USING DECONVOLUTION FOR CORRECTION OF NON-IDEAL STEP RESPONSE OF LIGHTNING IMPULSE DIGITIZERS AND MEASUREMENT SYSTEMS

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Abstract: Lightning impulse measurements can be highly influenced by measurement arrangement, characteristics of high voltage divider, digitizer (transient recorder) performance, and algorithms used for parameter evaluation. The main sources of measurement errors are the non-ideal step responses of digitizer and voltage divider. This paper discusses the use of deconvolution to correct for the non-ideal step response of a digitizer, and of a large mixed divider. Correcting the step response of the complete measurement system by one part at a time is desirable because it allows to evaluate the effectiveness of the correction with trustworthy methods.

Step response describes the output of a system as function of time when its input changes between two levels infinitely fast. Real life impulse digitizers and impulse voltage dividers have a finite rise time, and the response does not immediately settle to final value. Slow rise time is often the cause of error for front time parameter. Creeping response is often the cause of error for time to half-value parameter.

Step response of an instrument can be determined by applying a stable, known direct voltage, which is then shorted to ground by a mercury-wetted relay. The mercury-wetted relay is assumed nearly an ideal switch, which creates almost an ideal voltage step for input of the instrument. Convolving the derivative of the measured step response with an ideal input gives a measure of distortion caused by the non-perfect step response, and conversely deconvolving the measured step response with the measured signal gives the original input signal.

This paper presents an FFT-based method for step response correction using deconvolution. Deconvolution is a mathematical process, which is used to reverse the non-ideal effects of measuring instrument on recorded data. Effectiveness of the method is demonstrated by two examples. In the first example, the non-ideal step responses of the different ranges of an impulse digitizer are corrected. Functionality of the step response correction is evaluated by comparing the results against a calculable impulse voltage calibrator. Results showed that the step response correction reduced errors in lightning impulse parameters. Stability of the step response correction was analysed by studying several impulse calibration results that have been performed for the instrument within a year. The second example corrects the response of a 2400 kV impulse voltage divider. The effectiveness of the correction is evaluated by comparing its results to a 400 kV reference divider.

1 INTRODUCTION

Lightning impulse (LI) voltage test is a commonly used high voltage test specified in IEC 60060 1:2010 [1]. LI voltages are challenging to measure accurately because they are a combination of rapid phenomena (µs time scale) and high voltage (often several hundred kV). Important sources of measurement errors are usually the non-ideal step responses of digitizer and voltage divider, which together form the non-ideal step response of the complete measurement system.

Step response of a system describes the behaviour of its output as function of time when its input changes between two levels infinitely fast. In practice, impulse digitizers and impulse voltage dividers have a finite rise time, and the response does not immediately settle to final value. Rise time depends on ability of an instrument to react to fast voltage changes and slow rise time is often the cause of error for LI front time (T₁) parameter. Settling time depends on ability of an instrument to stabilize after fast voltage change and creeping response is often the cause of error for LI time to half-value (T₂) parameter.
The step response can be used to analyse measurement errors by applying convolution techniques [2] or to correct errors of a device by applying deconvolution techniques [3]. This paper presents an FFT-based method for step response correction using deconvolution. Method is used for correcting the non-ideal step response of an impulse digitizer and of a large mixed divider. Even though the main focus should be on correcting the non-ideal step response of a complete LI measurement system, separate corrections are explored due to practical considerations, such as the difficulty to apply a known step to a complete system because of the very low output voltage of the divider. A further advantage is that this facilitates accurate and verifiable corrections for both transient recorder and divider. Combining the two is not explored here, but is seen as a trivial extension.

2 CONVOLUTION AND DECONVOLUTION

Measuring instrument can be modelled as a four-terminal network as presented in Figure 1. In time domain, \( U_i(t) \) represents the input voltage of the instrument and the \( U_o(t) \) represents the output voltage of the instrument. For an impulse voltage divider, the \( U_i(t) \) is the applied impulse and the \( U_o(t) \) is the voltage that can be measured from its output. For an impulse digitizer, the \( U_i(t) \) is the voltage that is applied to its input and the \( U_o(t) \) is the recorded data.

\[ G(j\omega) = \frac{U_o(j\omega)}{U_i(j\omega)} \]  (1)

If we know the input voltage and the impulse response of a measuring instrument, then the output can be calculated as:

\[ U_o(j\omega) = G(j\omega) \cdot U_i(j\omega) \]  (2)

This above-mentioned process is called convolution and it is a way to analyze how the measuring instrument affects to the original signal [4]. For example, convolving the derivative of the measured step response with an ideal input gives a measure of distortion caused by the non-perfect step response.

On the contrary, deconvolution is a mathematical process, which is used to reverse the known effects of measuring instrument on measured signal [4]. That means it is possible to obtain the original input signal if we know the output signal and the impulse response of the instrument:

\[ U_i(j\omega) = \frac{U_o(j\omega)}{G(j\omega)} \]  (3)

Input signal and impulse response are usually based in time domain measurements and the output signal is also needed in time domain. This creates a need for FFT and IFFT. From (3) we can derive the deconvolution process using only time domain signals:

\[ U_i(t) = F^{-1}[U_i(j\omega)] = F^{-1}\left\{\frac{F[U_o(t)]}{F[G(t)]}\right\} \]  (4)

Step response correction presented in this paper is based on deconvolution in frequency domain and the corrected impulse is returned in time domain according to the (4).

3 CORRECTION FOR IMPULSE DIGITIZER

Step response correction was done for a commercial 12-bit digitizer with maximum sampling rate of 200 MS/s. The digitizer has 2 channels with 7 input ranges. VTT MIKES currently uses the digitizer in their reference LI measuring system. Without corrections, the digitizer TI errors are from +0.3 % to +0.4 % and T2 errors from -0.4 % to +0.3 %, compared with a calculable impulse voltage calibrator [5]. Test voltage Ua in the calibration varied between 0.18 V and 9 V depending on the used range so that the impulse peak was approximately 90 % of the used range.
3.1 Step response measurement

Step response of a digitizer was determined using a DC voltage source and a mechanical reed relay with mercury wetted contacts. Mercury-wetted relay, which is assumed almost an ideal switch because of bounce-free closing of contacts typically under 1 ns [6], was placed directly to the input of the digitizer. Steady DC voltage was applied to the input of a digitizer as presented in Figure 2. Voltage was set to approximately 90% of the digitizer range. Voltage was then shorted to ground with the mercury-wetted relay, which creates almost an ideal voltage step for input of the digitizer. The digitizer records the falling voltage step.

![Figure 2: Arrangement for the step response measurement.](image)

Average of 50 steps was used in order to increase the signal-to-noise ratio of the measurement, and the averaged step was normalized to a rising step from 0 to 1. Because step response depends on channel and range of a digitizer, the step response was repeated for all channels and ranges of the digitizer. In this case, 14 different step responses were determined. Same step response was used for both positive and negative impulses even though there might be some differences between the two cases.

Measurement time was approximately 500 µs with 10% trigger. Sample rate was 200 MS/s with a step length of 100 000 samples. The length of the derivated impulse response should be at least as long as the impulse signal to be corrected, and the sample rates should be equal for easy signal processing.

3.2 Step response correction

Correction for imperfect step response is implemented directly to the LI measurement and analysis software at VTT MIKES. Correction is done automatically right after the measured LI data has been fetched from the digitizer. This allows the user to see the wave shape and parameters of the corrected impulse in real time.

Procedure for step response correction is presented in Figure 3. Measuring software has all step responses stored in its memory. Depending on the used channel and range, the digitizer chooses the right step response for the upcoming measurement. LI measurement must have the same sample rate and length as the impulse response so that the deconvolution process is possible in the frequency domain [4]. In this case, 20 000 samples are recorded and the measuring time is 100 µs. The original raw measurement data is stored for possible post-processing. Before the correction can be made, the step response must be converted to impulse response \( G(t) \) by calculating the time derivative of the step response. Zero-padding is used both for input voltage and impulse response in order to avoid unwanted edge effects in the corrected signal [4]. Then both the LI measurement \( U_i(t) \) and the impulse response \( G(t) \) are converted to frequency domain by using FFT. After FFT, the deconvolution process is made by dividing every frequency component of the measured impulse \( U_i(\omega) \) with equivalent frequency component of the impulse response \( G(\omega) \) [4]. Resulting data \( U(\omega) \) is then converted to time domain by using IFFT. This time domain data is the step corrected impulse \( U_i(t) \).

![Figure 3: Procedure for step response correction.](image)

Deconvolution process usually degrades the signal-to-noise ratio of the corrected signal compared to the original measurement. That's because deconvolution is sensitive to noise and accuracy of the measured step response [4]. This degradation can be kept down by having the signal-to-noise ratio of the step response as good as possible. At first, an additional filtering was used for the deconvolved signal in the frequency domain in order to reduce noise. However, the standardized LI evaluation process also does digital filtering [1] so we observed that this double filtering loses some information from the measured signal. As a result, the digital filtering of the signal was only done by the LI evaluation process. In this case, the noise level after deconvolution has not been a problem.
The discrete convolution theorem assumes that the corrected input signal is periodic \[4\]. This causes the corrected impulse signal to fall to zero in the last datapoints. However, this effect can be neglected because these voltage points can be removed from the signal or they are filtered by the LI evaluation process.

### 3.3 Results

Visual evaluation of the step response correction was done by comparing the original step response with corrected one. Figure 4 presents a case, where a step response measurement was corrected by using over one-year-old step response measurement. The corrected step response reacts and stabilizes faster, however, there is some overshoot in the beginning.

![Figure 4: Original (red) and corrected (blue) step response. Note the logarithmic time scale.](image)

Another way to analyze the step correction is to calibrate the digitizer with the calculable impulse voltage calibrator \[5\], to analyze the \(U_t\), \(T_1\) and \(T_2\) errors of the digitizer. Results of the calibration for one digitizer channel with 0.84/60 µs impulses are presented in Figure 5. Both original and step corrected results are presented in the same figure.

![Figure 5: Impulse calibration results of the tested digitizer with positive 0.84/60 µs impulses. Uncertainty \(k = 2\) is 0.1 % for \(U_t\) calibration, 1.0 % for \(T_1\) calibration and 0.5 % for \(T_2\) calibration.](image)

As seen in Figure 5, the LI parameter errors of the digitizer were significantly reduced. \(U_t\) errors decreased in all ranges. \(U_t\) errors were less than ±0.1 % in all ranges except the two lowest ranges. Small signal levels, and subsequently lower signal-to-noise ratio of the recorded step response, might have caused this error for the lowest ranges. However, these lower ranges are not used for LI measurements. \(T_1\) and \(T_2\) errors were ±0.1 % or less in all the ranges, which indicates that the correction works well for the time parameters.

Calibration of the digitizer was performed 3 times within one year and the parameters \(U_t\), \(T_1\) and \(T_2\) have stayed within ±0.1 % on all ranges and channels (Figure 6). This indicates that the step response of the digitizer has stayed stable in time window of one year.

Good performance and stability in impulse calibrations has resulted in the use of the digitizer in the reference system of VTT MIKES. As National Metrology Institute (NMI) of Finland, VTT MIKES has used this digitizer successfully in customer calibrations \[7\]. The method has also been applied in processing of results for an ongoing world-wide LI intercomparison project \[8\], and for analysis of measurement data for an EU funded project “Metrology for Electrical Power Industry” \[9\].
The results from this divider-en power of a normalizing the step so that it goes from zero to one and then differentiating it into an impulse response. The flat part recorded before the step is discarded. At first, this part was kept during the deconvolution but that proved to produce an extreme noise level in the output.

To reduce edge artefacts, the impulse was zero-padded before and after the recording. The tail of the step response is padded with ones to achieve the same length as the recorded impulse. Finally, both records are zero-padded to an even power of two in order to take advantage of the FFT.

Since the recorded step response of the voltage divider is noisier compared with the step responses of the digitizer, some prefiltering before doing the lightning impulse evaluation was required. A Wiener deconvolution [4] is performed to reduce the noise added by the deconvolution according to following equation:

$$U_i(j\omega) = U_o(j\omega) \frac{G(j\omega)}{|G(j\omega)|^2 + \lambda^2}$$  \hspace{1cm} (5)

where $\lambda$ is the filter parameter.

### 4.3 Results

Comparison with the 400 kV reference digitizer was done by measuring both dividers with the same two-channel digitizer (12-bit, 150 MHz), which is different than the one used for step response measurement. Non-ideal step responses of the digitizer and the differences between the measuring channels might have some effect on the results, but it has been deemed negligible. Table 1 presents the step correction results with a filter parameter of 0.01. Additionally, the effect of different filter parameters was investigated for the 100 MS/s step response (Table 2).

**Table 1:** Results of the comparison ($\lambda = 0.01$). $\Delta$ is the relative difference compared with the reference voltage divider.

<table>
<thead>
<tr>
<th>Step response</th>
<th>$\Delta U_1$</th>
<th>$\Delta T_1$</th>
<th>$\Delta T_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Before deconvolution</td>
<td>-0.1 %</td>
<td>6.6 %</td>
<td>-0.5 %</td>
</tr>
<tr>
<td>100 MS/s</td>
<td>-0.1 %</td>
<td>1.3 %</td>
<td>-0.1 %</td>
</tr>
<tr>
<td>1 GS/s</td>
<td>0.0 %</td>
<td>1.3 %</td>
<td>-0.6 %</td>
</tr>
</tbody>
</table>

**Table 2:** Effect of the filter parameter to the deconvolved impulse.

<table>
<thead>
<tr>
<th>Filter parameter</th>
<th>$\Delta U_1$</th>
<th>$\Delta T_1$</th>
<th>$\Delta T_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 (no filtering)</td>
<td>-1.0 %</td>
<td>1.1 %</td>
<td>-1.4 %</td>
</tr>
<tr>
<td>0.01</td>
<td>-0.1 %</td>
<td>1.3 %</td>
<td>-0.1 %</td>
</tr>
<tr>
<td>0.05</td>
<td>-0.3 %</td>
<td>3.1 %</td>
<td>-0.1 %</td>
</tr>
<tr>
<td>0.1</td>
<td>-1.1 %</td>
<td>3.1 %</td>
<td>-0.1 %</td>
</tr>
</tbody>
</table>
Although the change in evaluated front time is significant between the different settings, a visual examination of the records is not as clear as seen in the Figure 7.

**Figure 7:** Comparison of the front of the (1.05/46.4 μs) impulse with different filter settings.

The evaluation of $U$ depends mainly on how the step response is normalized and should preferably be done using a measured scale factor of the divider together with the calibrated value of the step generator. This has, however, not been available and the results for $\Delta U$ are therefore not completely accurate. Errors in $T_1$ decreased significantly after the correction. However, since a step response of only 10 μs was used for 1 GS/s the results for $T_2$ are somewhat questionable.

## 5 CONCLUSIONS

FFT-based deconvolution was used to correct a tolerable step response of a digitizer and a poor step response of an impulse voltage divider. Good signal to noise ratio is required for the recorded step response in order to avoid adding excessive noise to deconvolved impulse. This can be achieved by averaging a large number (>50) of step responses.

Functionality of the digitizer correction was evaluated traceably with a calculable impulse voltage calibrator. The correction reduced systematically the errors in $U$, $T_1$ and $T_2$ to less than ±0.1 % in the higher ranges with positive 0.84/60 μs impulses. Correction has been in use for over one year and calibration results have been stable within ±0.1 % for all parameters.

Correction for a capacitive impulse voltage divider reduced the $T_1$ errors significantly. Remaining errors are just within the uncertainties of the reference system. Correction for a resistive reference voltage divider [10] of VTT MIKES is under development. The step responses of the digitizer and the divider will be corrected separately by software resulting a step response correction for the complete reference LI measurement system.

## ACKNOWLEDGMENTS

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## REFERENCES

[8] Bureau Intrenalional des Poids et Mesures (BIPM), Supplementary comparison EURAMET.EM-S42, Lightning impulse voltage measurement systems.