well as cost-effective. Sensor systems based on Bragg gratings have a large potential but are in many cases still too expensive to find their way to a large market.
However, in highly specialized applications with high demands on e.g. light weight and small size or when sensor points have to be distributed over a large area, solutions based on fibre Bragg gratings constitute an important alternative.

The field of fibre Bragg grating based applications is very large and a full treat-ment would easily fill a book by its own. I will in the following only take a closer look at one aspect, the tuning of gratings.

### 6.2 Grating tuning

There are a number of ways in which a fibre Bragg grating can be tuned after or during fabrication. The techniques include thermal heating, different mechanically induced stresses (compression or decompression) and chemical and electro-optical influences. All of these techniques have in common that they change the optical distance between adjacent grating fringes.

The most common methods today for grating tuning in an application involve heating or mechanical compression/decompression (see e.g. refs. 79–83). Optical fibres usually consist of fused silica glass that lacks a second-order nonlinear suscepti-bility $\chi^{(2)}$ (cf. eq. (2.11)). It is therefore not possible to obtain any electro-optical effect in ordinary optical fibres, since this is a second-order process. However, the process of thermal poling can be used to record an electric field in the glass, thereby permanently breaking the material symmetry resulting in an effective second order nonlinearity that can be used e.g. for tuning purposes [84].

One of the advantages with the electro-optic effect is that it is not limited by the rather slow relaxation processes associated with traditional techniques based on thermal or mechanical influences. While the latter usually have responses limited to the kilohertz region, electro-optical methods have potential to be able to work in the order of gigahertz. Research is proceeding in this area and techniques based on poling and the electro-optical effect are bound to grow in importance during the coming years.

There are many components based on fibre Bragg gratings on the market today that have some built-in tunability comprising small electrically controllable heaters. The tunability of these components is usually limited to one or very few parameters such as central wavelength of a filter or spectral delay rate of a chromatic dispersion compensator. It is still a challenge to design a component that is fully tunable in all relevant parameters, for instance a band pass filter with adjustable central wavelength, filter width and transmission strength.

Paper VI describes a design that has potential to give a band pass filter that is fully configurable in all these parameters by software. The idea is to heat a linearly
chirped grating with aid of an array of electrically controllable heaters. A localized heating of the grating induces a phase shift for the local reflected wavelength at that position, resulting in a narrow transmission peak within the filter stop band [80] (see fig. 3.5). It is shown by simulation that when kept within reasonable limits, the length of the heated region is not important for the width of the transmission peak. The transmission peak width is rather a function of the grating modulation strength in conjunction with its chirp rate.

For the experiments we used an ordinary thermal printer head consisting of 640 heater elements distributed over 8 cm. Any number of heater elements can be turned on or off individually, which opens for fully software controlled tunability in all parameters. So far, our experiments show a well controlled tunability in central wavelength and filter strength. Figure 6.2 shows the transmission central wavelength versus heater position over the entire tuning region.
Conclusions and Future Work

This thesis deals with methods for realization and characterization of fibre Bragg gratings, both from a theoretical and an experimental point of view. The wide range of applications is exemplified by temperature induced tuning of a grating filter.

A new very flexible and reliable technique to manufacture arbitrary long fibre Bragg gratings is presented. The fibre is sequentially exposed by an interference pattern from a continuous-wave UV source, and thus problems related to pulsed exposures vanish. As the experiments show, our fabrication method allows for a very good control of the exposed grating fringes with a minimum of phase errors. All parameters — including very large chirps — are controlled by software, which gives a very high and unprecedented flexibility. New grating designs are easily implemented and tested, which makes the fabrication system a perfect tool for research and development. Today, this method also serves as a basis for market-leading commercial production of high quality fibre Bragg gratings.

We have also introduced a new scheme for characterization of weak to fairly strong gratings by means of optical low-coherence reflectometry (OLCR). With aid of a doubly balanced interferometer, the differential phase throughout the grating is retrieved as well as the distribution of the modulation depth. Using differential measurements, noise due to thermal fluctuations and material vibration in the detection system are greatly reduced in comparison to previously published OLCR-based methods. As shown in our work, the complex impulse response comprising all information about the grating is determined. Since this includes the average refractive index, it is possible to perform measurements that provide information about phase errors due to a fluctuating average refractive index.

For strong gratings, there is a problem with OLCR, since reflections from further
within the grating will be very weak and much information may be lost due to noise. It is then necessary to use other characterization techniques. We have investigated the theory behind the interferometric side diffraction method, considering limitations and, to some extent, sensitivity to noise. We also implemented the method in a novel way by using UV beams in our grating fabrication setup as light source for the characterization. This is a very simple way of implementation, since the only thing that has to be added to an existing fabrication setup is a UV sensor and some data acquisition hardware and software.

Only geometrical entities are measured with the side diffraction method and no information about refractive index fluctuations is obtained. On the other hand, a side diffraction characterization system as described above is suitable for all applications where the geometrical position of the grating fringes has to be retrieved. For instance, this kind of setup is very suitable when stitching multiple gratings together to one long compound structure.

We have for the first time identified and investigated by simulation the different kinds of errors that may occur due to stitching processes in general. It is evident that even small errors may have a large impact on grating quality. In order to improve the quality of stitched gratings, a combination of the introduced realization and characterization methods can be used to precisely position the grating parts relative to each other.

The demand for tunable filter components is very high and we have investigated and further developed a recently proposed technique where transmission pass bands are created within the stop band of a Bragg grating filter by local heating. In our implementation we used a thermal printer head for a precise and programmable control of the heating positions. This technique has a potential to enable fabrication of a new kind of easily and completely software-configurable filters. Some further work is needed on refinement of the heat profile control in order to realize a high-quality component.

The art of realizing high-quality fibre Bragg gratings for a large number of application areas is nowadays not reserved to a few highly specialized research laboratories; there are numerous commercial companies offering grating-based components as well as different kinds of customized fabrication services. The realization techniques have reached such a maturity that most of the development has migrated from the academic world to commercial development laboratories.

However, there is still a lot of work to do in order to harness all aspects of fibre Bragg gratings and new applications demanding further refinement and control of the gratings are found every year. There are still unsolved issues, especially regarding grating characterization, that need to be addressed in the academic research.
While the side diffraction characterization technique is widely deployed to determine spatial grating properties and improve grating fabrication systems, this technique cannot reveal errors inflicted by a fluctuating average refractive index. OLCR-based techniques have the potential to deliver this information, but there is still no general solution how to handle the spatial parameter reconstruction for stronger gratings.

Future work needs to be focused on these problems as well as on new ways to integrate and combine characterization and realization techniques in order to increase the ability to account for both material and system imperfections. Furthermore, continued work on different tuning techniques will open for an ever wider range of application areas with flexible and cost-effective components.

To conclude, it is safe to say that fibre Bragg gratings will remain a hot research and development topic for years to come and future components will continue to impress the scientific community with a steadily increasing performance and flexibility.
Summary of Publications

Paper I

A novel characterization scheme for weak to fairly strong fibre Bragg gratings based on optical low-coherence reflectometry is presented. Measuring the differential phase with aid of a new compact interferometer design allows for a reduction of errors due to thermal fluctuations in the detection system. Several test gratings with simple phase profiles are investigated with good results. A comparison between a reflection spectrum calculated from characterization measurements and a spectrum obtained with a scanning DBR laser shows a very good agreement. In the appendix, the theoretical background of the method is deduced in the framework of linear systems theory.

Contribution: I built the main part of the interferometer, made most of the experiments and wrote the paper. The mathematical analysis in the appendix was developed by Johannes Skaar.

Paper II

In this paper, we propose and demonstrate a new fabrication technique for arbitrarily-shaped fibre Bragg gratings. The method uses a continuous-wave UV source for imprinting the gratings, which results in a very good control over the dose distribution along the grating. Moreover, all grating parameters are controlled by software, which makes the technique highly flexible. The setup is described in detail and a number of realized gratings displaying very good properties are shown.

Contribution: I took part in the hardware development, developed and implemented the main software algorithms, planned some of the experiments and wrote the paper.
Paper III

The theory behind the interferometric side scattering technique for characterization of fibre Bragg gratings is investigated. Using two-dimensional theory for wave propagation, it is shown that the phase of the detected interference pattern directly corresponds to the local phase of the interrogated grating and that the detected modulation depth is proportional to its refractive index profile. The sensitivity to noise is briefly considered and the results indicate that the method is suitable for a wide range of gratings with modulation depths from at least $\Delta n \approx 10^{-5}$ to $\Delta n \approx 10^{-3}$.

*Contribution*: I did most of the work on the calculations and wrote the paper.

---

Paper IV

This paper consists of an experimental evaluation and verification of the interferometric side scattering technique for fibre Bragg grating characterization. In order to simplify the measurements, the ultraviolet beams already aligned and present in our grating fabrication facility are used as probing light source. A chirped test grating is investigated using the technique and in its present configuration, the system has a typical reproduction accuracy of $10^\circ - 20^\circ$ for the phase and $2 - 5\%$ for the index profiles. With aid of different simulations, the importance of using a digital resolution in the analysis corresponding to the physical resolution of the fabrication system is pointed out. It is further experimentally verified that the signal-to-noise ratio is independent of probe beam power and that the phase profile has a larger impact on the grating reflection spectrum than the apodization profile.

*Contribution*: I built the characterization setup, made the experiments and analysis and wrote the paper.

---

Paper V

Grating errors originating from stitches in long chirped fibre Bragg gratings are identified and investigated by simulation. The anticipated response in a typical 10 Gbit/s transmission system is specifically considered. The effect of the errors on the transmission spectrum is also simulated and it is shown that the characteristics of this spectrum may help determining the limiting parameter for gratings with dominating stitch-induced errors.

*Contribution*: I identified the error sources, made all simulations and most of the analysis and wrote the paper.
Paper VI

A novel tunable filter based on a chirped fibre Bragg grating heated by a thermal printer head is presented. This solution allows for a simple and effective configuration of the filter parameters such as central wavelength, width and strength by software only. The technique is investigated both by simulations and experiments. It is shown that the width of the transmission peak due to heating is mainly determined by the refractive index modulation depth and chirp rate of the grating. The experimental results show transmission band widths of less than 5 pm.

Contribution: I designed, implemented and analysed most of the simulations. I also enhanced the software interface for the thermal printer head, made the experiments and wrote the paper.


Paper I

Characterization of fiber Bragg gratings by use of optical coherence-domain reflectometry


Authors: I. Petermann, J. Skaar, B. Sahlgren, R. Stubbe and A.T. Friberg
Characterization of Fiber Bragg Gratings by Use of Optical Coherence-Domain Reflectometry

E. Ingemar Petermann, Student Member, IEEE, Johannes Skaar, Student Member, OSA, Bengt E. Sahlgren, Raoul A. H. Stubbe, Member, OSA, and Ari T. Friberg, Fellow, OSA

Abstract—A method based on optical low coherence reflectometry for complete characterization of fiber Bragg gratings (FBG’s) is presented. It is shown that the measured signal corresponds to the impulse response of the grating filter, and the measurement therefore yields all information about the device. Experiments have been carried out with a novel dual-channel interferometer. The results are in excellent agreement with the theory, demonstrating the versatility of the method for characterization of fiber gratings.

Index Terms—Gratings, interferometry, optical fibers, optical fiber measurement, optical interferometry, reflectometry.

I. INTRODUCTION

Since the demonstration of the transverse holographic method in 1989 [1], many improvements have been realized for the fabrication of fiber Bragg gratings (FBG’s). In particular, new methods to produce long gratings with complex structures have been developed [2], [3]. Long fiber gratings are very sensitive to perturbations and require a practical characterization method for their actual refractive index envelope and local periodicity. One method of characterizing FBG’s is to measure the reflection spectrum with a spectrum analyzer. However, this does not yield a complete characterization unless the phase response is obtained as well. Another characterization method is optical coherence-domain reflectometry (OCDR), which in recent years has been used in several ways to perform measurements on fiber gratings [4], [5].

Using a frequency-domain method it has recently been established that optical low coherence reflectometry (OLCR) is directly related to the inverse Fourier transform of the complex reflection spectrum of the device under test [6]. In this paper, we restate this fact in the frame work of linear systems theory and use the result to fully characterize fiber Bragg gratings. The advantage of our method is its simplicity and generality.

Once the complex reflection spectrum is calculated from the measured OLCR data using the Fast Fourier Transform (FFT), the index modulation profile (both amplitude and phase), which is proportional to the coupling coefficient in coupled mode theory, may be obtained with the help of an inverse scattering method. In particular, a numerical solution to the Gel’Fand–Levitan–Marchenko (GLM) coupled equations is well suited for this problem [7]. However, the inverse scattering problem is ill-posed, and may give inaccurate results in the case of strong gratings and noisy data. In the case of weak gratings, the coupling coefficient and the reflection spectrum are simply a Fourier transform pair [8], so the complex envelope of the OLCR data directly corresponds to the complex coupling coefficient.

In order to verify the theory, measurements have been carried out with aid of a novel dual-channel interferometer. The two channels are used to produce differential data, thus reducing the system noise. A number of different gratings with easily predictable profiles have been analyzed, and for one grating, the reflection spectrum has also been obtained with a tunable DBR laser yielding the same result as with our method.

A rigorous mathematical analysis of the OLCR characterization method is given in the Appendix. Section II contains a description of the experiments and the obtained results. The main conclusions are summarized in Section III.

II. EXPERIMENTS

A. Experimental Setup

A verification of the theory was carried out using a two-channel interferometer capable of interrogating a test grating at two positions simultaneously. By using the information from two positions, it is possible to make differential measurements with the benefit of suppressing noise due to, e.g., thermal fluctuations in the system itself. Of course this somewhat limits the spatial (phase) measurements that can be performed in the front and rear parts of the grating, but this information is in most cases not so important.

A schematic two-dimensional (2-D) drawing showing the working principle of the interferometer is depicted in Fig. 1 (for a detailed description, see [9]). The light from the source and grating under study is split into two channels by beamsplitter $a$. A translatable retroreflector $b$ is used to alter the length of both reference paths simultaneously. A second retroreflector $c$ changes the distance between the two interrogation points in the grating by increasing the reference path length in one channel and the length of the test path in the other. Each channel is split into two complementary outputs that are fed into differential detectors. The electrical signal from each detector is then proportional to the corresponding interference.
signal. As shown in the Appendix, this signal is directly related to the real part of the impulse response of the investigated grating.

In all experiments, both interrogation points were scanned simultaneously along the grating and from the resulting data, the differential phase between these points throughout the grating was calculated. For a fiber Bragg grating the impulse response can be written on the form

\[ h_c(\tau) = h_{en}(\tau) e^{j\omega_B \tau} \]  

(1)

where

\[ h_{en}(\tau) = |h_{en}(\tau)| e^{j\Phi(\tau)} \]  

(2)

is the complex slowly-varying envelope of the impulse response and \( \omega_B \) the average Bragg frequency. In this context, it is convenient to view the impulse response as a function of scanning position instead of time, i.e.

\[ h_c(z) = h_{en}(z) e^{j\omega_B n_{eff} z/c} \]  

(3)

where \( c \) is the speed of light in vacuum and \( n_{eff} \) the effective refractive index in the fiber.

In a general case, the absolute value and phase of the envelope in (2) is obtained by first calculating the imaginary part of the impulse response from its experimentally determined real part using the Hilbert transform. Since the impulse response of a grating is approximately sinusoidal with the Bragg frequency \( \omega_B \), the absolute value of \( h_{en}(z) \) may in the present case instead be approximated by the average of the local maxima of the real part in the vicinity of \( z \). Accordingly, if the distance between the two interrogation points is denoted by \( d \) and the measured phase difference by \( \Delta \Phi(z) \), the phase \( \Phi(z) \) of the impulse response envelope is taken as

\[ \Phi(z) = \int^z \frac{\Delta \Phi(z)}{d} dz \]  

(4)

or, since we are dealing with sampled data

\[ \Phi(z_n) = \sum_{i=1}^{n} \frac{\Delta \Phi(z_i)}{d} \Delta z \]  

(5)

where \( z_i \) is the \( i \)th sample, \( n \) is the number of samples and \( \Delta z \) is the spatial distance between two samples.

The white light source used for the measurements consisted of two erbium-doped fiber amplifiers and a filter grating connected as shown in Fig. 2. The ASE of the first amplifier is guided to the filter through a circulator, is reflected, and then amplified by the second amplifier. With a filter grating band-width of 1 nm, the light eventually reaching the interferometer has a (vacuum) coherence length of approximately 1 mm and a power of several mW.

If only one channel is used, this coherence length determines the spatial resolution that can be obtained. When obtaining differential measurements, however, this is not the case. The spatial resolution is then rather determined by the sample distance \( \Delta z \) as well as by the resolution of the detectors. Furthermore, spatial variations with a period corresponding to a multiple of the distance between the two interrogation points will be suppressed.

The time it takes to obtain one measurement is only limited by the speed of the scanning movement and the sampling frequency for a given sample distance \( \Delta z \). In our experiments, we used a scanning speed of approximately 0.25 mm/s and a sampling rate near the Nyquist limit. For a typical 10 cm long grating the whole process of obtaining raw data and calculating the phase difference and envelope then took less than 10 minutes using a 200 MHz Pentium computer. Of course this time can be substantially shortened by using higher translation speeds and more powerful computer and data acquisition hardware.

B. Results

Some preliminary experiments confirming the functionality of the setup are given in [10]. Furthermore, a weak uniform grating containing three phase shifts of magnitudes \( -90^\circ \), \( +90^\circ \) and \( -50^\circ \) was fabricated. As can be seen in Fig. 3, a differential measurement with the interferometer yields three peaks corresponding to the phase shifts. Without the phase shifts, the curve would be a straight line with an offset
Fig. 3. Measured phase difference for a uniform grating with phase shifts $-90^\circ$, $+90^\circ$ and $-50^\circ$.

Fig. 4. Phase of the index modulation of the grating from Fig. 3.

depending on the distance between the interrogation points. In the present case, the phase difference decreases by $90^\circ$ as soon as the first interrogation point passes the first phase shift. When the second interrogation point follows, the difference returns to the initial value. As long as the distance between these points is larger than the coherence length of the source, the FWHM of these peaks can thus be used to estimate the distance. In the present case it is approximately 1.5 mm (cf. Fig. 3). A slow modulation of the phase that lowers the constant level by about 20° toward the end of the grating can be seen. The most probable cause of this is that the fiber was not perfectly aligned during the writing process.

Since the grating is weak, the phase of the index modulation is directly obtained by integrating the phase difference. The phase difference offset gives rise to an unwanted linear term that can be eliminated by choosing a suitable zero level. This corresponds to moving the reference plane for the reflection coefficient, or equivalently adding a constant time delay. The phase difference zero level is in this case set to $60^\circ$, which gives a modulation phase according to Fig. 4, where the three phase shifts once again clearly can be seen.

Fig. 5 shows the impulse response envelopes, which in this case are the same as the index modulation envelopes measured by the two channels. Since a phase shift works as a broad band reflector within the grating, there is a peak (positive or negative) for each phase shift in the envelope as well. As seen from (A11) and (A12) these unwanted peaks are obviously due to the fact that the source spectrum $S(\omega)$ is not constant for all frequencies at which the reflection spectrum $\tau(\omega)$ is significantly different from zero, i.e., the source is not entirely "white". The peaks can therefore be reduced by using a light source with broader spectrum.

Fig. 6 shows the phase difference data for a 10 cm long chirped grating. As can be seen, there is a slowly varying modulation on the signal here as well. As expected, the average derivative is not zero as was the case for a uniform grating. In order to simulate a simple sensor, another measurement was done on the same grating when heating one end with a
soldering iron, thus changing the chirp profile according to the heat gradient. The difference of these two measurements is plotted in Fig. 7. Obviously, this data could be used to determine the heat gradient along the fiber. This particular experiment was merely done to show the principle, though, and no proportionality constant between chirp and heat was calculated. The induced chirp happened in this case to be of the same order of magnitude, but with a different sign, as compared to the original chirp. The slow modulation disappears due to the subtraction in this plot and the noise in the measurement is revealed. Evidently, the fluctuation is typically $\pm 25^\circ$.

Performing a Fourier transform on the measured impulse response of the unheated grating yields the reflectance spectrum depicted in Fig. 8. The group delay is defined as the derivative of the phase spectrum with respect to frequency and can be seen in Fig. 9. As expected, the linearly chirped grating yields a group delay curve that is almost linear with a slope that corresponds to the chirp. As a last experiment, a weak uniform 10 cm grating with Bragg wavelength 1551.1 nm was fabricated. The reflectance spectrum as calculated from the white light interferometer data is given by the solid line in Fig. 10. The spectrum of the same grating was also measured with a tunable DBR-laser with a result corresponding to the dashed line in Fig. 10. As can be seen, the agreement between these two measurements is excellent. In Fig. 11, the group delay calculated from the interferometer data is shown. As opposed to the group delay spectrum for a chirped grating, this is more or less constant. The peaks are caused by the phase shifts that occur when the reflectance spectrum equals zero.

III. CONCLUSION

It has been shown, theoretically as well as experimentally, that low coherence interferometry is a versatile tool for characterizing fiber Bragg gratings. The output of the interferometer corresponds to the impulse response of the grating. From
this, we obtain the reflection spectrum and the group delay spectrum. In the case of a weak grating, the measured impulse response directly corresponds to the index modulation amplitude and phase (or the complex coupling coefficient). For stronger gratings it is necessary to use an inverse scattering method to obtain the same spatial information.

In the experimental verifications, a dual-channel interferometer has been used, giving the benefit of reducing noise originating from e.g. thermal fluctuations in the interferometer itself. The measurements show a noise corresponding to approximately ±25° in spatial phase. This value is by no means a limit; most of the noise probably comes from fluctuations in the velocity of the scanning interferometer arm and further improvements to this movement would probably also take away a substantial part of the noise.

**APPENDIX**

The basis of the theory is an ordinary Michelson interferometer used with a white light source as shown in Fig. 12. In the following, we apply a linear systems approach to derive the important result that the detector signal as a function of reference mirror position corresponds to the impulse response of the device under investigation.

At a given point the light from the source is represented by a random complex field \( E(t) \). It is characterized by the autocorrelation function

\[
\Gamma_{EE}(\sigma) = \langle E^\ast(t)E(t+\sigma) \rangle \quad (A1)
\]

where \( \langle \cdots \rangle \) denotes the time average. Light coming from the interferometer arm containing the fiber grating will interfere with light from the (broadband) reference mirror at the beamsplitter. Each arm is characterized by a respective impulse response, which for the reference arm is a simple time delay \( \tau \) corresponding to the time it takes for the light to travel twice the length of the arm. According to the theory of linear systems [11], the electric fields at the beamsplitter are thus given by

\[
E_2(t) = E(t - \tau) \quad (A2)
\]
Fig. 9. Group delay for the chirped grating in Fig. 6.

Fig. 10. Reflectance spectrum of a uniform grating, calculated from the OCDR data (solid curve), and measured by the DBR laser (dashed curve).

\[ E_\Delta(t) = E(t) \ast h_c(t) = \int_{-\infty}^{\infty} E(t') h_c(t - t') \, dt' \quad (A3) \]

where \( \ast \) denotes the convolution integral and \( h_c(t) \) is the complex impulse response of the interferometer arm containing the device under investigation. Note that \( h_c(t - t') = 0 \) for \( t < t' \) due to causality. The total field at the beam splitter becomes

\[ E_{12}(t) = E_1(t) + E_2(t). \quad (A4) \]

We are interested in the detector signal, which is proportional to the average power of \( E_{12}(t) \)

\[ I_{det}(\tau) = \langle |E_{12}(t)|^2 \rangle = \langle |E_1(t)|^2 \rangle + \langle |E_2(t)|^2 \rangle + 2 \Re \langle E_1^*(t) E_2(t) \rangle \quad (A5) \]

\( \Re \) denoting the real part. The first two terms in (A5) are trivial, since they are constant powers independent of the delay \( \tau \). We recognize the last interference term as a cross-correlation of \( E_1(t) \) and \( E_2(t) \), which will be dependent of \( \tau \), i.e.

\[ I(\tau) = \Re \langle E_1^*(t) E_2(t) \rangle = \Re \Gamma_{E_1 E_2}(0). \quad (A6) \]

This interference detector signal is determined by the properties of the source field \( E(t) \), impulse response \( h_c(t) \) and delay \( \tau \).

On substituting (A2) and (A3) into the expression for \( \Gamma_{E_1 E_2}(0) \) obtained in analogy with (A1), we find

\[ \Gamma_{E_1 E_2}(0) = \left\langle E^*(t - \tau) \cdot \int_{-\infty}^{\infty} E(t') h_c(t - t') \, dt' \right\rangle. \quad (A7) \]

Since the source has a finite power, the mean-square value \( \langle |E(t)|^2 \rangle \) is finite for all \( t \) and we may interchange the order
The same formula is also deduced in the frequency domain in [6]. Note that the result is general and valid for any optical filter and source spectrum.

Now, in OLCR the spectral bandwidth of the source is often much larger than the bandwidth of the device under investigation. Hence \( S(\omega) \) may be treated effectively as a constant near the Bragg frequency \( \omega = \omega_b \) characterizing the grating, yielding

\[
I(\tau) = 2 \Re \{ S(\omega) \cdot F^{-1} \{ r(\omega) \} \}, \tag{A12}
\]

Since \( S(\omega) \) is real, we obtain

\[
I(\tau) = \const \cdot \Im h_{\omega}(\tau), \tag{A13}
\]

Hence, ignoring a background power, the data from the white light interferometer corresponds to the real part of the impulse response of the device under investigation, provided the spectrum of the source is constant over the device bandwidth. The imaginary part of \( h_{\omega}(\tau) \) is generally obtained from the real part in terms of the Hilbert transform [12].

REFERENCES


E. Ingemar Petermann (S’98) was born in Österhaninge, Sweden, in 1972. He received the M.Sc. degree in engineering physics from the Royal Institute of Technology, Stockholm, Sweden, in 1997 and is currently pursuing the Ph.D. degree at ACREO AB, Stockholm, in cooperation with the Department of Physics—Optics at the same university. His main research interests are characterization of fiber Bragg gratings and modeling and simulation of wave propagation in periodical structures.

Johannes Skaar, photograph and biography not available at the time of publication.

Bengt E. Sahlgren was born in Stockholm, Sweden, in 1960. He received the M.Sc. degree in mechanical engineering from the Royal Institute of Technology, Stockholm, in 1987. He is currently employed as a Research Scientist at ACREO AB, Stockholm. He has ten years of scientific experience in the field of fiber optics. His main research interests are manufacturing techniques of fiber Bragg gratings and fiber optical sensor systems. Mr. Sahlgren is a member of the European Optical Society and the Swedish Optical Society.

Roua A. H. Stubbe was born in Norrköping, Sweden, in 1961. He received the M.Sc. and Ph.D. degrees in engineering physics from the Royal Institute of Technology, Stockholm, Sweden, in 1985 and 1991, respectively. He is currently employed as a Research Scientist at ACREO AB, Stockholm. He has more than ten years of scientific experience in fiber optics and holds several patents in the field. His main interest is applications of fiber Bragg gratings for optical sensor systems and telecommunication. Dr. Stubbe is a member of the European Optical Society, the Swedish Optical Society, and the Optical Society of America (OSA).

Ari T. Friberg received the Ph.D degree in optics from the University of Rochester, Rochester, NY, in 1980 and the Ph.D. degree in technical physics from the Helsinki University of Technology, Helsinki, Finland, in 1993. He has worked at universities in Helsinki, Rochester, London, U.K., and Berlin, Germany. In the 1990’s, he was with the Academy of Finland before being appointed Professor of Physics at the University of Joensuu, Finland. Since 1997, he has been Professor of Physics at the Royal Institute of Technology, Stockholm, Sweden. His main research interests deal with physical optics, coherence, and laser technology. Dr. Friberg is a Fellow of the Optical Society of America (OSA), as well as a Topical Editor of the Journal of the Optical Society of America and Associate Secretary of ICO.
Fabrication of advanced fiber Bragg gratings using sequential writing with a continuous-wave ultraviolet laser source


Authors: I. Petermann, B. Sahlgren, S. Helmfrid, A.T. Friberg and P.-Y. Fonjallaz
Fabrication of advanced fiber Bragg gratings by use of sequential writing with a continuous-wave ultraviolet laser source

Ingemar Petermann, Bengt Sahlgren, Sten Helmfrid, Ari T. Friberg, and Pierre-Yves Fonjallaz

We present a novel scheme based on sequential writing for fabrication of advanced fiber Bragg gratings. As opposed to earlier sequential methods this technique uses a cw UV laser source and allows for very precise control and repetitivity of the formation of the gratings. Furthermore it is possible to use high average irradiances without destroying the fiber, resulting in considerable reduction in fabrication time for complex gratings. The method has been applied to several test gratings, which proved its versatility and quality. © 2002 Optical Society of America

OCIS codes: 060.2310, 060.2340, 050.2770, 230.1480.

1. Introduction

There is a rapidly growing demand for high-quality optical Bragg gratings with arbitrary phase and index profiles, because these gratings are key elements in many components used in wavelength-division-multiplexing networks.1,2 In the past few years, several methods that improve quality and flexibility in the grating-fabrication process have been developed. A straightforward approach is to scan a UV beam over a long phase mask that has a fixed relative position to the fiber.3 Nonuniform profiles can in this case be fabricated either by postprocessing the illuminated region4 or by using a phase mask that contains the appropriate structure.5 Complex grating structures can also be synthesized by moving the fiber slightly relative to the phase mask during the scan.6

In 1995 a novel versatile sequential technique for writing long and complex fiber gratings was demonstrated.7,8 The idea was to expose a large number of small partially overlapping subgratings—each containing a few hundred periods or less—in sequence, where advanced properties such as chirp, phase shifts, and apodization were introduced by adjusting the phase offset and pitch of the subgratings. The flexibility of the method relies on the fact that all grating parameters are accessible in the control software, and no change of hardware, such as different kinds of phase masks, is needed to fabricate gratings of arbitrary shapes.

In the setup used in Refs. 7 and 8 each subgrating was created by exposing the fiber with a short UV pulse while the fiber itself was translated at a constant speed. The UV pulses were triggered by the position of the fiber relative to the UV beams, which was measured with a standard He–Ne-laser interferometer. There are several drawbacks in using pulsed operation of the UV source. The pulse energy exhibits fairly large fluctuations that introduce amplitude noise in the grating structure. Moreover it is necessary to use a low average pulse power since optical damage otherwise may be induced in the fiber. For strong gratings the fiber therefore has to be exposed several times with low energy instead of a single time with high energy. The velocity of the translation must additionally be kept low compared with the pulse length to maintain good visibility. Hence the entire writing process tends to become rather time-consuming. A slow velocity also results in an increase in noise due to temperature variations in the fiber. These problems can partly be alleviated by using a modulated cw laser to produce the pulses. Control of the pulse shape and pulse intensity be-
comes much better, but still most of the UV radiation is not effectively used in the grating-formation process.

In this paper we present and demonstrate a method that overcomes all these problems by using a setup with a cw UV source in which the pulses are replaced by a sawtooth movement of the interference pattern. The basic concept of this method was also proposed in Ref. 9.

In Section 2 we introduce the general setup and its basic operation, and in Section 3 we describe the generation of complex grating structures. In Section 4 we then display a few experimental verifications of the operation. Finally, in Section 5 we summarize the conclusions of the research.

2. Experimental Setup

The experimental setup is illustrated in Fig. 1. The fiber subject to exposure is placed in a holder mounted on an airbearing borne carriage that is translated by a feedback-controlled linear drive. The position of the translator stage relative to the UV interference pattern is measured with a heterodyne interference detection system utilizing a He–Ne laser as the light source. The resulting spatial resolution is \(~0.6\) nm, available over a translation length of approximately half a meter.

A frequency-doubled argon-ion laser launches 100-mW radiation of 244-nm wavelength into a double Sagnac interferometer, which generates the interference pattern forming the grating. Cylindrical lenses focus the two interfering beams into a line focus that coincides with the core of the fiber. Longitudinally, the focus extends over \(~100\) \(\mu\)m, which roughly corresponds to 200 fringes for a Bragg wavelength of 1550-nm resonance wavelength.

Since the fiber is moving at a constant speed, the pattern has to follow the fiber movement during exposure. To accomplish this two mirrors are mounted on piezocrystals in the UV interferometer, one in each beam path. A translation of the mirrors introduces a phase shift between the interfering beams, which shifts the fringe pattern under the envelope defined by the focus. The pattern thus moves with the fiber, continuously exposing the fiber. Since the piezoelements cannot move illimitably for long, they are fed with a sawtooth signal so that the fringes perfectly follow the fiber over some distance, typically one grating period. At the end of the voltage ramp in each period the fringes jump back to the original position. During this short moment the fiber is evenly exposed, giving a slight increase in index change. Each new period of the sawtooth signal results in a new subexposure of the fiber, corresponding to the new subgrating in earlier sequential methods. The properties of the piezocrystals limit the maximum writing speed, because it is essential that the extent of the ringing after the jump is much less than the period time of the signal. Otherwise the modulation depth of the generated grating may degrade. The maximum writing speed that could be obtained in our setup was of the order of centimeters per second. In practice the handling of the fiber before and after the exposure represents the actual time-limiting factor in the grating-fabrication process.

The entire grating structure is determined by the positions of the jumps, i.e., sign reversions of the sawtooth signal as described in detail below. These positions are calculated in advance by the PC and fed into the electronical control hardware that eventually performs the sign reversions when the desired positions are reached.

A step motor controlling the angle between the interfering beams is used to change the period of the interference fringes to match the desired local pitch of the grating. The angle change is performed symmetrically for both beams so that the center fringe does not move its position. In our setup the resolution for this pitch variation is \(~1.4\) pm in resonant wavelength. To prevent unwanted exposure outside the actual grating, the system also controls a shutter in the UV beam path that is open only within the grating region during the writing.

3. Realization of Complex Profiles

For simple unchirped and unapodized gratings all exposed fringes are in phase and the sawtooth jumps will appear at the same integral number of grating pitches throughout the writing process. Adding a (positive) phase shift, e.g., for distributed-feedback structures, merely corresponds to delaying the jump somewhat (Fig. 2). The interference pattern will then follow the fiber a little bit longer than otherwise at the position of the phase shift and make a correspondingly larger jump back. If the jump is delayed by \(\Delta \phi/2\), the resulting phase shift in the grating will be \(\Delta \phi\).

An important issue is the ability to apodize gratings to suppress unwanted sidelobes in the reflection spectrum. In our setup this is realized by so-called dithering. The visibility of the grating is changed by alternating the phase offset of the subexposures between two different values, thus in fact superposing two uniform gratings with the same properties but phase-shifted relative to each other. Choosing
phase offsets \( \pm \Delta \phi \) will give a total index variation of the form

\[
\Delta n(x) \propto \sin(kx - \Delta \phi) + \sin(kx + \Delta \phi) = \cos(\Delta \phi)\sin(kx),
\]

where \( x \) is the position along the fiber, \( k = 2\pi/\Lambda \), and \( \Lambda \) is the grating pitch. As can be seen the phase term directly determines the strength of the index variation.

This is the same principle as used in Ref. 8, but in the present setup, where moving fringes are used, the UV dose given in each of the two phase-shifted gratings is easily and precisely controlled. If the dose varies from subgrating to subgrating, which is the case when a pulsed UV light source is used, noise will be introduced in both the phase and the amplitude of the grating structure.

Figure 3(a) shows the sawtooth waveform for a phase offset of \( \pm \pi/3 \). As can be seen the waveform alternates between two phase-shifted versions of a uniform sawtooth wave. These waves result in two superposed gratings that are added according to the above equation, giving a fringe visibility of 50%.

The sign reversions at positions marked A in Fig. 3(a) sweep the interference pattern back to expose the next part of the grating, whereas the reversions at positions B cause the pattern to jump to the other phase offset. It is important that each of the two phase-shifted gratings be given the same exposure time; otherwise the apodization will have an additional variation following the local dose distribution between these gratings along the fiber. This is ensured by always letting the sawtooth signal follow each of the two phase-shifted waveforms the same number of periods before switching to the other. The gray lines indicate a waveform that would result in a maximum visibility (nonapodized) grating with the same phase as the one described above.

Since the piezocrystals have a finite response time, it is important to choose the switch positions between the two waveforms carefully. If, e.g., the positions marked C in Fig. 3(a) are used, the resulting sawtooth signal takes the form shown in Fig. 3(b). As before the reversions at the A positions correspond to sweeping back for the next subexposure, and at the B positions the pattern jumps to the other phase offset. In this case, however, the sign reversions turn out to be more closely spaced the smaller the phase dither \( \Delta \phi \) is. Hence the piezocrystals would not be able to respond to the signal as the visibility approaches 100%. Choosing the switch positions according to Fig. 3(a) instead results in reversions at an approximately constant rate regardless of current visibility.

By continually changing \( \Delta \phi \) for every new pair of subexposures, any apodization may be created and the further addition of single phase shifts as described at the beginning of this section gives any phase profiles.

Chirping a grating is equivalent to continually changing the distance between the fringes. A constant chirp, i.e., a linear increase/decrease in grating period and fringe distance, is the same as a quadratic increase/decrease in phase compared with a nonchirped grating. If the interference pattern were focused to contain only one single fringe, it would be possible to create any chirp throughout the grating. On the other hand, the fabrication time would increase since every overlapping fringe in the apodization scheme described above must be exposed individually. Furthermore the demand for precision of the fiber alignment increases the tighter the UV beam is focused. If the beam spot contains several fringes and the period of the interference fringes is fixed throughout the writing process, it is still possible to apply smaller chirps by merely adjusting the phase offset as long as the phase shift remains small over a length corresponding to one subexposure. Larger chirps will give an apodization effect due to overlapping of out-of-phase fringes since the actual interference fringe period differs too much from the desired grating pitch.

For larger chirps to be applied, the interference fringe period must be continually changed during the exposure to always match the desired local grating pitch. In our setup this is done by changing the
angle between the two interfering beams with the aid of the step motor described above. With this technique it is possible to create arbitrary chirps throughout the grating. It is imperative that the angle adjustment be symmetric with respect to the fiber normal in the plane of incidence; otherwise the overall position of the interference pattern also changes, thus introducing phase errors in the grating structure.

4. Experimental Results

To test the equipment, several gratings were written. The interference pattern has in each case been focused to a longitudinal size of $\sim 100 \mu m$.

It is often necessary to expose a grating several times at moderate power instead of one single time at high power, because the heat introduced by the UV beam otherwise becomes so large that the gratings degrade. It has also turned out to be hard to predict the UV power needed for a certain grating strength; it is much easier to fabricate gratings with a predefined reflectivity if the grating is exposed several times at small power until the required strength is obtained. It has proved important to use the same UV power and writing speed at each exposure and to keep the time between multiple writing passes approximately constant. If these requirements are not met, different temperatures due to different amounts of remaining heat in the fiber result in misalignment of the fringes from pass to pass, thus causing degradation of the grating rather than reinforcement. The effect is especially prominent when writing chirped gratings.

Figure 4 shows the reflection spectrum of a 1-cm uniform unapodized grating that was exposed once at 13 mW with a speed of 5 mm/s. Using a highly sensitive hydrogen-loaded fiber resulted in a grating with $\sim 20\%$ reflectivity and a modulation depth of roughly $2 \times 10^{-5}$. As can be seen the sinc-shaped spectrum is in perfect agreement with the simulated ideal response of such a grating. The resemblance with the simulation is still very good far away from the main peak, as shown in Fig. 5. The nearly perfect symmetry indicates that the number of phase errors is very low.

Adding an apodization to the grating may remove the sidelobes. This can be seen in Fig. 6, where the reflection spectrum of a 1-cm uniform grating with Hamming apodization is shown. The sidelobes are suppressed below the noise level at approximately $-35 \, dB$. This grating was exposed 5 times at 13 mW with a speed of 2 mm/s, and the smaller reflectivity compared with the unapodized grating is due to the less sensitive fiber, UVS-INT from CorActive, that was used.

Next a sinc-apodized 2-cm-long grating including 7 sidelobes on each side of the main peak was written in the same type of fiber as above. To suppress spectral sidebands, the grating was additionally apodized with a raised cosine profile. The resulting reflection spectrum is shown in Fig. 7. The grating was exposed 5 times at 100 mW, which gave a reflectivity of $\sim 20\%$. The inband noise is 10% and the sideband rejection at least 20 dB (Fig. 8).

A 2-cm-long, 10-nm chirped super-Gaussian apodized grating was written in a Redfern GF-5 fiber. The grating was exposed 10 times at 2 mm/s with a 100-mW beam power. Use was made of the step motor to optimize the local period continuously. As can be seen from Figs. 9 and 10, the filter response is very good with a sidelobe rejection of $\sim 22 \, dB$ and a reflectivity close to 100%. Note also the spectral
Finally, a 4-cm-long comb filter with 80 channels and a 50-GHz spacing was written in Redfern GF-5 fiber by superposing 80 uniform Hamming apodized gratings, each of which was exposed once at 6 mW with a speed of 2 mm/s, resulting in a reflectivity of \( \approx 10\% \). As can be seen from Fig. 11 the resemblance of the channels is very high with a typical sideband rejection of the order of 30 dB (Fig. 12).

5. Conclusions

We have developed and demonstrated a novel technique for fabrication of high-quality fiber Bragg gratings. By continuously moving the interference fringes of cw UV light with the moving fiber during the writing process, a maximum amount of light power is used for the actual exposure. As a result the fabrication time for all kinds of customized gratings is greatly reduced compared with earlier methods. Our method allows for precise control of grating formation, which is demonstrated by a number of realized gratings showing a very low number of phase errors. When this technique is used, the quality of the gratings is limited...
by fiber quality rather than by fabrication precision. All steps of the grating exposure are software-controlled so that any kind of phase and apodization profile can be realized by means of simple programmatic changes. The flexibility and speed of this technique make it a powerful tool for fabrication of arbitrarily shaped fiber Bragg gratings.

The authors thank Johannes Skaar, Norwegian University of Science and Technology, Trondheim, for fruitful discussions concerning the fiber grating design; Gilles Concas, Proximion AB, for writing the comb filter; and Gerry Meltz, OFT Associates, for providing the fiber used for the uniform and sinc-shaped gratings.

References
Paper III

Limitations of the interferometric side diffraction technique for fibre Bragg grating characterization

Optics Communications 201(4–6), pp. 301–308 (2002)

Authors: I. Petermann, S. Helmfrid and A.T. Friberg
Limitations of the interferometric side diffraction technique for fibre Bragg grating characterization

Ingemar Petermann\textsuperscript{a,}\textsuperscript{*}, Sten Helmfrid\textsuperscript{b}, Ari T. Friberg\textsuperscript{c}

\textsuperscript{a} Acreo AB, Electrum 236, SE 164 40 Kista, Sweden
\textsuperscript{b} Proximion AB, SE 164 40 Kista, Sweden
\textsuperscript{c} Royal Institute of Technology, Department of Microelectronics and Information Technology, Electrum 229, SE 164 40 Kista, Sweden

Received 19 September 2001; accepted 14 October 2001

Abstract

We make a quantitative investigation of the interferometric side diffraction technique for characterization of fibre Bragg gratings. As opposed to more commonly used non-interferometric alternatives, this method also allows for determination of the grating’s distributed phase. It is shown that for weak gratings the detected power and phase are directly proportional to the amplitude and phase of the refractive index modulation in the fibre. While the phase measurement is only limited by the amount of noise present, the relation between modulation amplitude and detected power becomes non-linear when using small probe beam wavelengths and gratings with a modulation depth $\Delta n \approx 10^{-3}$ and above. This behavior is inherent to the side diffraction and thus care has to be taken when performing apodization measurements on very strong gratings. The sensitivity to noise is briefly discussed and using a crude estimate for the noise level, the theory indicates that this method is suitable down to a grating modulation depth of at least $\Delta n \approx 10^{-5}$. © 2002 Elsevier Science B.V. All rights reserved.

1. Introduction

Realization of optimized fibre Bragg gratings is a complex process and it is therefore essential to have effective characterization methods at hand in order to determine and improve the grating quality. Following the rapidly growing demand for high-quality gratings, several characterization techniques have been developed during the past years [1–4]. In 1995, Krug et al. [3] presented a characterization method where the fibre grating is illuminated from the side by a probe laser beam. The first-order diffracted light can then be used to determine the refractive index variation of the grating. A later work [4] proposed an extension of this technique to include measurement of the grating phase by detecting the interference pattern between the zeroth and first diffraction orders.

In general, the noise introduced in side diffraction measurements originates from source fluctuations, detector noise and scattering due to surface roughness, impurities, and density variations within the fibre material. The roughness of the...
fibre surface is mainly caused by dust and residual parts of the coating and even though thorough cleaning will reduce this roughness, it is impossible to obtain a perfect smooth surface in an ordinary measurement environment. Microcracks introduced in and near the fibre surface during the fabrication process will also contribute to the optical roughness. The detector and source noise terms may be greatly reduced using lock-in detection techniques and the volume scattering effects can be expected to be orders of magnitudes lower than the contribution from the fibre surface roughness. In a recent work [5], the scattering noise has been reduced by a factor of approximately three by using a new scheme for lock-in modulation. Still, the surface scattering remains the largest source of noise and will be considered to be the only one in the following calculations.

In the present article, we focus on the interferometric side diffraction method described qualitatively in [4]. We make a quantitative investigation of this technique and determine its requirements for the properties of the grating under test as well as its sensitivity to noise. It is shown that this method is excellent for determining the phase distribution along the grating. The apodization is also easily measured, even though very strong gratings with modulation depths $\Delta n \approx 10^{-3}$ introduces a non-linearity that distorts the result. This effect turns out to be larger for smaller probe beam wavelengths.

In Section 2, the geometry of a possible experimental setup is described, in Section 3 a mathematical derivation of the diffracted fields is presented, and in Section 4 the details of the detection process are given, followed by concluding remarks in Section 5.

2. System geometry

In a possible experimental setup (cf. Fig. 1), the fibre with the Bragg grating under investigation is mounted on a motorized translator with precise control of the position. A (coherent) light beam with vacuum wavelength $\lambda$ is focused on the fibre. The transmitted wave (zero-order deflection) is then collimated and brought to interfere with the collimated first-order deflected beam. Subtracting the signal from two detectors as depicted in the figure allows for suppression of common signals in both interferometer arms.

It can be shown [3] that the diffraction is polarization dependent. In the following calculations s-polarization is assumed and further polarization effects are not taken into account.

The coordinate system for the fibre is chosen according to Fig. 2, i.e. $x$ along the fibre and $z$ perpendicular to the fibre in the plane of deflection. In the weak scattering limit, the light at each position $x$ directly after the fibre will have a phase proportional to the fibre’s local optical thickness. It should be mentioned that there is an additional
small shift along the (negative) $x$-axis due to refraction in the fibre, but since this does not change the shape of the wavefront, it can safely be neglected. A fibre grating will cause the optical path length to change periodically along the fibre core and thus form a diffraction phase grating, characterized by a transmission function $\tau(x,y)$. As a consequence of the cylindrical fibre shape, the optical thickness will vary depending on where the $xz$-plane intersects the fibre. This variation has only an impact on the strength of the deflected beams though, not on the deflection angle. Since furthermore the deflected beams of interest are all in one plane, it is possible to reduce the problem to two dimensions by using an average fibre (core) thickness $\rho_{av}$ in the calculations.

Assuming a non-slanted refractive index variation $\Delta n$ and a (local) geometrical grating period $\Lambda$, the transmission function takes the form

$$\tau(x) = \exp(i[kn\rho_{av} + \Delta \phi \sin(Kx + \phi)])$$  \hspace{1cm} (1)

where $k = 2\pi/\lambda$ is the wavenumber in free space, $n$ is the average refractive index of the fibre, $K = 2\pi/\Lambda$ is the grating “wavenumber” with (geometrical) period $\Lambda$ and $\phi$ is a phase factor reflecting the current position of the fibre. The phase modulation depth due to the grating is given by

$$\Delta \phi = k\rho_{av}\Delta n$$  \hspace{1cm} (2)

The parameters $\Lambda$ and $\Delta n$ are in general both dependent of fibre position.

Let us now describe the incident beam as propagating in $\zeta$-direction in a coordinate system $\zeta\xi$ rotated by the angle $\alpha$ and with the origin located directly in front of the fibre core as depicted in Fig. 2. In the vicinity of its waist, the time independent part of the beam (assuming time dependence $e^{-i\omega t}$) can be described by the expression

$$E_S(\xi, \zeta) = E_0 f(\xi)e^{i\zeta\zeta},$$  \hspace{1cm} (3)

where $E_0$ is the field amplitude and $f(\xi)$ the normalized transverse beam shape. The refractive index of the medium surrounding the fibre is assumed to be unity.

When focusing the beam such that its waist is located in the origin, the field directly in front of the fibre core, using the coordinate system of the fibre, takes the form

$$E_{in}(x) = E_S(x \cos \alpha, 0)e^{ikx \sin \alpha}$$

$$= E_0 f(x \cos \alpha)e^{ikx \sin \alpha}.$$  \hspace{1cm} (4)

Without loss of generality, the refracting effects of the fibre are omitted in the calculations.

3. Diffracted fields

The field $E$ in any point $P(x', z)$ in the Fraunhofer region beyond the fibre can be calculated using two-dimensional theory for wave propagation. With a near field $A(x)$ after the diffracting object, this theory gives [6]

$$E(\theta) = C \cos \theta \int_{-\infty}^{\infty} A(x) e^{-ikx \sin \theta} \, dx,$$  \hspace{1cm} (5)

where $\theta$ is the angle from the fibre normal to point $P$ as depicted in Fig. 2 and the constant $C$ is given by

$$C = \frac{e^{i(kR_0 - \pi/4)}}{\sqrt{2\rho_{av}}}.$$  \hspace{1cm} (6)

with $R_0$ being the distance between the origin and point $P$.

In the present case, the field $A(x)$ is given by the product of the incident field $E_{in}(x)$ and the grating transfer function $\tau(x)$. With the Fourier transform defined according to

$$\hat{A}(u) = \int_{-\infty}^{\infty} A(x) e^{-iux} \, dx,$$  \hspace{1cm} (7)

Eq. (5) then takes the form

$$E(\theta) = C \cos \theta \hat{E}_{in}(k \sin \theta) \ast \hat{\tau}(k \sin \theta),$$  \hspace{1cm} (8)

where the asterisk denotes convolution. The transmission function can be expanded in a series of Bessel functions according to

$$\tau(x) = e^{i\Delta \rho_{av}} [J_0(i\Delta \phi) + i2J_1(i\Delta \phi) \sin(Kx + \phi)$$

$$+ 2J_2(i\Delta \phi) \cos(2(Kx + \phi))$$

$$+ i2J_3(i\Delta \phi) \sin(3(Kx + \phi)) \ldots].$$  \hspace{1cm} (9)

Substituting Eqs. (4) and (9) into (8) and using the identity

$$J_\mu(i\gamma) = i^\mu I_\mu(\gamma),$$  \hspace{1cm} (10)
where \( I_\mu \) is the modified Bessel function of first kind and order \( \mu \) (see e.g. [7]), yields

\[
E(\theta) e^{-ikz\rho_m} = C_1 E_0 \cos \theta \left[ I_0(\Delta \varphi) \tilde{f}(k|\sin \theta + \sin x|/ \cos x) + 2\pi \sum_{\mu=1}^{\infty} i^{\mu} I_\mu(\Delta \varphi) \left\{ \tilde{f}(k|\sin \theta + \sin x|) - \mu K/ \cos x \right\} e^{i\mu \phi} + (-1)^{\mu} \tilde{f}(k|\sin \theta + \sin x|) + \mu K/ \cos x e^{-i\mu \phi} \right] ,
\]

(11)

where \( C_1 = C/ \cos x \). Suppose that the Fourier transform of the beam shape function, \( \tilde{f} \), obtains its maximum value for the argument \( a \). For the first term on the right-hand side this happens when \( \sin \theta = a \cos x / k - \sin x \), whereas the other terms have their maxima displaced by an integral number of \( K/ \cos x \). As long as this displacement is considerably larger than (half) the width of \( f \) and furthermore \( \tilde{f} \) decreases rapidly enough at its edges, only one term will contribute substantially to the field in any given direction \( \theta \). Each single term in the equation then corresponds to one single diffraction order. The angle of diffraction for an incident angle \( x \) is then given by the angle \( \theta_\mu \) that balances the corresponding \( \mu K \) to give the argument \( a \), i.e.

\[
\sin \theta_\mu = \frac{\mu K + a \cos x}{k} - \sin x. \tag{12}
\]

Since the grating is a weak scatterer, the zeroth order (transmitted) beam will be much stronger than the higher orders, which is also evident in the equation by the fact [7] that

\[
I_0(x) \approx 1, \quad x \ll 1 \quad \text{and} \quad I_\mu(x) \approx 0, \quad \mu > 0 \wedge x \ll 1. \tag{13}
\]

If we assume a Gaussian beam cross-section with FWHM \( l \), the condition for this “separability” of the summation takes the form

\[
A \ll \frac{\pi l}{\sqrt{2} \cos x} , \tag{14}
\]

i.e. the grating pitch has to be considerably smaller than the illuminated length \( l/ \cos x \) of the fibre. This can easily be understood, since in the case of an illuminated region of the same order or smaller than the pitch, the grating would be invisible to the probe and thus the field after the fibre would just consist of the diffraction pattern from a homogenous aperture.

A fraction of the incident light will be subject to scattering due to the surface roughness of the fibre. Without going into details about the origin of the noise we simply denote the fraction of incident irradiance that is scattered by the surface in the direction of the diffraction order \( \mu \) by \( \sigma_\mu \). The corresponding fields at the detector can then be expressed by

\[
E_{N,\mu} = \sqrt{\frac{\sigma_\mu}{J_0}} E_0 e^{i(kR_0 + \phi_{N,\mu})} , \tag{15}
\]

where \( \phi_{N,\mu} \) is the phase of the noise and the additional phase term \( kR_0 \) and amplitude factor \( 1/\sqrt{J_0} \) have been added for convenience. Both amplitude and phase of the noise will vary along the fibre, but we can assume that these variations are slow as compared to the light oscillation. It should be noted that the level of noise in general is wavelength dependent so that \( \sigma_\mu = \sigma_\mu(\lambda) \).

4. Interferometric detection

From Eq. (11) we find that the fields in the zeroth and first orders are given by

\[
E_{G,0} = C_1 E_0 \cos \theta_0 I_0(\Delta \varphi) \tilde{f}(k|\sin \theta_0 + \sin x|/ \cos x) ,
\]

\[
E_{G,1} = 2\pi i C_1 E_0 \cos \theta_1 I_1(\Delta \varphi) \tilde{f}(k|\sin \theta_1 + \sin x| - K)/ \cos x e^{i\phi} , \tag{16}
\]

where \( \theta_0 \) and \( \theta_1 \) are the deflection angles for the respective order. After diffraction, these orders are brought together to interfere. Omitting a possible phase constant due to different path lengths, the irradiance \( I_{\text{tot}} \) at the detector, including the noise, then takes the form
where \( \langle \ldots \rangle \) denotes time averaging in the detector and \( \Re \) the real part, \( I_m = ccE_0^2/2 \) is the irradiance of the field before the fibre, \( \Delta \theta \) is a small deviation angle from the diffraction directions and

\[
\tilde{f}_\mu(\Delta \theta) = \tilde{f}(a + k \Delta \theta \cos \theta_\mu / \cos \alpha).
\]

The first term on the right-hand side is the interference term between the zeroth and first orders of diffraction, i.e., the signal to be measured. The second term contains a combination of the noise field from each arm and the third consists of the noise mixed with the diffraction orders. The rest of the terms (denoted by “const.”) are combinations of contributions from the same interferometer arm and these can be easily removed by calculating the difference between the mutually phase shifted signals from two detectors, as depicted in Fig. 1.

Since all contributing fields have been generated by the same incident light beam, one can assume that they are correlated in time. The noise term involving \( I_0(\Delta \phi) \) will be much larger than the other two, which both contain a multiplication of small entities. Therefore it is sufficient to take this term into account in the following calculations. Omitting the constant terms, the last equation can now be simplified to

\[
I_{\text{tot}}(\Delta \theta) = 8\pi|C_1|^2 \cos \theta_0

\]

\[
\times \cos \theta_0 \Re \left[ \tilde{f}_\mu(\Delta \theta) \tilde{f}_1(\Delta \theta) \right]
\]

\[
\times I_m I_0(\Delta \phi) I_1(\Delta \phi) \cos \phi
\]

\[
+ 4I_m I_0(\Delta \phi) \frac{\sigma_1}{\lambda R_0} \cos \theta_0
\]

\[
\times \Re \left[ C_1 e^{-ikR_0} \tilde{f}_0(\Delta \theta) \right] \cos \phi_{N,1}.
\]

The energy \( U_\delta \) deposited in the detector by the light is finally found by integrating (19) over the detector’s acceptance angle \( \delta \theta \), i.e.

\[
U_\delta = \int_{-\delta \theta/2}^{+\delta \theta/2} I_{\text{tot}}(\Delta \theta) R_0 d(\Delta \theta).
\]

A change in the modulation depth of the grating, an apodization, causes the modulation depth of the detected signal to change accordingly. The argument \( \Delta \phi \) of the Bessel functions being quite small for weak to fairly strong gratings, we can write (cf. e.g. [7])

\[
I_0(\Delta \phi) I_1(\Delta \phi) \approx 1 + \left( \frac{\Delta \phi}{2} \right)^2 \frac{\Delta \phi}{2} \approx \frac{\Delta \phi}{2}.
\]

For a linear grating apodization change we thus have a linear change of the detected signal as well. When the phase change \( \Delta \phi \) is large, this is no longer the case and the response is non-linear. From Eq. (2) it is evident that this effect increases with decreasing probe wavelength. For a typical fibre core diameter of about 7–10 \( \mu \)m and a HeNe probe beam with \( \lambda = 632.8 \) nm, the response is approximately linear for index modulation depths of up to \( \Delta n \approx 10^{-3} \), above that the non-linearity results in a distortion of the signal (cf. Fig. 3). As a consequence, this method is unsuitable for apodization measurements of very strong gratings.

When the fibre under investigation is translated during the measurement, this will manifest itself in the equations as a change of the phase \( \phi \) introduced in the expression for the transmission function. Hence, according to Eq. (19), the detected fringes will change at the same rate as do the fringes of the grating. This is true independently of modulation depth and chirp and thus the possibility to perform phase measurements on weak gratings is only limited by the noise levels in the detection system.

As opposed to non-interferometric side diffraction characterization, the additional retrieval of the change of phase throughout the grating makes it possible to determine all geometrical parameters of the grating under test. It should be noted, that since the diffraction angle is insensitive to the average index of refraction, methods based on side diffraction cannot be used to measure optical entities.
The spatial resolution is determined by the width of the probe beam. A lower limit of this width is given through Eq. (14) by the pitch of the grating. As can be seen in this equation, the resolution is at least of the same order as the resolution offered by other techniques [1,2].

If the grating is chirped, the deflection angle for the first diffraction order will change slightly during the fibre translation. In a simple setup where the detection angle \( \theta_1 \) is fixed, the amplitude of the deflected beam in this direction will change with increasing Bragg detuning \( \Delta K \) and Eq. (19) takes the form

\[
I_{tot}(\Delta \theta) = 8\pi |C_1|^2 \cos \theta_0 \cos \theta_1 \\
\times \Re \left[ \tilde{f}_0(\Delta \theta)\tilde{f}_1^*(\Delta \theta + \Delta K/k \cos \theta_1) \right] \\
\times I_m I_0(\Delta \phi) I_1(\Delta \phi) \cos \phi \\
+ 4I_m I_0(\Delta \phi) \sqrt{\frac{\sigma_1}{kR_0}} \cos \theta_0 \\
\times \Re \left[ C_1 e^{-ikR_0} \tilde{f}_0(\Delta \theta) \right] \cos \phi_{N,1}.
\]

(22)

In order for the detectors to receive the same amount of light during the whole measurement, their acceptance angles must be enlarged to include this angular drift. Assuming a chirp width \( b \lambda \) we have that \( \Delta K = \pm bK / [2(1 + b)] \). The acceptance angle would then have to be at least

\[
\delta \theta = \delta \theta_0 + \frac{b \lambda}{1 + b \lambda}.
\]

(23)

where \( \delta \theta_0 \) is the angle corresponding to the width of the diffracted beam. A larger acceptance angle would in principle also mean an increase in the noise level. It should be noted, though, that the noise term in Eq. (22) vanishes when \( f \) approaches zero so that the only contributing additional noise originates from the second term in Eq. (17). Since this term is very small, it will still not contribute to any large extent even for reasonably large acceptance angles. In order to ensure the lowest level of noise for very large chirps, the detection angle has to be changed during the measurement (e.g. with a feedback system) so as to always correspond to the diffraction orders of the local grating period.

In the case of a detector that has the same acceptance angle as the irradiance angles of the diffracted beams, the signal-to-noise (S/N) ratio \( A \) is given by

\[
A = \frac{U_{d, \text{Signal}}}{U_{d, \text{Noise}}} = \frac{2\pi I_1(\Delta \phi) \cos \theta_1}{\sqrt{\sigma_1} \cos \alpha} \frac{Q_{01}}{Q_0},
\]

(24)

where

\[
Q_{01} = \int_{-\delta \theta_0/2}^{+\delta \theta_0/2} \Re \left[ \tilde{f}_0(\Delta \theta)\tilde{f}_1^*(\Delta \theta) \right] d(\Delta \theta),
\]

(25)

\[
Q_0 = \int_{-\delta \theta_0/2}^{+\delta \theta_0/2} \Re \left[ \tilde{f}_0(\Delta \theta) e^{-i\pi/4} \right] d(\Delta \theta).
\]

As can be seen, in the weak scattering approximation, the S/N ratio increases linearly with grating modulation depth and decreases with the square root of the noise level. In Fig. 4, a plot of the lowest grating modulation depth giving a S/N ratio of at least one as function of noise level is shown for a Gaussian beam shape with a width \( l = 100 \mu m \) and wavelength 632.8 nm. The level of the noise is dependent on many parameters, e.g. cleaning of the fibre before measuring, so it is hard to give a general estimate of its value without knowing the conditions for a specific measurement and setup. As reported in [5], the noise level can very well be of the same order of or even exceed the signal strength in the first diffracted order. Using this fact to give a crude estimate of \( \sigma_1/kR_0 \approx 5 \times 10^{-6} \), which corresponds to the first order diffracted energy of a grating with

Fig. 3. A plot of the ratio between \( I_0(\Delta \phi)I_1(\Delta \phi) \) and \( \Delta \phi/2 \) as function of modulation depth \( \Delta n \), with \( \lambda = 632.8 \text{ nm} \) and \( \rho_m = 10 \mu m \).
\[ \Delta n = 10^{-4}, \text{ the theory predicts a lowest detectable grating modulation depth of the order of magnitude of } \Delta n \approx 10^{-5}. \text{ Thorough cleaning of the fibre and additional suppression of the noise level with aid of lock-in techniques, e.g. as described in [5], should further lower this figure. In order to obtain a more exact figure, a more involved investigation concerning the origin and closer nature of the noise has to be carried out.} 

In the case of a Gaussian beam shape, the S/N ratio is increasing linearly with the probe beam width, and thus an increase in sensitivity can be traded for an equal decrease in spatial resolution.

5. Conclusions

We have shown that the interferometric side diffraction method is very useful for full characterization of gratings with modulation depths up to \( \Delta n \approx 10^{-5} \). Above this level, the phase is still measurable (at least as long as the weak grating approximation is valid), whereas the outcome of the apodization measurement degrades due to the inherent non-linearity of the scattering process.

The actual spatial resolution depends on the components of the specific setup that is used and on the choice of the probe beam width. In general, it can be assumed that this resolution is of the same order as that achievable by other characterization techniques.

Side diffraction methods determine the geometrical parameters rather than the optical. As a consequence, these methods are less suitable for measurements on gratings where the average refractive index variation is important, such as in polarization dependent fibres or in high-precision measurements that are to be used for e.g. group delay calculations. A better solution is then to use other techniques such as low-coherence reflectometry (see e.g. [2]) that also take these variations into account. On the other hand, in all applications where the grating properties have to be known as function of geometrical position in the fibre, the side diffraction method is superior. A possible application is e.g. to find the right offset position for each exposure when writing multiple exposure gratings.

The optical noise originating from the fibre surface roughness distorts the amplitude of the detected signal so that, for very weak gratings, neither phase nor apodization will be easily retrievable. Since most of the noise can be expected to originate from the surface roughness of the fibre, thorough cleaning of the fibre under test is essential when performing measurements on very weak gratings. The theory indicates that modulation depths of at least the order of \( 10^{-5} \) should be detectable.

References


Fibre Bragg grating characterization with ultraviolet-based interferometric side diffraction


Authors: I. Petermann, S. Helmfrid and P.-Y. Fonjallaz
Fibre Bragg grating characterization with ultraviolet-based interferometric side diffraction

Ingemar Petermann1,3, Sten Helmfrid2 and Pierre-Yves Fonjallaz1

1 Acreo AB, Electrum 236, S-164 40 Kista, Sweden
2 Proximion AB, Skalholstgatan 10B, S-164 40 Kista, Sweden
E-mail: ingemar.petermann@acreo.se

Received 23 May 2003, accepted for publication 6 June 2003
Published 1 July 2003
Online at stacks.iop.org/JOptA/5/437

Abstract
The interferometric side diffraction method for fibre Bragg grating characterization is evaluated using a set-up with an ultraviolet interrogation beam. In its present configuration, the reproducibility errors are 2–5% for the index and typically 10°–20° for the phase profile. The reflection spectra of chirped test gratings have been successfully reproduced with the aid of the acquired phase and refractive index envelope data. It is verified that the power of the interrogating beam, within some limits, is unimportant as regards the phase reproducibility, and the importance of relating the analysis to the fabrication parameters of the grating under investigation is stressed.

Keywords: Bragg grating, grating characterization, interferometric side diffraction

1. Introduction

The past decades have seen optical fibre Bragg gratings evolve from simple and fragile objects of study in research laboratories to highly sophisticated components incorporated in numerous commercial products for telecommunication and sensing. As part of this development, a need for characterization of such components has emerged. However, a straightforward spectral analysis is not sufficient for tracking fabrication errors and improving grating quality, which has led to the development of several techniques for direct spatial characterization of both phase and amplitude properties [1–5]. One of these is based on diffraction of sideways-incident light by the in-fibre Bragg grating [2]. In this method, the fibre grating works as a diffraction grating and deflects incident light with an efficiency determined by the grating’s modulation depth. The angle between the incident and deflected light is given by the grating pitch. Moreover, the interference pattern created by superposing the zeroth- and first-order deflected beams is proportional to the refractive index variation in the fibre [6, 7]. Thus, both the modulation depth and phase distributions can be retrieved while scanning the probe beam along the grating.

One way of bringing the zeroth and first diffracted orders together is to use two incident (coherent) beams satisfying the Bragg condition [8]. The zeroth diffraction order from one beam will then overlap and interfere with the first order arising from the other incident beam. This corresponds to the geometry of a grating fabrication system, and using the ultraviolet (UV) light beams present in such a system for the characterization allows for a very simple set-up that is easily implemented in most grating fabrication systems based on UV exposure [9]. In the present paper, we investigate the quality of characterization measurements obtained with a set-up based on this UV interferometric side diffraction method, and discuss some issues related to acquisition and analysis of interferometric side diffraction data in general.

2. Experiments

2.1. Experimental set-up
In our implementation of the interferometric side diffraction technique we use 244 nm UV light to probe the grating under investigation [9]. A UV detector is positioned below the fibre in our grating fabrication system, collecting the deflected and transmitted light from the same UV beams as are used for writing gratings (cf figure 1). The diameter of the beams at...
the fibre is approximately 100 μm, which also constitutes the spatial resolution. This configuration allows for a very simple set-up, since the functionality and aligning features of the already existing fabrication facility can be used. As described in [10], gratings in this facility are written by exposure to a continuous-wave fringe pattern moving with the fibre, which is translated with an interferometer-controlled translator stage. The movement of the fringes is realized by introducing a sawtooth-modulated phase shift between the two arms in the UV interferometer creating the fringe pattern.

When characterizing a grating, the fringe pattern is kept fixed and the power of the UV source turned down to a few percent of the power used when writing a grating. As a result, the grating will be evenly exposed to UV light, causing a slight change of the average refractive index and corresponding central wavelength of the grating’s reflective response. If the refractive index change in the fibre is close to saturation, an increased UV dose will also start erasing the grating. When interrogating gratings in highly sensitive fibres, it is therefore important to minimize the additional exposure by using a low UV power and a high scanning speed. In our experiments we successfully interrogated gratings written in hydrogen-loaded Ge-doped fibre. In the case of extremely sensitive fibres, though, a UV probe beam is probably not suitable.

The acquisition hardware is triggered by the position of the translator, ensuring equidistant sampling of the scattered field along the grating. An electronic low-pass filter was used to reduce the amount of noise from the broadband photomultiplier tube detector.

2.2. Results

The gratings in the experiments described here were written in hydrogen-loaded Ge-doped fibre; the characterization interrogation speed was set to 0.3 mm s⁻¹ and the sampling distance chosen to approximately 22 nm. The (relative) index modulation is determined from the acquired raw data by taking the level difference of consecutive peaks and valleys, and the phase is extracted by a time-domain algorithm localizing the position of the fringes along the grating. Figure 2 shows the result from a typical measurement. The grating was in this case 1 cm long and 2 nm chirped with a raised cosine apodization.

The laser power was 6 mW, which results in roughly 1.5 mW at the fibre due to losses in the interferometer. The data have been digitally averaged over a distance of roughly 5 μm only. As can be seen, the general shape of the measured data fits quite nicely to the computer-generated curves of an ideal grating also shown in the graphs. In the index modulation profile, a slight linear deflection from the reference can be seen; this is most probably due to misalignment of the fibre during the measurement.

Most of the discrepancies from the reference curves are mainly due to actual grating errors, but some arise from the uncertainty in the measurement. Repeating the experiment yields curves resembling the ones in figure 2 with reproducibility errors of 2–5% for the index and up to ±20° in phase for typical measurements. As long as the fibre is cleaned, the same reproducibility is obtained when performing scans directly after each other or with an interval of a few days. Especially for the phase, these errors are rather high, but bear in mind that the data are averaged over a very small distance only. Increasing the averaging window to the order of millimetres reduces these figures to roughly ±8° for the phase and ±1% or lower for the index. This reduction in spatial resolution is appropriate when determining the group delay ripple in chirped gratings, since this ripple is caused by fluctuation on the millimetre scale and above only [11, 12].

However, low-pass filtering the data in this manner may lead to a few problems when using the measurements for further calculations, since important information about the fine structure is lost. A simple way of illustrating this is to simulate the reflectance spectrum of the grating with the measured data as input and compare the result with the ‘real’ spectrum directly obtained by an optical spectrum analyser (OSA). Figure 3 shows this comparison for a 2 nm chirped and 1 cm long unapodized grating interrogated with a laser power of 2 mW. A simple matrix approach with a resolution of 200 fringes was used for the simulation of the reflective response [13]. The agreement between the two curves in 3(a) is very good, especially within the pass band. In this figure, an average window of approximately 5 μm has been employed for the measured data. In 3(b), the window is increased to 0.5 mm, which is above the physical resolution given by the width of the probe/writing beam. The resemblance of the curves is still quite good, but some of the fine structures start to disappear. This effect is even more prominent in 3(c) with a window size of 0.8 mm; and in 3(d), with averaging over 1 mm, the agreement is not much better than for a simulation based on generated ideal envelopes as shown in 3(e).

It is in most cases unnecessary to use a higher resolution than the physical width of the beam used when writing the grating, since grating errors on a smaller scale will be suppressed during fabrication. Lowering the digital resolution below this physical limit, however, will have an impact on the shape of the calculated amplitude spectrum, and it is therefore important to relate the analysis parameters to the fabrication resolution of the grating under investigation.

Using a wider beam when writing the grating will lower the spatial resolution during the fabrication process and also enable a correspondingly larger averaging window to be used in the characterization analysis. On the other hand, a low writing resolution makes it impossible to fabricate more advanced
Figure 2. The measurement result for a 1 cm long and 2 nm chirped grating with a raised cosine apodization. The index modulation envelope (left) has been normalized to its maximum value. Shown also are computer-generated reference curves for the respective envelope.

Figure 3. Comparison between the reflectance spectrum obtained by an OSA and simulated spectra for a 2 nm chirped 1 cm long unapodized grating. The measured phase and index envelope data used in the simulation have been averaged over (a) 5 µm, (b) 0.5 mm, (c) 0.8 mm, and (d) 1 mm. In (e) the simulation has been done with computer-generated ideal envelopes.

Grating structures such as large or non-linear chirps, phase shifts, and apodizations.

Despite the phase reproducibility errors, it is evident from comparing 3(a) and (e) that the characterization is good enough for reproducing the amplitude spectrum of chirped gratings. The variation of the calculated spectra for different measurements is of the order of per cent within the pass band. Outside the pass band, the difference between the simulated and directly measured curves is somewhat larger. Even though the positions of the side lobes are correct, their relative strengths are not reproduced correctly. The reason for this is that these parts of the spectrum are more sensitive to phase noise. Apart from measurement errors, errors originating from a fluctuation in average refractive index—which are not detectable by a side diffraction-based characterization method—may be of importance in this spectral region.

The phase reproducibility errors do not allow reproduction of e.g. group delay, where the variation between measurements proved to be of the order of ±100 ps. This is by no means a limitation of the technique; the reason is rather to be found in lack of precision in the positioning system.

When calculating the reflective response, the maximum modulation depth has been chosen so as to yield as high a resemblance to the actual reflectivity as possible. For the two gratings mentioned in this paper the modulation depth turned out to be approximately $4.3 \times 10^{-4}$, which also roughly
Figure 4. Comparison between the reflectivity spectrum obtained by an OSA and simulated spectra for the grating in 2. In (a), an ideal phase envelope and measured refractive index envelope has been fed into the simulation algorithm and in (b) vice versa.

Figure 5. Phase differences between different measurements with laser powers of 2, 6, and 18 mW. Each curve is the average of several measurements and displayed with an offset to increase clarity. In the upper part of the graph is the phase difference between two measurement series using the same power 2 mW displayed for reference.

3. Discussion

We have investigated the interferometric side diffraction technique for fibre Bragg grating characterization, utilizing UV light for probing the grating. The results are good enough for reproducing the grating’s amplitude reflective response, whereas the group delay cannot be correctly retrieved. The reason for this is probably to be found in the precision of the positioning system. While the electronic (relative) precision of our system is of the order of 0.6 nm, mechanical vibrations and air fluctuations cause much larger uncertainties in the position, resulting in phase errors in the measurements. This is not a large problem for determination of the modulation depth envelope, though, where we have noted a much better reproducibility. Furthermore, errors in the refractive index profile generally do not have a large impact on the reflective properties of the grating.

Since we are using the same translation system for writing and interrogating the grating, it may seem hard to separate writing and interrogation errors related to translation. This is not entirely the case though, since the writing process is more sensitive than the interrogation. Both processes are sensitive to differences in the actual fibre position as compared to the position of the mirror on the translator used in the interferometric positioning system, as well as air fluctuations in the path to this mirror. Moreover, small-scale cladding thickness and surface roughness of the fibre due to e.g. dirt particles or even fabrication errors contribute to the general noise level. These sources of errors will be exactly the same during writing and interrogation and cannot be separated in the measured data.

However, a possibly larger source of error during writing is a non-linear movement of the translator. The UV interference pattern will in principle always follow the fibre (or, rather, the aforementioned mirror) and thus expose the correct position on the fibre, but a non-linear noisy movement will cause some parts of the fibre to receive a higher UV dose than others. In the case of perfect plain non-chirped and non-apodized gratings this will only cause a fluctuating average refractive index, which is not detectable with a side diffraction technique. Gratings comprising more elaborate structures such as chirps, phase shifts, and apodizations are fabricated by superposing a large number of phase-shifted subgratings. A dose fluctuation in the exposure will then also result in geometrical phase errors, which can be detected by the side diffraction system. The same holds for noise originating from fluctuations in the UV source. Theoretically, it should be...
possible to detect changes in the local dose as variations in the refractive index envelope, but these variations will be hard to separate from other noise sources such as surface roughness of the fibre.

Since air fluctuations are rather slow, errors caused by this effect become more prominent as the time each measurement takes becomes longer. One way of achieving improvement is therefore to scan at a larger speed. A larger speed will probably also result in a more even translation, possibly causing less vibration between the mirror and fibre.

In order to improve the reproducibility, it might prove advantageous to average measurements performed for several azimuthal orientations of the fibre, which has not been done above. Simply increasing the probe beam power will not help with increasing the signal-to-noise ratio for the phase acquisition.

In conclusion, the results in this work show that fibre Bragg characterization with UV-based interferometric side diffraction is a simple and promising method for obtaining spatial data from gratings in connection with their fabrication, and that it is important to relate the analysis to the physical limits of the fabrication system. Some improvement of the positioning system in our set-up is still necessary to yield high-precision phase data for e.g. on-line compensation of high-performance dispersion compensating gratings.

References

Paper V

Stitch error effect on group-delay ripple for long chirped fiber Bragg gratings


Authors: I. Petermann and S. Helmfrid
Stitch error effect on group-delay ripple for long chirped fiber Bragg gratings

Ingemar Petermann and Sten Helmfrid

Different sources of stitch errors are identified and simulated in a 300 mm long chirped grating. In each case, the effect on the group-delay ripple and transmission spectrum is investigated. The anticipated response in a typical 10 Gbit/s transmission system is specifically considered. From the simulations, it is clear that information about error source and magnitude can be gained directly from the transmission spectrum. © 2005 Optical Society of America
OCIS codes: 060.2320, 060.2340, 230.1480.

1. Introduction
Chromatic dispersion compensation components are an important part of today's optical communication networks, and the quality demands grow along with increasing transmission rates. An efficient technical solution for this kind of component is given by linearly chirped fiber Bragg gratings. It is often necessary to stitch several gratings together to obtain grating lengths that can meet the high demands given by high-speed networks. As an example, dispersion compensation of an 80 km standard fiber over the entire C-band requires a grating length of the order of 6 m and a bandwidth of 35 nm. Even small errors in the stitching process that result in discontinuities in the combined grating may cause large errors in the group-delay characteristics, thus degrading the performance of the component. The maximum allowed group-delay ripple (GDR) amplitude for high-quality dispersion compensation is of the order of a few picoseconds, which sets a limit for the magnitude of the various kinds of stitch error. When optimizing a grating fabrication setup for small stitch errors it is important to be able to distinguish between the effect of different error sources to find the limiting parameter in the system.

While there are several investigations about profile-induced GDR in general, our study focuses on the effect of stitch imperfections, and in the following we will discuss what errors may appear as well as their effect on the GDR and transmission spectrum.

The first thing to do is to identify possible error sources. A perfect stitch means that all parameters such as chirp, apodization, and phase are continuous throughout the grating so that no stitch position can be recognized afterwards. In reality there are a number of things that can go wrong in the process. The most obvious one is that the fringes of the second grating are geometrically displaced with regard to the fringes of the first, resulting in a phase error. Another possibility is that the fringe visibility differs, giving a refractive-index modulation error. Furthermore, one might start the second grating with a pitch that does not match the last part of the first, resulting in a pitch error. A fourth source of error is the UV intensity, e.g., if the UV source in the grating exposing system is turned on too late when writing the second grating. This shutter error, as opposed to the modulation error, will cause a local change in the modulation depth at the stitch. If one of the gratings is exposed with a slightly different total UV dose, the modulation depth changes and the pitch profile will have an additional offset as compared with the other grating due to a different mean refractive index. This basically results in a combination of the modulation and pitch errors mentioned above.

In many cases, the aforementioned errors will also be accompanied by a fluctuation in the UV dose at the stitch, resulting in a dose error that causes a local change of the average refractive index and consequently the optical pitch.
2. Simulations

All simulations were carried out based on a chirped grating with a chirp rate of +5 pm/mm from 1551.5 to 1552.5 nm. This chirp rate is roughly what is needed to compensate an 80 km fiber link over the entire C-band. Without loss of generality, the grating length was limited to 200 mm to speed up the calculations. To suppress edge ripple, an 8 mm cos^2 apodization was applied at the start and end of the grating. The simulated grating consists of two 100 mm gratings stitched together. In the case of a perfect stitch, the GDR takes the form as shown in Fig. 1.

The simulation algorithm is based on the transfer-matrix method by use of the solution of the coupled mode equations for weak gratings. Each subgrating in the simulation has a length of 100 local periods, and the group delay and GDR are calculated with a resolution of 1.3 pm using Fourier analysis. The average refractive index is taken to be 1.4606 and the refractive index variation is 6.8 x 10^-4.

In all the following simulations, the GDR of the perfectly stitched grating in Fig. 1 is subtracted to illuminate the changes in response for different stitch errors.

A. Phase Error

It is obvious that a large phase error will affect the response of the grating, since it introduces a broadband reflection. It is also to be expected that the changes in group delay occur for wavelengths with a matching grating pitch after the stitch, since other wavelengths are mainly reflected before the stitch and thus do not see the disturbance.

The additional GDR due to a 10 deg phase error results in the graph shown in Fig. 2. All other errors are assumed to be negligible. As expected, the changes start after the center wavelength (1552 nm) of the grating. The ripple can be explained by considering that each wavelength will be reflected at the phase error as well as its original center of reflection. The resulting cavities, or etalons, change the time that the light is trapped within the grating as a function of cavity size, which in turn depends on the wavelength due to the linear chirp. The variation of this time as a function of wavelength manifests as a ripple in the group delay.

In telecommunication applications, component performance is dependent on the system bit rate. For example, GDR with a period shorter or much longer than the communication bandwidth will under normal circumstances not have any large effect on the performance. In the literature, the GDR is therefore often averaged over some distance in the wavelength domain corresponding to the bit rate. The resolution in the plot is 1.3 pm, and a plot averaged over 80 pm corresponding to a bit rate of approximately 10 Gbits/s is also shown in Fig. 2 for comparison. In this case, the high-frequency noise farther from the stitch is suppressed to a large extent. The larger the phase shift, the larger the induced GDR. Figure 3 shows the maximum induced GDR amplitude as a function of phase error with and without averaging. Note that, since the GDR variation is not necessarily symmetric around zero, we chose to plot the absolute maximum within the simulated wavelength interval from 1551.5 to 1552.5 nm instead of the more commonly used peak-to-peak value. It is evident that only errors of a few up to less than 10 deg in the averaged case are allowed to pass the demands for high-quality dispersion compensation.

The above simulations were carried out with the same chirp rate and refractive-index modulation depth. What happens when these are changed? The solid curve in Fig. 4 shows the maximum induced GDR for a phase error of 10 deg for different chirp rates but constant modulation depth. Apparently, the induced GDR is roughly proportional to the inverse chirp rate. For an unchirped grating, reflected light from the whole grating before and after the stitch has the same wavelength and thus the
Fig. 3. Maximum induced GDR amplitude versus phase error. The inset shows details for small error values.

Fig. 4. Maximum induced none-averaged GDR versus chirp rate for different error types.

Fig. 5. Maximum induced none-averaged GDR at 10 deg phase error versus refractive-index modulation depth. The GDR for wavelengths with main reflection centers before (solid curve) and after (dashed curve) the stitch are shown separately.

error has an effect on the response from the whole grating. Keeping the modulation depth at the same level and introducing a chirp will shrink the part of the grating over which each wavelength is reflected. The etalon effect between reflected light at the stitch and at the original reflection center in the grating will therefore be smaller in amplitude.

The induced GDR due to a 10 deg phase error versus modulation depth for a fixed chirp rate is shown in Fig. 5. The modulation magnitude of the refractive index determines the penetration depth of a wavelength for a given grating pitch. A large modulation means that a wavelength will be fully reflected at a shorter distance than in the case of a smaller modulation. If the modulation is weak enough, there will always be light of a specific wavelength present in the whole grating. The chirp rate then determines the spatial width of the main reflection center over which the light of that wavelength is partially reflected. When the mismatch to the local pitch is too large, the residual light will leak through and eventually end up as transmitted power. This also means that a wavelength with a matching local pitch before the stitch will see any errors present in the stitch, and thus GDR will be present for this wavelength as well (see dashed curve in Fig. 5). For wavelengths matching a pitch after the stitch, the GDR will decrease with decreasing modulation depth, since in this case a smaller amount of light is reflected at the phase error.

In the opposite case, when the modulation is strong, light of a specific wavelength will be fully reflected over a shorter distance than the main reflection center width stipulated by the chirp rate. No light will leak beyond its center of reflection, and consequently stitch errors will not be seen by wavelengths having a matching pitch before the stitch position (see solid curve in Fig. 5).

In Fig. 5 the dashed GDR curve for light with main reflection centers before the stitch virtually disappears for high values of the refractive-index modulation. This curve consists of two parts with a transition point at approximately $7.5 \times 10^{-1}$, which at the present chirp rate obviously corresponds to the limit where each wavelength is fully reflected within its main reflection center. Beyond this point no light from the reflection centers before the stitch will be able to leak through to see the stitch. In the limit of a very weak grating, the GDR before and after the stitch is of equal order, since then the approximation holds that the light field for all wavelengths is constant throughout the grating. The distinction between reflection centers before and after the stitch is thereby removed.

B. Modulation Error

Figure 6 shows an example of the GDR from a stitch where the fringes of the first grating have full visibility and those of the second 94% visibility. For the
current grating this corresponds to a refractive-index modulation difference of $4.1 \times 10^{-6}$. Note that the average refractive index is the same at both sides of the grating, the only parameter changed being the modulation depth. The discontinuity at the grating results in a broadband ripple that causes a ripple in much the same way as for the phase errors described above. The induced GDR versus modulation depth is shown in Fig. 7. Evidently, a few parts per million absolute difference in modulation depth before and after the grating is enough to cause a GDR of several picoseconds.

If the change is in the other direction, i.e., the first grating has 6% lower visibility than the second, the result is practically the same as above.

Keeping the error fixed at 6% and varying the chirp rate results in the dotted curve in Fig. 4. The relation is obviously similar to that of the phase error.

Figure 8 shows the maximum induced GDR for constant modulation error as a function of the GDR index modulation depth. It is clear that there is no linear relation between the GDR caused by relative modulation error and the general strength of the grating. For the absolute difference there is a linear relation, although from approximately $4 \times 10^{-5}$ and stronger. Again, the GDR curve for light reflected before the grating has a transition point at approximately $7.5 \times 10^{-5}$ (see dashed and dotted curves in Fig. 8). The GDR for a constant absolute error increases with decreasing refractive index in the limit of very weak gratings. This is due to the fact that the error approaches the same magnitude as the grating strength and would normally not occur in a real experiment.

C. Pitch Error

For pitch error, one should distinguish between two possible cases: positive and negative pitch jumps in the grating. In the former case (given the simulated ascending chirp) there will be a pitch region missing in the chirp structure, whereas the latter results in two closely spaced regions with the same pitch, allowing for Fabry–Perot effects in the reflective response from the region both before and after the grating.

Figure 9 shows the induced GDR for pitch jumps of $\pm 3.4$ pm corresponding to $\pm 10$ pm in reflected wavelength. As expected, an additional offset of the group delay occurs at the grating. The same kind of ripple as in the previous cases appears, since there is again a broadband ripple in the discontinuity at the grating. The fall of the induced group delay back to zero at 1552.5 nm in the graph is a result of the grating ending at this point and is not of any importance in the present context.
It seems reasonable that the induced GDR will be larger for larger pitch errors, which is verified in Fig. 10 where the induced GDR is plotted versus pitch error. As can be seen, the difference between positive and negative pitch jumps is considerable only for large errors. For an error of up to less than 1 μm, above which the demands for high-quality dispersion compensation are not met, the induced GDR is approximately the same in both cases.

Similar to the former cases, the pitch error is roughly proportional to the inverse chirp rate, as shown by the dashed curve in Fig. 4.

Figure 11 shows the induced GDR versus grating strength. Apart from an additional offset due to the group-delay offset, the behavior of this dependence is similar to the corresponding relations for phase and modulation errors.

**Fig. 10.** Maximum induced non-averaged GDR including offset versus pitch error. The inset shows details for small error values.

**Fig. 11.** Maximum induced non-averaged GDR at a 10 μm pitch error versus refractive-index modulation depth.

**D. Shutter Error**

Figure 12 shows the induced GDR due to a shutter error causing the exposure to start 100 μm too late for the second subgrating. The position of the second grating is correct, so the only difference from the perfectly stitched case is a hole in the grating modulation depth in the vicinity of the stitch.

For each wavelength there are three different centers of reflections: the end face of the first subgrating, the front face of the second, and at the matching local pitch within the second subgrating. The different possibilities of multiple reflections result in a GDR that is slowly increasing with increasing wavelength before leveling off to a constant amplitude. Making the same simulation with the stitch located earlier reveals that, for the current parameters, this level is approximately at ±10 μm, which is just at the end of the grating response in Fig. 12.

Figure 13 depicts the induced GDR versus shutter error. It seems that a few tens of micrometers is
enough to cause a noise of several picoseconds in GDR. However, since the lower perturbation frequencies have quite a small amplitude in this case, the effect of this error is also small when taking bit transmission rates of, e.g., 10 Gbits/s in account. Averaging over 80 pm in Fig. 12 takes away most of the ripple. For most gratings fabrication facilities it should be fairly easy to keep this kind of error to values of a few micrometers and below, so it is safe to say that other errors most likely will have a larger effect on the GDR.

As can be seen in the dashed-dotted curve in Fig. 4, the shutter error does not scale with chirp rate. The important factor is the size of the error, which is not wavelength dependent. Decreasing the grating strength causes light of any given wavelength to be reflected over a larger part of the grating. As a result, the effect of localized imperfections on the response is less prominent than for a stronger grating. Figure 14 shows the induced GDR versus grating strength, and, as expected, the same behavior as for the aforementioned errors can be seen.

Fig. 14. Maximum induced non-averaged GDR at 100 μm shutter error versus refractive-index modulation depth.

Fig. 15. Changes in the GDR as compared with the perfectly stitched case for a dose error at grating center corresponding to a pitch change of 36 pm or a total change in refractive index of 6.8 × 10⁻⁶. This corresponds to the modulation depth in a simulated grating. The change takes place over a distance of 50 μm, or roughly the length of a subgrating in the simulation.

E. Dose Error

Since the UV intensity is not the same over the width of the writing beam, there will be a small dose variation in a stitch if the overlapping of the edge sub-gratings is not optimal. This dose error causes the average refractive index to change and with it the local (optical) response wavelength of the grating. In a worst-case situation, the dose before the stitch will drop to zero before the first exposure after the stitch. Figure 15 shows the resulting GDR when the local pitch has decreased by 36 pm over a distance of 50 μm, or roughly the length of a subgrating. This corresponds to a dose reduction from maximum to zero in the simulated grating. In the simulation, the modulation depth was kept constant to illustrate the contribution of the UV dose alone, but in reality the modulation depth would decrease to zero as well, which in turn reduces the effect of this error.

Figure 16 shows the maximum induced GDR versus dose-induced pitch change. Even though the simulation describes the largest possible error for this grating—the dose will actually rarely reduce all the way to zero—it is clear that the error is quite small as compared with other errors, especially after averaging. The error scales with the region width over which the dose changes, but since this region corresponds to the (fixed) UV beam width of the fabrication system, this will usually not be a large problem.

There is also a possibility that the dose is raised by the same amount, e.g., if the second subgrating is overlapped with the first one. The effect on the GDR is then more or less the same as above.

3. Determination of Stitch Error from Spectral Response

It is possible to get a crude idea about the dominant error in a stitch by looking at the transmission (or
reflection) response, which is easily monitored during the grating fabrication process. Shutter and (positive) pitch errors both take away a part of the local period in the vicinity of the stitch, resulting in an increase in transmission for the corresponding wavelengths. In the case of phase errors, however, all local periods in the chirped grating are still present, and the change in the spectrum is rather a consequence of interferometric effects at the phase discontinuity. This change takes the form of a decrease followed by an increase in transmitted power (or vice versa, depending on the sign of the phase error), as can be seen in Fig. 17 where the transmission spectrum for a grating with a 10 deg stitch phase error is shown. This kind of error is therefore quite easily recognized in the spectrum. In principle it is even possible to get an idea of the magnitude of the phase error for a given chirp rate, since the amplitude of the central disturbance (as measured from top to bottom in decibels) changes with the phase error.

Figure 18 shows the transmission central ripple amplitude versus phase error for gratings of a few different strengths; it is clear that, within reasonable ranges, there is an approximately linear relation between ripple amplitude and error magnitude.

There is a difference in general behavior of the spectrum for modulation, pitch, and shutter errors as well. The transmission spectra for gratings with these errors are plotted in Fig. 19. A modulation error results in two distinct levels originating from reflection centers before and after the stitch. The disturbance due to the pitch error is concentrated around the missing wavelengths with little change from the perfectly stitched case farther from these values, whereas the shutter error causes a more evenly distributed disturbance along the whole spectrum. Without loss of generality, our pitch simulation was carried out by simply adding 30 pm in response wavelength for all pitches after the stitch. Since the grating length was kept constant, the spectrum is

![Graph showing maximum induced GDR versus dose-error-induced pitch change. The 36 pm corresponds to a dose reduction from maximum to zero in the simulated grating.](image)

![Graph showing maximum transmitted central ripple amplitude for different grating strengths versus stitch phase error.](image)

![Graph showing transmitted spectrum for a stitch with a 10 deg phase error at grating center.](image)

![Graph showing transmitted spectra for (positive) pitch, shutter, and modulation errors.](image)
broadened as evident from the graph. The behavior of the central part of the spectrum, which is of interest in the present context, is unaffected by this numerical simplification.

For the pitch error, there is a linear relation between error and central disturbance amplitude only for negative pitch jumps as can be seen in Fig. 20. The missing local periods for positive jumps result in a transmission increase for the matching light instead of a decrease. Since the transmitted amount of light is limited to the amount put into the grating (0 dB in Fig. 19), the positive part of the curve in Fig. 20 must therefore be limited as well and level off at a value corresponding to the general strength of the grating.

In the case of shutter errors, it is evident from Fig. 21 that the same relation is linear after 50–100 μm corresponding to the subgrating length. For a real experiment this is the same as the width of the writing beam, and in our simulation it corresponds to the length of each subgrating in the transfer-matrix algorithm. The reason for this behavior is that the modulation depth never decreases to zero between the stitched gratings for smaller shutter errors. Figure 22 shows the level difference in the transmission spectrum between wavelengths before and after the matching pitch versus error magnitude for a modulation stitch error. In this case, there is a more prominent quadratic term present, but for moderate modulation errors the relation will still be roughly linear.

The disturbance magnitude in the transmission spectrum scales with the chirp rate in much the same way as the corresponding maximum induced GDR.

Even though the spectral characteristics of the different stitch errors are distinct and easily recognizable in the simulated response, it is important to note that a mixture of different stitch errors along with distributed grating imperfections in a real experiment may mask some of these characteristics. The main use of this method is therefore to help indicate a dominating error.

4. Conclusions

In this study we have defined and investigated four main sources of errors that can occur when two linearly chirped gratings are stitched: phase misalignment, shift in modulation depth, pitch offset, and shutter delay.

Our simulations show that, for a 10 Gbit/s transmission system, phase, modulation, and pitch errors will in general be the most important contributors to the GDR, whereas accompanied variations in the UV dose as well as shutter inflicted errors are smaller and may in many cases be neglected. For our simulated 200 nm long grating with a chirp rate of 5 pm/mm and a strength of 6.8 × 10^{-5}, we found that, to comply with the demands of high-quality dispersion compensation components, phase errors should be less than 10 deg, modulation errors less than a few parts per million, and pitch errors less than 0.5–1 pm. These limits scale with the grating
strength and, apart from shutter inflicted errors, with the inverse chirp rate.

There is a distinct difference between the manifestation of phase, modulation, and pitch errors in the transmission spectrum. In the case of a dominant source of error, this fact may prove helpful to determine the limiting parameter in a given setup. Since there is an approximate proportionality between the characteristics of the transmission spectrum and the error magnitude for a given chirp rate, it is also possible to get a rough idea about the severity of the error.

References

Paper VI

Tunable and programmable optical bandpass filter

Submitted to Applied Optics (2007)

Authors: I. Petermann, S. Helmfrid, O. Gunnarsson and L. Kjellberg
We demonstrate a novel tunable and programmable optical bandpass filter with a transmission peak less than 5 pm wide that is tunable over 18.5 nm. The number of transmission windows within the stop band as well as each window’s position and strength are set by applying a heat profile to a chirped fibre Bragg grating with aid of a thermal printer head comprising a 640 pixel array. It is shown that the width of a transmission peak formed by local heating is mainly determined by the refractive index modulation depth and chirp rate and not by the width of the heated region. © 2007 Optical Society of America

**OCIS codes:** 060.2330, 060.2340, 230.1480.

### 1. Introduction

The number and size of optical networks are steadily growing and with them the need for optical switching and filtering components. One important component is the tunable bandpass filter, which makes it possible to tune in different carrier wavelengths with one single component.

A convenient and often used way to realize optical band-pass filters is to directly inscribe Bragg reflecting devices into an optical fibre.\(^1\) Such gratings can be tuned mechanically by stretching, pressing, or heating the fibre, or by electro-optical effects.\(^2,3,4\)

Li et al.\(^5\) recently demonstrated a tunable filter by heating a chirped grating with a movable wire. The same technique has been used to realize a switchable comb filter\(^6\) and was originally reported in conjunction with fibre Bragg grating characterization.\(^7\)

We investigate this technique in detail by simulations and present a tunable and programmable filter where the heater wires are replaced by a thermal printer head with 640 heater pixels. The result is a filter with no moving parts that can be fully switched, tuned and configured by software only.
2. Theory and simulations

One way of analysing chirped gratings that are heated by a point-like thermal source is to model the heat spot as a local phase shift.\(^5\) It is well known that a phase shift of 180 degrees results in a narrow transmission line inside the stop band due to negative interference in the reflected light. Other values of the phase shift only result in partial cancellation of the back reflection, which therefore reduces the magnitude of the transmission window. The phase shift is caused by the heat locally increasing the average refractive index in the fibre and thus increasing the optical path.

If the heat is applied over a length that is large compared to the grating pitch (which will usually be the case), we may also model the increased average refractive index as an increased optical pitch.\(^6\) Thus, a "hole" is created in the transmission stop band, since there are now reflection centres missing for some wavelengths. At the same time, reflection centres for other wavelengths now appear on several different locations in the chirped grating, causing multiple reflections and a modulation in the spectrum.

The reflective response of a perturbed grating is formed by complex interference of light reflected from different parts of the grating. While giving an approximate overall picture of the response, a simple phase shift-based simulation model will not be enough for an accurate analysis on a more detailed level.

We have chosen to model the applied heat as a local Gaussian increase in the pitch profile of the chirped grating. The width of this perturbation corresponds to the width of the heating pixels plus the width of the region around the pixels in which the heat penetrates. In the experiments, the heat penetration width can be minimized by mounting the fibre on a heat sink, thus leading away the excess heat not immediately under the active heater pixels. All simulations were carried out with the transfer matrix method based on the solution of the coupled mode equations for weak gratings.\(^8\)

The maximum value of the pitch perturbation corresponds to the largest shift in average refractive index (and thus pitch) that is induced and is dependent on the amount of heat that is applied.

Our aim is to create a narrow transmission band by introducing a temperature profile corresponding to a phase shift of \(180^\circ\). This results in a distributed feed-back (DFB) structure for light wavelengths with a reflection center at the heating point. Such structures are well-known for their ability to produce narrow transmission bands (see e.g. ref. 9).

The phase shift increases both with the maximum value and the width of the temperature profile. For a given maximum heat level, we should set the heated region to the shortest width resulting in full transmission at the target wavelength. Adjusting the transmission strength can be done by fine-tuning either the applied heat or the heater width from the optimum, resulting in slightly shifted, widened and weaker transmission peaks. In fact, starting from
zero and increasing the width or heat will cause a cyclic transmission/reflection of the target wavelength as the total phase shift rises to 180°, 360°, 540° and so forth with more or less the same width of the largest transmission peak (cf. Fig. 1). At the same time, a broad transmission window is built up from the noise level, degrading the performance of the grating as a transmission filter. This broad window is an effect of the phase shift being distributed over such a long part of the grating that the non-interferometric removal of reflection centres is significant for a larger range of wavelengths.

The total phase shift in the grating is proportional to the integral of the heat profile. One might therefore expect that the transmission curve is more or less unaffected by a constant increase in the width of the heated region and a simultaneous decrease of the maximum heat by the same factor. However, simulations show that the dependence between width and maximum heat level is not linear. The reason for this is that the reflected radiation that is cancelled by negative interference close to the transmission peak comes from a finite region in the grating close to the centre of the heat spot. If the heat spot is too wide, the phase shift over this region becomes less than 180°, which must be compensated by a higher temperature.

The DFB response is based on interferometric superposition of the reflected and transmitted contributions from the grating at each side of the phase shift. The actual width of the heating region, when kept within reasonable limits, should therefore be of little importance.
Fig. 2. Excerpt of the transmission spectrum of a heated grating with different heater widths. The maximum heat-induced reflected wavelength shift is in each case optimized to yield maximum transmission. All transmission peaks have a FWHM of 4 pm.

for the transmitted peak width as long as the maximum heat level is adjusted to give an overall phase shift of 180°. This is seen to be true in Fig. 2, where the transmission spectra for different heater widths are plotted, in each case with a maximum heat value adjusted to yield maximum transmission. Again, a rising base level is seen for the two widest heater widths. The fact that the transmission peak width is independent of the heater width is rather useful when incorporating the technique in a component, since the mechanical design and size of the heater elements is not crucial for the performance of the filter.

The smallest width of the transmission peak is determined by the refractive index modulation depth and chirp rate. This can be explained by the fact that a stronger modulation depth or a smaller chirp rate both result in a longer effective length over which light of each wavelength is reflected. A longer DFB structure, in turn, means a better defined resonance wavelength and thus a narrower transmission peak. This behaviour is verified in Figs. 3 and 4, respectively.

It is impractical to have very long grating components, both during grating manufacturing, component design, and assembly. It is therefore better to strive for a large modulation depth than for low chirp rates. In order to achieve large enough modulation depths, highly sensitized fibres have to be used for the grating. As an example, a 20 nm chirped and 80 mm long grating requires a modulation depth in the order of $8 - 10 \times 10^{-4}$ for a line width of less than 10 pm.
3. Experimental results

In the experiments, we used a 220 mm long and 60 nm chirped grating with an apodization of approximately 10 mm at each end. This corresponds to a chirp rate of 2.73 nm/cm. The grating modulation depth is in the order of $1 \times 10^{-3}$, yielding a stop band with a minimum
of 30 dB rejection (cf. Fig. 5). The grating was inscribed in highly sensitized fibre with the sequential writing method described in ref. 1 and then annealed for several days in 190°C.

A standard thermal printer head from Rohm Co., Ltd was used for the heating. This head has 640 heater elements spread over 80 mm, which means that we could only apply it over the middle third of the test grating, corresponding to a total tuning range of somewhat less than 20 nm. The control of the printer head does not include the ability to set an independent and continuous heat level for each element, as we ideally would like to have in our experiments. Instead, the heat is controlled by the duty cycle of a square wave that is sent at the same time to all elements that are currently activated. This is a rather crude method for our purpose and the measured spectra may be time dependent due to the cooling during the non-active part of the duty cycle. In the following experiments we adjusted the duty cycle until the result was stable over time. The fibre was mounted in a narrow cut on a piece of cardboard, which in turn was taped firm on a small table onto which the printer head was lowered. The spectra were then obtained with an optical spectrum analyser and a tunable laser set to continuous sweep. The spectral resolution is given by the laser line width of 1 pm.

We have in the present work focused on showing the potential of the technique and have not optimized the performance for all parameters. Developing a more precise control of the individual heater elements as well as mounting the fibre in good thermal contact to a heat sink will definitely improve the performance, at least with respect to stability and noise.6

Even with our crude set-up, the results look very promising. Figure 6 shows a close-up of the transmission spectra for a few different heater widths. A width of 5 elements

Fig. 5. Transmission spectrum of the test grating without any heating applied.
corresponding to 625 \( \mu m \) gives the cleanest peak with little noise and near 30 dB rejection outside the transmission band, whereas wider heated regions result in several peaks and a notably smaller rejection. The same behavior is also seen in the simulations in Fig. 1. It is expected that there are discrepancies in the details between the two figures, since phase errors in the grating will have a large impact on the spectral response close to the noise floor. Optimizing the heating duty cycle for a 625–\( \mu m \) wide spot resulted in a full width at half maximum for the spectral window of less than 5 pm (see Fig. 7). Applying other duty cycles
decreases the strength and moves the central wavelength of the window. Tuning the strength of the transmission peak therefore involves adjustment of both applied heat and position of the heater.

The linearity of the heating position versus central transmission peak wavelength is approximately 0.5 nm as shown in Fig. 8. This graph was obtained by measuring the peak position while heating the grating over the whole range, sampled every 20 pixels with a 10 pixel wide heating area. Even though the smallest distance between two adjacent transmission peaks is in the order of 30 pm while tuning a single peak, opening two transmission windows at the same time only allows for a resolution of 0.5 - 1 nm, since a closer spacing causes the windows to affect each other. However, this figure can probably be decreased by introducing a more advanced and independent heat control for each pixel including heat sinks. This will also enable more advanced heating profiles allowing e.g. for a better control of the transmission peak width.\textsuperscript{11}

It is a simple task to open up several windows with arbitrary positions within the tuning range, thus creating comb filters with individually tuned and switchable windows (cf. Fig. 9).

4. Conclusions

In conclusion, we have shown that the width of a transmission peak formed by local heating of a chirped grating mainly depends on the refractive index modulation depth and chirp rate. The width of the heated region is of less importance as long as it is kept within reasonable limits.
Fig. 9. Three transmission windows opened at the same time and spread out over the tuning range.

We have demonstrated a tunable transmission filter based on a chirped Bragg grating. The number of peaks as well as their strength and position over a range of 18.5 nm can be software configured by using a computer controlled array of 640 heater elements. Transmission peaks with full widths at half maximum of less than 5 pm have been demonstrated.

The advantage of this solution compared to other heater-based tuning techniques is that all parameters are easily controlled by software, allowing for a simple and effective design of all kinds of filters.

Our present setup does not allow for individual control of the amount of heat of each heater element. Applying more sophisticated control electronics for the heater array along with better thermal mounting of the Bragg grating fibre has the potential of yielding even better results and additionally enable a precise and individual control of position, width and strength of each transmission window.

Acknowledgements

The authors wish to thank Rohm Co., Ltd for providing the thermal printer head and Proximion Fibre Systems AB for providing the test grating. This work was partially funded by Ericsson Microwave Systems AB.

References


