Reactivity Assessment in Subcritical Systems

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Abstract

Accelerator-driven systems have been proposed for incineration of transuranic elements from spent nuclear fuel. For safe operation of such facilities, a robust method for reactivity monitoring is required. In this thesis, the most important existing reactivity determination methods have been evaluated experimentally in the subcritical YALINA-experiments in Belarus. It is concluded that the existing methods are sufficient for calibration purposes, but not for reactivity monitoring during regular operation of an accelerator-driven system. Conditions for successful utilization of the various methods are presented, based on the experimental experience.
List of Publications

Included Papers

The following papers constitute the thesis:


Author’s Contribution

All calculations of Paper I, as well as the writing of the paper, were performed by the author under the supervision of Dr. P. Seltborg. Concerning Paper II, the planning of the for the paper designated experimental program, the development of a data acquisition system, most of the data analysis and the writing of the paper were performed by the author.
Paper not Included

The following paper, written in parallel with the thesis work, is not included in this thesis:


Thesis Related Activities

Apart from the work resulting in the above listed papers the author has actively participated in the IAEA coordinated research project on “Analytical and Experimental Benchmark Analysis on Accelerator Driven Systems” and its collaborative activity “Low Enriched Uranium Fuel Utilization in Accelerator Driven Subcritical Assembly Systems”. This work has resulted in the YALINA-Booster benchmark specification [30]. Moreover, the author has participated in the preparation of the EUROTRANS experiments (European research programme for the TRANSmutation of high level nuclear waste in an accelerator driven system) related to the YALINA-Booster facility.
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Chapter 1

The Spent Nuclear Fuel Issue

1.1 Introduction

Today\(^1\), in total, 438 nuclear power reactors \([1]\) generate electricity constituting about 16% of the total world electricity production \([2]\). Although it is generally known that the spent nuclear fuel is a radiotoxic hazard that might burden future generations for hundred thousands of years, only one country, namely Finland, has decided upon a long-term plan for the spent fuel up to this date. Other countries have on-going negotiations and investigations on what to do with their spent nuclear fuel, as for instance Sweden and USA. In addition, there are currently 22 nuclear power plants under construction and 48 plants under planning globally \([1]\). Since the nuclear power does not contribute to the global warming it is likely that the amount of nuclear generated power will increase in future; thereby further raising the spent nuclear fuel issue and pushing nuclear power countries to find acceptable solutions for their spent nuclear fuel.

This thesis deals with a very specific problem related to spent nuclear fuel: reactivity assessment in subcritical systems dedicated to incineration of plutonium and minor actinides. To guide the reader into this subject, first the components of spent nuclear fuel will be described followed by proposed burners. That will lead to accelerator-driven systems and the need of a method to monitor the reactivity. In Chapter 2 reactivity determination methods used in this study are described followed by a description of the experimental facilities in Chapter 3. The simulation tool is briefly described in

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\(^1\) As of April 4 2007.
Chapter 4, followed by the experimental results in Chapter 5. Finally, the conclusions are presented in Chapter 6.

1.2 Spent Nuclear Fuel

Nuclear energy comes from neutron induced fission in, for instance, uranium. Natural uranium consists essentially of two isotopes: the fissile $^{235}\text{U}$ (0.7%) and the fertile $^{238}\text{U}$ (99.3%). In light-water reactors, most of the energy comes from fission in $^{235}\text{U}$ and the fuel is often enriched in this isotope. However, $^{238}\text{U}$ is also needed for safety reasons which will be discussed later. When a neutron is captured in $^{238}\text{U}$, the product will be $^{239}\text{U}$ that quickly $\beta$-decays, through the emission of an electron, to $^{239}\text{Np}$ followed by a second $\beta$-decay to $^{239}\text{Pu}$. With $^{239}\text{Pu}$ present in the core, subsequent neutron captures will lead to the build-up of heavier plutonium isotopes followed by a spectrum of americium and curium isotopes through further $\beta$-decays. Due to this process, the nuclear fuel after irradiation consists of, besides fission products and uranium left-overs, a non-negligible amount of transuranium elements (TRU). As can be seen in Table 1.1, the spent nuclear fuel consists of almost 95% of unused uranium, 1% of TRU and 4% of fission products. Apparently, 96% of the material can in principle be fissioned, and thus give energy, if there are enough incentives to develop the technology for doing so.
Table 1.1. Composition of UOX-fuel (uranium oxide) with 3.7% initial enrichment, burnt to 41.2 GWd/tHM², after four years of cooling [3]. The half-life and the effective ingestion dose coefficient [4] for each nuclide are also given.

<table>
<thead>
<tr>
<th>Element or Nuclide</th>
<th>Relative mass [%]</th>
<th>Half-life [a]</th>
<th>Effective dose coefficient [nSv/Bq]</th>
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<tr>
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<td>²³⁵U</td>
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² Gigawatt-days per tonne of heavy metal
1.3 Radiotoxicity of Spent Nuclear Fuel

Different radionuclides affect human tissues in different ways. The so called radiotoxicity of a nuclide depends on what tissues are exposed to the nuclide and for how long. In Table 1.1, the effective dose coefficients of the nuclides in spent fuel are shown. As can be seen, the effective dose coefficients of the actinides are generally much higher than those for the long-lived fission products. Figure 1.1 shows the radiotoxic inventory of the spent fuel of Table 1.1 as function of time, in relation to the radiotoxicity of natural uranium. Concerning the fission products, some short-lived nuclides, such as $^{90}$Sr and $^{137}$Cs, have been included. These fission products have very high activity during the first hundreds years and constitute the main part of the radiotoxic inventory during this period. When the activity of the fission products have declined, the radiotoxicity is dominated by americium and, after some thousands of years, plutonium becomes the main contributor. Clearly, the long-term radiotoxic issue is caused by plutonium and americium, although those elements constitute only slightly more than one percent of the spent fuel. Neptunium is less troublesome since it stays below the reference value. Curium, on the other hand, decays rather quickly to low levels, but would further irradiation of plutonium and americium be considered, the curium production will be an issue that must be taken into account. In an ideal recycling scenario, all plutonium, americium and curium will be transmuted resulting in a curve for the total radiotoxicity following that of the fission products. Consequently, in such scenario, it would be possible to reduce the final disposal time of the spent nuclear fuel from hundred thousands of years to thousands of years.
Figure 1.1. Radiotoxic inventory of UOX-fuel of 3.7% initial enrichment after a burnup of 41.2 GWd/tHM, normalized to the radiotoxicity of the amount of natural uranium needed to fabricate the enriched fuel (approximately 7-8 tons per ton 3.7% enriched fuel, depending on the amount of $^{235}\text{U}$ left in the tail).

### 1.4 Dedicated Burners

Conventional light-water reactors can apparently not burn all the fissionable nuclides present in the fuel. Mainly $^{235}\text{U}$ and other nuclides with an odd neutron number (N), such as $^{239}\text{Pu}$ and $^{242}\text{Am}$, will be fissioned. This effect comes from the fact that even-N nuclides are in energetically favored states and have therefore low probability for absorption of neutrons. Neutron absorption results in one of the two possibilities fission and capture. The fission probability is consequently defined as

$$p_f = \frac{\sigma_f}{\sigma_f + \sigma_c} \quad (1.1)$$

where $\sigma_f$ and $\sigma_c$ are the microscopic cross section for fission and capture respectively. The fission probability is depicted in Figure 1.2 as a function of incident neutron energy for an even-N nuclide and an odd-N nuclide. The
difference between the two types of nuclide can clearly be seen. The fission probability for the odd-N nuclide $^{235}$U is high for all energies, whereas the even-N nuclide $^{241}$Am is fissioned mainly for high energy neutrons above approximately 1 MeV. Therefore, in order to favor fission over capture for all nuclides in the spent fuel and thereby reach high transmutation rate of plutonium and minor actinides and decreased radiotoxicity, the neutron spectrum must be hard.

![Graph of fission probability vs. energy](image)

Figure 1.2. Fission probability as a function of incident neutron energy in $^{235}$U (odd-N) and $^{241}$Am (even-N) (data from ENDF/B-VI [5]).

### 1.4.1 Recycling in existing light-water reactors

In fact, recycling of plutonium is already taking place in thermal light-water reactors in countries such as France, Belgium and Great Britain. By combining plutonium and depleted uranium ($^{238}$U remnants from enrichment) a mixed oxide fuel (MOX) is fabricated. The main advantage of this fuel is that the amount of Pu, mainly $^{239}$Pu, can be reduced. A drawback is that since the spectrum is thermal, americium and curium will be built up through neutron captures in the plutonium and, in the end, increasing the burden of the radiotoxic inventory [3].

Recently, it has been proposed that the upper part of a boiling-water reactor fuel assembly, where the void hardens the neutron spectrum, can be used
for TRU transmutation. The study is, however, only a feasibility study, but indicates considerable performance [6].

1.4.2 Fast reactors and associated safety issues

The requirement on hard neutron spectrum rules out conventional use of light-water reactors as efficient TRU burners, since the water moderates the neutrons and thus decreasing the fission probability in even-N nuclides. In fast neutron reactors, other materials than water are used as coolants. Sodium has been used in several fast reactors, for instance Phoenix in France, BN-600 in Russia and JOYO in Japan, but other coolants such as lead-bismuth eutectic and helium have been proposed [7]. Due to the fast spectrum, the fast reactors are candidates for TRU burning, but when loading a critical fast core with minor actinides and plutonium instead of uranium some safety aspects of the core will change drastically. Safety aspects of fast reactors loaded with high fractions of americium will here be discussed.

Doppler feedback

The most fundamental reactivity feedback mechanism in a critical reactor is the Doppler feedback. When a material heats up, resonances in the reaction cross sections gets wider thus increasing the probability for the reaction to occur. This effect is especially pronounced in $^{238}$U. In a uranium fuelled core, this means that when there is a power increase, the resonances of the neutron absorption cross section of $^{238}$U widen, leading to increased neutron absorption of epi-thermal neutrons during moderation, thus preventing thermal fissions in $^{235}$U. This negative power feedback acts very fast on increased fuel temperature and makes the reactor operation stable.

When americium is introduced into the fuel, as burnup product or dedicated irradiation target, the core physics changes drastically. As can be seen in Figure 1.3, the capture cross section of $^{241}$Am is about one order of magnitude higher than the capture cross section of $^{238}$U in the energy region around 100 keV. During neutron slowing-down, the neutrons will be captured in the americium instead of the uranium. In case of a fuel temperature increase, a broadening of the absorption resonances of $^{238}$U in the energy region 0.1 – 10 keV will not decrease the reactivity to the same extent when there is americium present, thus deteriorating the Doppler feedback. Studies have shown that only even a small fraction of americium in sodium- or helium-cooled cores decreases the Doppler feedback essentially [8,9].
Effective delayed neutron fraction

In the fission process, in general, two or three neutrons are released immediately. These neutrons are referred to as prompt neutrons. The fission products, on the other hand, are in general unstable and may decay with a direct or subsequent emission of a neutron. For instance, consider the fission product “X”. This nuclide will, most probably, be neutron rich and will undergo one or several $\beta$-decays to reduce its neutron excess. The “X”-nuclide or one of the nuclei in its decay chain may then release a neutron, following the decay scheme below:

$$\ ^{\Lambda}X_N \overset{\beta^-}{\longrightarrow} ^{\Lambda+1}Y_{N-1} \longrightarrow ^{\Lambda+1}T_{N-2} + n .$$

The “X”-nucleus is called delayed neutron precursor and the “Y”-nucleus is the delayed neutron emitter (sometimes the precursor and the emitter is the same nucleus). The time from the fission event to the emitting of the delayed neutron depends on the half-life of the delayed neutron precursor and possible intermediate steps. However, important is the extremely large difference in time compared to the prompt neutrons, which are released within less than $10^{-15}$ s after the fission event. The delayed neutrons are in general released within seconds or minutes. An example is the reaction below:
Fission products generated by thermal fission in $^{235}$U typically have a mass number in the regions around 90 and 130. Consequently, the most important delayed neutron precursors are found in these regions. Examples are isotopes of the elements Br, I, Rb and Kr. In total, there are approximately 40 known delayed neutron precursors among the fission products [10].

The total number of neutrons released after a fission event, $\nu$, is the sum of the prompt fission neutron yield, $\nu_p$, and the delayed neutron yield, $\nu_d$:

$$\nu(E) = \nu_p(E) + \nu_d(E).$$  \hspace{1cm} (1.2)$$

As indicated in Eq. (1.2), the neutron yields are in fact energy dependent. The prompt neutron yield increases in general linearly with energy [11] and as an effect of that, the delayed neutron yield is altered since the mass number distribution of the fission product changes due to the changed incident neutron energy. The delayed neutron fraction is defined as

$$\beta(E) = \frac{\nu_d(E)}{\nu_p(E) + \nu_d(E)} = \frac{\nu_d(E)}{\nu(E)}$$  \hspace{1cm} (1.3)$$

and its value is dependent both on the fissioning nuclide and the incoming neutron energy. In general it can be stated that the value of the delayed neutron fraction increases with the mass number $A$ for a certain element (constant $Z$) and decreases for increasing $Z$ (heavier elements). The delayed neutron fraction for $^{235}$U, $^{239}$Pu and $^{241}$Am is depicted in Figure 1.4 as a function of incident neutron energy. It can be noted that the delayed neutron data is known to much less accuracy for the higher actinides, such as $^{241}$Am, compared to $^{235}$U and $^{239}$Pu. The delayed neutron fraction is in general constant up to approximately 0.5 MeV, where it drops to less than 50%. This means that for fast systems, the delayed neutron fraction will be lower than for thermal systems.

The energy of the delayed neutrons depends on the reaction from which the neutron originates. In general, this energy is in the order of some hundred keV and is, consequently, less than the average energy of the fission Watt spectrum (1-2 MeV). Therefore, in a thermal system, the delayed neutrons are more efficient in inducing further fissions than the fission neutrons since they have less probability for absorption in $^{238}$U during slowing-down. Taking this efficiency into account, the effective delayed neutron fraction, $\beta_{eff}$, has been introduced. In general, for a thermal nuclear reactor $\beta_{eff} > \beta$, and for a fast reactor $\beta_{eff} \leq \beta$.

The effective delayed neutron fraction plays an important role in reactor kinetics and control. If the inserted reactivity is larger than $\beta_{eff}$, the reactor becomes prompt supercritical. In this state, the self-sustained chain reactions
rely on prompt neutrons only and the power of the core will increase several orders of magnitude in a fraction of a second. In this time-scale, no mechanical control rod system can manage the core power and the prompt critical state must therefore always be avoided. If the core consists of large fractions of TRU, the effective delayed neutron fraction will be small, thus decreasing the margin to prompt criticality.

As will be clear later, the effective delayed neutron fraction plays an important role in reactivity measurements, since it is the connecting parameter when calculating the effective multiplication factor from the reactivity expressed in the unit of dollars ($).

![Graph showing delayed neutron fractions for 235U, 239Pu, and 241Am](image)

**Figure 1.4.** Delayed neutron fractions for 235U, 239Pu, and 241Am in the unit of pcm$^3$ as given by three different nuclear data libraries [5].

**Coolant temperature coefficient and coolant void worth**

The coolant does not only cool the core, but also affect the core neutronics. Neutrons are absorbed in the coolant, which thereby acts as a poison. If there is a coolant density decrease, caused by a temperature increase, fewer neutrons will be absorbed in the coolant and the power increases further.

---

3 pro cent milles, $10^{-5}$. 
Another effect comes from the fact that the coolant moderates the neutrons, in fast reactors to some extent, causing a softening of the neutron spectrum. In a core loaded with a large fraction of even-N nuclides, such as $^{240}$Pu, $^{242}$Pu, $^{241}$Am and $^{243}$Am, this becomes troublesome. As could be seen in Figure 1.2, the fission probability for even-N nuclides increases with incident neutron energy. If there is a neutron spectrum hardening, caused by for instance sudden coolant boiling, the fission rate will increase followed by a power increase. Such positive feedback mechanisms must be avoided, thus leading to a limitation to the fraction of even-N nuclides that may be loaded into the core.

The positive feedback can in some designs be balanced by relying on increased neutron leakage due to the lower coolant density and changed lattice geometry due to the temperature increase. A positive coolant temperature coefficient is allowed if it is compensated by the Doppler feedback from the fuel.

In case of total voiding of the core, the situation is more severe. The neutron spectrum becomes very hard and an excess of neutrons will suddenly be available due to the strongly decreased absorption. In fast reactors, this implies a cumbersome situation that may cause an overall reactivity insertion of several dollars (units of $\beta_{eff}$) [12].

1.4.3 Subcritical Systems

In previous sections it has been concluded that fast neutron systems must be employed for efficient transmutation of plutonium and minor actinides, and for radiotoxicity reduction. However, fast reactors suffer from deteriorated safety parameters when loaded with large fractions of these elements. Therefore, it has been proposed to employ fast subcritical source-driven cores as burners of uranium-free fuels [13,14]. The subcriticality makes the core less sensitive to positive reactivity feedbacks, thereby allowing the use of fuels based on plutonium, americium and curium. The margin to criticality must be chosen large enough to withstand any reactivity increase that can make the core critical. On the other hand, a large subcriticality level requires a strong source. An effective multiplication factor of approximately 0.97 is foreseen in full-scale designs [15]. However, core voiding may still be a concern even at this subcriticality level [12].

A constant power level will be retained by coupling a strong external neutron source to the subcritical core. This source will most likely consist of a proton accelerator coupled to a spallation target. Therefore these systems are referred to as accelerator-driven systems (ADS).
1.5 Reactivity Assessment

When operating an ADS loaded by plutonium and minor actinides, criticality must under all circumstances be avoided. Therefore, monitoring of the subcriticality is essential for maintaining safe operation. The aim of this thesis is to identify existing methods for reactivity assessment and to investigate their applicability in source-driven systems in order to achieve a deeper understanding of their reliability and stability.

Measurements of subcriticality in reactors have been performed since the fifties [16,17]. However, the interest of subcritical systems for spent nuclear fuel incineration has increased the need of a stable and reliable subcriticality determination method. Since important parts of the safety system of a future ADS will be based on this designated method, new requirements on the performance are raised, the most important being:

- Capability of online monitoring of the reactivity, i.e. short measurement time.
- Low spatial dependence.
- High accuracy.
- Detector type independence.
- Neutron source independence.

One major study in this field has been performed at the MASURCA facility in Cadarache, France, within the MUSE project [18]. In that study, a number of reactivity methods were investigated and compared to each other in a fast neutron spectrum at low power. Some further studies were performed within the TRADE [19] and the RACE [20] programs. These studies aimed at higher power levels to include thermal feedbacks in a thermal neutron spectrum, but both projects were cancelled at early stages. In the YALINA-experiments in Belarus, studies on reactivity determination are performed in a thermal and a coupled fast-thermal spectrum. There are two subcritical cores, here referred to as YALINA-Thermal and YALINA-Booster that are coupled to a neutron generator. Paper I deals with experimental results from YALINA-Thermal, whereas Paper II concerns YALINA-Booster.
Chapter 2

Reactivity Determination Methods in Subcritical Systems

In this chapter, the various reactivity methods investigated in the two papers will be described. First, the underlying theory is briefly presented.

2.1 Basic Concepts in Neutron Transport

The ultimate goal in reactor studies is to determine the neutron distribution in space, energy and time in the reactor. This is achieved by describing the motion of neutrons in the reactor and their interactions with the present materials. The most fundamental quantity in nuclear reactor theory is the neutron density as a function of space, energy and time: \( n(r, E, t) \). The expected number of neutrons of energy \( dE \) around \( E \) in an infinitesimally small volume \( d^3r \) at time \( t \) around \( r \) is \( n(r, E, t)d^3r dE \).

Another important quantity in reactor theory is the reaction rate density \( F_x(r, E, t) \), where \( x \) denotes symbolically the occurring interaction. From this quantity, the corresponding macroscopic cross section, \( \Sigma_x \), can be introduced:

\[
F_x(r, E, t) = \Sigma_x(r, E) \cdot v n(r, E, t),
\]

(2.1)

where \( v \) is the neutron velocity.

If having \( n(r, E, t) \), a complete picture of the neutron density distribution in the reactor is known. Unfortunately, no equation is satisfied exactly by \( n(r, E, t) \). Therefore, the angular neutron density, \( N(r, \Omega, E, t) \), must be introduced. The parameter \( \Omega = v/v \) describes the direction of a neutron leaving the position \( r \). Consequently, \( N(r, \Omega, E, t)d^3r d\Omega dE \) is the expected number of
neutrons in the volume \( d^3r \) about \( r \), with energy \( dE \) about \( E \), moving in direction \( \Omega \) in solid angle \( d\Omega \) at time \( t \). The products \( vn(r,E,t) \) and \( vN(r,\Omega,E,t) \) occur very frequently in reactor theory and they have therefore been given special names. Thus we introduce the neutron flux

\[
\phi(r, E, t) \equiv vn(r, E, t)
\]

and the angular neutron flux

\[
\Phi(r, \Omega, E, t) \equiv vN(r, \Omega, E, t).
\]

The neutron flux and the angular neutron flux are related through

\[
\phi(r, E, t) = \int_{4\pi} \Phi(r, \Omega, E, t) d\Omega.
\]

### 2.2 The Point Reactor Model

The distribution of neutrons in a reactor obeys the neutron transport equation, sometimes called the linear Boltzmann equation:

\[
\frac{1}{v} \frac{\partial \Phi(r, \Omega, E, t)}{\partial t} = -\Omega \cdot \nabla \Phi(r, \Omega, E, t) - \Sigma_a(r, E) \Phi(r, \Omega, E, t) + \\
\int_{0}^{\infty} \int_{4\pi} \Sigma_s(r, \Omega', E' \rightarrow \Omega, E) \Phi(r, \Omega', E', t) d\Omega' dE' + \\
+ \frac{\Sigma_f(E)}{4\pi} \int_{0}^{\infty} \int_{4\pi} \left[ 1 - \beta(E') \right] v(E') \Sigma_f(r, E') \Phi(r, \Omega', E', t) d\Omega' dE' + \\
+ \sum_{i} \frac{\chi_i(E)}{4\pi} \lambda_i C_i(r, t) + \\
+ S(r, \Omega, E, t)
\]

Here, \( \Sigma_a \) is the macroscopic absorption cross section, \( \Sigma_f \) is the macroscopic fission cross section, \( \chi \) and \( \chi' \) are the energy spectrum of the prompt and delayed neutrons respectively; \( \Sigma_s \) is the differential macroscopic scattering cross section describing the transfer probability that an incident neutron of initial direction \( \Omega' \) and energy \( E' \) emerges from a possible collision with direction \( \Omega \) and energy \( E \). The delayed neutron precursor density is represented by \( C_i \) with decay constants \( \lambda_i \). Finally, the external source is here given as \( S \). The delayed neutron precursor densities follow the relation

\[
\frac{\partial C_i(r, t)}{\partial t} = \int_{0}^{\infty} \beta(E') v \Sigma_f(r, E') \Phi(r, \Omega', E', t) d\Omega' dE' - \lambda_i C_i(r, t)
\]
where $\beta_i$ is the delayed neutron fraction for the delayed neutron precursor group $i$ [21]. The two loss terms of Eq. (2.5) come from streaming and neutron absorption, whereas the gain terms arrive from scattering, fissions, delayed neutron precursors and from the external source. Eqs. (2.5) and (2.6) are very difficult to solve, hence a number of simplifying models and assumptions have been proposed. Several common models are as follows:

- multi-group model postulating that the energy $E$ may assume only a discrete number of energy levels,
- one-group model characterized by a single neutron energy and energy independent cross sections,
- diffusion model postulating Fick’s law between the neutron current and the neutron flux, and
- point reactor model assuming separation of space and time variables according to

$$\phi(r, t) = v n(t) \psi(r) \quad (2.7)$$

and

$$C_i(r, t) = c_i(t) \psi(r). \quad (2.8)$$

This is also valid for a homogeneous infinite reactor where the shape function is constant.

From now on, the point reactor model will be assumed. One can then derive the following equations [10,21,22]:

$$\frac{d n(t)}{d t} = \rho(t) - \beta_{\text{eff}} n(t) + \sum_{i=1}^{6} \lambda_i c_i(t), \quad (2.9)$$

where the neutron density, $n$, and the delayed neutron precursor densities, $c_i$, have been introduced through averaging over space, energy and solid angle. The reactivity, $\rho$, is defined by

$$\rho = \frac{k_{\text{eff}} - 1}{k_{\text{eff}}} \quad (2.10)$$

where $k_{\text{eff}}$ is the ratio of neutron production to neutron absorption. Further, the parameter $\Lambda$, the neutron reproduction time$^4$, has been introduced, de-

$^4$ The traditional name of $\Lambda$ is the mean neutron generation time [23]. However, it was recently proposed to rename this parameter the neutron reproduction time [24].
scribing the inverse production rate of neutrons in the system. In Eqs. (2.9), the space dependence has been removed, which means that all points in the reactor are described by the same equations, thereby carrying the name \textit{the point kinetic equations}. Despite the simplicity of these equations, they are in most cases sufficient to describe the reactor behavior in a satisfactory way.

2.3 The PNS Slope Fitting Method

Assuming that the delayed neutrons can be neglected, Eqs. (2.9) read

\[
\frac{dn(t)}{dt} = \alpha n(t),
\]

(2.11)

where

\[
\alpha = \frac{\rho - \beta_{\text{eff}}}{\Lambda}
\]

(2.12)

is the prompt neutron decay constant. By introducing short pulses of source neutrons in a subcritical core, this decay constant can be found [17]. With the knowledge of $\beta_{\text{eff}}$ and $\Lambda$, the reactivity, and thereby $k_{\text{eff}}$, can be found. One source neutron pulse is not sufficient for finding $\alpha$, thus a pulsed neutron source (PNS) is needed. The flux as a function of time is recorded after each pulse in an accumulating histogram that, after the experiment, shows the collected flux from all pulses. From this histogram, the prompt neutron decay constant can be found through function fitting. An example of a PNS histogram can be found in Figure 2.1.

2.4 The PNS Area Method

By separating the total neutron density, $n(t)$, in Eqs. (2.9) into prompt and delayed neutron densities and thereafter integrating over time, the prompt and delayed neutron areas, $A_p$ and $A_d$, can be obtained. These areas are depicted in Figure 2.1. Even though the neutron source is pulsed, both the integrated prompt and delayed neutrons densities will be in equilibrium. It has been shown [16] that the reactivity in dollars is given by

In this thesis, as well as in \textbf{Paper II}, the latter is used, but in \textbf{Paper I} the former name is used.
\[ \frac{\rho}{\beta_{\text{eff}}} = -\frac{A_p}{A_d}. \] (2.13)

This method of determining reactivity was developed by the Swedish reactor physicist Nils Göran Sjöstrand, thus the method is sometimes referred to as the Sjöstrand method but also as the area method.

Eq. (2.12) can be rewritten on the form

\[ \frac{\Lambda}{\beta_{\text{eff}}} = \frac{1}{\alpha} \left( \frac{\rho}{\beta_{\text{eff}}} - 1 \right) \] (2.14)

and thus giving an experimental value of the fraction \( \Lambda/\beta_{\text{eff}} \) based on a PNS experiment and the assumption that the point kinetic approximation holds.

---

![Figure 2.1](image-url)

Figure 2.1. Prompt and delayed neutron areas used in the area method, as well as the exponential decay of prompt neutrons (time discretization 5 μs).

### 2.5 The Source Jerk Method

Assuming a subcritical core at constant power driven by an external source, a neutron flux level somewhere in the core will be \( n_0 \). Suddenly, the external neutron source is shut-down or removed very quickly. Then, the neutron flux
changes rapidly to a semi-stable level $n_t$. It can be shown [21] that the reactivity in dollars is given by

$$\frac{\rho}{\beta_{\text{eff}}} = \frac{n_t - n_0}{n_t}. \quad (2.15)$$

### 2.6 The Rossi-α Method

Due to the fission chain reactions in a multiplying medium, detected neutrons may be correlated to each other in space and time. It might happen that two neutrons, detected by the same neutron detector, originate from the same fission chain. In such case, it is likely that these two detections, or events, are close in time. It can be shown that the probability to detect one more neutron after the first neutron decreases exponentially in time with the prompt neutron decay constant $\alpha$. The probability, $p(t)dt$, to detect a second neutron within the time interval $dt$ around $t$, assuming that the first neutron was detected at $t = 0$, is

$$p(t)dt = -\frac{eD_\nu}{2\alpha\Lambda^2} e^{\alpha t} dt + F \varepsilon dt. \quad (2.16)$$

In this equation, $\varepsilon$ is the detector efficiency measured in counts per fission occurred and $F$ is the fission rate of the system [25,26]. Consequently, the term $F\varepsilon$ is the mean count rate during the measurement. $D_\nu$ is the Diven factor defined as

$$D_\nu = \frac{\nu(\nu - 1)}{\nu^2} \quad (2.17)$$

where $\nu$ is the number of prompt neutrons per fission [27]. It should be noted that in this thesis, as well as in Paper II, the sign convention of $\alpha$ has been conserved. In most references, $\alpha$ is defined with the opposite sign for the noise methods. Since the result will be compared to the PNS measurements a consequent use of the sign has been chosen.

Finally, by plotting the probability density $p(t)$ obtained experimentally, the prompt neutron decay constant can be found from the correlated part through fitting of an exponential function. Figure 2.2 shows how to obtain the Rossi-α histogram. All time intervals from an event $t_1$, $t_2$, $t_3$... $t_n$, less than a certain interval length $T$, are calculated and added to a histogram, shown to the right in the figure. This procedure is repeated for each event and the subsequent time intervals are accumulated in the histogram. After long time, it will be visible that shorter time intervals will dominate the histogram if the events are correlated.
2.7 The Pulsed Rossi-α Method

The Rossi-α technique can be applied in the same way to data from a pulsed neutron source experiment as for a continuous source. It has been shown that the pulsed Rossi-α histogram consists of three terms [28]:

\[ p(t)dt = A_1 e^{\alpha t} dt + A_2 dt + A_3 \sum_{n=1}^{\infty} (a_n^2 + b_n^2) \cos(\omega_n t) dt \]  \hspace{1cm} (2.18)

The first term is the ordinary correlated exponential term, also found in the classical formula for a steady state source, and the last two terms are the uncorrelated terms. The last term is a non-decaying oscillating term consisting of a Fourier series of the pulsed neutron source characteristics. The constants of Eq. (2.18) are given in Ref. [28].

2.8 The Feynman-α Method

In a nuclear reactor, the counts, \( c \), of a neutron detector will deviate from a true Poisson distribution due to the presence of fissile material. The deviation is denoted \( Y \):

\[ \frac{\sigma(c)^2}{c} = \frac{\bar{c}^2 - c^2}{c^2} = 1 + Y. \]  \hspace{1cm} (2.19)

It can be shown that \( Y \) is a function of \( \Delta T \), the time base used in the measurement. This relation is often referred to as the variance-to-mean or the Feynman-α formula [25,26]:

Figure 2.2. Description of how the Rossi-α histogram is obtained. The detection of a neutron (event) is represented by a vertical line.
\[ Y(\Delta T) = \frac{\varepsilon D_v}{N^2 \alpha^2} \left( 1 + \frac{1 - e^{\alpha \Delta T}}{\alpha \Delta T} \right). \]  

(2.20)

The statistical error in \( Y \) depends on the number of detections according to

\[ \sigma_Y = Y \left\{ \frac{1}{N} \left[ \frac{4 + \frac{3}{c}}{Y} \frac{1}{Y} + \left( \frac{2 + \frac{3}{c}}{c} \right) + \frac{Y}{c} \right] \right\}^{1/2} \]  

(2.21)

where \( N \) is the number of detections for a given \( \Delta T \) [26]. Consequently, the statistical accuracy will be higher if the measurement is running for a long time and for low \( \Delta T \).

The dead time effect is taken into account by correcting the Feynman-\( \alpha \) formula, Eq. (2.20), according to

\[ Y(\Delta T) = \frac{\varepsilon D_v}{N^2 \alpha^2} \left( 1 - \frac{d}{\Delta T} \right) \left( 1 + \frac{1 - e^{\alpha(\Delta T - d)}}{\alpha(\Delta T - d)} \right) - 2Cd + \frac{Cd^2}{\Delta T} \]  

(2.22)

where \( C \) is the count rate of the detector with dead time \( d \) [29].
Chapter 3

The YALINA Experiments

At the Joint Institute for Power and Nuclear Research in Sosny outside Minsk in Belarus, two subcritical cores have been constructed: YALINA-Thermal and YALINA-Booster. These two cores are the major subject for testing and validation of the reactivity determination methods presented in the previous chapter. YALINA-Thermal (referred to as Yalina in Paper I) started operation in the beginning of this century. In 2005 the fuel of YALINA-Thermal was moved to the new core YALINA-Booster. Paper I covers experiments performed in October 2004 based on the YALINA-Thermal core and Paper II treats experiments performed on the YALINA-Booster core in June 2006.

Yalina (Яаліна) is the Belarusian word for spruce.

3.1 YALINA-Thermal

The YALINA-Thermal core is loaded with uranium dioxide of 10% enrichment in $^{235}\text{U}$. The fuel pins are situated in a lattice of quadratic geometry, depicted in Figure 3.1. The region closest to the central deuteron target is filled with lead in order to obtain a more spallation-like neutron spectrum. Outside the lead zone, there is a moderating region, filled by polyethylene ($\text{C}_2\text{H}_4$). The reflector is made of graphite with a thickness of about 40 cm. Five experimental channels (EC) are placed at different positions at different radial distances. The experimental channels are positioned in such a way that their relative influence on each other is minimized. As can be seen in Figure 3.1, EC1 is close to the source, EC2 is penetrating the lead zone, EC3 is located in the moderating thermal zone and EC5 and EC6 are located in the reflector. There are in total 280 fuel
elements, each of them with a diameter of 11 mm. The spacing between two adjacent elements is 20 mm and the total length of the active fuel is 500 mm.

![Figure 3.1. Vertical cross-sectional view of the YALINA-Thermal core.](image)

### 3.2 YALINA-Booster

YALINA-Booster is a subcritical core with two zones employing a fast and a thermal neutron spectrum respectively. The core consists of a central lead zone, a polyethylene zone, a radial graphite reflector and a front and back biological shielding consisting of borated polyethylene (Figure 3.2). The loading is 132 fuel pins, containing 90% enriched metallic uranium, 563 fuel pins containing uranium dioxide of 36% enrichment and a maximum of 1141 EK-10 fuel pins containing uranium dioxide of 10% enrichment. The zero-power core is cooled by natural convection of the surrounding air.

The fast-spectrum lead zone and the thermal-spectrum polyethylene zone are separated by a so called thermal neutron filter, which consists of one layer of 108 metallic uranium pins and one layer of 116 boron carbide (B₄C) pins, which are placed in the outermost two rows of the fast zone. Thermal neutrons diffusing from the thermal zone to the fast zone will either be absorbed by the boron or by the natural uranium, or they will be transformed into fast neutrons through fission in the natural uranium. In this way, a coupling of only fast neutrons between the two zones is maintained.
There are seven axial experimental channels (EC1B-EC4B and EC5T-EC7T) in the core and two axial experimental channels (EC8R and EC9R) and one radial experimental channel (EC10R) in the reflector. Moreover, there is one neutron flux monitoring channel in each corner of the core. Three B$_4$C-control rods, with a total reactivity worth of approximately -300 pcm can be inserted into the thermal zone. A detailed description of the core is available in the YALINA-Booster benchmark description [30].

In these measurements two configurations were studied. These configurations have a fully loaded fast zone, as described above, and 1132 and 1061 fuel pins of 10% enrichment in the thermal zone respectively. The loading of these fuel pins was made based on cylindrical symmetry (Figure 3.2).

Figure 3.2. Schematic cross-sectional view of YALINA-Booster (the 1132-configuration).
3.3 The Neutron Source

For the PNS and source jerk experiments, a so-called neutron generator was used. The neutron generator is a deuteron ion accelerator coupled to a Ti-T or Ti-D target, located in the center of the core. The D-T or D-D fusion reactions give neutrons with energy around 14 MeV and 2.5 MeV respectively. The neutron generator can be operated in both continuous and pulsed mode and provides the possibility to generate pulses with frequencies from 1 Hz to 7 kHz with pulse duration of 2 – 130 μs. The maximum beam current in continuous mode is 2 mA, with a beam diameter of about 20 mm, giving a maximum neutron yield of approximately $2 \cdot 10^{11}$ neutrons per second for the Ti-T target and $2 \cdot 10^9$ neutrons per second for the Ti-D target. The deuteron energy is around 250 keV.

For the Rossi-α and Feynman-α measurements, a $^{252}$Cf-source was used.

3.4 Detectors and Data Acquisition

For all measurements in YALINA-Thermal, a $^3$He-detector of 10 mm active length was used. This detector type relies mainly on the (n,p)-reaction in $^3$He, thus being sensitive mainly to thermal neutrons. Data was collected using a multi-scaler (Turbo-MCS). The total dead time of the electronic chain has been estimated to 0.8 μs.

In the YALINA-Booster experiments, a $^3$He-detector with an active length of 250 mm was used. This large detector made it possible to perform noise measurements, which require an efficient detector by means of detecting a large number of neutrons per fission. Moreover, it shortened the measurement times considerably. Data were collected by two systems: a multi-scaler and a counter/timer card. The counter/timer card has been programmed by the author to register the arrival time of each detection using a time stamping routine with an accuracy of 12.5 ns. By doing so, a complete record of the experiment is collected and all analysis can be performed afterwards. This is suitable for the noise analysis, since it allows for both a Rossi-α and a Feynman-α analysis on the same data. The total dead time of the electronic chain was in the YALINA-Booster experiments estimated to 3.3 μs.
Chapter 4

The Monte Carlo Simulation Tool

4.1 MCNP

In order to support the analysis of the experimental results, the experimental setups were analyzed by a Monte Carlo simulation tool. For YALINA-Thermal, the code MCNP4c3 [31] was used, and for YALINA-Booster MCNP5 [32]. The basic principle of a Monte Carlo code is that based on a very detailed three-dimensional model and a nuclear interaction data library, a huge amount of particle stories is simulated. In this case, neutrons are transported through a model of YALINA-Thermal [33] or YALINA-Booster [34]. Information concerning interactions with nuclides, such as scattering, capture, fission etc, is given by nuclear data libraries. In the YALINA-Thermal study, the nuclear data libraries ENDF/B-VI, JEFF3.0 and JENDL3.3 were used, whereas in the YALINA-Booster study the libraries JEFF3.1 and JENDL3.3 were used [5]. After a large amount of transported neutrons, quantities such as effective multiplication factor, thermal flux, and reaction rates can be determined.

4.2 Errors

Results from the Monte Carlo calculation method are always accompanied by a statistical error. On top of that, there are errors from the nuclear data libraries and modeling errors.

By simulating a large number of neutron histories, \(N\), the statistical error can be reduced since the relative error, \(\epsilon_{rel}\), follows
\[ e_{rel} = \frac{1}{\sqrt{N}}. \]  

For this reason some of the calculations were performed on a computer cluster with up to ten processors operating in parallel. In most cases the statistical error is smaller than other sources of error and can in those cases be negligible.

Errors from nuclear data libraries were in these studies only investigated by changing data library. The identification of uncertainties from individual nuclides is outside the scope of this thesis. Deviations between different libraries were found to be small. Significant differences were found only for JENDL3.3 in the calculation of the effective delayed neutron fraction for YALINA-Thermal.

Modeling errors were a major concern for YALINA-Booster, but not for YALINA-Thermal. In YALINA-Thermal only a few materials were used in relatively small amounts, whereas in YALINA-Booster other construction materials had to be used to support the much heavier construction. Moreover, the materials used in YALINA-Booster that also was used in YALINA-Thermal were used in larger amounts. These materials (stainless steel, lead, aluminum, polyethylene) contain traces of neutron absorbing nuclides. These traces had to be taken into account to achieve reliable results for YALINA-Booster. Sensitivity studies on these traces have been performed on both YALINA-Thermal and YALINA-Booster, but the influence was significant only for YALINA-Booster. In fact, the MCNP calculations of YALINA-Thermal in Paper I were performed without the trace materials, since the traces were discovered later during the work with YALINA-Booster. However, after inserting the trace materials into the YALINA-Thermal model it was found that the contribution to the reactivity was only 160±20 pcm. A complete description of the materials including the trace materials can be found in the YALINA-Booster benchmark specification [30].

### 4.3 Calculation of Integral Kinetic Parameters in MCNP

When calculating integral kinetic parameters, one must keep in mind that these parameters were originally defined for critical systems, since the corresponding critical adjoint function is used when defining them. This makes their definitions ambiguous in source-driven systems [35]; however, the classical definitions [21] will here be considered as sufficient.
4.3.1 The effective multiplication factor

The effective multiplication factor, $k_{\text{eff}}$, can easily be calculated by MCNP using the KCODE card. In this calculation, $k_{\text{eff}}$ is calculated as

$$k_{\text{eff}} = \lim_{n \to \infty} \frac{N_{n+1}}{N_n},$$

(4.2)

where $N_n$ is the number of neutrons in the $n$th neutron generation. To get a reliable result, one must omit the first neutron generations, while a well-distributed fission source has not yet been established.

4.3.2 The effective delayed neutron fraction

MCNP does not have any built-in routine that calculates the effective delayed neutron fraction. Klein Meulekamp and van der Marck have developed a routine that can be used for this purpose [36,37]. This routine calculates $\beta_{\text{eff}}$ as

$$\beta_{\text{eff}} = \frac{N_d}{N_{\text{tot}}},$$

(4.3)

where $N_d$ is the number of fissions induced by delayed neutrons and $N_{\text{tot}}$ is the total amount of fissions occurred. They show, however, that the same result can be achieved from two consecutive KCODE-runs:

$$\beta_{\text{eff}} \approx 1 - \frac{k_p}{k_{\text{eff}}}. $$

(4.4)

In this equation, $k_p$ is the multiplication factor when transporting prompt neutrons only, given by the MCNP cards TOTNU NO and PHYS:n 3j -1 together with KCODE. The ordinary $k_{\text{eff}}$ is calculated with TOTNU and PHYS:n 3j -1. This method requires 40 times more CPU-time than the first method above [36,37], but, on the other hand, the source code does not have to be changed.

4.3.3 The prompt neutron decay constant

In the older version of MCNP, version 4c3, there is a function to calculate the prompt neutron decay constant, $\alpha$. However, this function, acode, works only for critical or near critical systems and was removed in MCNP version 5. Instead, to determine $\alpha$, the neutron flux after a source neutron pulse insertion as a function of time was calculated. For this calculation, it is preferable to omit the delayed neutrons. When the exponential flux decay is known, it is straightforward to determine the prompt neutron decay constant. The drawback of this method is that it requires a large amount of CPU-time to achieve sufficient statistics.
4.3.4 The neutron reproduction time

In Monte Carlo calculations, there is no direct access to either the adjoint neutron flux or the importance function. This means that all results from MCNP are non-adjoint weighted. Unfortunately, this has large influence on time parameters, such as the neutron reproduction time. As an example, for YALINA-Thermal the difference between the expected value and the direct value from MCNP was almost 200%. Instead, in these studies, the neutron reproduction time has been calculated from the reactivity, the effective delayed neutron fraction and the prompt neutron decay constant based on Eq. (2.12).
Chapter 5

Experimental Results

5.1 YALINA-Thermal

The results presented here originate from measurements performed in October 2004.

5.1.1 PNS slope fitting

Pulses of 2 μs duration were introduced in the core and the detector was located in EC1 – EC6. PNS histograms for EC3 and EC6 are displayed in Figure 5.1. Unfortunately, in the data from EC1 – EC3 there are no single exponential decays, making the analysis difficult. Therefore, a sum of many exponentials,

\[ f(t) = \sum_{i=1}^{\infty} A_i e^{\alpha_i t}, \quad (5.1) \]

was fitted to the experimental data. This method was first adopted to the reflector channels with satisfactory results. Thereafter, the same procedure was carried out on the core channels data. In all cases, not more than three exponentials and a constant level were required to reach a satisfactory reduced \( \chi^2 \). This pure mathematical method of fitting a sum of exponentials is questionable since there is no physical model behind. For deeper subcriticalities it is not recommended to adopt this method. However, in this case, the decay of the fundamental mode is clearly visible in the reflector channels thus indicating the expected slope of the other curves. The results can be found in Table 5.1 and indicates a spatial spread of approximately 7%.
Figure 5.1. PNS histograms for YALINA-Thermal (time discretization 50 μs).

Table 5.1 Results from the PNS experiment in YALINA-Thermal.

<table>
<thead>
<tr>
<th>Experimental Channel</th>
<th>α [s⁻¹]</th>
<th>ρ/β_{eff} [$\mu$]</th>
<th>Λ/β_{eff} [ms]</th>
</tr>
</thead>
<tbody>
<tr>
<td>EC1</td>
<td>-675±13</td>
<td>-13.9±0.1</td>
<td>22.1±0.4</td>
</tr>
<tr>
<td>EC2</td>
<td>-722±19</td>
<td>-13.7±0.1</td>
<td>20.3±0.6</td>
</tr>
<tr>
<td>EC3</td>
<td>-711±11</td>
<td>-12.9±0.1</td>
<td>19.5±0.3</td>
</tr>
<tr>
<td>EC5</td>
<td>-634±21</td>
<td>-13.0±0.1</td>
<td>22.1±0.7</td>
</tr>
<tr>
<td>EC6</td>
<td>-653±2</td>
<td>-13.5±0.1</td>
<td>22.2±0.2</td>
</tr>
<tr>
<td>Weighted Mean</td>
<td>-656</td>
<td>-13.4</td>
<td>21.6</td>
</tr>
<tr>
<td>Spread⁵</td>
<td>45 (6.9%)</td>
<td>0.4 (3.3%)</td>
<td>1.3 (5.9%)</td>
</tr>
</tbody>
</table>

5.1.2 PNS area method

The PNS area method, Eq. (2.13), was applied to the same data as in the previous section and the results are shown in Table 5.1. The spatial spread is 0.4 $\mu$ or 3.3%. The ratio Λ/β_{eff} has been calculated from Eq. (2.14).

Further analysis may be performed with the knowledge of the effective delayed neutron fraction. Values both for β_{eff} and k_{eff} from MCNP are found in Table 5.2. Concerning β_{eff}, one can observe that ENDF/B-VI and JEFF3.0 give similar values and JENDL3.3 a slightly lower value. For k_{eff} the libraries give results around 0.92 not more than about 300 pcm apart. Based on the

⁵ Spatial spreads are calculated relatively the weighted mean.
values in Table 5.1, inferred semi-experimental values of $k_{\text{eff}}$ and $\Lambda$ can be found through

$$k_{\text{eff}} = \frac{1}{1 - \left[ \frac{\rho}{\beta_{\text{eff}}} \right]^{\exp}_{\text{MCNP}}}$$  \hspace{1cm} (5.2)

and

$$\Lambda = \frac{1}{\alpha^{\exp}} \left( \left[ \frac{\rho}{\beta_{\text{eff}}} \right]^{\exp} - 1 \right) \beta_{\text{eff}}^{\text{MCNP}},$$  \hspace{1cm} (5.3)

where “exp” refers to experimental data. By using the weighted mean values of $\alpha$, $\rho/\beta_{\text{eff}}$ and $\beta_{\text{eff}}$ and their spread as uncertainties, the result yields $k_{\text{eff}} = 0.906\pm0.004$ and $\Lambda = 170\pm14$ $\mu$s. Apparently, the $k_{\text{eff}}$ calculated by MCNP deviates from the experimental value from the area method. In Paper I, the neutron reproduction time was estimated by MCNP through the calculation of $\alpha$ and a value around 140 $\mu$s was found. This comes from the different values of $k_{\text{eff}}$.

Table 5.2. Results from MCNP (YALINA-Thermal).

<table>
<thead>
<tr>
<th>Library</th>
<th>$\beta_{\text{eff}}$ [pcm]</th>
<th>$k_{\text{eff}}$ (MCNP)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ENDF/B-VI</td>
<td>788±9</td>
<td>0.91803±0.00005</td>
</tr>
<tr>
<td>JEFF3.0</td>
<td>793±9</td>
<td>0.92010±0.00007</td>
</tr>
<tr>
<td>JENDL3.3</td>
<td>742±9</td>
<td>0.92114±0.00006</td>
</tr>
</tbody>
</table>

5.1.3 Source jerk

The source jerk experiment was performed by measuring the neutron flux with a $^3$He-detector in EC2 when discharging the ion feeder. This procedure gives a very quick shut-down (on the microsecond scale). During the writing of this thesis, it was found that some of the calculations related to the source jerk experiment in Paper I were incorrect. The numbers presented here are correct and the result presented in Paper I is unfortunately wrong. In particular, this concerns the error estimation.

As indicated in Eq. (2.15), the neutron flux levels before and immediately after the source jerk must be found in order to estimate the reactivity. The neutron flux level before the source jerk can easily be found by evaluating the mean value of the count rate the seconds before the shut-down. When performing this, possible source fluctuations were not considered. The second level, on the other hand, is more difficult to obtain since it is not constant in time (Figure 5.2). However, during a very short time-scale, the neutron flux can be regarded as semi-stable and a mean value can be evaluated. The length
of this semi-stability was estimated by means of the reduced $\chi^2$-value relative to the mean value. In this way it was found that $n_0 = 100970 \pm 165$ s$^{-1}$ and $n_1 = 9518 \pm 380$ s$^{-1}$, which finally yields $\rho/\beta_{\text{eff}} = -9.6 \pm 0.4$ $\%$. This value corresponds to $k_{\text{eff}} = 0.931 \pm 0.004$, which is higher compared to the PNS measurements. A reason for that might be unstable source operations.

![Graph](image)

Figure 5.2. Source jerk experiment in YALINA-Thermal (EC2). The time discretization is 20 ms.

### 5.1.4 Summary of YALINA-Thermal Results

The most important results from the YALINA-Thermal measurement and analysis are summarized in Table 5.3.

---

6 In Paper I, the value $\rho/\beta_{\text{eff}} = -8.9 \pm 1.9$ $\%$ was given based on a fitting of an exponential function after the source jerk.
<table>
<thead>
<tr>
<th>Experimental Methods</th>
<th>Simulation</th>
<th>Exp. + Sim.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slope fitting</td>
<td>MCNP keff</td>
<td>(\beta_{\text{eff}}) [pcm]</td>
</tr>
<tr>
<td>Area + Slope fitting</td>
<td>MCNP keff</td>
<td>(\beta_{\text{eff}}) [pcm]</td>
</tr>
<tr>
<td>Area + MCNP</td>
<td>(\Lambda) [ms]</td>
<td>(k_{\text{off}})</td>
</tr>
<tr>
<td>(a_s) [s(^{-1})]</td>
<td>(\rho / \beta_{\text{eff}}) [$\mu$]</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>EC</th>
<th>(a_s)</th>
<th>(\rho / \beta_{\text{eff}})</th>
<th>(k_{\text{off}})</th>
<th>(\Lambda)</th>
<th>(k_{\text{off}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>EC1</td>
<td>(-675\pm13)</td>
<td>(-13.9\pm0.1)</td>
<td>-</td>
<td>-</td>
<td>(-13.9\pm0.1)</td>
</tr>
<tr>
<td>EC2</td>
<td>(-722\pm19)</td>
<td>(-12.9\pm0.1)</td>
<td>-</td>
<td>-</td>
<td>(-13.7\pm0.1)</td>
</tr>
<tr>
<td>EC3</td>
<td>(-711\pm11)</td>
<td>(-9.6\pm0.4)</td>
<td>-</td>
<td>-</td>
<td>(-9.6\pm0.4)</td>
</tr>
<tr>
<td>EC4</td>
<td>(-634\pm21)</td>
<td>(-13.0\pm0.1)</td>
<td>-</td>
<td>-</td>
<td>(-13.0\pm0.1)</td>
</tr>
<tr>
<td>EC5</td>
<td>(-653\pm2)</td>
<td>(-12.9\pm0.1)</td>
<td>-</td>
<td>-</td>
<td>(-12.9\pm0.1)</td>
</tr>
<tr>
<td>EC6</td>
<td>(-653\pm2)</td>
<td>(-13.5\pm0.1)</td>
<td>-</td>
<td>-</td>
<td>(-13.5\pm0.1)</td>
</tr>
</tbody>
</table>

Table 5.3. Summary of the most important YALINA-Thermal results.
5.2 YALINA-Booster

Results from measurements performed in June 2006 are presented below.

5.2.1 PNS slope fitting

In contrast to YALINA-Thermal, the PNS response in YALINA-Booster was much easier to analyze due to the single exponential behavior. PNS histograms for detectors in the core, EC6T, and the reflector, EC8R, are shown in Figure 5.3 and Figure 5.4. For comparison, histograms for both configurations in EC6T are shown in Figure 5.5. The prompt neutron decay constants were found by fitting a function of the form

\[ f(t) = A_1 e^{\alpha t} + A_2. \]  

(5.4)

The first part of each histogram was omitted and a sensibility analysis was performed to find the best starting point of the fitting. In all cases, a reduced \( \chi^2 \)-value less than 1% from unity was obtained. Results for both configurations and for all experimental channels are shown in Table 5.4 and Figure 5.6 indicating small spatial spreads of 1.1% and 1.5% respectively.

![Figure 5.3. PNS histogram for the 1132-configuration (normalized to the constant level and with time discretization 5 \( \mu s \)).](image-url)
Figure 5.4. PNS histogram for the 1061-configuration (normalized to the constant level and with time discretization 5 μs).

Figure 5.5. PNS histogram for the central core channel, EC6T, for the 1061- and 1132-configurations (time discretization 5 μs).
Table 5.4. Results from PNS measurements.

<table>
<thead>
<tr>
<th>Conf.</th>
<th>EC</th>
<th>$\alpha$ [s$^{-1}$]</th>
<th>$\rho/\beta_{eff}$ [$%$]</th>
<th>$\Lambda/\beta_{eff}$ [ms]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1132</td>
<td>EC5T</td>
<td>-654.1±1.6</td>
<td>-3.60±0.03</td>
<td>7.04±0.04</td>
</tr>
<tr>
<td></td>
<td>EC6T</td>
<td>-662.6±1.8</td>
<td>-3.36±0.02</td>
<td>6.58±0.04</td>
</tr>
<tr>
<td></td>
<td>EC7T</td>
<td>-649.4±1.4</td>
<td>-3.37±0.02</td>
<td>6.73±0.03</td>
</tr>
<tr>
<td></td>
<td>EC8R</td>
<td>-648.7±2.0</td>
<td>-3.61±0.02</td>
<td>7.11±0.03</td>
</tr>
<tr>
<td></td>
<td>EC9R</td>
<td>-644.0±3.3</td>
<td>-3.92±0.03</td>
<td>7.65±0.05</td>
</tr>
<tr>
<td>Weighted mean value</td>
<td>-652.7</td>
<td>-3.54</td>
<td>6.96</td>
<td></td>
</tr>
<tr>
<td>Spread</td>
<td>7.1 (1.1%)</td>
<td>0.23 (6.5%)</td>
<td>0.42 (6.0%)</td>
<td></td>
</tr>
<tr>
<td>1061</td>
<td>EC5T</td>
<td>-875.0±1.9</td>
<td>-5.09±0.03</td>
<td>6.96±0.03</td>
</tr>
<tr>
<td></td>
<td>EC6T</td>
<td>-892.1±2.7</td>
<td>-4.71±0.03</td>
<td>6.40±0.03</td>
</tr>
<tr>
<td></td>
<td>EC7T</td>
<td>-866.7±2.2</td>
<td>-4.75±0.03</td>
<td>6.64±0.03</td>
</tr>
<tr>
<td></td>
<td>EC8R</td>
<td>-863.4±3.2</td>
<td>-5.07±0.03</td>
<td>7.03±0.03</td>
</tr>
<tr>
<td></td>
<td>EC9R</td>
<td>-860.1±3.6</td>
<td>-5.33±0.03</td>
<td>7.36±0.04</td>
</tr>
<tr>
<td>Weighted mean value</td>
<td>-872.7</td>
<td>-4.95</td>
<td>6.82</td>
<td></td>
</tr>
<tr>
<td>Spread</td>
<td>12.9 (1.5%)</td>
<td>0.26 (5.3%)</td>
<td>0.38 (5.5%)</td>
<td></td>
</tr>
</tbody>
</table>

Figure 5.6. Prompt neutron decay constants from the PNS fitting method for different detector positions. The weighted mean values are indicated as straight lines.

5.2.2 PNS area method

The constant level of delayed neutrons was found from the parameter $A_2$ in Eq. (5.4). The results are shown in Table 5.4 and Figure 5.7 and indicate a spatial spread of approximately 6%. An MCNP analysis was performed based on the two nuclear data libraries JEFF3.1 and JENDL3.3. Results concerning $k_{eff}$ and $\beta_{eff}$ can be found in Table 5.5. In the analysis, the value $\beta_{eff} = 733±21$ pcm has been used for both configurations. From the experi-
mental values of the reactivities in dollars and the prompt neutron decay constants, $k_{\text{eff}}$ and $\Lambda$ have been calculated in the same way as for YALINA-Thermal:

\[
k_{\text{eff}}^{1132} = 0.975 \pm 0.002
\]

\[
k_{\text{eff}}^{1061} = 0.965 \pm 0.002
\]

\[
\Lambda^{1132} = 51.0 \pm 3.4 \mu s
\]

\[
\Lambda^{1061} = 50.0 \pm 3.1 \mu s.
\]

The values for $k_{\text{eff}}$ are in good agreement with those from MCNP. There is only a small difference in neutron reproduction time for the two configurations.

From the measured reactivities, the reactivity difference can easily be calculated. According to Figure 5.7 both configurations obey the same spatial dependence. Therefore, it is expected that the difference between the two reactivity levels has smaller spatial dependence. In fact, the spatial spread is small: $\Delta \rho / \beta_{\text{eff}} = 1.41 \pm 0.05$ $\mu s$.

Figure 5.7. Reactivity (in dollars) from the area method for different detector positions. The weighted mean values are indicated as straight lines.
Table 5.5. Results from MCNP (YALINA-Booster).

<table>
<thead>
<tr>
<th>Conf.</th>
<th>Library</th>
<th>$\beta_{\text{eff}}$ [pcm]</th>
<th>$k_{\text{eff}}$ (MCNP)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1132</td>
<td>JEFF3.1</td>
<td>734.6±6.5</td>
<td>0.97602±0.00004</td>
</tr>
<tr>
<td></td>
<td>JENDL3.3</td>
<td>738.4±16</td>
<td>0.97646±0.00010</td>
</tr>
<tr>
<td>1061</td>
<td>JEFF3.1</td>
<td>728.2±8.9</td>
<td>0.96267±0.00005</td>
</tr>
<tr>
<td></td>
<td>JENDL3.3</td>
<td>737.0±17</td>
<td>0.96343±0.00010</td>
</tr>
</tbody>
</table>

5.2.3 Rossi-α

Figure 5.8 and Figure 5.9 show Rossi-α histograms normalized to the mean count rate\(^7\). When analyzing the Rossi-α histograms it appeared that they did not perfectly obey Eq. (2.16). During the first millisecond a fast decay was observed, followed by a single exponential decay decay and a constant level. This behaviour is not described by the traditional approach and, consequently, these data points were avoided in the fitting process. The prompt neutron decay constant was found by removing the unit constant level from the normalized histogram and then a single line was fitted to the logarithm of the histogram. It was found that the result was very sensitive to the time bin and the starting point of the fitting but not to the end point or the width of the histogram. For each measurement, a large number of linear fittings with deviation from unity of the reduced $\chi^2$ less that 1% could be obtained for different starting points and time bins. Consequently, for each measurement a distribution of prompt neutron decay constants and their spread was obtained. The result represents a mean value of these candidate prompt neutron decay constants and the standard deviation is taken as their spread. In this way it was possible to obtain an acceptable representation of the numerous solutions and their individual error.

Results for $\alpha$ are shown in Table 5.6. A comparison between prompt neutron decay constants from the Rossi-α measurement and those from the PNS slope fitting, shows that the values are in general, but not perfectly, in agreement for the 1132-configuration. However, for the 1061-configuration the results diverge strongly. Similar fitting problems were found in other studies [38, 39]. An explanation might come from the fact that for small time-scales, and deep subcriticalities, effects from the fast-thermal region coupling as well as reflector effects may become important. It was recently suggested that a two-region model may be applied to describe the Rossi-α distributions with satisfactory results [38], but it has not been verified in this study.

\(^7\) The label "Auto-correlation" in the figures relates to the fact that events from the same detector are correlated to each other, hence they are auto-correlated.
Table 5.6. Results from the Rossi-α analysis.

<table>
<thead>
<tr>
<th>Conf.</th>
<th>EC</th>
<th>$\alpha$ [s$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1132</td>
<td>EC5T</td>
<td>-693±37</td>
</tr>
<tr>
<td></td>
<td>EC6T</td>
<td>-743±20</td>
</tr>
<tr>
<td></td>
<td>EC7T</td>
<td>-628±15</td>
</tr>
<tr>
<td></td>
<td>EC9R</td>
<td>-673±48</td>
</tr>
<tr>
<td>1061</td>
<td>EC5T</td>
<td>-1076±75</td>
</tr>
<tr>
<td></td>
<td>EC6T</td>
<td>-1008±78</td>
</tr>
<tr>
<td></td>
<td>EC7T</td>
<td>-1022±53</td>
</tr>
</tbody>
</table>

Figure 5.8. Rossi-α histograms for the 1132-configuration.

Figure 5.9. Rossi-α histograms for the 1061-configuration.
5.2.4 Pulsed Rossi-α

Kitamura et al. have described how to analyze the data from a pulsed Rossi-α measurement [28]. First the Rossi-α histogram is calculated as usual, as displayed in Figure 5.10. Since the source is pulsed, the histogram looks very different from traditional Rossi-α histograms.

![Figure 5.10. Rossi-α histogram for YALINA-Booster with a pulsed source.](image)

The assumption that the first term of Eq. (2.18) has decayed completely after 80 ms is then made. At that time, the signal only consists of the two uncorrelated terms. Since these terms do not decay in time, experimental data can be chosen from the interval 80-93 ms (the last U-form) and then be deleted subsequently from the previous U-formed intervals. By doing so, only the correlated exponential term will be left, as depicted in Figure 5.11. In an ideal case, the constant level after removal will be zero. Finally, the exponent is determined in the same as was done for the classical Rossi-α case. An example of the final solution is depicted in Figure 5.12.

All results are presented in Table 5.7. It is found that the results from the pulsed Rossi-α analysis are in agreement with the results from the PNS fitting method. The errors are larger for the Rossi-α case, due to the more complicated data treatment required to arrive at the result. Compared to the traditional Rossi-α approach, the pulsed method gives results with much higher accuracy. This comes from the fact that the amplitude of the correlated term is much higher in the pulsed experiment, thus increasing the signal-to-noise ratio.
Figure 5.11. Rossi-$\alpha$ histogram with the oscillation term removed.

Figure 5.12. Fitting of exponential to the Rossi-$\alpha$ histogram after the removal of the oscillation term.
Table 5.7. Results from the pulsed Rossi-α analysis.

<table>
<thead>
<tr>
<th>Conf.</th>
<th>EC</th>
<th>α [s⁻¹]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1132</td>
<td>EC5T</td>
<td>-671±9</td>
</tr>
<tr>
<td></td>
<td>EC9R</td>
<td>-618±44</td>
</tr>
<tr>
<td>1061</td>
<td>EC5T</td>
<td>-854±4</td>
</tr>
<tr>
<td></td>
<td>EC6T</td>
<td>-881±9</td>
</tr>
<tr>
<td></td>
<td>EC7T</td>
<td>-890±11</td>
</tr>
<tr>
<td></td>
<td>EC8R</td>
<td>-868±11</td>
</tr>
<tr>
<td></td>
<td>EC9R</td>
<td>-842±36</td>
</tr>
</tbody>
</table>

5.2.5 Feynman-α

Feynman-α distributions were calculated for measurements performed in EC5T-EC7T for both configurations (Figure 5.13 and Figure 5.14). In the reflector channels, the statistical accuracy was too low to give reliable results. Function fitting based on Eq. (2.22) was performed to obtain the prompt neutron decay constants shown in Table 5.8. As can be seen, the results from the Feynman-α analysis are in agreement with the Rossi-α results but fail to predict the PNS slope fitting results. During the fitting process it was noticed that the result was strongly dependent on the choice of fitting interval. The influence on α from the fitting interval was much larger than the statistical error from the fitting procedure itself. Therefore, the spread of the α-results from many different fitting intervals, all giving reduced χ² equal to unity, was used as the uncertainty of the final value. The final value was considered to be the mean value of the decay constants obtained from these fittings. In this context one must remember that Eq. (2.20) and Eq. (2.22) are valid only if ΔT is much smaller than the time constant of the fastest decaying delayed neutron group [26]. Therefore, in the analysis, no data points after 30 ms were considered.

Table 5.8. Results from the Feynman-α analysis.

<table>
<thead>
<tr>
<th>Conf.</th>
<th>EC</th>
<th>α [s⁻¹]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1132</td>
<td>EC5T</td>
<td>-746±72</td>
</tr>
<tr>
<td></td>
<td>EC6T</td>
<td>-787±45</td>
</tr>
<tr>
<td></td>
<td>EC7T</td>
<td>-825±46</td>
</tr>
<tr>
<td>1061</td>
<td>EC5T</td>
<td>-1057±42</td>
</tr>
<tr>
<td></td>
<td>EC6T</td>
<td>-1035±10</td>
</tr>
<tr>
<td></td>
<td>EC7T</td>
<td>-1194±23</td>
</tr>
</tbody>
</table>
Figure 5.13. Feynman-\(\alpha\) plots for the 1132-configuration.

Figure 5.14. Feynman-\(\alpha\) plots for the 1061-configuration.
5.2.6 Summary of YALINA-Booster Results

The results from the YALINA-Booster measurement and analysis are summarized in Table 5.9.
Table 5.9. Summary of results from the YALINA-Booster experiments.

<table>
<thead>
<tr>
<th>Conf.</th>
<th>EC</th>
<th>Area</th>
<th>Slope fitting</th>
<th>Rossi-α</th>
<th>Pulsed Rossi-α</th>
<th>Feynman-α</th>
<th>MCNP</th>
<th>Area+ MCNP</th>
<th>Area+ Slope fitting+ MCNP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>ρ/β&lt;sub&gt;eff&lt;/sub&gt;</td>
<td>α [s&lt;sup&gt;-1&lt;/sup&gt;]</td>
<td>k&lt;sub&gt;eff&lt;/sub&gt;</td>
<td>β&lt;sub&gt;eff&lt;/sub&gt; [pcm]</td>
<td>k&lt;sub&gt;eff&lt;/sub&gt;</td>
<td>Λ [ms]</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>[$]$</td>
<td>[s&lt;sup&gt;-1&lt;/sup&gt;]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>EC5T</td>
<td>-3.60±0.03</td>
<td>-654.1±1.6</td>
<td>-693±37</td>
<td>-671±9</td>
<td>-746±72</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>EC6T</td>
<td>-3.36±0.02</td>
<td>-662.6±1.8</td>
<td>-743±20</td>
<td>-</td>
<td>-787±45</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1132</td>
<td>EC7T</td>
<td>-3.37±0.02</td>
<td>-649.4±1.4</td>
<td>-628±15</td>
<td>-</td>
<td>-825±46</td>
<td>0.976</td>
<td>0.975±0.002</td>
<td>51.0±3.4</td>
</tr>
<tr>
<td></td>
<td>EC8R</td>
<td>-3.61±0.02</td>
<td>-648.7±2.0</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>EC9R</td>
<td>-3.92±0.03</td>
<td>-644.0±3.3</td>
<td>-673±48</td>
<td>-618±44</td>
<td>-</td>
<td>733±21</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>EC5T</td>
<td>-5.09±0.03</td>
<td>-875.0±1.9</td>
<td>-</td>
<td>-1076±75</td>
<td>854±4</td>
<td>-1057±42</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>EC6T</td>
<td>-4.71±0.03</td>
<td>-892.1±2.7</td>
<td>-</td>
<td>-1008±78</td>
<td>881±9</td>
<td>-1035±10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1061</td>
<td>EC7T</td>
<td>-4.75±0.03</td>
<td>-866.7±2.2</td>
<td>-</td>
<td>-1022±53</td>
<td>890±11</td>
<td>-1194±23</td>
<td>0.963</td>
<td>0.965±0.002</td>
</tr>
<tr>
<td></td>
<td>EC8R</td>
<td>-5.07±0.03</td>
<td>-863.4±3.2</td>
<td>-</td>
<td>-868±11</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>EC9R</td>
<td>-5.33±0.03</td>
<td>-860.1±3.6</td>
<td>-</td>
<td>-842±36</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Chapter 6

Conclusions

Through the YALINA-experiments, reactivity determination methods have been investigated experimentally in the region $0.90 < k_{\text{eff}} < 0.98$. In this chapter, some specific conclusions from YALINA-Thermal and YALINA-Booster are presented followed by some general conclusions and future applications of the results.

6.1 YALINA-Thermal

Through the YALINA-Thermal experiments, it was found that the PNS slope fitting method is less reliable when there is no fundamental mode. However, in this study, the prompt neutron decay constant could be estimated through multiple exponential fitting since a fundamental mode was visible in the reflector channels. Hence the PNS slope fitting method is preferable closer to criticality. In complete absence of a fundamental mode, a theoretical model describing the neutron flux after a neutron pulse insertion must be applied. Based on this model, the kinetic parameters can be adjusted to find the best fit to the measured neutron flux.

It was shown that the area method gives a value of the reactivity with a certain spatial spread. However, this value deviated slightly from the value obtained through simulations. Compared to MCNP, the effective multiplication factor was underestimated by the area method. However, the area method delivers a result with high statistical accuracy.

In this study, the source jerk method was just demonstrated. Although it did not reproduce the result of the other methods for one source interruption, it has probably a potential for reactivity monitoring if used in repeated mode.
6.2 YALINA-Booster

In the YALINA-Booster experiments it was found that the PNS slope fitting technique is very stable closer to criticality. The area method still suffer from spatial dependence also around $k_{\text{eff}} = 0.97$ as it did around $k_{\text{eff}} = 0.90$ in YALINA-Thermal. However, with the area method it is possible to measure reactivity differences with low spatial spread.

In these experiments, the continuous source Rossi-$\alpha$ and Feynman-$\alpha$ methods were not reliable in their classical definitions for low $k_{\text{eff}}$. The agreement with the PNS measurements was better for the less subcritical configuration.

The pulsed Rossi-$\alpha$ is reliable but unnecessarily complicated, since the result can be achieved directly through slope fitting of the PNS histogram.

6.3 General Conclusions

By reconsidering the list of requirements of an online reactivity meter in Section 1.5, one can find that actually none of the studied method fulfills all the criteria (short measurement time, low spatial dependence, high accuracy, detector type and neutron source independence). The requirement of short measurement time was fulfilled only by the source jerk method, but this method suffers from low accuracy. The PNS slope fitting method has low spatial dependence, but is more difficult to apply if there is no fundamental mode. On the other hand, the area method delivers results with high accuracy but with a certain spatial dependence. Further, the measurement time is long for both methods. Long measurement times are experienced also when applying the noise methods and the results from those methods are generally not trustworthy for deep subcriticalities. Moreover, the noise methods will probably not work for higher power levels, when passing the neutron noise threshold [40]. The last two features, detector type and neutron source independence, were not investigated in this study.

6.4 Future Applications

This thesis has provided deeper understanding in the most frequently used methods for subcriticality measurements, but apparently there is more work to be performed in order to find a suitable reactivity meter for subcritical systems. Concerning ADS, there are essentially two regions of applications for reactivity determination methods: fuel loading and regular operation. During loading of the core, most probably the inverse multiplication method will be used, as when loading any core. However, this method does only tell the
operator when the core will become critical and does not tell how subcritical
the core is. At some point \( k_{\text{eff}} \) must be known in order to find a suitable op-
eration configuration. In this case, some of the methods used in this study
may be used; the area method, which seems to be reliable in the region for
ADS operations, is probably a good choice. A problem is, however, that \( \beta_{\text{eff}} \)
must be known in order to translate the result of the measurement, given in
dollars, to \( k_{\text{eff}} \). For uranium and plutonium based systems this parameter can
easily be calculated. Moreover, it has been measured for several systems his-
torically [41,42]. However, for cores loaded with high fractions of americium
and curium, this is not as straight forward. The quality of the delayed neutron
data for these isotopes is not as good as for uranium and plutonium, which is
indicated in Figure 1.4, and measurements of \( \beta_{\text{eff}} \) for such systems have, to the
author’s knowledge, never been performed.

For the regular operation of an ADS, the current-to-flux measurement
technique has been proposed [18]. By measuring the proton beam current, \( I_p \),
and the neutron flux somewhere in the core a reactivity dependent ratio, \( R \), is
obtained:

\[
R(\rho) = c \frac{I_p}{\phi}, \quad \rho < 0.
\]  

(6.1)

Any change in \( R \) without a preceding change in beam current, may be inter-
preted as resulting from a reactivity change. The factor \( c \) may be calibrated
with the traditional reactivity determination methods and it is here assumed
that the neutron source strength is directly proportional to the beam current.
The current-to-flux method seems to be a reliable tool for online monitoring,
but it has not yet been verified.

Another related method, which probably can be performed in parallel with
the current-to-flux measurement, is the repeated source jerk experiment [43].
The source operates with short interruptions of length \( t_I \) with frequency
\( f = 1/T \) according to Figure 6.1. After the source interruption the beam starts
again and operates during the time \( t_B \), obeying

\[
T = t_I + t_B.
\]  

(6.2)

The expected corresponding neutron flux is depicted in Figure 6.2. Starting
from neutron flux level \( n_0 \), the neutron flux will decay following the prompt
neutron decay constant, \( \alpha \), to a semi-stable flux level \( n_I \). The level \( n_I \) will de-
crease slowly following the decay of the delayed neutron precursors. The
neutron flux level \( n_B \) indicates the inherent source of spontaneous fissions etc.
After several beam interruptions the level \( n_I \) can be estimated with high accu-
rance and the reactivity in dollars can be calculated as
\[
\rho = \frac{\left(n_1 - n_B\right) - \left(n_0 - n_B\right)}{\left(n_1 - n_B\right)} = \frac{n_1 - n_0}{n_1 - n_B} \tag{6.3}
\]

In addition, through this experiment the prompt neutron decay constant, \(\alpha\), can be found. When using this method in a full power ADS, \(t_I\) must be in the scale of a millisecond, not to induce thermal variations accompanied by induction of stress in the construction materials. Also this method has not yet been evaluated experimentally.

![Figure 6.1. Neutron source characteristics for the repeated source jerk experiment.](image)

![Figure 6.2. Expected neutron flux characteristics following the source variations in Figure 6.1.](image)

Although the methods studied in this thesis do not fulfil the requirements for being candidates for online reactivity monitoring, they will serve as tools for calibration of the possible future techniques presented here; in particular the combined PNS slope fitting and area methods.
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My warmest gratitude goes to my fiancée Elin who always supports me in the ups and downs of life. And last but not the least, my family is honored for their never ending commitment to my well being.

Thank you all!
Analysis of reactivity determination methods in the subcritical experiment Yalina

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Abstract

Different reactivity determination methods have been investigated, based on experiments performed at the subcritical assembly Yalina in Minsk, Belarus. The development of techniques for on-line monitoring of the reactivity level in a future accelerator-driven system (ADS) is of major importance for safe operation. Since an ADS is operating in a subcritical mode, the safety margin to criticality must be sufficiently large. The investigated methods are the Slope Fit Method, the Sjöstrand Method and the Source Jerk Method. The results are compared with Monte Carlo simulations performed with different nuclear data libraries. The results of the Slope Fit Method are in good agreement with the Monte Carlo simulation results, whereas the Sjöstrand Method appears to underestimate the criticality somewhat. The Source Jerk Method is subject to inadequate statistical accuracy.

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Keywords: Yalina; Reactivity determination; Subcritical; MCNP; ADS

1. Introduction

In order to reduce the radiotoxic inventory of the nuclear waste, accelerator-driven systems (ADS) have been suggested to transmute the accumulated transuranic elements [1]. In a subcritical core it is possible to use large fractions of “exotic” fuels, apart from plutonium also consisting of the minor actinides americium and curium. Intensive research programs investigating the physics and technology of proton accelerators, spallation targets and subcritical cores are required...
for the development of full-scale ADS in the future. This present work is dedicated to the study of reactivity determination methods of a subcritical zero-power core. The development of reliable methods for reactivity determination is essential for the safe operation and for the licensing of a future ADS.

Recently, the comprehensive MUSE program (multiplication with an external source), performed at the MASURCA facility in Cadarache, France, was completed [2]. In the MUSE experiments, a neutron generator, mainly consisting of a deuteron accelerator and a tritium target, was coupled to a subcritical core operating with a fast neutron energy spectrum. A major part of the experiments in MUSE was devoted to the investigation of methods for reactivity determination of different subcriticality levels [2].

Parallel with the MUSE program, another European program devoted to ADS studies has been running at the Yalina facility outside Minsk, Belarus [3,4]. This facility has the same basic construction with a neutron generator coupled to a subcritical core, but the neutron spectrum of Yalina is thermal. The fact that the fission chain process relies mainly on reactions induced by thermal neutrons, implies that the neutronic time scales are in the order of a factor $10^3$ larger than in a fast system. The methods used in this study, earlier evaluated in a fast spectrum in the MUSE experiments, will now be investigated in a thermal spectrum. A pulsed neutron source (PNS) experiment has been analyzed by the Slope Fit Method [5] and the Sjöstrand Method [6], and a source jerk experiment [5] has been performed and analyzed. The results have been compared with the results from the MUSE experiments and Monte Carlo simulations.

2. The Yalina facility

Yalina is a subcritical assembly operating with a thermal neutron energy spectrum. The external source coupled to the core is provided by a neutron generator consisting of a deuterium accelerator and a Ti–D or Ti–T target (Table 1). In the experiments, the Ti–T target was used, situated in the center of the core. As fuel, EK-10 type fuel rods with 10% enriched uranium oxide are used in polyethylene blocks for moderation. Under normal conditions, the core is loaded with 280 fuel rods, but this number can easily be modified. The very low power of the core makes the natural convection of the surrounding air sufficient for cooling.

The core (Fig. 1) consists of subassemblies surrounding the target and the ion channel up to the side dimensions of $400 \times 400 \times 576$ mm. Each subassembly is made of nine blocks ($80 \times 80 \times 63$ mm) of polyethylene (density $0.927 \text{ g/cm}^3$) with 16 channels for the fuel rods, with a diameter of 11 mm. At the sides and behind the target, the polyethylene blocks are replaced by lead blocks of the same geometry. The purpose of these lead blocks is to diffuse the energy spectrum of the (D,T)-neutrons into a more spallation-like spectrum, by $(n,xn)$- and scattering reactions. The core is surrounded by a graphite reflector with a thickness of approximately 400 mm. Only at the side facing the accelerator, and its opposite side, no shielding or borated polyethylene is used, for ease of handling. Five axial experimental channels (EC) with 25 mm diameter are located inside the core and reflector [7].

3. Initial Monte Carlo simulation

The experimental setup has been analyzed using MCNP version 4c3 [8]. The effective multiplication factor, $k_{eff}$, and the effective delayed neutron

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Main parameters of the neutron generator (NG-12-1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deuteron energy</td>
<td>100–250 keV</td>
</tr>
<tr>
<td>Beam current</td>
<td>1–12 mA</td>
</tr>
<tr>
<td>Pulse duration</td>
<td>0.5–100 μs</td>
</tr>
<tr>
<td>Pulse repetition frequency</td>
<td>1–10 000 Hz</td>
</tr>
<tr>
<td>Spot size</td>
<td>20–30 mm</td>
</tr>
<tr>
<td>Ti–T target Maximum neutron yield</td>
<td>$\sim 2.0 \times 10^{12} \text{ ns}^{-1}$</td>
</tr>
<tr>
<td>Reaction Q-value</td>
<td>17.6 MeV</td>
</tr>
<tr>
<td>Ti–D target Maximum neutron yield</td>
<td>$\sim 3.0 \times 10^{10} \text{ ns}^{-1}$</td>
</tr>
<tr>
<td>Reaction Q-value</td>
<td>3.3 MeV</td>
</tr>
</tbody>
</table>
fraction, $\beta_{\text{eff}}$, have been calculated for three different data libraries; ENDF/B-VI, JEFF3.0 and JENDL3.3 [9]. The effective delayed neutron fraction was calculated from the following relation:

$$\beta_{\text{eff}} = \frac{N_d}{N_{\text{Tot}}}$$

where $N_d$ is the number of fissions induced by delayed neutrons and $N_{\text{Tot}}$ is the total number of fissions [10]. The results for the different libraries are displayed in Table 2. For the following analysis, the mean generation time, $\Lambda$, is needed. Since MCNP calculates non-adjoint-weighted time parameters, not suitable for kinetic calculations, another approach must be adopted. This is described in the next section.

### 4. Reactivity determination

#### 4.1. Pulsed neutron source experiment

When a neutron pulse enters a subcritical core, a number of fission chains start to propagate in the fuel. Because of the subcriticality, every fission chain will die out rapidly, which is characterized by a global exponential decay of the neutron flux.

By studying the prompt neutron decay after a neutron pulse, it is possible to determine the reactivity of the core in two different ways, either by applying the Slope Fit Method or the Sjöstrand Method, frequently also called the Area Method. These two methods have been used to analyze the data collected from the pulsed neutron source experiment. During the experiment, the neutron generator was operating at 43 Hz emitting deuterium pulses of duration 2 $\mu$s. The $^3$He-detector was situated in different experimental channels at the core mid-plane. Fig. 2 shows the accumulated detector counts after 40,000 source pulses. The inherent source can in all experiments be neglected.

#### 4.1.1. Slope fit method

This method was first introduced already in Ref. [11], but the method has achieved increased interest during the last years, due to its possible applicability to ADS [2,12–14].

Neglecting the delayed neutrons, the point-kinetic equations take the following form for a reactor without sources [5,15]:

$$\frac{dn(t)}{dt} = \alpha n(t).$$

This equation has an exponential solution given by

$$n(t) = n_0 e^{\alpha t},$$

where $\alpha$ is the prompt neutron decay constant:

$$\alpha = \frac{\rho - \beta_{\text{eff}}}{\Lambda}.$$  

In a subcritical reactor, $\alpha$ is negative, which gives rise to an exponential decrease of the neutron flux. In the short time-scale after a neutron pulse
injection, the neglecting of the delayed neutrons is legitimate and thus, the decay is mainly described by the prompt neutron decay constant. By measuring \( \rho \) experimentally, the reactivity, \( \rho \), can be found if \( \beta_{\text{eff}} \) and \( A \) are known.

From Fig. 2 it is evident that one single exponential is not enough to describe the complete behavior of the neutron flux decay after a neutron pulse insertion. During the first milliseconds, there is an injection and an adjustment period. During the injection period, the flux decreases rapidly in the three innermost experimental channels, located inside the core, and at the same time in the reflector channels, the neutron population is subject to a fast increase. After the injection period follows the adjustment period, when the neutron flux approaches a common decay rate in all channels. After approximately 4 ms, the neutron flux reaches a fundamental decay mode, characteristic of the inherent reactor properties and described by the point-kinetic equations. During this process, the neutron flux decreases with approximately the same rate in all channels. Finally, the neutron flux reaches a constant level, due to the delayed neutron background.

Due to the exponential behavior of neutron flux changes in the reactor according to Eq. (3), it is appropriate to describe the neutron pulse response mathematically by a series of exponentials. By fitting a function of the form

\[
    f(t) = \sum_{i=1}^{\infty} A_i e^{\lambda_i t}
\]

(5)
to all data points, using the function fitting code MINUIT [16], it is possible to determine the exponential component which represents the fundamental decay mode. During the fitting procedure, all data points describing each pulse are used. It turns out that four terms are necessary to describe the response function with satisfactory statistical agreement. Two fast exponentials are required for the injection and adjustment periods, one exponential for the fundamental decay mode and finally, one constant for the delayed neutron background. The \( \lambda \)-values, describing the fundamental decay mode, are summarized in Table 3 and displayed visually for EC3 and EC5 in Fig. 3. The values are followed by a one standard deviation statistical error. Throughout this study, only statistical errors are considered.

The neutron pulse and the subsequent neutron flux have also been simulated with MCNP, relying on the nuclear data library ENDF/B-VI. The reaction rate with \(^3\text{He}\) has been tracked in each experimental channel during a period of 10 ms after the neutron pulse. The results for EC 2, 3 and 5 are displayed in Fig. 4. These data can be analyzed in the same way as the experimental data, to find the \( \lambda \)-values. In this case, the reactivity is already known, through earlier simulations, which means that the mean generation time can be found from Eq. (4).

In the simulated case, three exponentials are sufficient to describe the response function with

<table>
<thead>
<tr>
<th>( \lambda (\text{s}^{-1}) )</th>
<th>( \rho ) (pcm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>EC1</td>
<td>(-675 \pm 13)</td>
</tr>
<tr>
<td>EC2</td>
<td>(-722 \pm 19)</td>
</tr>
<tr>
<td>EC3</td>
<td>(-711 \pm 11)</td>
</tr>
<tr>
<td>EC5</td>
<td>(-634 \pm 21)</td>
</tr>
<tr>
<td>EC6</td>
<td>(-653 \pm 2)</td>
</tr>
</tbody>
</table>

\( \beta_{\text{eff}} \) and \( A \) were calculated using ENDF/B-VI.

\( \beta_{\text{eff}} \) and \( A \) were calculated using ENDF/B-VI.
satisfactory statistical agreement, since the delayed neutrons are not included in the simulation. Two fast exponentials are required for the injection and adjustment periods and one exponential for the fundamental decay mode. The $\alpha$-values and their corresponding mean generation time, for each experimental channel, are summarized in Table 4.

By using the simulated values of the mean generation time in Table 4, it is possible to calculate the reactivity from the experimental values of $\alpha$ (Table 3). The maximum difference between the experimental $\alpha$-values for different channels is approximately 10%. The differences are caused by statistical errors from the fitting procedure and different physical properties around the different experimental channels. The slope is found to be a little lower in the reflector channels. In Table 4, it can be seen that the simulated values deviate somewhat from the experimental values in Table 3, but they follow more or less the same pattern.

4.1.2. Sjöstrand method

Considering the much shorter time scale of the decay of the prompt neutron flux compared to the delayed neutron precursor lifetimes, the delayed neutron flux contribution can be regarded as constant during the studied time interval. If the area under the response function is divided into a
prompt neutron area, $A_p$, and a delayed neutron area, $A_d$, as illustrated in Fig. 5, the reactivity in dollars can be expressed as [6]
\[
\frac{\rho}{\beta_{\text{eff}}} = - \frac{A_p}{A_d}.
\] (6)

The prompt area is obtained by trapezoidal numeric integration and the delayed neutron area is obtained by averaging the values from the last milliseconds where the curve has flattened out. The results are listed in Table 5 and the maximal difference between the values is approximately 7%.

4.2. Source jerk experiment

The idea behind the Source Jerk Method is to operate the subcritical reactor at steady state, at flux level $n_0$, and then suddenly remove the neutron source. At this point, the system will make a prompt jump to a lower level, $n_1$, determined by the delayed neutron background. This level is only quasistatic and will decay according to the decay rate of the delayed neutron precursor groups [5]. The reactivity in dollars is given by
\[
\frac{\rho}{\beta_{\text{eff}}} = \frac{n_1 - n_0}{n_1}.
\] (7)

During the source jerk experiment, the neutron flux is measured with a $^3$He-detector in EC2 (Fig. 6). The neutron flux levels are estimated by the flux values before and immediately after the prompt jump, which gives the reactivity $\rho = -8.9 \pm 1.7\%$. The statistical error is large due to the low count rate after the source jerk.

4.3. Experimental estimation of the mean generation time

It is of interest to verify the Monte Carlo-based values of the mean generation time (Table 4) experimentally. An estimation of the ratio $\Lambda/\beta_{\text{eff}}$ can be found by rewriting Eq. (4) as
\[
\frac{\Lambda}{\beta_{\text{eff}}} \approx \frac{1}{2} \left( \frac{\rho}{\beta_{\text{eff}}} - 1 \right)
\] (8)

![Fig. 5. Illustration of the prompt and the delayed neutron areas utilized in the Sjöstrand Method.](image)

| Table 5 |
| Results from the Sjöstrand Method |
| $\rho$ (S) |
| EC1 | $-13.9 \pm 0.1$ |
| EC2 | $-13.7 \pm 0.1$ |
| EC3 | $-12.9 \pm 0.1$ |
| EC5 | $-13.0 \pm 0.1$ |
| EC6 | $-13.5 \pm 0.1$ |

![Fig. 6. Application of the Source Jerk Method in Yalina.](image)
and combining the values of $\alpha$ obtained from the Slope Fit Method with the values of $\beta/\beta_{\text{eff}}$ obtained from the Sjöstrand Method or the Source Jerk Method. By using the calculated values of $\beta_{\text{eff}}$ (Table 2), it is possible to find an estimate for $L$. From the results listed in Table 6, obtained by combining the Slope Fit Method and the Sjöstrand Method, it can be expected that the true value of $L$ is in the range 140–180 $\mu$s. The combination of the Slope Fit Method and the Source Jerk Method gives a lower value. The results diverge since the different methods give different values of the reactivity. However, this simple method shows that the values obtained by simulation (Table 4) should be good estimations of the real value. As a comparison, it can be mentioned that the non-adjoint-weighted mean generation time calculated by MCNP, which is a very poor estimation of the true value, is approximately 370 $\mu$s.

### 4.4. Validity of the point-kinetic model

Since all three methods used in this study are based on the point-kinetic model, the validity of the model when applied to this system has been investigated.

---

The point-kinetic equations were developed for critical reactor calculations, and do not describe a subcritical system driven by an external neutron source. From the Slope Fit Method with the values of $\beta/\beta_{\text{eff}}$ obtained from the Sjöstrand Method or the Source Jerk Method. By using the calculated values of $\beta_{\text{eff}}$ (Table 2), it is possible to find an estimate for $L$. From the results listed in Table 6, obtained by combining the Slope Fit Method and the Sjöstrand Method, it can be expected that the true value of $L$ is in the range 140–180 $\mu$s. The combination of the Slope Fit Method and the Source Jerk Method gives a lower value. The results diverge since the different methods give different values of the reactivity. However, this simple method shows that the values obtained by simulation (Table 4) should be good estimations of the real value. As a comparison, it can be mentioned that the non-adjoint-weighted mean generation time calculated by MCNP, which is a very poor estimation of the true value, is approximately 370 $\mu$s.

---

### Table 6

Experimental estimation of the mean generation time, $A$

<table>
<thead>
<tr>
<th>$A/\beta_{\text{eff}}$ (10$^{-3}$ s)</th>
<th>$A$ (µs)</th>
<th>ENDF/B-VI</th>
<th>JEFF3.0</th>
<th>JENDL3.3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Slope Fit+Sjöstrand</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EC1 22.1±0.4</td>
<td>174±4</td>
<td>175±4</td>
<td>164±4</td>
<td></td>
</tr>
<tr>
<td>EC2 20.3±0.6</td>
<td>160±5</td>
<td>161±5</td>
<td>151±5</td>
<td></td>
</tr>
<tr>
<td>EC3 19.5±0.3</td>
<td>154±3</td>
<td>155±3</td>
<td>145±3</td>
<td></td>
</tr>
<tr>
<td>EC5 22.1±0.7</td>
<td>175±6</td>
<td>175±6</td>
<td>164±6</td>
<td></td>
</tr>
<tr>
<td>EC6 22.2±0.2</td>
<td>175±3</td>
<td>176±3</td>
<td>164±2</td>
<td></td>
</tr>
<tr>
<td><strong>Slope Fit+Source Jerk</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EC2 14.5±1.1</td>
<td>114±9</td>
<td>115±9</td>
<td>108±8</td>
<td></td>
</tr>
</tbody>
</table>

---

1The (non-adjoint-weighted) neutron mean generation time is not given explicitly by MCNP. It must be calculated as the ratio between the prompt removal lifetime, $l$, and $k_{\text{eff}}$, or as the ratio between the prompt fission lifetime, $\tau$, and the number of prompt neutrons per fission, $n_p$ [15,17,18].
5. Discussion of results

5.1. Comparison between the methods

The Slope Fit Method, the Sjöstrand Method and the Source Jerk Method have been applied to the same configuration of the Yalina experiments. In comparison with the MCNP calculations, the Slope Fit Method shows similar results, whereas the Sjöstrand Method and the Source Jerk Method underestimates and overestimates \( k_{\text{eff}} \), respectively. Both the Sjöstrand Method and the Slope Fit Method produce results with low statistical errors. The Source Jerk Method, on the other hand, is connected with larger errors due to the large uncertainty in the lower neutron flux level, which makes it difficult to make comparisons with the other methods. All results are summarized in Table 7 and Fig. 7.

When applying the Slope Fit Method on deep subcritical configurations, as in the present case, it can sometimes be difficult to find the correct slope through a fitting procedure. The effects from the injection and adjustment periods have disappeared after about 4 ms, as mentioned in the previous section, and the delayed neutron background starts to influence the shape after approximately 7 ms (Fig. 3). Consequently, the fundamental decay mode is the dominating mode during a relatively short time period. The situation is most problematic in the experimental channels in the core, where it is very difficult to visually distinguish a single slope. In the reflector, on the other hand, the fast exponentials have opposite sign, due to the fast increase in neutron flux during the injection period, which makes the fundamental decay mode evident.

According to the results, the Sjöstrand Method has a tendency to give lower values of \( k_{\text{eff}} \) than other methods. Especially the difference between the Sjöstrand Method and the Slope Fit Method is worth to notice, since they are based on the same measurement. The same tendency was also observed in the MUSE-4 experiments [2,12]. The main probable reason for the discrepancies is the use of the point-kinetic approximation. When analyzing the experimental data with the Sjöstrand Method, all data from the pulse insertion to the end of the pulse response are used. However, only the data representative for the fundamental decay mode, used in the Slope Fit Method analysis, is valid according to the point-kinetics. Since the fundamental decay mode will be more and more

<table>
<thead>
<tr>
<th>Method</th>
<th>EC1</th>
<th>EC2</th>
<th>EC3</th>
<th>EC5</th>
<th>EC6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_{\text{eff}} )</td>
<td>0.9012 ± 0.0013</td>
<td>0.9027 ± 0.0013</td>
<td>0.9076 ± 0.0011</td>
<td>0.9069 ± 0.0011</td>
<td>0.9040 ± 0.0013</td>
</tr>
<tr>
<td>ENDF/B-VI</td>
<td>0.9088 ± 0.0013</td>
<td>0.9022 ± 0.0013</td>
<td>0.9072 ± 0.0011</td>
<td>0.9064 ± 0.0011</td>
<td>0.9036 ± 0.0013</td>
</tr>
<tr>
<td>JEFF3.0</td>
<td>0.9065 ± 0.0013</td>
<td>0.9079 ± 0.0013</td>
<td>0.9126 ± 0.0011</td>
<td>0.9119 ± 0.0011</td>
<td>0.9091 ± 0.0012</td>
</tr>
<tr>
<td>JENDL3.3</td>
<td>0.9072 ± 0.0011</td>
<td>0.9079 ± 0.0013</td>
<td>0.9126 ± 0.0011</td>
<td>0.9119 ± 0.0011</td>
<td>0.9091 ± 0.0012</td>
</tr>
</tbody>
</table>

![Fig. 7. Summary of all \( k_{\text{eff}} \) obtained through different methods applied on data from five experimental channels (Sjöstrand Method and Slope Fit Method). The error bars correspond to one standard deviation in each direction. The different libraries refer to the chosen value of \( b_{\text{eff}} \).](image)
dominating closer to criticality, the gap between the two methods is expected to decrease when approaching criticality. This effect was also observed in the MUSE-4 experiments [12].

The differences between the calculations based on the different nuclear data libraries are small; approximately 4% for the reactivity and 6% for $\beta_{\text{eff}}$. In comparison, the data library ENDF/B-VI gives a somewhat lower value of $k_{\text{eff}}$ and JENDL3.3 gives a lower value of $\beta_{\text{eff}}$. However, the choice of nuclear data library does not induce prominent effects on the result.

5.2. Comparison between the PNS experiment and MCNP

As mentioned before, there is no delayed neutron background when simulating the pulsed neutron source experiment in MCNP. In order to compare the simulation with the experiment, this background must be subtracted from the experimental data. The result for EC5 is depicted in Fig. 8 and indicates good agreement.

6. Conclusions

Three reactivity determination methods, the Slope Fit Method, the Sjöstrand Method and the Source Jerk Method, have been investigated by applying them to the Yalina experiments. Two types of experiments were performed at the facility; a pulsed neutron source experiment (PNS) and a source jerk experiment. The PNS experiment has also been simulated with MCNP. The simulations provided parameters, such as effective delayed neutron fraction and mean generation time, which were necessary for the evaluation of the reactivity and the effective multiplication constant from the experiments.

From the measurements it can be concluded that:

- The Sjöstrand Method underestimates the criticality slightly in comparison with MCNP and the other methods, but gives low statistical error.
- The Slope Fit Method is inconvenient to apply to deep subcritical configurations, but gives reliable results in comparison with MCNP. The cleanest response functions are achieved in the reflector channels.
- The Source Jerk Method is connected with large uncertainties.
- Although the neutron spectrum of Yalina is thermal and has different kinetic parameters than in MUSE, the results show many similarities.

Moreover, from the simulations it can be concluded that:

- MCNP gives reliable results, which was shown by the simulations in comparison with the experiments. However, care must be taken when calculating time parameters.
- The point-kinetic approximation describes the system well under the studied circumstances.

In this work, differences between different reactivity levels were not studied. Even if the methods do not predict the absolute value of the reactivity absolutely correct, they may be able to predict reactivity changes with good precision.

None of the methods investigated in this study have the capability of measuring the reactivity without disturbing a running system. However, the
methods can be used to estimate the subcriticality during loading and for calibration of other possible measurement techniques, for example the current-to-flux reactivity indicator. In the profound study of different measurement techniques for reactivity monitoring in an ADS performed within the MUSE-4 program, the method based on the current-to-flux reactivity indicator has appeared to be the major candidate for on-line monitoring [24].

Acknowledgements

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References

Neutron Kinetic Characterisation of the Subcritical ADS Experiment YALINA-Booster

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Abstract – A subcritical zero-power source-driven coupled core, the YALINA-Booster, has been constructed for experimental investigations of neutron kinetics of source-driven systems. In this study, the reactivity of two subcritical configurations has been determined by the pulsed neutron source area method. The prompt neutron decay constants have been evaluated through the pulsed neutron source slope fitting method as well as by the pulsed and continuous Rossi-α method and the Feynman-α method. Results show satisfactory stability for the pulsed neutron source area and slope fitting methods, whereas the continuous noise methods fail to predict the prompt neutron decay constant for the deep subcriticalities studied. On the other hand, the pulsed Rossi-α method gives reliable results. Further, it is concluded that the neutron reproduction time of the YALINA-Booster is characterised by the thermal zone.

KEYWORDS: YALINA-Booster, Accelerator-driven systems, reactivity determination, pulsed neutron source measurements, noise measurements

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I. INTRODUCTION

Accelerator-driven systems (ADS) may play an important role in future nuclear fuel cycles to reduce the long-term radiotoxicity and volume of spent nuclear fuel [1]. In order to guarantee safe operation of reactor systems based on fuels containing high fractions of plutonium and minor actinides, they must operate in subcritical mode driven by an external neutron source. The subcriticality monitoring of such systems is a necessity, but it is still an open question how it will be performed [2]. Recent studies [3,4,5], as well as this study, have been devoted to determining reactivity at low power in thermal and fast systems. Since this study is based on a coupled fast-thermal core without access to a critical reference configuration, it differs from the previously mentioned studies. The experiments reported here were carried out in order to characterise the core for further experiments and to fine-tune the data acquisition system and the calculation tools. Moreover, the performance and reliability of some various reactivity determination methods were investigated.

Without a critical reference level, where the physics is well known, there is no possibility of determining the reactivity of subcritical configurations by conventional methods based on, for instance, control rod movements. On the other hand, this is the situation that will be faced when a full-scale facility is commissioned in future. In this study, the pulsed neutron source (PNS) area method, or the Sjöstrand method [6], has been chosen as the reference method for reactivity determination, based on its stability shown in the previous studies.

First a theoretical background is given, describing the various reactivity determination methods used. Then the experimental facility is described, followed by the results from the PNS measurements. Having obtained the reference values of the reactivity, values from PNS $\alpha$-fitting are compared to those from noise analysis.

II. THEORETICAL BACKGROUND

II.A.1 PNS Slope Fitting

The idea of using the prompt neutron decay constant, defined as

$$\alpha \equiv \frac{1}{n} \frac{dn}{dt} = \frac{\rho - \beta_{eff}}{\Lambda},$$

(1)

where $n$ is the neutron density, for subcriticality determination purposes, was proposed by Simmons and King [7]. The prompt neutron decay constant, $\alpha$, can be used solely as reactivity indicator, being zero for a prompt critical core and negative for reactivities below one dollar. However, since the zero reactivity level is the important level both from safety point of view and for proper comparison of reactivity levels of different cores, the reactivity, $\rho$, or the effective multiplication factor, $k_{eff}$, are the preferred quantities for comparisons. As can be found from Eq. (1), a drawback of this method is that for the determination of the reactivity from a measurement of the prompt neutron decay constant, one must know the effective delayed neutron fraction, $\beta_{eff}$, and the neutron reproduction time, $\Lambda$ (traditionally named the mean neutron generation time [8]).

The prompt neutron decay constant can easily be determined by fitting a line to the detector response of a neutron pulse insertion in logarithmic scale to a straight line. However, only one pulse is not enough to find the slope with sufficient accuracy, hence raising the need for a pulsed neutron source. Therefore, this method of finding $\alpha$ for reactivity determination is here referred to as the PNS slope fitting method, but different names are used in other papers.

II.A.2 PNS Area Method

When source neutrons are introduced into the subcritical core, numerous fission chains start to propagate. Due to the subcriticality, each chain will die out causing a decrease in the number of prompt fission neutrons. The number of delayed neutrons, on the other hand, will stay approximately constant over time since their time characteristics are much slower than that of the prompt neutrons. By considering the fraction of prompt
to delayed neutron areas in a PNS histogram, the reactivity in dollar can be obtained [6]:

\[ \frac{\rho}{\beta_{\text{eff}}} = -\frac{A_p}{A_d}. \]  

(2)

The prompt neutron area, \( A_p \), and the delayed neutron area, \( A_d \), are depicted in Figure 1. If there is a non-negligible contribution from an inherent source, e.g. spontaneous fissions, it must be taken into account. However, in these measurements it was found that the inherent source was completely negligible.

This method, as well as the PNS slope fitting method, is based on the assumption that the point-kinetics approximation is valid. If so, an estimate of the ratio \( \Lambda / \beta_{\text{eff}} \) can be found by rewriting Eq. (1) and including experimental data from the area method:

\[ \frac{\Lambda}{\beta_{\text{eff}}} = \frac{1}{\alpha} \left( \frac{\rho}{\beta_{\text{eff}}} - 1 \right). \]  

(3)

II.A.3 Rossi-\( \alpha \)

Due to chain reaction processes in multiplying media, the probability \( p(t)dt \) of detecting a neutron in the interval \( dt \) around time \( t \) after the detection of a neutron at \( t = 0 \) consists of two parts: an exponentially decaying correlated part and a constant uncorrelated part:

\[ p(t)dt = -\frac{\epsilon D}{2\alpha\Sigma} e^{\alpha t}dt + F e^{\alpha t}dt. \]  

(4)

In this equation, \( \epsilon \) is the detector efficiency in counts per fission occurred and \( F \) is the fission rate of the system [9,10]. Consequently, the term \( F e^{\alpha t} \) is the mean count rate during the measurement. \( D \) is the Diven factor defined as

\[ D = \frac{\nu (\nu - 1)}{\nu^2}, \]  

(5)

where \( \nu \) is the number of prompt neutrons per fission [11]. By plotting the probability density \( p(t) \), the prompt neutron decay constant can be found from the correlated part through the fitting of an exponential function.

II.A.4 Rossi-\( \alpha \) with pulsed source

The pulsed Rossi-\( \alpha \) formula has been derived based on the backward master equation technique by Kitamura et al. [12]. In its most general form, the pulsed Rossi-\( \alpha \) formula can be expressed as

\[ p(t)dt = A_1 e^{\alpha t}dt + A_2 dt + A_3 \sum_{n=1}^{\infty} \left( a_n^2 + b_n^2 \right) \cos(\omega_n t)dt. \]  

(6)

The constants \( A_i, i = 1,2,3, \) are given in ref. [12]. Depending on the shape and frequency of the neutron source pulse, the parameters \( A_i, a_n, b_n \) and \( \omega_n \) will be different (based on a Fourier series of the neutron source time characteristics). However, the main structure of Eq. (6) is maintained independently of the specific source characteristics. The first two terms have the same meaning as in the traditional Rossi-\( \alpha \) formula: one exponentially decreasing correlated term and a constant uncorrelated term. The third oscillating term is uncorrelated and non-decaying, caused by the pulsed source.

II.A.5 Feynman-\( \alpha \)

Due to the presence of multiplying media in a nuclear reactor, the count rate, \( c \), of a detector will deviate from a true Poisson distribution. This deviation is denoted \( Y \):
It can be shown [9,10] that $Y$ is dependent on the time gate width, $\Delta T$ that is used in the measurement, according to

$$Y(\Delta T) = \frac{\varepsilon D_v}{\Lambda^2 \alpha^2} \left( 1 + \frac{1 - e^{\alpha \Delta T}}{\alpha \Delta T} \right).$$  (8)

The statistical uncertainty of $Y$ is given as

$$\sigma(Y) = \sqrt{\frac{1}{N} \left\{ \left( 4 + \frac{3}{\varepsilon} \right) \frac{1}{Y} + \left( 2 + \frac{3}{\varepsilon} \right) \frac{Y}{\varepsilon} \right\}}^{1/2}$$  (9)

where $N$ is the number of time bins [10].

The dead time effect is taken into account by correcting the Feynman-$\alpha$ formula, Eq. (8), according to

$$Y(\Delta T) = \frac{\varepsilon D_v}{\Lambda^2 \alpha^2} \left( 1 - \frac{d}{\Delta T} \right) \left( 1 + \frac{1 - e^{\alpha (\Delta T-d)}}{\alpha (\Delta T-d)} \right)$$

$$-2Cd + \frac{Cd^2}{\Delta T}$$  (10)

where $C$ is the count rate of the detector with dead time $d$ [13].

III. THE YALINA-BOOSTER

The YALINA-Booster is a subcritical fast-thermal core coupled to a neutron generator (NG-12-1). The main parts of the neutron generator are the deuteron ion accelerator and a Ti-T or Ti-D neutron production target, with a diameter of 45 mm, located in the centre of the core. The neutron energy is around 14 MeV for the DT- and 2.5 MeV for the DD-reaction. The neutron generator can be operated in both continuous and pulse mode and provides the possibility to generate pulses with frequencies from 1 Hz to 7 kHz with pulse duration of 2 – 130 $\mu$s. The maximum beam current in continuous mode is 2 mA with a beam diameter about 20 mm giving a maximum neutron yield of approximately $2 \times 10^{11}$ neutrons per second for the Ti-T target and $2 \times 10^{9}$ neutrons per second for the Ti-D target.

The core consists of a central lead zone, a polyethylene zone, a radial graphite reflector and a front and back biological shielding of borated polyethylene. The loading is 132 fuel pins containing metallic uranium of 90% enrichment, 563 fuel pins containing uranium dioxide of 36% enrichment and a maximum of 1141 EK-10 fuel pins containing uranium dioxide of 10% enrichment. The zero-power core is cooled by natural convection of the surrounding air.

The fast-spectrum lead zone and the thermal-spectrum polyethylene zone are separated by a so called thermal neutron filter, which consists of one layer of 108 pins with metallic natural uranium and one layer of 116 pins with boron carbide (B$_4$C) which are placed in the outermost two rows of the fast zone. Thermal neutrons diffusing from the thermal zone to the fast zone will either be absorbed by the boron or by the natural uranium, or transformed into fast neutrons through fission in the natural uranium. In this way, a coupling of only fast neutrons between the two zones is maintained.

There are seven axial experimental channels (EC1B-EC4B and EC5T-EC7T) in the core and two axial experimental channels (EC8R and EC9R) and one radial experimental channel (EC10R) in the reflector. Moreover, there is one neutron flux monitoring channel in each corner of the core. Three B$_4$C-control rods, with a total reactivity worth of approximately -300 pcm, can be inserted in the thermal zone. A more detailed description is available in the YALINA-Booster benchmark description [14].

In these measurements two configurations were studied. These configurations have a fully loaded fast zone, as described above, and 1132 and 1061 fuel pins of 10% enrichment in the thermal zone respectively. The loading of these fuel pins was made based on cylindrical symmetry (Figure 2).
IV. EXPERIMENTAL RESULTS

For the measurements based on a pulsed neutron source, the TiD-target was used, located in the core centre. For the continuous source noise methods a $^{252}$Cf-source was used, located 62 mm in front of the TiD-target, which was kept in the same position. All measurements were performed with a $^3$He-detector, sensitive mainly to thermal neutrons.

Data were recorded using a counter/timer card that stores the arrival time of each detection event from the $^3$He-detector and the trigger signal from the pulsed neutron source. The accuracy of the time stamping is 12.5 ns and a total dead time of 3.3 $\mu$s must be taken into consideration. Having access to all events from the experiment, any data analysis can be performed off-line, after the experiment has been performed. This procedure is well suited for the noise part of the analysis, but can also be used for the PNS measurements. In parallel, data were recorded with a multi-scaler to ensure the reliability of the two systems.

IV.A PNS Slope fitting

PNS measurements were performed in EC5T-EC9R. A typical pulse response histogram is shown in Figure 1. In contrast to the previous YALINA experiment [4], a clear fundamental mode decay was visible in all experimental channels, making the analysis much simpler. The function minimising tool Minuit [15] was used to fit a function of the type

$$ f(t) = A_1 e^{\alpha t} + A_2 $$

(11)

to the obtained histograms. This procedure gives both the prompt neutron decay constant, $\alpha$, and the constant level, $A_2$, which is used for the area method when separating the prompt neutron area from the delayed neutron area. In the fitting procedure, the first part of the signal was not considered and a sensitivity analysis on the fitting start-point was performed. For all cases, the reduced $\chi^2$ deviated less than 1% from unity. As can be seen in Table 1 and Figure 3, the resulting prompt neutron decay constants can be achieved with high accuracy and small spatial spread (less than 2%).

IV.B PNS Area Method

From the fitting of Eq. (11) to the histograms the constant level of delayed neutrons was obtained. Knowing this level, the reactivity in dollars was obtained after calculating the prompt and delayed neutron areas (Table 1). In the core region, the area method gives rather stable results, but in the outermost detector position, in the reflector, the result is slightly different. The reactivity as a function of distance from the booster zone follows the same pattern in both configurations (Figure 4). The reactivity difference between the two configurations has, on the other hand, very low spatial spread: $\Delta \rho/\beta_{\text{eff}} = 1.41 \pm 0.05 \$.
In Table 1, the ratio $\Lambda/\beta_{\text{eff}}$ is shown, calculated from Eq. (3). These values are used later to estimate the neutron reproduction time.

In the further analysis the weighted mean values of the obtained parameters will be used as reference values. Since the spread in all cases is much larger than the statistical error in each point, the spread will be used as the uncertainty of the weighted mean value. The spread is evaluated relative to the weighted mean, rather than the arithmetic mean as in the original definition.

### Table 1. Results from PNS measurements. $\Lambda/\beta_{\text{eff}}$ is calculated from the experimental values of $\alpha$ and $\rho/\beta_{\text{eff}}$ relying on Eq. (3). The spread is evaluated relative to the weighted mean value.

<table>
<thead>
<tr>
<th>Conf.</th>
<th>EC</th>
<th>$\alpha$ [s$^{-1}$]</th>
<th>$\rho/\beta_{\text{eff}}$ [S]</th>
<th>$\Lambda/\beta_{\text{eff}}$ [ms]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1132</td>
<td>EC5T</td>
<td>-654.1±1.6</td>
<td>-3.60±0.03</td>
<td>7.04±0.04</td>
</tr>
<tr>
<td></td>
<td>EC6T</td>
<td>-662.6±1.8</td>
<td>-3.36±0.02</td>
<td>6.58±0.04</td>
</tr>
<tr>
<td></td>
<td>ECT7</td>
<td>-649.4±1.4</td>
<td>-3.57±0.02</td>
<td>6.73±0.03</td>
</tr>
<tr>
<td></td>
<td>EC8R</td>
<td>-648.7±2.0</td>
<td>-3.61±0.02</td>
<td>7.11±0.03</td>
</tr>
<tr>
<td></td>
<td>EC9R</td>
<td>-644.0±3.3</td>
<td>-3.92±0.03</td>
<td>7.65±0.05</td>
</tr>
<tr>
<td>Weighted mean value</td>
<td>7.1</td>
<td>0.23</td>
<td>0.42</td>
<td></td>
</tr>
<tr>
<td>Spread</td>
<td></td>
<td>(1.1%)</td>
<td>(6.5%)</td>
<td>(6.0%)</td>
</tr>
<tr>
<td>1061</td>
<td>EC5T</td>
<td>-875.0±1.9</td>
<td>-5.09±0.03</td>
<td>6.96±0.03</td>
</tr>
<tr>
<td></td>
<td>EC6T</td>
<td>-892.1±2.7</td>
<td>-4.71±0.03</td>
<td>6.40±0.03</td>
</tr>
<tr>
<td></td>
<td>ECT7</td>
<td>-866.7±2.2</td>
<td>-4.75±0.03</td>
<td>6.64±0.03</td>
</tr>
<tr>
<td></td>
<td>EC8R</td>
<td>-865.4±3.2</td>
<td>-5.07±0.03</td>
<td>7.03±0.03</td>
</tr>
<tr>
<td></td>
<td>EC9R</td>
<td>-860.1±3.6</td>
<td>-5.33±0.03</td>
<td>7.36±0.04</td>
</tr>
<tr>
<td>Weighted mean value</td>
<td>-872.7</td>
<td>4.95</td>
<td>6.82</td>
<td></td>
</tr>
<tr>
<td>Spread</td>
<td></td>
<td>(1.5%)</td>
<td>(3.5%)</td>
<td>(5.5%)</td>
</tr>
</tbody>
</table>

By normalising the Rossi-$\alpha$ histogram to the count rate, the probability distribution will follow a function of the form

$$p(t) = Ae^{\alpha t} + 1$$  \hspace{1cm} (12)

and the number of fitting parameters is thereby reduced from three to two, since the count rate of the measurement is known to high accuracy. Moreover, this procedure makes it possible to compare Rossi-$\alpha$ histograms from different measurements.

Analysis of the Rossi-$\alpha$ histograms revealed that the correlated part of the probability time-distribution did not follow a single exponential. The first millisecond had to be omitted to make it possible to fit an exponential with satisfactory $\chi^2$-test results, as depicted in Figure 5 and Figure 6. Similar effects were found in other studies [16,17]. An explanation might come from the fact that for small time-scales and deep subcriticalities effects from the fast-thermal region coupling as well as reflector effects may become important.

Results for $\alpha$ are shown in Table 2. Comparison between prompt neutron decay constants from the Rossi-$\alpha$ measurement and those from the PNS slope fitting, showed that the values are in general, but not perfectly, in agreement for the 1132-configuration. However, for the 1061-configuration the results diverge strongly. This indicates that a two-region model should be applied to describe the Rossi-$\alpha$ distributions with satisfactory results [17].

### Figure 4. Reactivity (in $\$) from the area method for different detector positions. The weighted mean values are indicated as straight lines.

### Figure 5. Rossi-$\alpha$ histograms for the 1132-configuration.
IV.D Rossi-α with pulsed source

For large \( t \), the correlated term of Eq. (6) is negligible. By considering a period of the oscillation term at large \( t \) and the constant term as time-independent, this uncorrelated part can be removed from the total signal to get the correlated part alone, as described in ref. [12]. Figure 7 shows the Rossi-α histogram for the 1061-configuration in EC5T from 0 to 100 ms. After 80 ms, the correlated part is assumed to be negligible and the U-formed shape from 80 ms to 93 ms (13 ms source period) is subtracted from each period. The resulting exponential correlated part is depicted in Figure 8. This procedure has been applied to most of the pulsed measurements of the two configurations. As for the traditional Rossi-α method, some data points in the beginning of each Rossi-α histogram had to be omitted in the function fitting procedure in order to achieve a satisfactory reduced \( \chi^2 \) value.

When comparing this pulsed Rossi-α study to that of ref. [12], one can notice that in this study the total correlation function for small \( t \) is almost as large as the uncorrelated part. This may cause problems when the uncorrelated part is subtracted. However, this study shows that the small difference is enough for the recovery of the prompt neutron decay constant with satisfactory accuracy.

As can be seen in Table 2, the results from the pulsed Rossi-α analysis are in good agreement with the PNS slope fitting results. It should be mentioned that the uncertainties in the pulsed Rossi-α results are much larger than for the corresponding PNS slope fitting results, although the measurement time was the same. This comes from the much more complicated data analysis that must be performed when utilising the pulsed Rossi-α method in comparison to the PNS slope fitting. However, the accuracy of the pulsed Rossi-α method is better than for the traditional Rossi-α method for similar measurement time. This comes from the fact that the amplitude of the correlated term is much higher in the pulsed experiment, thus increasing the signal-to-noise ratio.

IV.E Feynman-α

Feynman-α distributions were calculated for measurements performed in EC5T-EC7T for both configurations. In the reflector channels the statistical accuracy was too low to give reliable results. Function fitting based on Eq. (10) was
performed to obtain the prompt neutron decay constants shown in Table 2. Investigations were performed to see whether the finite measurement time had an influence on the result [18], but no effect could be found. As can be seen in Table 2, the results from the Feynman-α analysis are in agreement with the Rossi-α results but fail to predict the PNS slope fitting results.

During the fitting process it was noticed that the result was strongly dependent on the choice of fitting interval. The influence on α from the fitting interval was much larger than the statistical error in α itself from the fitting procedure. Therefore, the spread of the α-results from many different fitting intervals, all giving reduced χ² equal to unity, was used as the uncertainty of the final value. The final value was considered to be the mean value of the decay constants obtained from these fittings. In this context one must remember that Eq. (8) and Eq. (10) are valid only if ΔT is much smaller than the time constant of the fastest decaying delayed neutron group [10]. Therefore, in the analysis, no data points above 30 ms were considered.

Figure 9. Feynman-α plots for the 1132-configuration.

Figure 10. Feynman-α plots for the 1061-configuration.

Table 2. Prompt neutron decay constants as obtained from various methods at various detector positions.

<table>
<thead>
<tr>
<th>Conf.</th>
<th>EC</th>
<th>Rossi-α</th>
<th>Feynman-α</th>
<th>Pulsed Rossi-α</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1132</td>
<td>EC5T</td>
<td>-693±37</td>
<td>-746±72</td>
<td>-671±9</td>
</tr>
<tr>
<td></td>
<td>EC6T</td>
<td>-743±20</td>
<td>-787±45</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>EC7T</td>
<td>-628±15</td>
<td>-825±46</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>EC8R</td>
<td>-</td>
<td>-</td>
<td>-618±44</td>
</tr>
<tr>
<td></td>
<td>EC9R</td>
<td>-673±48</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1061</td>
<td>EC5T</td>
<td>-1076±75</td>
<td>-1057±42</td>
<td>-854±4</td>
</tr>
<tr>
<td></td>
<td>EC6T</td>
<td>-1008±78</td>
<td>-1035±10</td>
<td>-881±9</td>
</tr>
<tr>
<td></td>
<td>EC7T</td>
<td>-1022±53</td>
<td>-1194±23</td>
<td>-890±11</td>
</tr>
<tr>
<td></td>
<td>EC8R</td>
<td>-</td>
<td>-</td>
<td>-868±11</td>
</tr>
<tr>
<td></td>
<td>EC9R</td>
<td>-</td>
<td>-</td>
<td>-842±36</td>
</tr>
</tbody>
</table>

V. MONTE CARLO ANALYSIS

The Monte Carlo code MCNP [19] was used to analyse the experiment. An input deck was prepared, describing the conditions of the facility during the measurements on a very detailed level. During the preparation of the input deck it was found that traces of different materials in the construction materials (stainless steel, lead and cladding) could have large influence on the calculation results. After receiving results from chemical analysis of these materials, a proper material definition was created [14]. The reliability of the input deck was ensured through comparisons based on various flux measurements at various locations in the core.

Based on the nuclear data libraries JEFF3.1 and JENDL3.3 [20] the effective multiplication factor for the two configurations was calculated. As can be seen in Table 3 both libraries suggest values around 0.976 and 0.963 for the two configurations respectively. The difference between
the two libraries is only a few tens of pcm in both cases.

The effective delayed neutron fraction, $\beta_{eff}$, has been estimated based on the so-called prompt method:

$$\beta_{eff} = 1 - \frac{k_p}{k_{eff}}. \quad \text{(13)}$$

In this equation, $k_p$ is the multiplication factor when transporting prompt neutrons only and $k_{eff}$ is the effective multiplication factor with all neutrons included. Calculations were performed relying on the nuclear data libraries JEFF3.1 and JENDL3.3 and the results are shown in Table 3. No difference in effective delayed neutron fraction between the two configurations could be found. In the following analysis, the value 733±21 pcm will be used for both configurations representing both libraries. This value has been confirmed [21] based on the more exact method developed by Klein Meulekamp and van der Marck [22]. In this context, one must keep in mind that the definition of the integral kinetic parameters for source-driven systems is ambiguous [23]; however, the classical definitions [24] will here be considered as sufficient.

Table 3. Effective multiplication factor and effective delayed neutron fraction in pcm for the two configurations based on two nuclear data libraries.

<table>
<thead>
<tr>
<th>Conf.</th>
<th>JEFF3.1</th>
<th>JENDL3.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1132</td>
<td>$k_{eff}$ 0.9760±0.00004</td>
<td>$k_{eff}$ 0.9764±0.00010</td>
</tr>
<tr>
<td></td>
<td>$\beta_{eff}$ 734.6±6.5</td>
<td>$\beta_{eff}$ 738.4±16</td>
</tr>
<tr>
<td>1061</td>
<td>$k_{eff}$ 0.9626±0.00005</td>
<td>$k_{eff}$ 0.9634±0.00010</td>
</tr>
<tr>
<td></td>
<td>$\beta_{eff}$ 728.2±8.9</td>
<td>$\beta_{eff}$ 737.0±17</td>
</tr>
</tbody>
</table>

VI. ESTIMATION OF KINETIC PARAMETERS

VI.A Effective Multiplication Factor

Knowing the reactivity in units of dollar and the effective delayed neutron fraction from MCNP, an experimental estimation of the effective multiplication constant can be obtained. By using the relation

$$k_{eff} = \frac{1}{1 - \left(\frac{\rho}{\beta_{eff}}\right)_{\text{MCNP}}} \quad \text{(14)}$$

and the spread of $\rho/\beta_{eff}$ as its uncertainty the result is $k_{eff} = 0.9747±0.0018$ for the 1132-configuration and $k_{eff} = 0.9650±0.0020$ for the 1061-configuration. When comparing these values with those obtained by MCNP one notices that the MCNP values are within the uncertainty of the inferred $k_{eff}$ values.

VI.B Neutron Reproduction Time

In the same way as for $k_{eff}$, an experimental estimation of the neutron reproduction time can be obtained based on Eq. (3) with the knowledge of $\beta_{eff}$:

$$\Lambda = \frac{1}{\alpha_{\text{exp}}} \left(\left[\frac{\rho}{\beta_{eff}}\right]_{\text{exp}} - 1\right) \beta_{eff}^{\text{MCNP}}. \quad \text{(15)}$$

The result is $\Lambda = 51.0±3.4$ μs for the 1132-configuration and $\Lambda = 50.0±3.1$ μs for the 1061-configuration. These values are typical for a thermal core rather than a fast core, indicating that the neutron multiplication process is mainly governed by the thermal part of the core. In fact, MCNP analysis of the booster zone alone with no fuel pins in the thermal zone shows that $k_{eff}$ is around 0.6 for the fast zone.

VII. CONCLUSIONS

Two subcritical configurations of the coupled fast-thermal core YALINA-Booster have been characterised based on neutron kinetic measurements. First of all, it must be stressed that the only direct measurements have been performed on $\alpha$ and $\rho/\beta_{eff}$. The values of $k_{eff}$ and $\Lambda$ obtained from Eq. (14) and Eq. (15) are only inferred values, based on the assumption that the simulated $\beta_{eff}$ is correct within the uncertainty. Moreover, the result relies on the validity of the point-kinetics approximation. In this study, this validity was not investigated, as was done in ref. [4],
since the very clear response function of the neutron pulse insertion did not raise such concerns.

The PNS slope fitting method showed excellent stability: less than 2% spread for all experimental channels. In the core region, the area method appeared to be rather stable, but in the reflector some deviation was found. Keeping in mind that this result may be reactivity dependent, this is in contrast to other studies, where the area method has shown to be more stable than the PNS slope fitting [25].

The $^{252}$Cf-source based noise methods Rossi-$\alpha$ and Feynman-$\alpha$ did not reproduce the result from the PNS slope fitting, indicating the need of two-region models. The deviation increased with the subcriticality level.

Prompt neutron decay constants obtained from the pulsed Rossi-$\alpha$ formula are in good agreement with those from the PNS slope fitting method. However, it is much simpler to directly find the prompt neutron decay constant from the PNS histogram than performing the Rossi-$\alpha$ analysis. Thus, the pulsed Rossi-$\alpha$ method is less attractive for reactivity monitoring of pulsed systems.

Finally it was concluded that the neutron reproduction time of the YALINA-Booster core is comparable to that of a thermal system.

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