Development of Accurate Reduced Order Models in a Simulation Tool for Turbomachinery Aeromechanical Phenomena

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If you want to find the secrets of the universe, think in terms of energy, frequency and vibration.

-Nikola Tesla-
Abstract

Modern gas turbines are still vulnerable to vibrations when operated at certain speeds. This unstable environment can lead to high cycle fatigue (HCF) and damage several of the components inside the turbine. Since engineers are striving to increase the turbines’ efficiency with thinner and more complex blade shapes, these critical speeds will always be present. For these reasons, aeromechanical analyses that is the study of structural and aerodynamic forces need to be assessed with a high level of accuracy. Since this type of analysis are very computational expensive, reduced order models (ROMs) are utilized to decrease the degrees of freedom (DoF) for a faster computation without compromising the accuracy. The present work focuses on cyclic and noncyclic ROMs implemented in an already existing aeroelastic tool, with different characteristics in their condensation and ease of usage depending on the analysis.

The AROMA (Aeroelastic Reduced Order Model Analysis) tool has been previously developed to predict the fatigue life of turbomachinery blades with the use of ROMs. The aim of this work has been to improve the tool in terms of accuracy, flexibility and speed, by employing additional reduction methods capable to predict forced responses analysis of large industrial-size models.

The understanding of an aeroelastic phenomena would not be complete if mistuning is not considered in the analysis. A mistuned bladed-disk means that all its sectors do not share the same mass and stiffness properties, which in reality this is the case. Mistuning can be addressed as probabilistic, taking into account the manufacturing tolerances and wear of the bladed disk, or it can be assessed as deterministic, also known as intentional mistuning. The latter is achieved to increase the flutter stability by breaking the circumferential traveling waves modes due to energy confinement, and also to have a certain understanding of the forced response amplitude, which helps in designing for worst and best case scenarios.

The ROMs that have been incorporated in the AROMA tool are known as the component mode synthesis (CMS) and subset nominal mode (SNM) approaches. The CMS is split into two branches, these are the fixed- and free-interface methods known as Craig-Bampton (CB) and Craig-Chang (CC), respectively. An intensive study with numerical and experimental validation has been performed for these three reduction methods. The outcome of the study is that each of these methods have their own drawbacks and benefits depending on the aeromechanical analysis problem. The SNM showed that it produces fast computations, with high level of accuracy when the mistuning level is low. On the other hand, a novel and unique approach, Craig-Chang multisubstructuring (CCMS), demonstrated fast computations and high accuracy when the mistuning level is high.
Sammanfattning

Moderna gasturbiner är fortfarande sårbara för vibrationer under drift vid vissa hastigheter. Denna instabila miljö kan leda till högcykelutmattning (HCF) och skada flera av komponenterna inuti turbinen. Eftersom ingenjörer strävar efter att öka turbinens verkningsgrad med tunnare och mer komplexa skovelformer, kommer dessa kritiska hastigheter alltid att finnas. Av dessa skäl måste aeromekaniska analyser, vilka är studier av strukturella och aerodynamiska krafter, bedömas med hög noggrannhet. Eftersom denna typ av analys är väldigt beräkningsmässig dyr, används reducerade ordermodeller (ROM) för att minska antalet av frihetsgrader (DoF) för en snabbare beräkning, utan att kompromissa med noggrannheten. Nuvarande arbete fokuserar på cyklika och icke-cyklika ROM som implementeras i ett redan existerande aerelasticiskt verktyg, med olika egenskaper i deras kondens och användarvänlighet beroende på analysen.

AROMA (Aeroelastic Reduced Order Model Analysis) verktyget har tidigare utvecklats för att förutse utmattning hos turbomaskinskolvare genom att använda reducerade ordermodeller. Syftet med detta arbete har varit att förbättra verktyget när det gäller noggrannhet, flexibilitet och beräkningshastighet genom att använda ytterligare reduktionsmetoder som kan prediktera påtvingad aerelasticisk respons för storskaliga modeller som används i industrin.

Förståelsen av ett aerelasticiskt fenomen skulle inte vara fullständigt om asymmetrier (mistuning) inte beaktas i analysen. Mistuning i det här fallet innebär en störning av strukturella egenskaper hos en rotorskiva, så att olika sektorer av skivan innehar olika mass- och styvhets parametrar. Mistuning kan hanteras som probabilistisk, genom att ta hänsyn till tillverkningstoleranser och slitage på skovlar, eller det kan bedömas som deterministisk, som är även känd som avsiktlig mistuning. Det senare används för att öka fladerstabilitet genom att bryta energin i de löpande vågornas modformer och också för att ha en viss förkänsla av tvångsresponsamplituden som hjälper till att utforma för värsta och bästa fallscenarier.

De ROM som har integrerats i AROMA verktyget är kända som metoder för component mode synthesis (CMS) och subset nominal mode (SNM). CMS är vidare uppdelat i två granar: fasta gränssnitts metoder som kallas för Craig-Bampton (CB) och fria gränssnitts metoder under namnet Craig-Chang (CC). En intensiv studie med numerisk och experimentell validering har utförts för dessa tre reduktionsmetoder. Utfallet av studien är att varje av dessa metoder har sina för- och nackdelar beroende på den aeromekaniska analysen. SNM visade att det producerar snabba beräkningar, med hög noggrannhet när störningsnivåer är låga. Ett nytt och unikt tillvägagångssätt Craig-Chang multisubstructuring (CCMS) demonstrerade snabba beräkningar och hög noggrannhet även för höga störningsnivåer.
Preface

This thesis has been performed at the Heat and Power Technology Division at the Royal Institute of Technology (KTH) with financial support from the Swedish Energy Agency via the Turbopower COMP10 program, with the collaboration of the industrial partners GKN Aerospace and Siemens Industrial Turbomachinery AB. Also it has been partly funded from the Swedish National Space Board (Rymdstyrelsen) and Vinnova, via the NRFP-AROMA Space program in a consortium between KTH and GKN Aerospace. A visit to Duke University took place as part of a mobility funded from the European Institute of Innovation and Technology through the Innoenergy PhD School. The GIInde 5 Consortium provided the test data during the mobility period. All the financial and technical support throughout the different projects is gratefully acknowledged.

This dissertation is a compilation of four scientific papers (two journal and two conference papers) and complemented with theoretical background and additional work that has been done in the different projects.

List of Appended Papers


**Contribution:** M. Gutierrez developed the idea of using mistuned modes in the noncyclic approach and coded all the mathematical formulations, performed the analysis and the writing. R. Bladh and H. Mårtensson were involved in the methodology and discussion. T. Fransson and D. Vogt supervised and reviewed the work.


**Contribution:** M. Gutierrez originated the idea of the noncyclic method, developed and coded the mathematical formulations, performed the analysis and the writing. R. Bladh and H. Mårtensson were involved in the methodology and discussion. P. Petrie-Repar and D. Vogt supervised and reviewed the work.


**Contribution:** M. Gutierrez performed the structural analysis and the writing. T. Gezork, and S. Yan performed the aerodynamic analysis. C. Andersson gave insights from an industrial perspective. T. Fransson and D. Vogt supervised and reviewed the work.


**Contribution:** M. Gutierrez coded the formulations, and performed the analysis and the writing. R. Kielb was involved in problem formulation and methodology. N. Key gave insights from the experimental data and reviewed the work. P. Petrie-Repar supervised and reviewed the work.

**Additional Papers, Reports and Presentations**


viii P. Petrie-Repar, **M. Gutierrez**, AROMA Synthesis - Reduced Order Model at Industry Level, Presented at Turbopower Program Conference, Linköping University, Linköping, 2015.
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Nomenclature

c aerodynamic damping matrix in physical DoF
\(c\) reduced damping matrix
C damping matrix in physical DoF
\(\dot{C}\) damping matrix in cyclic DoF
E complex Fourier matrix
F force in physical DoF
\(\dot{F}\) force in cyclic DoF
h harmonic index
I identity matrix
\(j\) \(\sqrt{-1}\)
K stiffness and aerodynamic matrix in physical DoF
k aerodynamic stiffness matrix in physical DoF
\(\dot{K}\) stiffness matrix in cyclic DoF
M amount of DoF in a sector
\(\dot{M}\) mass matrix in physical DoF
\(\dot{M}\) mass matrix in cyclic DoF
N number of blades
P modes prescribed in the linearized CFD solver
Q pseudo-inverse matrix built with \(P\) modes
q reduced coordinate
R real Fourier matrix
T transformation matrix
x physical coordinate
\(\ddot{x}\) cyclic or modal coordinates
\(\otimes\) Kronecker product

Greek Symbols
\(\alpha\) inter blade phase angle (IBPA)
\(\gamma\) reduced external force
\(\zeta\) critical damping ratio
\(\kappa\) reduced stiffness matrix
\(\lambda\) eigenvalues
\(\mu\) reduced mass matrix
\(\Phi\) normal or dynamic modes
\( \Psi \)  static/constraint or attachment modes

\( \omega \)  frequency

**Subscripts**

*aero*  related to aerodynamic matrix

\( b \)  set of boundary DoF

*CB*  Craig-Bampton

*CC*  Craig-Chang

*coupled*  aerodynamic coupled force

*exc*  aerodynamic excitation force

*Guyan*  related to Guyan reduction

\( i \)  set of interior DoF

\( k \)  set or related to kept normal modes

\( m \)  set of master DoF

\( r \)  set of rigid body DoF

\( s \)  set of slave DoF

*SNM*  subset nominal mode

*struc*  related to structural matrix

**Superscripts**

\( B \)  blade number

\( b \)  backward TWM

\( D \)  disk number

\( f \)  forward TWM

\( s \)  sector number

\( T \)  transpose or complex conjugate transpose

**Abbreviations**

*AMM*  asymptotic mistuning model

*AROMA*  aeroelastic reduced order model analysis

*CB*  Craig-Bampton

*CBC*  Craig-Bampton cyclic

*CBMS*  Craig-Bampton multisubstructuring

*CC*  Craig-Chang

*CCC*  Craig-Chang cyclic

*CCMS*  Craig-Chang multisubstructuring

*CFD*  computational fluid dynamics

*CMS*  component mode synthesis

*CWB*  chord-wise-bending

*DoF*  degrees of freedom

*EO*  engine order

*EoM*  equation of motion

*FEM*  finite element model

*FMM*  fundamental mistuning mode

*HCF*  high cycle fatigue

*HL*  high loading

*IBPA*  inter blade phase angle
<table>
<thead>
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<th>Acronym</th>
<th>Description</th>
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<td>INFC</td>
<td>influence coefficient domain</td>
</tr>
<tr>
<td>LL</td>
<td>low loading</td>
</tr>
<tr>
<td>MAC</td>
<td>modal assurance criteria</td>
</tr>
<tr>
<td>ND</td>
<td>nodal diameter</td>
</tr>
<tr>
<td>PE</td>
<td>peak efficiency</td>
</tr>
<tr>
<td>ROM</td>
<td>reduced order model</td>
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<td>TWM</td>
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<td>ZZENF</td>
<td>zig-zag engine-order nodal-diameter frequency</td>
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Chapter 1

Introduction

Gas turbines, such as the one in Figure 1.1, are susceptible to blade vibrations that can lead to failure and possibly a catastrophic event. One of the main causes of these vibrations are aeromechanical phenomena, which are due to the interaction of aerodynamic and structural forces. In the past, blade vibrations have been suppressed with robust blade designs (thicker/longer/connected blades) with lower efficiencies. However, in today competitive market, manufacturers are striving for higher efficiencies, lower emissions and lower noise by using complex and innovative blade-shapes that are thinner and more highly loaded. These modern blades are more susceptible to aeromechanical problems. To assure the structural integrity of these new designs, an accurate assessment of the aeromechanical behavior has to be performed.

Figure 1.1: Rolls Royce Trent 7000 (Picture courtesy of Rolls-Royce plc)

The two main aeromechanical problems that affect gas turbines are flutter and forced response. Flutter is a self-induced and self-sustained vibration, whereas a forced response problem occurs when an excitation force matches the natural frequencies of the structure. Both phenomena take into account the inertial and
elastic forces from the structure and the aerodynamic force from the airflow. Within an aeromechanical analysis, the motion of the structure is determined by considering these forces. If the amplitude of the vibration is high enough, this can lead to elastic strain on the material, known as high cycle fatigue (HCF). These phenomena play an important role in every gas turbine development program, as expressed by El-Aini et al. [1]: "while over ninety percent of the potential HCF problems are uncovered during development testing of a new engine, the remaining few accounts for nearly thirty percent of the total development cost and are responsible for over 25% of all engine distress events".

The study of flutter and forced response is challenging due to the many variables that are present in this type of analysis. In the past, aeromechanical analyses have been treated in a simplified way, using lumped mass models such as in Griffin and Hoosac [2] and by Óttarsson and Pierre [3], and using very robust or low fidelity models where the accuracy has been compromised. However, in order to push the aeromechanical limits, accurate analyses are required. These high fidelity analyses require high computational effort, therefore the use of reduced order models (ROMs) has been a common practice for industry and academia.

1.1 Fundamentals in Aeromechanics

Aeromechanical phenomena occur by the interaction of the inertial, elastic and aerodynamic forces, as described by the Collar’s triangle [4] in Figure 1.2. When the system is subjected to only the elastic and aerodynamic forces, it can result in static aeroelasticity, where a twist due to a steady aerodynamic influence can push the strength capabilities of the structure. On the other hand, dynamic aeroelasticity is when the three forces act simultaneously, such as in flutter and forced response. In a bladed-disk or blisk the inertial force is related to its mass, the elastic force is due to its stiffness, whereas the aerodynamic force is an external unsteady force on the blade due to a stator-rotor interaction, such as wakes and potential fields, and also by the unsteady pressure produced by the blade motion itself.
The three forces can also be represented in the equation of motion (EoM) as observed in Eq. (1.1),

\[
\begin{align*}
\mathbf{M}_{\text{struc}} \ddot{x} + \mathbf{C}_{\text{struc}} \dot{x} + \mathbf{K}_{\text{struc}} x &= \{\mathbf{F}_{\text{exc}}\} + \{\mathbf{F}_{\text{coupled}}(x(t), \dot{x}(t))\} \\
(1.1)
\end{align*}
\]

where the structural forces are located on the left side of the equation and the aerodynamic forces on the right side. A qualitative description of where these forces come from can be observed in Figure 1.3. Here, it is clear the importance of performing a stage analysis to obtain the excitation force (\(\mathbf{F}_{\text{exc}}\)), since it takes into consideration the wakes, potential fields and possible shocks from a rotor-stator interaction. Instead, it is considered as a common practice to only use one blade row to obtain the aerodynamic coupled force (\(\mathbf{F}_{\text{coupled}}\)), which is a function of the displacement and velocity of the modes.

\[
\begin{align*}
\mathbf{M}_{\text{struc}} \ddot{x} + \mathbf{C}_{\text{struc}} \dot{x} + \mathbf{K}_{\text{struc}} x &= \{\mathbf{F}_{\text{exc}}\} + \{\mathbf{F}_{\text{coupled}}(x(t), \dot{x}(t))\} \\
(1.1)
\end{align*}
\]

Usually, the aerodynamic coupled force is moved to the left side and split into aerodynamic damping and stiffness terms, expressed as:
\[
[M_{\text{struc}}] \ddot{x} + [C_{\text{struc}} + C_{\text{aero}}] \dot{x} + [K_{\text{struc}} + K_{\text{aero}}] x = \{ F_{\text{exc}} \}.
\] (1.2)

This coupled EoM is utilized for a coupled forced response analysis and it requires high computational effort to be solved. Thus, ROMs are employed to overcome this problem by decreasing the size of the matrices. It is assumed that the unsteady coupled and external forces on the right-hand side are harmonic.

1.1.1 Structural Dynamics

The vibrational characteristics of a bladed-disk are determined with an eigenvalue or modal analysis including only the inertial and stiffness terms in a undamped EoM, expressed as,

\[
[M_{\text{struc}}] \ddot{x} + [K_{\text{struc}}] x = \{ 0 \},
\] (1.3)

and without any aerodynamic term. The EoM is solved in the frequency domain by introducing the assumption of \( x = \tilde{x} e^{\lambda t} \) and stated as,

\[
[\lambda^2 M_{\text{struc}} + K_{\text{struc}}] \tilde{x} = \{ 0 \},
\] (1.4)

where \( \tilde{x} \) is the modal coordinate or modeshape and \( \lambda = j \omega \), where \( \omega \) the angular frequency. Thus, this equation represents a bladed-disk in a vacuum environment. Only real frequencies and real modeshapes are obtained after solving this equation. In turbomachinery, a bladed-disk can be made from an integral single structure, which is known as blisk, or from two different structures composed of the disk and the blade part. The vibration of any of these bladed-disk configurations can be considered as blade- or disk-dominated, depending if it is the blade or the disk driving the frequencies. The blade modes are classified by families (e.g. first bending, first torsion, second bending) as observed in Figure 1.3, but when the blade has a complex geometry, the mode number is sufficient, as seen in Figure 1.4.

![Figure 1.3](image1.jpg)

**Figure 1.3: Blade modes**

The disk modes are classified depending on the nodal diameter (ND) and nodal circle (NC) as shown in Figure 1.5 which are considered as inflection lines where the vibration changes direction.
The ND line’s location depends on the inter blade phase angle (IBPA), which is calculated as $\frac{2\pi ND}{N}$, and $N$ is the number of blades. It is common that at low ND, the bladed-disk modes shapes behave as disk-dominated, and blade-dominated at high ND, as observed in the zig-zag engine-order nodal-diameter frequency (ZZENF) diagram of Figure 1.6. It is important to remark that the frequency increases for each family, which means that the second mode is stiffer than the first one. In addition, the maximum amount of ND is given by $\frac{N}{2}$ when $N$ is even, and $\frac{N-1}{2}$ when $N$ is odd.

Figure 1.6: ZZENF diagram for three mode families of a blisk. Two speed-lines plotted (red and orange), crossing four different resonant crossings at different ND
1.1.2 Flutter

To solve for flutter or stability problems, the excitation force and the structural damping terms are omitted from the coupled EoM, such as,

$$\begin{align*}
[M_{\text{struc}}]\ddot{x} + [C_{\text{aero}}]\dot{x} + [K_{\text{struc}} + K_{\text{aero}}]x &= \{0\}.
\end{align*}$$

(1.5)

The equation is then transformed to the frequency space as an eigenvalue problem with the same substitution used in the modal analysis, such as,

$$\begin{align*}
[\lambda^2 M_{\text{struc}} + \lambda C_{\text{aero}} + K_{\text{struc}} + K_{\text{aero}}] \tilde{x} &= \{0\}.
\end{align*}$$

(1.6)

where $C_{\text{aero}} = \Re(F_{\text{coupled}}/\omega)$ and $K_{\text{aero}} = \Re(F_{\text{coupled}})$, $\omega$ is the frequency of oscillation and $\text{Amp}$ is the amplitude of oscillation. This analysis can be achieved in the influence coefficient (INFC) or in the traveling wave modes domains (TWM). From the unsteady aerodynamics side, the matrices $C_{\text{aero}}$ and $K_{\text{aero}}$ are determined by calculating the unsteady pressure field on adjacent blades due to oscillation of one blade, this occurs in the physical coordinates and known as the INFC domain, as shown in Figure 1.7. The INFC domain gives a clear physical insight of the unsteady aerodynamics in the blade row, where the force acting on a blade due to the motion of another one can be easily derived, as stated by Crawley [5].

![Figure 1.7: Superposition of INFC from blades -1, 0 and +1 projected on the TWM domain at different IBPA. Left: INFC domain/Right: TWM domain](image)

On the other hand, the unsteady pressure field can also be calculated in the TWM domain, where the oscillation occurs in all the blades at a certain mode and
prescribing one ND at a time. In other words, it is solved in an uncoupled harmonic manner by a linear combination of cosine and sine standing waves. Thus, the unsteady pressure is described in the frequency or cyclic space, with an amplitude and a phase. Moreover, when solving the EoM, the structural side should also be transformed to the cyclic space. It is common to transfer the unsteady force from the INFC domain to the TWM domain or vice-versa as following:

\[
\tilde{F}_{TWM} = F_{(-1,0)}e^{-j\alpha} + F_{(0,0)} + F_{(1,0)}e^{j\alpha} = \sum_{n=-N/2}^{+N/2} F_{(n,0)}e^{nj\alpha} \quad (1.7)
\]

where \(\alpha\) denotes the IBPA, \(\tilde{F}\) expresses the force in cyclic form and \(F\) represents the force in the influence coefficient domain, the subscripts \((-1,0)\) denotes the unsteady force subjected on blade -1 due to the motion of blade 0, as observed in Figure 1.7. Then, depending on the IBPA of concern, each of the forces can be mapped to the TWM domain. The advantage of this domain is that the EoM has a decoupled form, where each harmonic can be solved separately. The imaginary part of the TWM at each IBPA is used to obtained the stability diagram, shown in Figure 1.8, and assess if the bladed-disk is prone to flutter.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{stability_diagram.png}
\caption{Stability diagram indicating unstable regions}
\end{figure}

### 1.1.3 Forced Response

As mentioned earlier, a coupled forced response analysis is described with Equation 1.2. The EoM can also be transferred to the frequency domain with the same assumption of \(x = \tilde{x}e^{\lambda t}\), and can be expressed as,

\[
[\lambda^2 M_{\text{struc}} + \lambda C_{\text{aero}} + K_{\text{struc}} + K_{\text{aero}}]\tilde{x} = \{F_{\text{exc}}\}. \quad (1.8)
\]

To solved for \(\tilde{x}\) (vibration amplitude), the terms are arranged as follow:

\[
\tilde{x} = [\lambda^2 M_{\text{struc}} + \lambda C_{\text{aero}} + K_{\text{struc}} + K_{\text{aero}}]^{-1}\{F_{\text{exc}}\}. \quad (1.9)
\]
In this equation, the computational cost is very expensive when the inverse needs to be solved for all the DoF, which is the reason of why ROMs are important in the forced response analysis. The excitation force ($F_{exc}$) depends on an engine order (EO), that it is also harmonic in time and discrete in space around the bladed-disk. Thus, in the relative frame of reference, the force moves circumferentially with a fixed amplitude around the blades. The EO is also known as excitations per revolution (EO/rev), and it is analogous to the ND in a structure. Hence, resonance can occur leading to HCF problems, if the EO of the excitation force matches the ND of the structure at a similar frequency. This resonance points are indicated in Figure 1.6 at the places where the speed-lines crosses the natural frequencies. For example, the red line crosses the first mode family at 8ND, which means that an 8EO can excite this mode, since it matches the vibrational pattern and the excitation frequency at around 700Hz. The range of EO from 1 to 5 are very common in gas turbines and needs to be considered for forced response. Another diagram that has a clear view on the resonance crossings with respect to the rotor speed is the Campbell diagram, shown in Figure 1.9. Here, the first five EO and also the higher EO due to stator-rotor interactions are plotted. The upstream stator wakes excite the rotor depending in the number of blades, for example, 20 stator vanes excite the rotor with 20EO. The same occurs with the downstream stator due to the potential fields and depending on the number of blades. The resonance crossings can be avoided by modifying the structure’s natural frequency

![Campbell Diagram](image-url)
or by changing the amount of stator vanes. However, in some designs it is impossible
to avoid all the resonance crossing within the desired operating range. For this
reason, forced response analyses are required to predict the vibration amplitudes
on these crossings and then obtain the alternating stress. Combined with the mean
stress that is calculated in a static analysis, these two will determine if the bladed-
disk fails or not due to HCF. Both of these stresses vary along the bladed-disk, as
observed in Figure 1.10.

![Figure 1.10: Stresses](image)

The bladed-disk is then assessed with a Goodman or Haigh diagram. An exam-
ple of a Haigh diagram is shown in Figure 1.11. The diagram considers the mean
and alternating stresses, and the combination of both determines if it is under a
safe or unsafe region. The safe area is limited by the maximum allowable fatigue
and yield stresses of the material. An infinite life is achieved, if all the different
points around the bladed-disk are located inside this region. Important to remark
that the alternating stress margin decreases, as the mean stress increases towards
the maximum yield stress. Thus, highly loaded blades have less HCF margin in
their vibration amplitude.
1.1.4 Structural Mistuning

The HCF margin from the previous section usually narrows when mistuning is taken into account. Mistuning will occur due to minor imperfections in a bladed-disk, due to wear, deterioration, damage, corrosion, and manufacturing tolerances. These imperfections are unavoidable in a real bladed-disk, breaking the harmonic patterns of the TWM. Mistuning can be addressed in a probabilistic way, such as in Sinha and Chen [8], where a simplified bladed-disk model was solved with a higher order technique. Moreover, mistuning can be achieved intentionally by modifying the structural characteristics of the bladed-disk. The latter is intended when a more stable design in terms of flutter is needed, as shown by Kaza and Kielb [9] with the use of a cascade, and also to design for the best case-scenario with respect to the forced response amplitudes as in Castanier and Pierre [10], leading to an increase in the fatigue life.

1.2 Reduced Order Models

Several approaches exist in literature to analyze the dynamic characteristics of a bladed-disk with their own benefits and drawbacks. One of the alternatives is to analyze a full model of the 360° bladed-disk, as in Figure 1.12(a). Another approach is to use a sector model with cyclic symmetry boundary conditions, as in Figure 1.12(b). Certainly, the use of these two approaches implies high computational effort, and sometimes it is even impossible when all the aeromechanical forces have to be taken into account. Thus, the reduction methods have to be applied to either the full or sector model.

When using the sector model with cyclic symmetry boundary conditions, a harmonic description is satisfied for the generalized coordinates. These coordinates
describe perfectly the vibrational pattern for a tuned undamped bladed-disk. A linear combination of these tuned modes can serve as a basis for the mistuned models, as stated in Bladh and et al. [11], [12]. The basis is accurate enough for bladed-disks with low level of mistuning; however, it is not as accurate for large mistuning, since the traveling waves starts breaking when the level of mistuning increases.

Several reductions methods have been used in the past, the most common ones have been Guyan, component mode synthesis (CMS) that was first introduced by Hurty [13], and subset nominal mode (SNM) by Yang and Griffin [14]. All these ROMs have their own transformation matrices ($T$) that are built with different set of modes. These matrices aim to span the physical DoF of a bladed-disk to a reduced space, which is expressed as

$$x = Tq,$$  \hspace{1cm} (1.10)

where $x$ and $q$ imply the physical and reduced DoF, respectively. The difference between each of the reduction techniques rely on the basis or set of modes that build each of their transformation matrix. The reduction of the EoM when applying the transformation is stated as,

$$T^T [M_{struc} ] \ddot{q} + T^T [C_{struc} + C_{aero}] \dot{q} + T^T [K_{struc} + K_{aero}] q = T^T \{ F_{exc} \}. \hspace{1cm} (1.11)$$

The projection of the transformation matrix generates the EoM in the reduction space as follows:

$$[\mu_{struc}] \ddot{q} + [c_{struc} + c_{aero}] \dot{q} + [\kappa_{struc} + \kappa_{aero}] q = \{ \gamma_{exc} \}, \hspace{1cm} (1.12)$$

where $\mu$ and $\kappa$ define the reduced mass and stiffness, respectively. The $c$ stands for the reduced damping and $\gamma$ for the reduced force.
1.2.1 Cyclic Symmetry Transformation

Rotational periodicity in bladed-disks is a characteristic that can be exploited with the use of cyclic symmetry, which means that the DoF need to be transformed from the physical (e.g. Cartesian or cylindrical) to the cyclic space that is defined with TWM, this transformation is expressed as,

\[ x = (E \otimes I)\tilde{x}, \quad (1.13) \]

where \( I \) and \( E \) stand for identity and Fourier matrices, respectively. The basis or vectors of the Fourier matrix have the characteristics that depends on the TWM harmonic index, which is limited by the number of blades \( N \), such that,

\[
E = \begin{bmatrix}
\frac{1}{\sqrt{N}} & \frac{1}{\sqrt{N}} e^{j\alpha} & \frac{1}{\sqrt{N}} e^{j2\alpha} & \cdots & \frac{1}{\sqrt{N}} e^{j(N-1)\alpha} \\
\frac{1}{\sqrt{N}} & \frac{1}{\sqrt{N}} e^{j2\alpha} & \frac{1}{\sqrt{N}} e^{j4\alpha} & \cdots & \frac{1}{\sqrt{N}} e^{j2(N-1)\alpha} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\frac{1}{\sqrt{N}} & \frac{1}{\sqrt{N}} e^{j(N-1)\alpha} & \frac{1}{\sqrt{N}} e^{j2(N-1)\alpha} & \cdots & \frac{1}{\sqrt{N}} e^{j(N-1)(N-1)\alpha}
\end{bmatrix}, \quad (1.14)
\]

where \( \alpha \) defines the IBPA of the TWM. The matrix \( E \) has a complex form, which means that \( \tilde{x} \) is also complex; however, there exist another Fourier formulation that is represented in a real form, such as,

\[
R = \begin{bmatrix}
\frac{1}{\sqrt{N}} & \frac{1}{\sqrt{N}} \sqrt{\frac{2}{N}} & 0 & \cdots & \frac{1}{\sqrt{N}} \\
\frac{1}{\sqrt{N}} & \frac{1}{\sqrt{N}} \sqrt{\frac{2}{N}} \cos \alpha & \frac{1}{\sqrt{N}} \sqrt{\frac{2}{N}} \sin \alpha & \cdots & \frac{-1}{\sqrt{N}} \\
\frac{1}{\sqrt{N}} & \frac{1}{\sqrt{N}} \sqrt{\frac{2}{N}} \cos 2\alpha & \frac{1}{\sqrt{N}} \sqrt{\frac{2}{N}} \sin 2\alpha & \cdots & \frac{1}{\sqrt{N}} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\frac{1}{\sqrt{N}} & \frac{1}{\sqrt{N}} \sqrt{\frac{2}{N}} \cos (N-1)\alpha & \frac{1}{\sqrt{N}} \sqrt{\frac{2}{N}} \sin (N-1)\alpha & \cdots & \frac{-1^{(N-1)}}{\sqrt{N}}
\end{bmatrix}, \quad (1.15)
\]

where the last column is included just if \( N \) is an even number. Here, the basis for the matrix \( R \) are real standing waves and the same occurs with \( \tilde{x} \), when this \( R \) transformation is instead used in Eq. (1.13). The real form of the Fourier matrix is commonly applied in commercial software, and basically this is the reason of why they use two isolated sectors (i.e. cosine and sine) in their analyses.

The advantage of using cyclic symmetry is the low computational cost when the reduction takes place, since only one sector model is needed. Cyclic symmetry boundary conditions strictly mean that the right-hand interface of a sector shares a symmetry with the left-hand interface. Thus, both interfaces have the same vibration amplitude, but one interface is lagging in phase with respect to the other, depending on the harmonic index of the TWM. As described in Bladh [15], the
1.2. REDUCED ORDER MODELS

cyclic symmetry formulation can be achieved from an integral structure like Figure 1.12(a) or from an isolated sector as in Figure 1.12(b). The stiffness matrix of an integral structure can be expressed as,

\[
K = \begin{bpmatrix}
K_1 & K_2 & 0 & \cdots & 0 & K_2^T \\
K_2^T & K_1 & K_2 & \cdots & 0 \\
0 & K_2^T & K_1 & K_2 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & K_1 & K_2 \\
K_2 & 0 & 0 & \cdots & K_2^T & K_1
\end{bpmatrix},
\]  
(1.16)

here \( K_1 \) refers to the DoF of one sector that are not connected with any DoF of the adjacent sector. On the contrary, \( K_2 \) indicates the coupling terms between adjacent sectors. As it can be observed, this stiffness matrix has a circulant form and can be transformed to a block decoupled matrix when projected with the Fourier matrix \( E \), as defined in Olson [16],

\[
\tilde{K} = (E \otimes I)^T K (E \otimes I) = \begin{bpmatrix}
\tilde{K}_1 & 0 & 0 & \cdots & 0 & 0 \\
0 & \tilde{K}_2^f & 0 & \cdots & 0 & 0 \\
0 & 0 & \tilde{K}_3^f & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & \tilde{K}_3^b & 0 \\
0 & 0 & 0 & \cdots & 0 & \tilde{K}_2^b
\end{bpmatrix}.
\]  
(1.17)

In here, \( f \) and \( b \) mean forward and backward TWM, whereas the subscript (i.e. 1,2,3) represents the harmonic index and the tilde (~) refers to cyclic coordinates as in Castanier and Pierre [17]. The entire matrix is \( MN \times MN \), where \( M \) is the amount of DoF per sector, and every block diagonal matrix (\( \tilde{K}_1 \)) is a \( M \times M \) matrix. The Fourier matrix \( R \) can be used in the same fashion as \( E \) to decouple the matrices, but the outcome is a pseudo-block diagonal matrix with \( 2M \times 2M \) blocks that refers to the double harmonics, except when the harmonic equals one and for the last harmonic if \( N \) is even.

Moreover, another approach is to use the isolated sector instead and assemble it altogether as in Eq. (1.16), but without the coupling terms, since all the sectors are isolated now. Then after the transformation to the cyclic form, the matrix can be constrained with displacement compatibility equations. For the sake of brevity, only the stiffness matrix has been shown; however, the mass matrix is analogous to the stiffness matrix. The advantage of decoupling the stiffness and mass matrices is
that the EoM can be solved for every block-diagonal separately, which significantly reduces the computation effort. Note that this applies only to tuned bladed-disk.

**Aerodynamic Forces from TWM to INFC**

As it has been mentioned, the $K_{aero}$ and $C_{aero}$ can be described in the INFC or TWM domain. However, it is a best practice to obtain the unsteady forces in the TWM domain using a 3D linearized CFD solver. The approach consists in using the modeshapes of interest, as the ones shown in Figure 1.6, to prescribe a motion in the CFD analysis. This means, that all the modes within the frequency spectrum of concern need to be solved to obtain the unsteady forces, including every nodal diameter in backward and forward TWM. After the analyses, the unsteady forces are gathered and formulated in a TWM matrix that is decoupled for every harmonic. As it has been mentioned in Crawley [5] and presented in brevity in Equation 1.7, the Fourier matrix $E$ can be utilized to transform between domains as in,

$$F_{aero} = (E \otimes I)\hat{F}_{TWM}(E \otimes I)^{-1}$$

(1.18)

where $(E \otimes I)^{-1}$ equals to $(E \otimes I)^T$, since $(E \otimes I)$ is an orthogonal matrix. This transformation is possible if $C_{aero}$ is a square matrix, where the same amount of modes as DoF need to be included in the matrix. This is impractical when the bladed-disk is modeled with a large amount of DoF. For this reason, one of the solution is to use a pseudo-inverse, as in Adhikari [18], or referred to a multimode least square (MLS) method as in Mayorca [19]. For the MLS method, first the TWM forces need to be projected to the INFC domain for every IBPA on the reference blade. Then, these forces are projected with a basis that is built with the MLS method, as follows,

$$Q = [P_{j,i}^T P_{i,j}]^{-1}[P_{j,i}^T] \text{ where, } i = 1, 2, ..., M \text{ } j = 1, 2, ..., m$$

(1.19)

where $P$ are the eigenvectors or modes ($m$) used in the CFD calculations and taking into account all the NDs. The matrix $Q$ has dimensions of $m \times M$ and it is used to project back the unsteady forces that are solved in the linearized solver to the physical coordinate. Thus, the $K_{aero}$ term indicating the real part of the unsteady forces that are project back as follow:

$$k_{aero,blade} = [\Re(\hat{F}_{INFC})][Q]$$

(1.20)

and for the $C_{aero}$ term is obtained by,

$$c_{aero,blade} = \frac{[\Im(\hat{F}_{INFC})]}{\omega_j}[Q]$$

(1.21)
1.2. REDUCED ORDER MODELS

where $\hat{F}_{INFC}$ has dimension of $M \times m$. Thus, the dimensions for $K_{aero}$ and analogous for the $C_{aero}$ is of $MN \times MN$, which can be expressed as,

\[
K_{aero} = \begin{bmatrix}
k_{aero,0} & k_{aero,1} & 0 & \cdots & 0 & k_{aero,-1} 
k_{aero,-1} & k_{aero,0} & k_{aero,1} & \cdots & 0 & 0 \\
0 & k_{aero,-1} & k_{aero,0} & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & k_{aero,0} & k_{aero,1} \\
k_{aero,1} & 0 & 0 & \cdots & k_{aero,-1} & k_{aero,0}
\end{bmatrix}.
\]  

(1.22)

1.2.2 Guyan

The Guyan method represents a static condensation without considering the inertial term, which makes it one of the less accurate reduction methods. Its transformation matrix is described as,

\[
T^{(s)}_{\text{Guyan}} = \begin{bmatrix}
I_{m,m} \\
-k_{s,s}^{-1}k_{s,m}
\end{bmatrix} = \begin{bmatrix}
I_{m,m} \\
\Psi_{s,m}
\end{bmatrix} 
\]  

(1.23)

where $m$ and $s$ denote the master and slave DoF, respectively. Both, master and slave DoF represents the total amount of the physical DoF. The difference is that the master DoF are kept after the Guyan reduction, whereas the slave DoF disappear. The advantage of the Guyan method is that when the EoM is in the reduced space, a physical definition of the master nodes is still valid. The drawback is that first, the master DoF are selected manually depending on the modes of concern, which means it does not allow a one size-fits-all strategy. Second, it accounts for a significantly lack of accuracy, since not every DoF is reflected in the reduced space and it only takes into account the static behavior.

1.2.3 Component Mode Synthesis (CMS)

The CMS approach developed by Hurty has inspired different authors to introduce new methods, such as the fixed-interface and free-interface, known as Craig-Bampton [20] (CB) and Craig-Chang [21] (CC) respectively. These approaches deals with the reduction of one component at a time, and then all the reduced components are assembled together. If a cyclic formulation is imposed, the components are split into blade and disk; however, if the multisubstructuring formulation is imposed, each sector represents a component.
CHAPTER 1. INTRODUCTION

Fixed-Interface Formulation (Craig-Bampton)

This type of reduction uses two set of modes, which can be categorized as static and dynamic modes. The static or constraint modes are similar to the Guyan basis, but in this case the master DoF are the ones located at the interface of each of the components, whereas the slave DoF are the remaining ones. In other words, the static modes can be explained as a static deformation of the slave DoF when all the master DoF are moved a unit displacement one by one. The CB transformation is described as,

$$\mathbf{T}_{CB}^{(s)} = \begin{bmatrix} \Phi_{b,k} & I_{b,b} \\ \Phi_{i,k} & \Psi_{i,b} \end{bmatrix} \begin{bmatrix} \Phi_{i,k} \\ \Psi_{i,b} \end{bmatrix} = ,$$ (1.24)

where $b$ stands for the boundary or interface DoF, $i$ denotes the interior DoF, and $k$ expresses the kept modes. These kept modes are a set of truncated dynamic modes that have been obtained from an eigenvalue problem. These modes are calculated by fixing the interface DoF, which allows the assembly of all the components by enforcing displacement compatibility at these interfaces. The amount of static modes is the same as the number of interface DoF.

Free-Interface Formulation (Craig-Chang)

The free-interface formulation is known as the CC reduction. As a CMS approach, it also takes into consideration a set of dynamic modes, and instead of static modes it is complemented with attachment modes. The transformation differs when a component is constrained or unconstrained, the basis for the former is described as,

$$\mathbf{T}_{CC}^{(s)} = \begin{bmatrix} \Phi_{b,k} & \Psi_{b,b} \\ \Phi_{i,k} & \Psi_{i,b} \end{bmatrix},$$ (1.25)

whereas for unconstrained components, a different basis is used to take into account for rigid body modes, as expressed in,

$$\mathbf{T}_{CC}^{(s)} = \begin{bmatrix} \Phi_{b,k} & \Psi_{b,b} & \Phi_{b,r} \\ \Phi_{i,k} & \Psi_{i,b} & \Psi_{i,r} \\ \Phi_{r,k} & \Psi_{r,b} & \Psi_{r,r} \end{bmatrix},$$ (1.26)
where \( r \) stands for the rigid body DoF. The reason of using another basis is because the attachment modes are computed from the flexibility matrix, which is the inverse of the stiffness matrix. Since the inverse does not exist when the component is unconstrained, a set of rigid body modes are needed to make the stiffness matrix non-singular.

### 1.2.4 Subset Nominal Mode

The transformation matrix is built with only dynamic modes, such as,

\[
T_{SNM} = \begin{bmatrix} \Phi_k \end{bmatrix}
\]

It is a remarkable advantage when building each harmonic in a block-diagonal form, as shown in Eq. (1.17). After applying the cyclic symmetry and solving an eigenvalue problem within the cyclic space, the set of eigenmodes are truncated depending on the amount of DoF required in the reduction.

Other reduction methods that are considered branches of the SNM, is the fundamental mistuning model (FMM) developed by Feiner and Griffin [22], it considers the excitation of only a single family of modes that are closed together in frequency. Then a similar approach is taken by Martel et al. [23] with the asymptotic mistuning model (AMM). Another branch is the modified modal domain analysis (MMDA) by Sinha [24], which applies non-nominal modes that are obtained by proper orthogonal decomposition.

### 1.3 State-of-the-Art

In the past, there have been two dominant methods to address the prediction of forced response. These are the decoupled and the coupled methods. The former one, solves the structural and the aerodynamic forces independently from each other, and then these are gathered together to be solved in the frequency domain. It relies on fast computational times and high flexibility since the parameters can be modified independently when solving for either the structural or the aerodynamic parts. The coupled method is usually solved in the time domain when the nonlinearities are considered, or in the frequency domain with low computational cost. This method solves interchangeably the structural and aerodynamic forces, where one serves as an input to the other at every time step. Two of the pioneers
for the decoupled method has been Kaza and Kielb [9], where two rigid body modes of a blade in a cascade were considered. Mistuning was applied by perturbing the frequencies by a random factor, and the aerodynamic side was solved using an incompressible unsteady flow. The study helped to understand the beneficial effects of mistuning on flutter. Then, another decoupled method was employed by Griffin and Hoosac [2] by using a lumped mass model. The model was capable of representing the disk and blade masses, and the structural coupling effect of the disk. Later, Crawley [5] introduced a detailed explanation between the traveling wave, individual blade and standing wave decoupled formulations that can be applied to an actual bladed-disk, where frequency mistuning, structural and aerodynamic coupling effects were considered. In the early nineties, Chiang and Kielb [25] presented one of the first reduction methods applied to bladed-disks in gas turbines. A truncation of modes has been applied in the traveling wave domain. Moreover, a three-dimensional aeroelastic effect was also introduced by incorporating a decoupled method, using two-dimensional aerodynamic calculations stacked together and a three-dimensional finite model. One of the first coupled methods was performed by Sayma and et al. [26], where the cyclic symmetry approach was used to reduced the amount of DoF. The method was able to treat friction damping, and with aerodynamic and structural coupling effects. In addition, a three-dimensional mesh motion was considered by prescribing the modeshape of interest. The detailed features from the structural and the aerodynamic simulations, led to advanced reduced order models to lessen the computational efforts. Thus, it was Green [27] that described an aeroelastic decoupled method using reductions such as Guyan and the CB. It was able to include friction and aerodynamic damping, frequency mistuning, and a detailed description of the structural behavior. However, it still had a significant amount of DoF due to the low reduction capabilities of these two ROMs. During the same year, Moffat and He [28] implemented two coupled method where the structural and aerodynamic equations were solved simultaneously, one in the frequency-domain and the other one using a time marching approach. The intention was to include the forced prediction of a bladed-disk during the design phase, by using a modal decomposition ROM. These two methods have been compared to a decoupled one, which was preferred since the coupled methods were altered by a frequency shift when predicting the forced response. This decoupled method was built for only single modes and using a modal decomposition as the only ROM.

Another decoupled method was implemented by Mayorca [7], which was the previous version of the aeroelastic tool named AROMA (Aeroelastic Reduced Order Model Analyses) that has been developed in the past at KTH. The tool was able to consider aerodynamic coupling from the interaction of various modes using a multimode least square method [19]; however, it was lacking in flexibility and accuracy due to only having the Guyan reduction [29], as its only ROM. Moreover, it was not able to solve for industry-size parent structural models.

Currently, a one-size-fits-all strategy does not exist in industry, where it is common to combine commercial software with their own in-house tools to solve for
flutter and forced response problems. There is no tool that can address a more complete aeromechanical chain during their design phase, which can include accurate ROMs, probabilistic and deterministic mistuning, friction damping, aerodynamic force and damping altogether. In addition, it is common to only have one reduction method within their best practices to solve for all the aeromechanical analysis, without differentiating about the characteristics of the reduction.
1.4 Objectives

The main objective of this thesis is to apply current and modified ROMs (reduced order model) to an existing aeromechanical design tool (AROMA), so that AROMA can be used to determine accurately and efficiently the forced response of a typical industrial case, which includes large mistuning and friction damping.

The following tasks have been performed to achieve the main objective:

- Compare different ROMs that are currently used in industry and academia.
- Implement the SNM method to include small mistuning in a probabilistic way.
- Develop noncyclic ROMs to analyze large and geometric mistuning on bladed-disk.
- Implement the ROMs in the AROMA tool.
- Validate the forced response predictions numerically and experimentally.

1.4.1 Methodology

This work has been structured to address the aeromechanical needs from an industrial perspective. The requirements were determined by the partners of the Turbopower program. The main concern among industry is to be able to accurately predict forced response with deterministic and probabilistic mistuning. Moreover, their desire is an aeromechanical tool that can achieve fast computational times, so it can be incorporated during the design phase.

The current work was complemented into the AROMA tool that was previously developed at KTH. In order to meet with the requirements from industry, the work has been focused on the implementation of efficient and accurate ROMs. A detailed literature survey has been performed to understand the benefits and drawbacks of various ROMs due to the different parameters involved in an aeromechanical simulation, such as mistuning, friction damping, aerodynamic coupling, structural coupling and forcing functions. Several ROMs have been chosen and coded in AROMA. In order to address large deterministic mistuning, a novel ROM approach from the CMS family is developed. The cyclic symmetry formulation has been omitted, since it is known that when mistuning is encountered the traveling wave patterns are broken. Therefore, the novel approach focus on a multishubstructuring alternative, focused on building the basis with mistuned modes already.

Unsteady aerodynamic simulations were performed to obtain the forcing functions and the aerodynamic coupling forces that are essential in the aeroelastic equation. The forcing functions are the results of an unsteady Reynolds averaged Navier Stokes (URANS) time-marching simulations that are run in ANSYS CFX, whereas the aerodynamic coupling forces are calculated using GKN Aerospace in-house code.
LINNEA, which is a 3D linearized solver where different modes and nodal diameters are solved. Then, these unsteady forces have been transferred or mapped to the FE domain.

A numerical validation is achieved to understand the ROMs capabilities, where different industrial-like components such as turbine and compressor bladed-disk have been used. The ROMs are compared against full 360° bladed-disk which have been built using the commercial FEM software ANSYS.

An experimental validation is carried out in order to understand the uncertainties that propagates in the aeromechanical chain when using the AROMA tool.

1.5 Summary of Appended Papers

A brief description of the papers is given here. A more detailed description of the results is given later in chapter 4.

I  Forced Response Analysis of a Mistuned, Compressor Blisk Comparing Three Different Reduced Order Model Approaches

Description: This paper presents a comparison of three reduced order models (ROMs) for the structural modeling of blisks. Two of the models assume cyclic symmetry, these are the subset nominal mode (SNM) and Craig-Bampton cyclic (CBC), while the third model is free of this assumption, which has been named as Craig-Bampton multisubstructuring (CBMS).

Contributions: A unique comparison between cyclic and noncyclic ROMs has been performed. It has been shown the importance of using a noncyclic ROM, when the bladed-disk accounts for large mistuning. A novel way of using the mistuned modes in the CBMS basis has been described.

II Forced Response Analysis of a Mistuned Blisk Using Noncyclic Reduced-Order Models

Description: In this paper, a unique way to address the ROM is presented, where each blisk sector is attached as individual substructures using the free-interface approach, which has been named as Craig-Chang multisubstructuring (CCMS).

Contributions: A unique and novel CCMS approach has been developed, which can include not only mistuning on the blade and disk parts of a sector, but also geometry changes.

III Forced Response Analysis of a Transonic Turbine Using a Free Interface Component Mode Synthesis Method
CHAPTER 1. INTRODUCTION

**Description:** A transonic high pressure turbine is investigated featuring large ranges of disk and blade dominated modes. The free-interface approach CC cyclic (CCC) has been assessed for all these modes, with a detailed study of the frequencies and modeshapes. A forced response analysis has been performed on a resonance crossing of interest.

**Contributions:** A numerical validation of the CCC approach has been addressed. In addition, an artificial constraint technique has been developed when working on the project related to this paper.

IV A Mistuned Forced Response Analysis of an Embedded Compressor Blisk using a Reduced Order Model

**Description:** This paper shows a detailed mistuned forced response analysis of a compressor blisk. The blisk belongs to the Purdue Three-Stage(P3S) Compressor Research Facility.

**Contributions:** The experimental validation of the SNM ROM has been achieved. It has been shown the importance of the boundary conditions applied at the drum end-points. The tuning of the model is an important factor to assimilate the response to the experiments.

1.6 Thesis Outline

The thesis is comprised of the introduction chapter stating a brief background on aeromechanics, the state-of-the-art, the objectives and a summary of the appended papers. The second chapter is a description of the simulation tool, AROMA. The third chapter gives a detailed description of the ROMs, and includes a subsection about mistuning. Then the result chapter quantifies the characteristics of the ROMs when these are validated numerically and experimentally, it also denotes the benefits and drawbacks of each ROM and discusses the ROMs industrial usage. The main outcomes are emphasized in the conclusion, complemented with the contributions to the state-of-the-art and the future work.
Chapter 2

AROMA Tool

2.1 Description

The programming of the AROMA tool was initiated with the PhD work of Mayorca [7]. The main outcome was that the tool was capable of integrating most of the relevant tasks for an aeromechanical analysis; however, modifications were still required to be capable of analyzing larger models, and also additional reduction methods were needed apart from the Guyan reduction, to improve the flexibility and accuracy of the tool. Hence, during the present work, further implementations have been achieved to make the tool less computational expensive and more accurate.

The purpose of the AROMA tool is to simplify and gather all the required steps when solving for the complete aeromechanical chain. These steps are shown in Figure 2.1. The pre-processing stage is based on in-house tools or commercial software that are utilized to obtain the unsteady forces and the structural information from a computational fluid dynamics (CFD) and a finite element model (FEM) solver, respectively. The unsteady forces or forcing functions can be calculated by URANS time-marching simulations or by an alternative harmonic solver, for the former case the forces are then projected with a Fourier transformation to obtain the necessary harmonic forces. Another source of information needed is the total damping, which can be either the structural damping, the aerodynamic damping or both. The aerodynamic damping is usually obtained by linearized CFD solvers for every mode and IBPA, whereas the structural damping is often obtained by experiments. The steps surrounded by the red box are the ones performed by AROMA. The tool takes the input from the pre-processing stage and the first step is to map the unsteady forces from the CFD to the FEM mesh. The mapping consists in transferring the information from every CFD node to the 5 closest nodes on the FEM mesh. The information is transferred in a weighted fashion, which depends on the distance between the CFD and FEM nodes. After the mapping, all the information is arranged in the physical degrees of freedom (DoF), meaning that all the matrices and vectors that conform the coupled aeromechanical EoM are in physical coordinates,
as in Eq. (1.2). The next step is an essential one in AROMA, which is using a basis to reduce the matrices of the EoM. The quality of the basis determines the accuracy of the analysis. At this stage, mistuning can be introduced to the ROMs before solving the equation for a forced response analysis or for a stability analysis. The stability analysis solves for the complex eigenfrequency, where the real part defines the vibration frequency and the imaginary part the damping term. On the other hand, the vibration amplitudes are obtained if a forced response analysis is performed. These amplitudes can be taken out of AROMA and be introduced as displacement boundary conditions on a FEM solver to obtain the alternating stress. Then the stress can be introduced back into AROMA to plot the Haigh diagram, as the one in Figure 1.11 to assess the fatigue life of the bladed-disk.

Figure 2.1: AROMA description (Adapted from Mayorca [7])
2.2 Programming Language

AROMA has been developed in MATLAB, but there have been efforts during the current work to transfer the entire program to C++, making it more robust and capable to handle large amounts of DoF. At present, this new AROMA++ is able to import the FEM model and run a generalized eigenvalue analysis to obtain the frequencies and modeshapes. This is one of the first steps and it is still an ongoing work. A comparison of the computational time to solve a generalized eigenvalue problem can be observed in Figure 2.2. It is clear the advantage of using AROMA++ for large amounts of DoF. This has been a case-specific test; however a similar outcome is expected for other models.

Figure 2.2: Eigenvalue problem comparison between AROMA (Matlab) and AROMA++(C++)

2.3 AROMA-PF

An additional task has been achieved in the Turbopower COMP10 project within another work package. It has been the development of AROMA-PF to assess HCF in a probabilistic way by computing the failure probability, as described in Figure 2.3. A detailed description can be found in Sandberg et al. [30]. AROMA-PF is now currently integrated to AROMA, so it can be accessed after calculating the forced response analysis.
## 2.4 Nonlinear Solver

Another task related to the same COMP10 project has been the development of a nonlinear solver in order to address friction damping, which is described in Afzal [31]. The solver has the capability to consider friction contact problems in bladed disks, such as the ones located in shrouds, snubbers or under-platform dampers.

Within the current thesis work, the nonlinear solver has been already integrated to AROMA and it has been tested without the use of ROMs, as shown in Figure 2.4. The implementation of the reduction methods on the non-linear solver is still an ongoing effort.
Figure 2.4: AROMA friction damping computation without reduction methods
Chapter 3

Reduced Order Models
Implementation

3.1 Fixed-Interface Formulation (Craig-Bampton)

In Figure 3.1, the component is represented by an isolated sector, and the interface DoF connect the sector with its adjacent one. For this case, the cyclic symmetry conditions are not present, and instead several isolated sectors are expanded and attached to build the reduced 360° bladed-disk. This technique has been named CB multisubstructuring (CBMS), since various sectors or substructures are attached together. A similar approach has been introduced by Moyroud et al. [32], using tuned normal modes and by Sternchuss and Balmes [33] using a quasi-cyclic method, both using this fixed-interface approach. The benefit of the current developed multisubstructuring method is that mistuned modes can be already included in its basis, which makes it far more accurate in the calculation of the vibrational modes.

\[ T^{(s)}_{CB} = \begin{bmatrix} \end{bmatrix} \]

Figure 3.1: CBMS basis

The other formulation is the CB cyclic (CBC), it uses the Fourier matrix to enforce the cyclic symmetry conditions. This formulation splits the bladed-disk into the blade- and disk parts. Hence, a different transformation basis is required for each of them. As mentioned earlier, this cyclic symmetry approach can be achieved by an integral substructure, where the sectors are coupled to each other.
and then these are projected to the cyclic space, or it can be built with the isolated sectors already in the cyclic space followed by the application of cyclic boundary conditions. Either way, the dynamic modes obtained are a representation of a 360° disk calculated for every harmonic. On the other hand, the constraint modes for the disk part are solved by enforcing a unit displacement for every DoF at the interface between the blade and disk. The transformation basis is represented in Figure 3.2.

\[ T^{(s)}_{CB} = \begin{bmatrix} \end{bmatrix} \]

Figure 3.2: CBC basis for the disk

The basis for the blade-part involves only the blade tuned modes, and no cyclic symmetry is applied directly, which means that each blade shares the same dynamic behavior. Then for the constraint modes, a static computation of the blade is achieved by enforcing the unit displacement on each DoF at the interface. The basis for the blade with the dynamic and constraint modes can be observed in Figure 3.3.

\[ T^{(s)}_{CB} = \begin{bmatrix} \end{bmatrix} \]

Figure 3.3: CBC basis for the blade

The mathematical formulation for the CBMS and CBC methods applied to bladed-disk is given in Paper I, and a more detailed description of the CBMS in Paper II.

### 3.2 Free-Interface Formulation (Craig-Chang)

As far as the author is concerned, the multisubstructuring technique has never been done in open literature using the free-interface approach. The difference compared to the aforementioned fixed-interface, is that now the interface DoF are not being fixed during the eigenvalue problem and so these are free to move. Moreover, instead of a unit displacement on the interface to build the attachment modes, a
unit force load is enforced on each of the DoF, one by one prior to being multiplied by the flexibility matrix. The CC multisubstructuring (CCMS) formulation is shown in Figure 3.4. The interface of the dynamic modes is held free for every sector, whereas the attachment modes are built from a unit force load on this interface. As mentioned before, the mistuned dynamic and attachment modes can be used in the basis. Note that the flange-part is held fixed for every sector as in a real bladed-disk.

\[
T^{(s)}_{CC} = \begin{bmatrix}
T(s)_{CC} \\
T(s)_{CC} \\
T(s)_{CC}
\end{bmatrix}
\]

Figure 3.4: CCMS basis

The CC cyclic (CCC) symmetry formulation for the disk-part is analogous to the fixed-interface approach. The only difference is that the interface DoF between the blade and the disk are kept free to move when solving for the dynamic modes, and the unit force acts on these interface DoF to obtain the attachment modes.

On the other hand, the blade-part subjected to a cyclic symmetry formulation represents an unconstrained component. Thus, besides the dynamic and attachment modes, six rigid body modes need to be considered in its basis as shown in Figure 3.6.

\[
T^{(s)}_{CC} = \begin{bmatrix}
T(s)_{CC} \\
T(s)_{CC} \\
T(s)_{CC}
\end{bmatrix}
\]

Figure 3.5: CCC basis for the disk
The advantage of the free-interface approach towards the fixed-interface one, is that the former not only uses displacement compatibility, but it also applies force equilibrium on the interface DoF during the synthesis of the components. This leads to a further reduction of the ROM, since the interface DoF disappear.

The detailed mathematical formulation for the CCMS method applied to bladed-disks is given in Paper II, and for the CCC method in Paper III.

### 3.2.1 Artificial Constrained Components

During the thesis work, a novel technique has been developed that applies an artificial constraint to the unconstrained components, excluding the need of rigid body modes in their basis. First, the isolated sector with the blade- and disk-parts are obtained with their corresponding mass and stiffness matrices, as shown in Figure 3.7.

![Figure 3.7: Isolated sector constrained at the flange](image)

The mathematical representation of the stiffness of the isolated sector is expressed as,

\[
K_{sector} = \begin{bmatrix}
K_{i1,i1} & K_{i1,i2} & K_{i1,b} \\
K_{i2,i1} & K_{i2,i2} & K_{i2,b} \\
K_{b,i1} & K_{b,i2} & K_{b,b}
\end{bmatrix},
\]  

(3.1)
where $i_1$ and $i_2$ denotes for the DoF of the blade and the disk components respectively. As mentioned earlier, $b$ stands for the interface DoF, which in this case is the coupling between these components. The trick of this technique is to split these two in a way that the unconstrained part is not left with a singular matrix. The strategy is to divide by half the coupling term ($K_{b,b}$) of the mass and stiffness matrix of each component, as it can be observed in Figure 3.8.

Figure 3.8: The blade- and the disk-parts are split to prevent singular matrices in each component

The stiffness matrix of each component, which is analogous to the mass matrix, can be expressed as,

$$
K_{\text{blade}} = \begin{bmatrix}
K_{i_1,i_1} & K_{i_1,b} \\
K_{b,i_1} & \frac{K_{b,b}}{2}
\end{bmatrix}, \quad K_{\text{disk}} = \begin{bmatrix}
K_{i_2,i_2} & K_{i_2,b} \\
K_{b,i_2} & \frac{K_{b,b}}{2}
\end{bmatrix}. \quad (3.2)
$$

This technique allows the components to have a basis such as in Equation 1.25. The artificial constraint then disappears when the synthesis or the assembly of the reduced components takes actions. In other words, when the reduced blade-part is assembled together with the reduced disk-part by enforcing displacement compatibility and force equilibrium, the artificial constraint disappears in both components.

### 3.3 Subset Nominal Mode

The simplicity of the SNM method makes it as one of the easiest to implement. As mentioned earlier this method is built in the cyclic space, where the set of eigenmodes are truncated. As observed in Figure 3.9, these truncated tuned modes represented as 360° bladed-disk, serve as the basis of the transformation matrix. The detailed mathematical formulation of this SNM method is described in Paper I and Paper IV.
CHAPTER 3. REDUCED ORDER MODELS IMPLEMENTATION

\[ T_{SNM} = \begin{bmatrix} \text{SNM basis} \end{bmatrix} \]

Figure 3.9: SNM basis

3.4 Mistuning

In AROMA, mistuning has been addressed in a physical and artificial way, and it has been incorporated by a modification of the structure’s mass or stiffness. The physical way is performed by modifying the Young’s modulus or the density of the material, to change the stiffness or mass respectively as in Figure 3.10(c). Also, mistuning can be achieved by adding or subtracting material on a sector, by designing a lump as in Figure 3.10(b)(d) or a cut-off on the bladed-disk.

![Tuned and mistuned sectors](image)

(a) Tuned (b) Mistuned (c) Mistuned (d) Mistuned

Figure 3.10: Tuned and mistuned sectors b) Mass added c) Material properties modified d) Mass with modified material properties added

The mistuned sector is then subtracted from the tuned one to obtain the difference in mass or stiffness, described as,

\[ \Delta M = M_{tuned} - M_{mistuned} \ (physical). \]  \hfill (3.3)

The artificial way simplifies the mistuning process, where instead of modifying the material properties, the matrices are directly perturbed inside AROMA. Therefore, only one tuned sector is needed, which is multiplied by a mistuned factor to obtain the difference in mass or stiffness, expressed as,

\[ \Delta M = (\text{fac})M_{tuned} \ (artificial). \]  \hfill (3.4)
The difference is then added to the EoM in the physical space, such as,

\[ \mu_{\text{struc}}^{(s)} = T^T(M + \Delta M)T, \]  

(3.5)

and the same for the stiffness,

\[ \kappa_{\text{struc}}^{(s)} = T^T(K + \Delta K)T. \]  

(3.6)

Here, the \( s \) represents each of the sectors, meaning that each sector around the bladed-disk can have its own difference in mass or stiffness. Depending on the ROM, \( \mu_{\text{struc}} \) and \( \kappa_{\text{struc}} \) can be obtained for one sector at a time, and then apply constrained equations to obtain the entire assembly, or \( T \) can be applied directly to the entire bladed-disk all at once. The \( \Delta K \) and \( \Delta M \) is built in the same manner as Equation 1.16 and contain the same coupling terms \( M_2 \) and \( K_2 \), but since each sector can be independently mistuned, the coupling matrices and the \( M_1 \) and \( K_1 \) might differ from row to row and column to column.
Chapter 4

Results

This chapter addresses the main objective of section 1.4 by presenting a summary of the results for the different ROMs, in terms of accuracy, amount of DoF, and mistuning approach. The first section 4.1 shows a comparison between these ROMs when using an industry-size bladed-disk, and it is complemented with a numerical validation of a parent (i.e. all the DoF taken into consideration) FEM 360° bladed-disk. The second section 4.2 presents the experimental validation of a compressor blisk model using the SNM ROM. Then the last section describes the applicability of AROMA to different industrial scenarios.

The methodology of the analyses is the same as specified in Figure 2.1, where standard industrial methods for calculating the aerodynamic forces and the structural information have been used. The forcing function ($F_{exc}$) is obtained from an unsteady CFD simulation, and the aerodynamic coupled force ($F_{coupled}$) is calculated with GKN Aerospace in-house code LINNEA, which is a 3D linearized frequency-domain solver, then the coupled forces are transformed from the TWM to the physical INFC domain. Both aerodynamic forces have been mapped to the FEM following a weighted criterion that depends on the distance between the CFD and FEM nodes. The coupled equation is then gathered together, as represented in Equation 1.2. This equation is then projected with the different reduction bases to decreased the amount of DoF.

The terms $K_{aero}$ and $C_{aero}$ from Equation 1.2 are not taken into consideration in the performed analyses of sections 4.1.1, 4.1.2, 4.1.3 and 4.2. Only in 4.1.4 is when the entire coupled forced response analysis was performed. Moreover, $C_{struc} = \frac{2\zeta K_{struc}}{\omega}$ for all the analyses.

To understand the level of mistuning, the amplification factor that is the ratio between the peak amplitude of the maximum mistuned response curve and the peak amplitude of the tuned response curve was calculated for all the mistuned analyses. The ratios are compared to the theoretical maximum amplification limit due to Whitehead [34] $(\frac{1+\sqrt{N}}{2})$.
4.1 Numerical Validation

The numerical validation was performed by comparing results calculated for a parent FEM 360° bladed-disk without ROMs, with results calculated using ROMs. This numerical validation has the advantage that it directly exposes the accuracy of the ROMs, since the only two main uncertainties that drive the error is the quality and quantity of the ROM’s transformation basis. The calculated frequency and the modeshapes are assessed to determine the error after a performed eigenvalue analysis for each reduced space, and also the calculated vibration amplitude is assessed after the forced response analysis. The equation that is used to obtain the relative error has been the following:

\[
\text{Relative Error} = \frac{\text{Result}_{360} - \text{Result}_{ROM}}{\text{Result}_{360}} \times 100, \tag{4.1}
\]

where the term Result can be the frequency or the Modal Assurance Criteria (MAC) number. The relative error calculation applies to the mistuned cases by using a mistuned parent FEM 360° bladed-disk as well. With respect to the MAC number, the closer to unity the better, which represents a perfect match between the ROM and the parent FEM 360° bladed-disk. The MAC number can be obtained by using two different modeshapes, such as \(x_1\) and \(x_2\) in

\[
\text{MAC} = 100 \cdot \frac{(x_1^T x_2)^2}{(x_1^T x_1)(x_2^T x_2)}. \tag{4.2}
\]

4.1.1 Comparison Between Cyclic and Noncyclic ROMs (Paper I)

The goal is the comparison of three different ROMs approaches. The subset nominal mode (SNM), the CB cyclic (CBC), and the CB multisubstructuring (CBMS), which only the CBMS does not apply cyclic symmetry. The study object is the rotor Hulda, which is a high-pressure ratio transonic 1\(\frac{1}{2}\) stage fan for high-speed aircraft propulsion, discussed by Mårtensson and et al. [35]. The rotor can be seen in Figure 1.12 whereas the entire cross section can be observed in Figure 4.1[a]. The structural characteristics are presented in Figure 4.1[b] where the frequencies of the first four families are shown for 11 ND due to its 23 blades. These frequencies are obtained after running a modal analysis of the full 360° blisk with and without prestress. With prestress the blisk becomes stiffer since it accounts for the centrifugal force and spin softening, where the former is driving the frequencies in this case.

The transformation matrices of the three ROMs have been built with different amount of dynamic modes as described in Table 4.1. The bases of CBMS, CBC and SNM are obtained as described in Figure 3.1, Figure 3.2 and Figure 3.9.
4.1. NUMERICAL VALIDATION

![Cross section (rotor marked with R)](image)

(a) Cross section (rotor marked with R)  

![Frequency of four families vrs ND](image)

(b) Frequency of four families vrs ND

Figure 4.1: Hulda

respectively. The amount of kept modes dictates the size and accuracy of the reduction, where a compromise between these two is required.

Table 4.1: Description of the amount of DoF

<table>
<thead>
<tr>
<th>ROM</th>
<th>Kept(^a) modes</th>
<th>CBC(^b) DoF</th>
<th>CBMS(^c) DoF</th>
<th>SNM(^d) DoF</th>
</tr>
</thead>
<tbody>
<tr>
<td>CBC2 - CBMS2 - SNM2</td>
<td>2</td>
<td>5704</td>
<td>16468</td>
<td>46</td>
</tr>
<tr>
<td>CBC4 - CBMS4 - SNM4</td>
<td>4</td>
<td>5750</td>
<td>16514</td>
<td>92</td>
</tr>
<tr>
<td>CBC8 - CBMS8 - SNM8</td>
<td>8</td>
<td>5842</td>
<td>16606</td>
<td>184</td>
</tr>
<tr>
<td>CBC16 - CBMS16 - SNM16</td>
<td>16</td>
<td>6026</td>
<td>16790</td>
<td>368</td>
</tr>
<tr>
<td>CBC32 - CBMS32 - SNM32</td>
<td>32</td>
<td>6394</td>
<td>17158</td>
<td>736</td>
</tr>
<tr>
<td>CBC50 - CBMS50 - SNM50</td>
<td>50</td>
<td>6808</td>
<td>17572</td>
<td>1150</td>
</tr>
<tr>
<td>CBC100 - CBMS100 - SNM100</td>
<td>100</td>
<td>7958</td>
<td>18722</td>
<td>2300</td>
</tr>
</tbody>
</table>

\(^a\) The kept modes stand for every sector.
\(^b\) The total DOF includes the constraint DOF from the blade-disk interface.
\(^c\) The total DOF includes the constraint DOF from the sector-sector interface.
\(^d\) The kept modes multiplied by the number of sectors.

A study with the different amounts of DoF has been achieved with a tuned blisk, by assessing the frequencies and modes shapes. The outcome was that the three ROMs converged to an accurate result with 50 kept modes, where the relative error is considered negligible, as observed in Figure 4.2. For the CBC ROM, 50 kept modes stand for 25 for the disk and 25 for the blade, as described in Figure 3.2.
and Figure 3.3. The same trend in convergence occurs with the other family modes in terms of their frequency and MAC error.

Figure 4.2: Relative error of the tuned third mode family MAC between ROM cases and the parent FEM $360^\circ$ blisk without prestress

Several forced response analyses have been performed to compare the three ROM’s by using the mistuned blisk shown in Figure 4.3. The purpose was to excite the third mode family on its 2ND. This resonant point is not the blisk’s real one, but it was used to excite a veering location, where the ROM is susceptible to inaccuracies. The forcing function is calculated from a CFD analysis with an unsteadiness of 15EO due to 15 variable inlet guide vanes. However, this forcing function was expanded as a 2EO excitation instead of the 15EO, to excite the area of concern. The critical damping ratio of 0.4% was used in all the cases. Different mistuned cases have been considered, where the mass or the stiffness has been modified in a physical or artificial way. Moreover, the amount of mistuning applied was of 5% or 30%, these amount is reflected from the forced response results in terms of standard deviation of the frequency and amplitude, which can be observed in Table 4.2 for the physical mistuned cases. The standard deviation for the mistuned frequency is not significantly high, since the resonant mode is a disk-dominated one, where the vibrational energy is being transferred around the disk. It can be seen that the modified mass generates higher level of mistuning than the stiffness, which is due to the mistuned location. Additionally, it can be seen that the maximum
amplification factor is of 1.794 and occurs with the case of 30% mass mistuning, and still below of the Whitehead limit of 2.89 due to the 23 blades.

![Figure 4.3: Hulda mistuned (darker areas at the tip indicate the mistuned regions)](image)

Table 4.2: Standard deviations of the mistuned frequencies and amplitudes of the forced response cases with prestress

<table>
<thead>
<tr>
<th>Cases</th>
<th>Modification$^a$</th>
<th>Amount$^b$</th>
<th>Std. freq.$^c$</th>
<th>Std. amp.$^d$</th>
<th>Amp. factor$^e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>M</td>
<td>5%</td>
<td>0.0450%</td>
<td>4.5637%</td>
<td>1.09</td>
</tr>
<tr>
<td>2</td>
<td>S</td>
<td>5%</td>
<td>0.0421%</td>
<td>1.9912%</td>
<td>1.014</td>
</tr>
<tr>
<td>3</td>
<td>M</td>
<td>30%</td>
<td>0.0605%</td>
<td>26.4122%</td>
<td>1.794</td>
</tr>
<tr>
<td>4</td>
<td>S</td>
<td>30%</td>
<td>0.0416%</td>
<td>9.5633%</td>
<td>1.08</td>
</tr>
</tbody>
</table>

$^a$ The modification refers if the Mass or the Stiffness are modified.
$^b$ The amount refers always as an increased in 5% or 30%.
$^c$ Standard deviation of the mistuned frequencies obtained on the location of max displacement.
$^d$ Standard deviation of the maximum mistuned amplitudes of each blade.
$^e$ Amplification factor is the ratio between the peak amplitude of the maximum response curve, and the peak amplitude of the tuned response curve.

The major discrepancies between the ROMs occur when considering the artificial mass mistuning. It makes sense since the coupling terms between the mistuned and non-mistuned DoF that belongs to the mass matrix, are not modified as it happens with the physical mistuning. Besides those cases, the 30% physical mistuning in Figure 4.4 represents the major differences between the three ROMs, which are observed in Figure 4.5. Here, an overview of the accuracy in terms of relative
error has been summarized for all the cases, except for the modified stiffness with artificial mistuning since the relative error was considerably high due to the same coupling error.

![Figure 4.4: Steady-state forced response for the 30% physical mass mistuning case with prestress, the two blades with max. deflection amplitude for each of the ROMs are shown](image)

The outcome of the study has been that the CBMS method is the only ROM, from the ones studied, suitable for aeromechanical analyses when considering a high level of mistuning since its accuracy is not compromised when increasing mistuning. This is due to the advantage of including the mistuned modes in its basis, as seen in Figure 3.1. However, the drawback is the high amount of DoF in the reduced space, as shown in Table 4.1. On the other hand, the SNM approach is the best ROM from the ones studied to be used for small mistuning, since its low amount of DoF requires less computational effort and it is still significantly accurate. One of the reasons of the different results between the cases with and without prestress, is due to the static analysis run on the prestress cases. The analysis takes into consideration the centrifugal and aerodynamics loads, and for mass mistuning it takes an important role since after prestress, not only the mass of the intended area is modified, but also the entire stiffness of the sector. Moreover, the way to employed artificial mass mistuning is very practical and still accurate enough when considering small mistuning, such as when the mass or stiffness was modified by 5%. Therefore, this type of mistuning can be utilized for the probabilistic mistuning, where several Monte-Carlo simulations are performed with stochastic mistuned patterns or frequencies.
4.1. NUMERICAL VALIDATION

Figure 4.5: Highest ROM amplitude relative error compared to the mistuned parent FEM 360 for each case

4.1.2 A Novel CC Multisubstructuring (CCMS) Method (Paper II)

The aim of this study was to find a reduction method that is able to accurately simulate high levels of mistuning, such as the CBMS that has been shown in the previous section, but also to further decrease the number of DoF of the reduced space. The alternative was to develop and implement the free-interface method CCMS into AROMA. The ROM has been validated to the parent 360° blisk, Hulda test case, and also compared with the CBMS. In Table 4.3 it can be seen the difference in number of DoF per sector of these two ROMs, where the CB approach has considerably more DoF.

Table 4.3: Description of the number of DoF for CBMS and CCMS ROMs for the full 360° blisk

<table>
<thead>
<tr>
<th>Cases</th>
<th>Kept modes(^a)</th>
<th>CBMS DoF(^b)</th>
<th>CCMS DoF</th>
</tr>
</thead>
<tbody>
<tr>
<td>CBMS2 or CCMS2</td>
<td>2</td>
<td>2*23+16422=16468</td>
<td>2*23=46</td>
</tr>
<tr>
<td>CBMS4 or CCMS4</td>
<td>4</td>
<td>4*23+16422=16514</td>
<td>4*23=92</td>
</tr>
<tr>
<td>CBMS10 or CCMS10</td>
<td>10</td>
<td>10*23+16422=16652</td>
<td>10*23=230</td>
</tr>
<tr>
<td>CBMS20 or CCMS20</td>
<td>20</td>
<td>20*23+16422=16882</td>
<td>20*23=460</td>
</tr>
<tr>
<td>CBMS50 or CCMS50</td>
<td>50</td>
<td>50*23+16422=17572</td>
<td>50*23=1150</td>
</tr>
<tr>
<td>CBMS100 or CCMS100</td>
<td>100</td>
<td>100*23+16422=18722</td>
<td>100*23=2300</td>
</tr>
</tbody>
</table>

\(^a\) The kept modes for each sector.

\(^b\) The 16422 value comes from the interface DoF.
In the paper it was shown how the CCMS approach is more accurate than the CBMS in terms of frequency; however, the opposite happens when the MAC is compared for these two ROMs. The maximum relative error for both ROMs in terms of frequencies and MAC for the tuned and mistuned analyses, is less than 0.0175%, which is considered as a negligible error. Also, there is not a clear difference in the accuracy for the blade- or disk-dominated family modes, as it has been expected that the CC would be more accurate because the interface DoF for the CC are free to move, whereas in the CB these are fixed.

Four different mistuned patterns have been used in the analyses, and the one with the highest response amplitude is shown in Figure 4.6(b) whereas Figure 4.6(a) shows the pattern where the largest relative error for the CCMS ROM occurs. The resonance response is due to a 15 engine order (EO) coming from the upstream VIGV that excites the fourth mode at 8ND at around 3396 Hz, as observed in Figure 4.6(b). An additional resonance condition was assessed at the same fourth mode but at 3ND instead, which is located at a disk-dominated region and it has been considered as a virtual crossing, adjusting the forcing function and not representing the blisk’s real response. The mistuned 2 pattern shows the highest relative error again for this case. The relative error comparing these two ROMs for the actual and virtual resonance can be observed in Figure 4.7. For all the cases, the relative error between the CBMS and CCMS ROMs is considered negligible. This demonstrates that both ROMs are suitable to assess a large mistuned industry-size model, with the advantage that CCMS approach has a considerably low number of DOF, and hence, lower computational expense.

The effect of the mistuning on the forced response can be seen in the standard deviations of these patterns (shown in Table 4.4). The mistuned 2 real case shows
4.1. **NUMERICAL VALIDATION**

higher standard deviation in terms of the mistuned frequency, and higher standard deviation in terms of the forced response amplitude for the mistuned 4 real case. There is a considerable difference between the standard deviations with respect to the real and virtual cases, more variation can be seen on the real cases which are considered blade-dominated responses, indicating an impact if the resonant point is on a blade- or disk-dominated region. This is shown qualitatively in Figure 4.8 for the mistuned 2 pattern. Moreover, in Table 4.4 it can be seen that the maximum amplification factor is of 1.467 for the mistuned 4 pattern and still below the Whitehead limit of 2.89 due to the 23 blades.

**Figure 4.7**: Relative amplitude error between each of the ROMs against the mistuned parent FEM 360° blisk from the mistuned forced response analysis excited within a range of ±3% of the resonant frequency

**Figure 4.8**: Forcet responses for the mistuned 2 cases when exciting the fourth mode family

(a) Mistuned 2 Real (8ND excitation/blade-dominated)   (b) Mistuned 2 Virtual (3ND excitation/disk-dominated)
Table 4.4: Standard deviations of the mistuned forced response for the real and virtual cases

<table>
<thead>
<tr>
<th>Cases</th>
<th>Resonance</th>
<th>Std. dev. freq.</th>
<th>Std. dev. amp.</th>
<th>Amp. factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mistuned 1</td>
<td>Real</td>
<td>0.706%</td>
<td>18.29%</td>
<td>1.213</td>
</tr>
<tr>
<td>Mistuned 2</td>
<td>Real</td>
<td>1.312%</td>
<td>33.23%</td>
<td>1.106</td>
</tr>
<tr>
<td>Mistuned 3</td>
<td>Real</td>
<td>1.263%</td>
<td>32.56%</td>
<td>1.303</td>
</tr>
<tr>
<td>Mistuned 4</td>
<td>Real</td>
<td>0.929%</td>
<td>44.11%</td>
<td>1.467</td>
</tr>
<tr>
<td>Mistuned 1</td>
<td>Virtual</td>
<td>0%</td>
<td>5.86%</td>
<td>1.226</td>
</tr>
<tr>
<td>Mistuned 2</td>
<td>Virtual</td>
<td>0.097%</td>
<td>11.08%</td>
<td>1.24</td>
</tr>
<tr>
<td>Mistuned 3</td>
<td>Virtual</td>
<td>0.072%</td>
<td>7.75%</td>
<td>1.224</td>
</tr>
<tr>
<td>Mistuned 4</td>
<td>Virtual</td>
<td>0.043%</td>
<td>6.36%</td>
<td>1.165</td>
</tr>
</tbody>
</table>

a Standard deviation of the mistuned frequencies obtained on the location of max displacement of each blade.
b Standard deviation of the maximum mistuned amplitudes of each blade.
c Amplification factor is the ratio between the peak amplitude of the maximum response curve, and the peak amplitude of the tuned response curve.
d Due to the frequency sweep interval, there is no quantified difference between the blades.

4.1.3 CC Cyclic (CCC) Method and the Novel Artificial Constraint Method (Paper III)

This section examines the forced response of a transonic high pressure turbine stage MT1, described in Beard et al. [36]. Several modifications of the actual blisk were made, and the structural behavior was changed, however, the aerothermodynamic characteristics remain the same. The unsteady aerodynamic analyses were performed by Gezork et al. [37]. One of the modifications was to add a shroud to the MT1 rotor, however, the unshrouded case was also analyzed. These two models are shown in Figure 4.9.

The CCC method has been applied in this study for all the forced response analyses. The cyclic symmetry boundary conditions have been enforced on the periodic interfaces, and even on the shroud interface as shown in Figure 4.10. As it has been described in section 3.2, the disk and the blade are split into two different components. For the unshrouded case, the blade is unconstrained after the split. Therefore, the artificial constraint technique developed during this project and explained in 3.2.1 has been used.
4.1. NUMERICAL VALIDATION

Figure 4.9: Two different FEM geometries

(a) Unshrouded case
(b) Shrouded case

Figure 4.10: Disk and blade sector components

A convergence study for the number of dynamic of kept modes has been assessed for the forced response analysis. The results are shown in Table 4.5. The case of using 100 kept modes for the disk and another 100 kept modes for the blade is taken as the reference to judge convergence. There is already a good convergence when using 2 disk modes (DM) and 2 blade modes (BM) for the shrouded and unshrouded cases. The case with 50 DM and 50 BM has been selected to complete
all the analyses, where the error difference is negligible.

Table 4.5: ROM uncertainty propagated in a forced response calculation as maximum absolute amplitude (DM=Disk Modes/BM=Blade Modes)

<table>
<thead>
<tr>
<th>ROM kept modes [-]</th>
<th>Unshrouded</th>
<th>Rel. Error a</th>
<th>Shrouded</th>
<th>Rel. Error a</th>
</tr>
</thead>
<tbody>
<tr>
<td>1DM+1BM</td>
<td>4.541E-05</td>
<td>4.2%</td>
<td>3.637E-05</td>
<td>3.7%</td>
</tr>
<tr>
<td>2DM+2BM</td>
<td>4.423E-05</td>
<td>1.4%</td>
<td>3.792E-5</td>
<td>0.36%</td>
</tr>
<tr>
<td>15DM+15BM</td>
<td>4.361E-05</td>
<td>0.003%</td>
<td>3.777E-5</td>
<td>0.031%</td>
</tr>
<tr>
<td>50DM+50BM b</td>
<td>4.361E-05</td>
<td>0.002%</td>
<td>3.779E-5</td>
<td>0.011%</td>
</tr>
<tr>
<td>100DM+100BM c</td>
<td>4.361E-5</td>
<td>0%</td>
<td>3.778E-5</td>
<td>0%</td>
</tr>
</tbody>
</table>

a Relative error using the (100DM+100BM) case as reference  
b Baseline case  
c Reference case to judge convergence

Figure 4.11: Modal deflections for the first mode family

The modeshapes that are observed in Figure 4.11 are the ones excited with a 32EO due to the 32 upstream stator blades. Both cases have been designed to match the resonance crossing at the first mode family at different rotational speeds, the unshrouded case at 4149Hz and the shrouded case at 7965Hz.

After running the forced response analyses, the amplitudes have been taken as inputs for a fatigue assessment. The amplitudes have been taken to ANSYS APDL 16.0 to obtain the alternating principal stresses and then these have been marched in time by AROMA to obtain the maximum octahedral stress. The mean stresses were calculated before in ANSYS as well, considering prestress. The mean and alternating stresses for the unshrouded case are shown in Figure 4.12.
4.1. NUMERICAL VALIDATION

(a) Mean Stresses

(b) Octahedral Stresses

Figure 4.12: Stresses for the unshrouded case

For this unshrouded case, the Haigh Diagram in Figure 4.13 shows acceptable fatigue margins. The maximum stress factor is around 0.53 and it is below the allowable fatigue limit. Therefore, this design is able to withstand HCF failure for this crossing.

Figure 4.13: Haigh diagram for the unshrouded case
4.1.4 ROMs Comparison in a Flutter and Coupled Forced Response Analysis

In this section, a detailed comparison between the Guyan, SNM, CBC, CBMS, CCC, and CCMS reductions is performed, introducing the aerodynamic coupling forces in the analyses. In the previous sections, all the analyses were performed assuming only a single damping ratio value, hence, the aerodynamic coupling terms of $K_{aero}$ and $C_{aero}$ from Equation 1.11 were not taken into account. The aim is to differentiate in terms of accuracy and computational cost for the different reductions for the Hulda test case described in subsection 4.1.1. The bladed-disk used for this comparison is shown in Figure 4.14. The resonant point investigated is the 3ND of the second mode family and only considering tuned analyses. The aerodynamic coupled force ($F_{coupled}$) is calculated with GKN Aerospace in-house code LINNEA, where the vibration of the first two mode families for every ND is prescribed on the rotor. This means that due to the 23 blades, 46 different analyses that include the backward and forward traveling waves have been calculated. Then the coupled forces are transformed from the TWM to the physical INFC domain with Equation 1.19 to Equation 1.22.

![Figure 4.14: Hulda rotor](a) Mesh (b) Second mode family 3ND)

A stability analysis that is used to determine the flutter margins, has been performed for every reduction method and they have been compared to the parent FEM 360° bladed-disk. From the analyses, eigenfrequencies with a real and an imaginary component are obtained. The real part stands for the frequency of oscillation, whereas the imaginary part is associated with the aerodynamic damping and can be expressed as a damping ratio. These eigenfrequencies are shown in
4.1. NUMERICAL VALIDATION

Figure 4.15 for mode 1 and mode 2. It can be seen qualitatively that most of the reductions share the same eigenfrequencies, except for the Guyan one, and the cyclic reductions (SNM, CBC, CCC) for low frequencies. The relative error between every reduction against the Full360° is quantified in Table 4.6. The relative error has been calculated as in Equation 4.1 for all the cases, except for the error of the imaginary eigenfrequencies. Instead, the difference between the calculated and true values has been normalized against the damping range of all the NDs (distance from max to min). Here, it can be seen that the CCMS is the most accurate reduction in terms of the real and imaginary components.

Figure 4.15: Eigenvalues from a stability analysis for the different ROMs, normalized to damping ratios

The accuracy due to the mapping of the forces can be obtained by comparing the damping ratio before the mapping, hence, the one calculated directly from the linearized CFD computation in the TWM, against the one after projecting from TWM to INFC domain and after the CFD to FEM mapping, which is considered as the Full360° case without any reduction. Thus, the reduction uncertainty is not included in the comparison. The qualitatively difference can be observed in Figure 4.16. Main differences are presented on the low NDs, and significantly for Mode 2. The error is mainly due to the fact of using a FEM that is coarser in some of the critical locations. For example, in the disk-dominated (i.e. low ND) regions, where the modeshapes moves away from not only being a blade motion.

Another comparison has been performed for the forced response analyses, with and without aerodynamic damping. The Hulda blisk has been subjected to a 3EO unsteady excitation force, to excite the second mode with a 3ND forward traveling wave, as observed in Figure 4.14(b). The response with and without aerodynamic coupling can be seen in Figure 4.18. For the uncoupled case, the damping ratio of 0.5262% has been assigned for each response, which is obtained directly from the CFD analysis, shown in Figure 4.16(b), and also represented with a star in Figure 4.17(b). It is clear that
for the uncoupled response, all the reduction methods are qualitatively matching; however, for the coupled one, the Guyan reduction and the cyclic reductions (SNM, CBC, CCC) are not matching the Full360° case. The same can be deduced from Table 4.6 where these four reductions have a relative error above 2%. It seems that after the mapping and the transformation from TWM to INFC of $K_{aero}$ and $C_{aero}$, the cyclic bases are not longer the best suit for a tuned forced response analyses, but further studies need to be made.
4.2 EXPERIMENTAL VALIDATION OF AROMA

Table 4.6: Generalization of different ROMs characteristics based on performed analyses

<table>
<thead>
<tr>
<th>ROM</th>
<th>DoF</th>
<th>Stability Error [%]</th>
<th>Forced Response Error [%]</th>
<th>Analysis Time [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Real</td>
<td>Imag</td>
<td>C</td>
</tr>
<tr>
<td>Guyan</td>
<td>39537</td>
<td>0.0356</td>
<td>4.759</td>
<td>3.612</td>
</tr>
<tr>
<td>SNM</td>
<td>2300</td>
<td>0.0264</td>
<td>2.095</td>
<td>2.455</td>
</tr>
<tr>
<td>CBC</td>
<td>6578</td>
<td>0.0267</td>
<td>2.095</td>
<td>2.456</td>
</tr>
<tr>
<td>CCC</td>
<td>2300</td>
<td>0.0267</td>
<td>2.095</td>
<td>2.454</td>
</tr>
<tr>
<td>CBMS</td>
<td>16100</td>
<td>0.001</td>
<td>0.0265</td>
<td>0.0085</td>
</tr>
<tr>
<td>CCMS</td>
<td>2300</td>
<td>0.0001</td>
<td>0.0265</td>
<td>0.0234</td>
</tr>
<tr>
<td>Full360</td>
<td>141588</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

- A low level is preferred for faster computations.
- Coupled EoM without the right-hand excitation force.

![Graph](image)

Figure 4.18: Forced response predictions for the different ROMs

4.2 Experimental Validation of AROMA

In this section, an experimental validation of AROMA is presented. The results of a forced response analysis of a blisk in the Purdue Three-Stage (P3S) Compressor Research Facility (Li et al. [38]) are compared with experimental results. These results were presented in Paper IV. The ROM employed was the SNM, because random mistuning and only small imperfections were considered, as observed in Table 4.7. The study was focused on three different resonant points, that were in-
vestigated in the experiments. These resonant crossing can be seen in Figure 4.19.

The unsteady forces have been obtained by CFD simulations performed at different corrected speeds (Nc) of 68%, 74% and 86% speed. Two different stage configurations have been examined, one includes an upstream stator of 44 vanes, and the second one includes an upstream stator of 38 vanes, leading to a 44EO and 38EO respectively. For each speed, the blisk has been subjected to a low loading (LL), peak efficiency (PE) and high loading (HL) forcing conditions.

Table 4.7: Standard deviation of the mistuned forced response cases

<table>
<thead>
<tr>
<th>Cases</th>
<th>Std. dev. freq. a</th>
<th>Std. dev. amp. b</th>
<th>Amp. factor c</th>
<th>Exp. Amp. factor d</th>
</tr>
</thead>
<tbody>
<tr>
<td>Torsion 44EO</td>
<td>0.32%</td>
<td>44.1%</td>
<td>-1.932/2.162</td>
<td>-2.283/2.056</td>
</tr>
<tr>
<td>CWB 88EO</td>
<td>0.45%</td>
<td>34.303%</td>
<td>1.704/1.692/1.565</td>
<td>1.675/1.919/1.535</td>
</tr>
<tr>
<td>Torsion 38EO</td>
<td>0.352%</td>
<td>38.68%</td>
<td>1.781/1.771/2.136</td>
<td>1.812/1.982/2.047</td>
</tr>
</tbody>
</table>

a Standard deviation of the mistuned frequencies obtained on the location of max displacement (The maximum of the LL, PE and HL cases).
b Standard deviation of the maximum mistuned amplitudes of each blade (The maximum of the LL, PE and HL cases).
c Amplification factor calculated from the analyses for three loading conditions.
d Amplification factor from the experimental results for three loading conditions.

As stated in Eq. (1.27), the basis of the transformation matrix for SNM is built with dynamic modes. These modes can be selected for only one family or for a specific frequency spectrum with multiple mode families, the latter one has been done for this paper. The damping for the forced response analysis has been introduced as a critical damping ratio (ζ) calculated for each ND in a tuned manner by unsteady CFD simulations. Thus, the damping matrix is built by block-diagonal terms for every harmonic in the cyclic space calculated as,

\[
\tilde{C}_{h,m} = \frac{2 \cdot \zeta_{h,m}}{\omega} \cdot \tilde{K}_{h,m}, \text{ where } h = \left\{ \begin{array}{ll} 1, 2, ..., \frac{N}{2}, ..., -2 & \text{if even} \\ 1, 2, ..., \frac{N-1}{2}, \frac{N-1}{2}, ..., -2 & \text{if odd} \end{array} \right. \text{ and } m = 1, 2, 3, ..., \#DOF
\]  

where \(\omega\) stands for the angular frequency of the mode of interest, and \(h\) the harmonic number.

First, several forced response analyses were studied to compare AROMA with the results performed by the in-house program MISER [9], [39] used at Duke University. Currently, MISER uses the FMM reduction that is based on a single DoF per ND approach. The results from MISER using the P3S Compressor blisk have been described in Li et al. [38] and in Besem et al. [40]. Both programs, AROMA
4.2. EXPERIMENTAL VALIDATION OF AROMA

Figure 4.19: ZZENF diagram indicating three resonant points

and MISER, showed similar results as shown in Figure 4.20 for the first CWB family excited by the second harmonic of the stator passing frequency (88EO) at PE. The maximum response shows minor differences, almost insignificant in these types of analyses, as in the magnitude of the peak around 4953Hz, whereas the mean response shows a good qualitatively match. The same trends were observed for the other resonant crossings.

As part of the study, the FEM was modified to investigate its influence on the results. The paper discusses two domain configurations, as observed in Figure 4.21. The long domain was the one expected to be close to the real blisk’s response; however, it was observed that the drum had to be stiffer in order to achieve a better match with experimental data. Thus, the short domain has been used instead to capture the frequency range as in the experiments. The difference between these two domains in terms of their structural behavior is compared in Figure 4.22. The long domain has a disk-dominated region from 0ND to 6ND, which has changed to a blade-dominated region on the short domain. The response of the two domains when excited by a 38EO is shown in Figure 4.23, and it is clear that the short domain response presents a better match to the experimental one.

Additional results for the HL condition and for only the short domain are presented in Figure 4.24. Here, the maximum and mean responses for both families show a good agreement when compared with experiment. The rest of the analyses
CHAPTER 4. RESULTS

(a) AROMA (SNM) calculation
(b) MISER (FMM) calculation

Figure 4.20: First CWB family excited by 88EO(11ND) at 68%Nc for the long domain

(a) Long domain
(b) Short domain

Figure 4.21: Domains with different boundary conditions

(a) Long domain
(b) Short domain

Figure 4.22: First torsion family (1T) excited by a 38EO(-5ND) at 86%Nc and by a 44EO(-11ND) at 74%Nc

with similar trends are shown in the paper. In Table 4.4 it can be seen that all the maximum amplification factors are below the Whitehead limit of 3.3723 due to the
33 blades, and that the maximum is of 2.283 for the PE of the Torsion 44EO case. Sources of inaccuracy are present in the experimental and predicted results. The experimental measurements faced inaccuracies due to noise in the data, whereas the boundary conditions in the predicted model was an important driver to calculate the disk-blade modal interactions, where mistuning takes an important role.

Figure 4.23: Mistuned amplitude response for the first torsion family excited by a 38EO(-5ND) at 86%Nc

Figure 4.24: Mistuned amplitude response
4.3 AROMA Industrial Usage

The new and modified reduction methods have upgraded AROMA to treat with industrial aeromechanical problems within the design phase of a turbine program. The usage of the tool depends on the type of analysis that it is desired. The following are recommendations depending on these types:

**Tuned**

For tuned analyses, the best approach is to use the SNM reduction, since the basis of the traveling wave modes is exactly the same as the eigenmodes of the aeroelastic system. From Table 4.6, it can be observed that the SNM is the most efficient for forced response analyses compared with the other reductions, where the average sweep time per frequency for the studied bladed-disk is only 0.3 seconds. The other three alternatives are the CCMS, CCC, and CBC. However, the preprocessing for CBC and CCC are less straightforward since the blade- and the disk- parts have to be split beforehand.

**Mistuned**

The mistuned analyses need to be separated into deterministic and probabilistic. The probabilistic mistuning accounts for small levels of mistuning, such as when the goal is to include the mass and stiffness differences due to wear, manufacturing tolerances, minor damage, and corrosion. This type of mistuning is considered random and extremely difficult to measure and this is the reason why it is assessed in a probabilistic way (Monte-Carlo simulations), which could involve 1000 forced response calculations. When small mistuning is encountered, the bladed-disk still vibrates in patterns similar to the tuned case, meaning that the traveling waves serve as an accurate basis for this type of analysis. For this reason, the SNM method is a good alternative, since it is accurate enough and involves low computational cost for multiple analyses.

A different approach needs to be taken when mistuning is deterministic. Deterministic mistuning refers to intentional mistuning, or when a large portion of the bladed-disk is modified, like a blade tip loss. Moreover, intentional mistuning can be used to design for worst or best case scenarios by imposing conditions so that the response can fall on a safe or unsafe region. This type of mistuning narrows the aeroelastic design margins by predicting in advance the energy confinement (i.e. blades with the highest amplitudes). For deterministic cases, the vibration modes are no longer behaving as traveling waves. Thus, a reduction method that uses cyclic conditions is not the most suitable for these types of analyses. It is also common that only a few forced response analyses are necessary. Therefore, the best options are the CBMS and the CCMS, but due to the low amount of DoF and still with a very high accuracy, the CCMS is considered the optimum method.
4.3. AROMA INDUSTRIAL USAGE

It is also common to study both mistuning types together, which means that an intentional mistuned bladed-disk can also be analyzed probabilistic, since it is obvious that the random differences, such as, manufacturing tolerances and wear, are always present in a real bladed-disk. For this reason, the reduced method should be accurate and efficient when these two mistuning types are involved. The deterministic mistuning is the one driving the selection, since most of the reductions involve high accuracy for low mistuning. Hence, the best option is the CCMS method, since it is highly accurate for deterministic mistuning without compromising the computational cost, and suitable for multiple analyses. The second best reduction is the SNM, where the accuracy is slightly compromised but it is still fast enough to use in the design phase.

Friction Damping

If the bladed-disk has friction damping, the best options are the CB methods, since the physical DoF where the damping is imposed, can be retained after the reduction. It is important to remark that the choice of reduction method also depends on whether the friction damping is under-platform, snubbers, interlocked-shrouds or another type of damping. The best option for most cases is the CBC, due to the fast computations as indicated in Table 4.6. The Guyan reduction can also be an alternative, since it also retains physical DoF, but keeping in mind that the accuracy is less and it is not as fast if multiple analyses are required.

Physical or Artificial Mistuning

It has been discussed that mistuning can be addressed as artificial or physical, as shown in Figure 3.10. The accuracy when using these two ways is examined in Figure 4.5. The implementation of the artificial way is faster than the physical one, since it is just a perturbation of the mass and stiffness matrices by a certain factor. On the other hand, every mistuned sector needs to be read in AROMA when using the physical mistuning, which takes more time to build the transformation basis. In addition, when assembling different patterns every reduction needs to be initiated from scratch when using the physical option, except for CBMS and the CCMS that has the advantage that each sector is reduced independently. In this case, already reduced sectors can be assembled in various patterns. The best options for a deterministic and probabilistic analysis are the physical and artificial mistuning, respectively. Although, the CBMS and CCMS can be used in a physical way if a higher accuracy is wanted and the computational time is not critical.

A ranked recommendation of every reduction method depending on the characteristics of the bladed-disk and the type of forced response analysis is shown in Table 4.8.
Table 4.8: ROM application depending on the characteristics of the bladed-disk, evaluated from 1-6 (1=best choice)

<table>
<thead>
<tr>
<th>ROM</th>
<th>Tuned</th>
<th>Mistuning</th>
<th>Mistuning</th>
<th>Mistuning</th>
<th>Friction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Prob.</td>
<td>Deter.</td>
<td>Prob. &amp; Deter.</td>
<td>Damping</td>
</tr>
<tr>
<td>Guyan</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>SNM</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>CBC</td>
<td>4</td>
<td>4</td>
<td>5</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>CCC</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>CBMS</td>
<td>5</td>
<td>5</td>
<td>2</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>CCMS</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

*a* Both mistuning approaches are applied on the bladed-disk.

*b* Assessed by the physical DoF after reduction and overall amount of DoF.
Chapter 5

Conclusions

An extensive study of accurate reduced order models (ROMs) for turbomachinery forced response analyses has been performed. Each different reduction method has its own benefits and drawbacks, and a one-fits-all approach is not possible. However, they can be individualized for specific cases based on their characteristics.

Mistuning has been an important parameter that these ROMs must address in an accurate way, and they have been tested with small and large mistuning. The outcome has been that the reduction methods that involves cyclic symmetry, are not capable of breaking the traveling waves modes when large mistuning is encountered. The alternative is to use the Craig-Bampton multisubstructuring (CBMS) and Craig-Chang multisubstructuring (CCMS) approaches, these two novel approaches are built without cyclic symmetry. It has been discussed that these approaches can deal with large mistuning, such as a sector with a lump, or a cut-off tip, since their transformation matrices are built with a mistuned basis, which makes it ideal for them to span the reduced space. It has been shown, that the CC (i.e. free-interface method) can further reduce the amount of degrees of freedom (DoF) without compromising the accuracy of the results, which makes it preferable than the CB (i.e. fixed-interface method). On the other hand, the subset nominal mode (SNM) approach is the ideal one for small mistuning. Its ease of implementation, and the simplicity in its usage, makes it one of the favorites among the aeroelastic community.

Several flutter and coupled forced response analyses have been performed for the Guyan, CB cyclic (CBC), CC cyclic (CCC), CBMS, CCMS and SNM reduction methods. A comparative study has been assessed regarding their computational time, accuracy, and their number of DoF.

The AROMA tool is capable of assessing the fatigue life of industrial bladed-disk by using accurate reduction methods, through a series of steps involving the mapping of the aerodynamic force, employing mistuning, and applying the coupled aerodynamic and structural damping. Three real bladed-disk, one turbine and two compressors, have been utilized to validate the analyses numerically and against
CHAPTER 5. CONCLUSIONS

experimental data.

5.1 State-of-the-Art Contributions

This thesis has made a contribution to the field in the following ways.

- The implementation of the CBMS approach is unique in the sense that the mistuned modes are used in the basis.

- The developed CCMS approach has proven satisfactory results to assess a mistuned bladed-disk. This novel approach can include not only large mistuning on the blade and disk parts of a sector, but also geometry changes as in the loss of the tip or outer portion of the blade.

- An artificial constraint technique has been developed to split the blade and disk components. It can be used on the free-interface approach when a component is unconstrained. The technique eliminates the rigid body modes from component and therefore from its basis. Moreover, the attachment modes are obtained for a constraint component instead.

- It has been shown the importance of the boundary conditions when validating against experiments. The tuning of the model is necessary to assimilate the response to the experiments. In the experiments, the modes can behave differently due to not well understood factors. The degree of importance of these factors on the response is case-specific.

- A detailed comparison of the different ROMs characteristics has been performed and this is of great use within the field. Recommendations are given depending on the industrial applicability (i.e. tuned, deterministic/probabilistic mistuning, friction damping).

5.2 Future Work

Based on the current work done and the results, several tasks are suggested as future work:

- Implement the ROMs on the nonlinear solver that is already incorporated in AROMA, and validate the results to experimental data. A bladed-disk contact in a slip, stick or separation state can be solved by using a time-discrete friction contact model, which is transformed back and forth from the frequency to the time domain. The most convenient ROMs are the ones that are capable to retain physical DoF after their condensation, which are the CBC, the CBMS and the Guyan reduction. The first being the most efficient due to its low number of DoF.
5.2. **FUTURE WORK**

- Keep the ongoing effort of transferring AROMA from MATLAB to C++, which will lessen the computational cost.

- Find innovative solutions, such as best- or worst-case scenarios in forced response analysis for real industrial bladed-disk by using different mistuning patterns. For example, a certain level of mistuning helps in breaking the traveling waves and move away from the flutter margins, and if possible without a significant increment on the forced response amplitude.

- Further validation of forced response predictions with different mistuning levels and ROMs. More bladed-disk analyses can be performed to study the accuracy of the cyclic and noncyclic ROMs when subjected to different levels of mistuning at several resonant crossings.

- Assess the probabilistic fatigue life of a real bladed-disk, using the entire aeromechanical chain in AROMA. Solving for multiple coupled forced response analyses and using AROMA-PF to assess for the probability of failure.
Bibliography


