Linear Acoustic Modelling
And Testing of Exhaust Mufflers

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By

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Abstract

Intake and Exhaust system noise makes a huge contribution to the interior and exterior noise of automobiles. There are a number of linear acoustic tools developed by institutions and industries to predict the acoustic properties of intake and exhaust systems. The present project discusses and validates, through measurements, the proper modelling of these systems using BOOST-SID and discusses the ideas to properly convert a geometrical model of an exhaust muffler to an acoustic model. The various elements and their properties are also discussed.

When it comes to Acoustic properties there are several parameters that describe the performance of a muffler, the Transmission Loss (TL) can be useful to check the validity of a mathematical model but when we want to predict the actual acoustic behavior of a component after it is installed in a system and subjected to operating conditions then we have to determine other properties like Attenuation, Insertion loss etc.,.

Zero flow and Mean flow (M=0.12) measurements of these properties were carried out for mufflers ranging from simple expansion chambers to complex geometry using two approaches 1) Two Load technique 2) Two Source location technique. For both these cases, the measured transmission losses were compared to those obtained from BOOST-SID models.

The measured acoustic properties compared well with the simulated model for almost all the cases.

Key Words: Acoustic Modelling, Exhaust System, Muffler, End Corrections, Transfer Matrix, Acoustical two port, Acoustical one port, Two Microphone Method (TMM), Transmission Loss, Attenuation, Reflection Co-efficient, AVL Boost
Acknowledgements

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Thanks to Hans Bodén my guide at KTH and Mats Åbom for being really helpful in appreciating the intricacies in the later stages of the thesis. Thank you both of you for allowing me to use the flow test rig at MWL for the mean flow measurements. I am also thankful to the lab technicians who helped me during the course of work at ACC & MWL.

Special thanks to Eric for your help and support in and out of this project. Finally, I would like to thank my family and my friends for giving me the moral support that helped me to carry out the thesis with the same zeal and enthusiasm with which I had started it.

Vielen Dank!
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1. Introduction

Road traffic noise is caused by the combination of rolling noise (arising from tyre road interaction) and propulsion noise (comprising engine noise, exhaust system and intake noise). Controlling these noise sources, which contributes to the globally emitted engine noise is the subject of stringent road noise regulations which is being updated year by year.

The propulsion noise comprises of combustion, mechanical noise and the noise radiated from the open terminations of the intake and exhaust systems which is caused by

I. The pressure pulses generated by the periodic charging and discharging process, which propagates to the open ends of the duct system (Pulse noise), and

II. The mean flow in the duct system, which generates significant turbulence and vortex shedding at geometrical discontinuities (Flow generated noise).

Figure 1. Schematic of Engine Noise Sources.
Mufflers play an important role in reducing the exhaust and intake system noise and as a result, a lot of research is done to designing these systems effectively. The traditional “build & test” procedure which is time consuming and expensive, can nowadays be assisted by numerical simulation models which are able to predict the performance of several different muffling systems in a short time. A number of numerical codes have been developed in the past few decades, based on distinct assumptions.

Considering only one-dimensional models, the two types of simulation models may be distinguished as

I. Linear Acoustic models: This is based on the hypothesis of small pressure perturbations within the ducts, and

II. Non-linear gas dynamics models: This describes the propagation of finite amplitude wave motion in the ducts.

Linear acoustic models are frequency domain techniques which for instance use the four-pole transfer matrix method to calculate the transmission loss of mufflers. This approach is very fast but the predicted results may be unreliable because the propagating pressure perturbations generally have finite amplitude in an exhaust system.

On the other hand, non-linear gas dynamic models are able to simulate the full wave motion in the whole engine intake and exhaust system and are based on time domain techniques. This simulation follow the gas flow from valves to open terminations and so is suited to deal with finite amplitude wave propagation in high velocity unsteady flows. The excitation source can be modeled by means of appropriate boundary conditions for the flow in these simulations.

AVL BOOST is a 1D- gas dynamic tool which predicts engine cycle and gas exchange simulation of the entire engine. It also incorporates the linear acoustic prediction tool SID (Sound In Ducts) [Appendix 1] so it is possible to simulate both the non-linear and linear acoustic behavior of the system.
2. Muffler and Properties

2.1 Muffler:
A muffler is a device used to reduce the sound from systems containing a noise source connecting to a pipe or duct system such as combustion engines, compressors, air-conditioning systems etc., In internal combustion engines, mufflers are connected along the exhaust pipe as a part of the exhaust system. There are two main types of mufflers, reactive and dissipative.

![Diagram of Muffler Types]

\[\text{Muffler} \rightarrow \text{Active (reflective)} \leftarrow \text{Area Change} \rightarrow \text{Resonators} \]

\[\text{Main Types: Reactive, Dissipative} \rightarrow \text{Active (reflective)} \rightarrow \text{Flow Constriction} \rightarrow \text{Porous Materials} \]

2.1.1 Reactive:
Reactive mufflers are usually composed of several chambers of different volumes and shapes connected together with pipes, and tend to reflect the sound energy back to the source, they are essentially sound filters and are mostly useful when the noise source to be reduce contains pure tones at fixed frequencies or when there is a hot, dirty, high-speed gas flow. Reactive muffler for such purpose can be made quite inexpensively and require little maintenance.
2.1.2  **Dissipative:**

Dissipative mufflers are usually composed of ducts or chambers which are lined with acoustic absorbing materials that absorb the acoustic energy and turn it into heat. These types of mufflers are useful when the source produces noise in a broad frequency band and are particularly effective at high frequencies, but special precautions must be taken if the gas stream has a high speed and temperature and if it contains particles or is corrosive.

Some mufflers are a combination of reactive and dissipative types. Selection of these mufflers will depend upon the noise source and several environmental factors.

2.2  **Acoustic properties of Mufflers:**

There are several parameters which describe the acoustical performance of a muffler. These include noise Reduction (NR), Insertion Loss (IL), Attenuation (ATT), and the Transmission Loss (TL). Noise Reduction is the sound pressure level difference across the muffler. It is an easily measurable parameter but difficult to calculate and a property which is not reliable for muffler design since it depends on the termination and the muffler. The Insertion loss is the sound pressure level difference at a point usually outside the system, without and with the muffler present. Insertion loss is not only dependent on the muffler but also on the source impedance and the radiation impedance. Because of this insertion loss is easy to measure and difficult to calculate, however insertion loss is the most relevant measure to describe the muffler performance. Transmission loss is the difference in sound power between the incident wave entering and the transmitted wave exiting the muffler when the muffler termination is anechoic (no reflecting waves present in the muffler). TL is a property fully dependent on the muffler only. Since it is difficult to realize a fully anechoic termination (at low frequencies) TL is difficult to measure but easy to calculate. Attenuation is the difference in the sound power incident and the transmitted through the muffler but the termination need not be anechoic.

The acoustic properties measured to validate the models in this thesis are Transmission Loss (TL), Attenuation (ATT) and Reflection Coefficient (REF).
Why Transmission Loss (TL)?

1. It is a property of the muffler alone and it is independent of the source (its position and strength)
2. It is easy to predict but difficult to measure since it is very difficult to achieve an anechoic termination

\[ TL = 20 \log_{10} \left( \frac{P_1}{P_2} \right) \]

Figure 3. Why Transmission Loss?

It is a property independent of the inlet and outlet pipe length and solely dependent on the geometry of the muffler itself.

Why Attenuation (ATT)?

1. It is a property dependent on the muffler and also the termination, therefore attenuation predicts the actual behavior of the muffler after it is installed in a system
2. Helps to verify the acoustic length of the muffler

\[ ATT = 20 \log_{10} \left( \frac{P_1}{P_2} \right) \]

Figure 4. Why Attenuation?

Attenuation is a property dependent on the outlet pipe length so it is helpful to validate the models and their lengths.
2.3 Wave Reflection in Flow Ducts

All duct systems consist of sections of uniform duct separated by area and other discontinuities where some of the incident wave energy is reflected, some energy is dissipated, while the remainder is transmitted to the adjacent sections. These area discontinuities can be classified according to their observed behavior and they normally include terminations of length of uniform duct and sudden expansion or contractions in duct cross-section such as those found at the ends of the expansion chambers and side branches. It should be realized that the one-dimensional models for simulation of plane wave motion in ducts does not take account of the three-dimensional waves arising at these discontinuities.

Evaluation of the pressure reflection and transmission coefficients for each discontinuity involves satisfying the boundary conditions associated with it, conservation of mass, conservation of energy and momentum flux across the discontinuity. The reflective property is expressed by a pressure reflection coefficient $R$, defined in terms of the component pressure wave amplitude as

$$R = \frac{p^-}{p^+}$$

(2.1)

Since the wave amplitudes $p^-$ & $p^+$ are complex valued, the reflection coefficient $R$ is also normally, a complex quantity.

A close approximation to the calculated reflection coefficient for an unflanged pipe of radius $a$ is expressed by [Ref-4 Davies (1988)]

$$R_0 = 1 + 0.0133ka - 0.59079(ka)^2 + 0.33576(ka)^3 - 0.6432(ka)^4$$

(2.2)

in the useful range $0 < ka < 1.5$

Where $R_0$ corresponds to the zero flow Mach number $M$

A close approximation for $R$ on $R_0$ for an outflow Mach number into a duct is expressed in [Ref-10 Munt (1990).]
2.3.1 End Correction:
To take the three dimensional effects into account at these junctions an end correction, \( l \) is added to the length of the duct adjoining the discontinuity. The introduction of the end correction locates the plane of wave reflection, shifted by a distance \( l \) away from the geometrical discontinuity plane.

![Diagram](image)

Figure 5. Wave reflection coefficient \( R \) and end correction \( l \), at the open outflow end.

In other words, the end correction corresponds to the extended length required to obtain a phase shift corresponding to the three dimensional effects between the incident \( (p^+) \) and reflected wave \( (p^-) \).

2.3.2 Determination of End correction:
The above defined end correction can also be defined as the extra length added to the pipe to produce a reflection coefficient of

\[
R = \left| R \right| e^{i\theta} = \left| R \right| e^{i\left(\frac{\pi + 2kl}{1-M^2}\right)},
\]

Where, \( R \) is the reflection coefficient at the opening.

The end-correction \( l \) is normalized with respect of the pipe radius is given by,

\[
\frac{l}{a} = \frac{(\theta - \pi)(1-M^2)}{2ka}
\] (2.4)
Where \( \theta \) is the phase of the reflection coefficient \( R \) obtained theoretically Munt’s model (Ref-10 Munt (1990)) or experimentally from the cluster technique and \( a \) is the pipe radius.

### 2.3.3 Open Outflow End Corrections:

For an open end termination, the correction length \( l \) is dependent on the value of \( ka \), on the geometry of the open end with its surroundings and on the Mach number \( M \) of the mean flow. There exists many theoretical models to calculate this open outflow end correction and a few end corrections at zero flow are given in **Table 1**.

![Figure 6. Open Outflow End Correction](image)

<table>
<thead>
<tr>
<th>Proposed by</th>
<th>End Correction Value</th>
<th>Valid for</th>
</tr>
</thead>
<tbody>
<tr>
<td>P.O.A.L Davies</td>
<td>( l_o = 0.6133 - 0.1168(ka)^2 )</td>
<td>( ka &lt; 0.5 )</td>
</tr>
<tr>
<td>P.O.A.L Davies</td>
<td>( l_o = 0.6393 - 0.1104ka )</td>
<td>( 0.5 &lt; ka &lt; 2 )</td>
</tr>
<tr>
<td>Norris and Sheng</td>
<td>( l_o = \frac{[0.6133 - 0.027(ka)^2]}{[1+0.19(ka)^2]} )</td>
<td>( 0 &lt; ka &lt; 3.8 )</td>
</tr>
<tr>
<td>Dalmont et. al</td>
<td>( l_o = \frac{0.6133 - 0.027(ka)^2}{[1+0.19(ka)^2]} - 0.012 \sin^2(2ka) )</td>
<td>( ka &lt; 1.5 )</td>
</tr>
</tbody>
</table>
Munjal & Peter et.al. \( \frac{l_0}{a} = 0.6 \)

Levine and Schwinger \( \frac{l_0}{a} = 0.6133 \)

Low Frequency

Onorati \( \frac{l_0}{a} = 0.6 - 0.6 \left( \frac{K}{K_{2000}} \right)^2 \)

Table 1. Open Outflow End Corrections-Zero Flow.

A close approximation for these end corrections with mean outflow with Mach M, is given by [Ref-23 Davies (1980)]

\[ l_M = l_0 \left[ 1 - M^2 \right] \quad \& \quad l_M = l_0 \]

End corrections calculated from both these equations are plausible. Therefore an open outflow end correction value between these two equations is likely.

Some Open outflow End corrections at zero flow are plotted in Figure 7 as a function of Helmholtz number (ka)

![Figure 7 Outflow End Corrections-Zero Flow.](image)
### 2.3.4 Expansion and Contraction End correction:

The correction length for abrupt area changes as in the case of an expansion or contraction in ducts are not frequency dependent but a function of geometry only and is valid for both zero flow and non-zero mean flow. Table 2 below gives a few end corrections which are widely used,

![Figure 8. Expansion and Contraction End Correction.](image)

<table>
<thead>
<tr>
<th>Proposed by</th>
<th>End Correction Value</th>
<th>Valid for</th>
</tr>
</thead>
<tbody>
<tr>
<td>Karal</td>
<td>$\frac{l_c}{r_1} = \frac{8}{3\pi} \left[ 1 - 1.238 \frac{r_1}{r_2} \right]$</td>
<td>$0 &lt; \frac{r_1}{r_2} &lt; 0.5$</td>
</tr>
<tr>
<td>Karal</td>
<td>$\frac{l_c}{r_1} = \frac{8}{3\pi} \left[ 0.875 \left( 1 - \frac{r_1}{r_2} \right) \left( 1.371 - \frac{r_1}{r_2} \right) \right]$</td>
<td>$0.5 &lt; \frac{r_1}{r_2} &lt; 1$</td>
</tr>
<tr>
<td>P.O.A.L Davies</td>
<td>$l_c = 0.63r_1 \left[ 1 - e^{-\left( r_1 / r_2 \right)^{1.5}} \right]$</td>
<td>$0 &lt; \frac{r_1}{r_2} &lt; 1$</td>
</tr>
<tr>
<td>Torregrosa</td>
<td>$\frac{l_c}{r_1} = 2 \left( 0.26148 - e^{-1.31906r_1 / r_2} \right)$</td>
<td>$0 &lt; \frac{r_1}{r_2} &lt; 1$</td>
</tr>
</tbody>
</table>

**Table 2. Expansion and Contraction End Corrections.**

$r_1$ and $r_2$ are the radii of the small and large ducts respectively.
Some Expansion and Contraction End corrections are plotted in Figure 8 as a function of a ratio of the duct radii.

![Figure 9](image_url)  
**Figure 9** Expansion and Contraction End Corrections

### 2.3.5 Resonator Neck End Correction:

The three dimensional effects occurring at the area discontinuity (neck-tube and neck-cavity) of a Helmholtz resonator has to been taken into account so there are two different end corrections applied in the case of the Helmholtz resonator. They are given in the table below,

![Figure 10](image_url)  
**Figure 10**. Resonator Neck End Correction.
<table>
<thead>
<tr>
<th>Proposed by</th>
<th>End Correction Value</th>
<th>Valid for</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ingard (neck-cavity)</td>
<td>( \frac{l_c}{r} = \left( \frac{8}{3\pi} \right) \left( 1 - 1.24 \frac{r}{R} \right) )</td>
<td></td>
</tr>
<tr>
<td>Onorati (neck-tube)</td>
<td>( l_c = 0.3r )</td>
<td>( f &lt; 500 )</td>
</tr>
<tr>
<td>Rayleigh (neck-tube)</td>
<td>( l_c = 0.85r )</td>
<td>( 500 &lt; f &lt; 1500 )</td>
</tr>
<tr>
<td>Onorati (neck-tube)</td>
<td>( l_c = r ) or ( \text{more} )</td>
<td>( 1500 &lt; f )</td>
</tr>
<tr>
<td>Rayleigh-Empirical</td>
<td>( \frac{l_c}{r} = 0.5 \ln f - 2.68 )</td>
<td>( 400 &lt; f &lt; 1200 )</td>
</tr>
</tbody>
</table>

**Table 3. Various Resonator Neck End Corrections.**

The end correction lengths given in the table are suitable also for the calculations with strong mean flow, since the side resonators do not experience a mean flow.
2.4 Modes of wave propagation in Linear Acoustic theory:

The propagation of three dimensional modes in a cylindrical duct systems, are related to a specific pressure distribution in the duct as shown in figure below [Ref-21 Eriksson (1980)]. Even though only plane waves are permitted in the inlet and outlet pipes, one or more higher order modes gets cut on in the chamber in the higher frequencies depending on the diameter of the chamber.

![Higher Order Modes Diagram](image)

**Figure 11. Higher Order Modes.**

When m or n take a nonzero value then there exist higher order modes in the ducts. Each higher order mode has an associated cutoff frequency below which it is not possible for that mode to propagate. These frequencies may be determined by noting that the radial component of the particle velocity must go to zero at the wall duct. The cutoff frequencies within a given duct may be calculated using:
\[ f_c = \frac{x_{mn} c}{\pi d} \] (2.5)

Where \( d \) is the duct diameter and \( c \) is the sound velocity, with the values of \( x_{mn} \) as given in the above figure. At very low frequencies only plane wave mode can propagate, and the pressure distribution across the duct is uniform.

The first circumferential higher order mode in a circular duct is the \((1, 0)\) mode, which has a cutoff frequency of

\[ f_c = \frac{1.84 c}{\pi d} \] (2.6)

And the cutoff frequency of the first radial mode \((0, 1)\) is

\[ f_c = \frac{3.83 c}{\pi d} \] (2.7)

The cutoff frequencies are reduced by a factor \((1 - M^2)^{1/2}\) in the presence of mean flow, where \( M \) is the flow Mach number.
3. Measurement of Acoustic properties:

The standard technique today for measuring the acoustic plane wave properties in ducts, such as absorption coefficient, reflection coefficient and impedance is the Two Microphone Method (TMM) (Ref-7 Bodén & Åbom (1984)). The sound pressure is decomposed into its incident and reflected waves so that the input sound power can be calculated. Transmission Loss can in principle be determined from measurement of the incident and transmitted power using two-microphone method on the upstream and downstream side of the test object provided that a fully anechoic termination can be implemented on the outlet side, which is practically very difficult in low frequency region and especially with flow. Instead the two load technique has been used where the sufficient information for determining the two-port matrix is obtained from two sets of measurements with different loads on the outlet side and for the mean flow measurements carried out at MWL,KTH the two-source location technique was employed by placing the source on the upstream and downstream side of the test object.

3.1 Two-Microphone wave decomposition:

The sound field in a straight hard walled duct below the first cut-on frequency will consist only of plane propagating waves. The sound field can be written as [Ref-2 Munjal 1987]

\[
p(x,t) = p_+(t-x/c) + p_-(t+x/c)
\]  

(3.1)

Where  
\[ p = \text{acoustic pressure} \]
\[ c = \text{Speed of sound} \]
\[ x = \text{spatial coordinate along the duct axis.} \]

The idea behind the two-microphone wave decomposition is that in the low frequency region the sound field can be completely determined by simultaneous
pressure measurements at two axial positions along the duct. In the frequency domain, the sound field can be written as

\[
\hat{p}(x, f) = \hat{p}_+(f) \exp(-ik_+x) + \hat{p}_-(f) \exp(ik_-x)
\]

\[
\hat{u}(x, f) = \frac{1}{\rho c} \left[ \hat{p}_+(f) \exp(-ik_+x) - \hat{p}_-(f) \exp(ik_-x) \right]
\]

Where, \( \hat{p} \) = Fourier transform of the acoustic pressure,

\( \hat{u} \) = Fourier transform of particle velocity averaged over the duct cross-section,

\( x \) = Length coordinate along the duct axis,

\( f \) = Frequency,

\( k_\pm \) = Complex wave number for waves propagating in the positive or negative \( x \)-direction,

\( \rho \) = Density and

\( c \) = Speed of sound.

The complex wave numbers can be calculated using the results from Howe [Ref-18 Howe (1995)] or measured [Ref-1 Allam (2004)] if two microphones on either side are available. When the complex wave numbers are known the incident \( (p_+) \) and reflected \( (p_-) \) wave amplitude can be calculated using pressure measurements at two microphone positions.

Figure 12. Measurement Configuration of Two-Microphone Method.
\[ \hat{p}_1(x, f) = \hat{p}_+(f) + \hat{p}_-(f) \]  \hspace{1cm} (3.4)

\[ \hat{p}_2(x, f) = \hat{p}_+(f) \exp(-ik_+s) + \hat{p}_-(f) \exp(ik_-s) \]  \hspace{1cm} (3.5)

Where, \( s \) represents the microphone separation as shown in figure, using the above equations \( \hat{p}_+ \) and \( \hat{p}_- \) can be expressed by

\[ \hat{p}_+(f) = \frac{\hat{p}_1(f) \exp(ik_-s) - \hat{p}_2(f)}{\exp(ik_-s) - \exp(-ik_+s)} \]  \hspace{1cm} (3.6)

And

\[ \hat{p}_-(f) = \frac{-\hat{p}_1(f) \exp(-ik_-s) + \hat{p}_2(f)}{\exp(ik_-s) - \exp(-ik_+s)} \]  \hspace{1cm} (3.7)

According to [Ref-1 Allam (2004)] the following conditions should be fulfilled for successful use of the method

- The measurement must take place in the plane wave region.
- The duct wall must be rigid in order to avoid higher order mode excitation.
- The test object should not be placed closer than 1-2 duct diameters the nearest microphone. This is due to the fact that spatially non uniform test objects could excite higher order modes and therefore create near field effects at the microphones.
- The propagation of the plane wave must be unattenuated. However in practice it is not true even for no flow case because of various mechanisms, mainly associated with viscosity, heat conduction, will cause deviations from the ideal behavior.

Bodén and Åbom [Ref 3 Bodén & Ref 8 Bodén] showed that the two microphone method has the lowest sensitivity to errors in the input data in a region around \( ks = \pi(1 - M^2)/2 \).

Åbom and Bodén [Ref 8 Åbom] stated that to avoid large sensitivity to errors in the input data, the two microphone method should be restricted to the frequency range.

\[ 0.1\pi(1 - M^2) < ks < 0.8\pi(1 - M^2) \]  \hspace{1cm} (3.8)
3.2 Acoustical one-ports:

Reflection co-efficient at the open end (x=0) of an unflanged pipe is given by Holland and Davies [Ref-19 Holland (2000)] as

\[ R_0(f) = \frac{H_{12} - \exp(-i k_s s)}{\exp(i k_s s) - H_{12}} \]  

(3.9)

Where, \( H_{12} \) = Transfer function between microphone 1 and microphone 2,

\[ k_+ = \frac{(2\pi f / c - i\delta)}{(1 + M)} \]

\[ k_- = \frac{(2\pi f / c + i\delta)}{(1 - M)} \]

\( \delta \) = Term representing attenuation.

\( s \) = Microphone spacing.

Therefore the Reflection coefficient at an arbitrary cross section along the duct is given by

\[ R_L(f) = \frac{H_{12} - \exp(-i k_+ s)}{\exp(i k_- s) - H_{12}} \exp\left[2ikL/(1 - M^2)\right] \]  

(3.10)

Ignoring propagation losses and flow, the two pressure signals are identical when \( ks = n\pi \), where \( n \) is any integer; this method will yield poor results when the distance between the microphone is close to multiples of half an acoustic wavelength. Therefore the spacing between the microphones must be kept to within a half wavelength of the highest frequency of interest.
3.3 Acoustical Two-ports:

A two-port is a linear system with an input and output. Assuming plane wave propagation at the inlet and outlet port, the properties of these acoustical two-ports can be determined from theory or by measurements by assuming two state variables at each port. A number of different choices of state variables are possible. However, some state variables are more convenient to use than others, for example, to measure fluctuating density may not be easy. Two state variables which are frequently used is pressure ($p$) and volume flow ($q$). This type of formulation is common when having systems with one preferred energy transport and it is called the transfer matrix formulation and the relation between the input and output of a time-invariant and passive two-port can be written as

$$\hat{y} = H\hat{x}$$  \hspace{1cm} (3.11)

Where, $\hat{x}$ and $\hat{y}$ = state vectors at the input and output as shown in figure

$H$ = is a [2x2]-matrix

![Figure 13. Black Box relating two pairs of state Variable x, y.](image)

To determine the two-port matrix $H$ from measurements, four unknowns must be determined. To get the four equations needed for complete experimental determination of properties of an acoustical two-port two independent test states (‘and “) must therefore be created. The matrix equation obtained is

$$\begin{bmatrix} \hat{y}' & \hat{y}'' \end{bmatrix} = H \begin{bmatrix} \hat{x}' & \hat{x}'' \end{bmatrix}$$  \hspace{1cm} (3.12)
The unknown two-port matrix $H$ can be determined from this equation if and only if

$$\det(X) \neq 0 \tag{3.13}$$

Where, $X$ is the matrix containing the two-port state vectors.

Depending on the coupling of the duct system either the Transfer-matrix (If Cascade) or the mobility-matrix (If Parallel) of the two-port is used.

The transfer-matrix form uses the acoustic pressure ($p$) and the particle velocity ($q$) i.e.

$$X = \begin{bmatrix} \hat{p}_a & \hat{q}_a \end{bmatrix} \quad \text{And} \quad Y = \begin{bmatrix} \hat{p}_b & \hat{q}_b \end{bmatrix}.$$ If there is not internal sources inside the two-port element the transfer-matrix could be written in the following form

$$\begin{bmatrix} \hat{p}_a \\ \hat{q}_a \end{bmatrix} = \begin{bmatrix} T_{aa} & T_{ab} \\ T_{ba} & T_{bb} \end{bmatrix} \begin{bmatrix} \hat{p}_b \\ \hat{q}_b \end{bmatrix} \tag{3.14}$$

The transfer matrix can be solved if the below equation is satisfied

$$\det \begin{bmatrix} \hat{p}'_b & \hat{p}''_b \\ \hat{q}'_b & \hat{q}''_b \end{bmatrix} \neq 0 \tag{3.15}$$

Here “a” and “b” represents two different duct cross-sections. Three basic assumptions concerning the sound field inside the transmission line are made

- Only plane waves are allowed to propagate at the inlet and outlet section of the system
- The field is assumed to be linear, i.e. the acoustic pressure is typically less than one percent of the static pressure [Ref-20 Åbom (1991)] so that the analysis can be done in the frequency domain.
- The two-port system is passive, i.e. no internal sources are allowed. This is a problem concerning flow generated noise.
### 3.3.1 Two-Source Location Technique:

The two source location method is based on the above mentioned transfer matrix approach. The two-port data is determined by having two independent test states by two source location method as shown in figure below.

![Measurement Configuration for the Two-Source Location Technique.](image)

**Figure 14. Measurement Configuration for the Two-Source Location Technique.**

The first state is obtained by turning loudspeaker \( A \) on and \( B \) off and the second independent test state is obtained by turning loudspeaker \( B \) on and \( A \) off.

From 3.12 and 3.14 we have

\[
\begin{bmatrix}
\hat{p}_a'

\hat{q}_a'

\end{bmatrix} =
\begin{bmatrix}
T_{aa} & T_{ab}

T_{ba} & T_{bb}

\end{bmatrix}
\begin{bmatrix}
\hat{p}_b

\hat{q}_b

\end{bmatrix}
\]

\( (3.16) \)

If the input and output vector of the transfer matrix are measured, we obtain the following matrix equation from the definition of the transfer matrix using the two-port conditions. It is also possible to use this method using a single loudspeaker and by reversing the muffler during the second test state.

The transmission loss of the test object can be obtained from

\[
TL(n) = 20 \log_{10} \left[ \frac{1}{2} \left( \frac{Z_a}{Z_b} \right)^2 + \frac{T_{aa}(n)}{\sqrt{(Z_a * Z_b)}} + T_{ab}(n) * \sqrt{(Z_a * Z_b)} + T_{bb}(n) \left( \frac{Z_b}{Z_a} \right) \right]
\]

\( (3.17) \)
Where $Z_a = \rho c_0/A_a$ and $Z_b = \rho c_0/A_b$ are the characteristic impedances at the duct cross section at “a” and “b”.

### 3.3.2 Two-Load technique:

This transfer matrix based approach realizes the two independent test states needed to determine the two-port data by changing the loads at the termination instead of moving the sound source to the other end thus generating four equations with four unknowns. The setup is as shown in the figure below,

![Measurement Configuration for the Two-Load Technique.](image)

In this two-load method, if the loads are very similar, the results will be unstable. Generally, two loads can be two different length tubes, a single tube with and without absorbing material, or even two different mufflers.
In this thesis, two loads are achieved by having an open termination and a closed termination. The different impedance ratios of the two loads used in this thesis is as shown in the figure below.

![Impedance Ratio for the Two Loads](image)

**Figure 16. Impedance Ratio of the Two Loads used in the Two-Load Technique**

The transmission loss of the test object can be obtained from

\[
TL(n) = 20 \log_{10} \left| \frac{1}{2} T_{aa}(n) \sqrt{\frac{Z_a}{Z_b}} + \frac{T_{ab}(n)}{\sqrt{Z_a Z_b}} + T_{ba}(n) * \sqrt{Z_a Z_b} + T_{bb}(n) \sqrt{\frac{Z_b}{Z_a}} \right|
\]

3.18

Where \( Z_a = \rho c_0 / A_a \) and \( Z_b = \rho c_0 / A_b \) are the characteristic impedances at the duct cross section at “a” and “b”
4. Test Set-up

4.1 One Port Measurement:

All one-port experiments were carried out at room temperature in the test setup at the Acoustic Competence Centre (ACC). The setup consists of one loudspeaker as an acoustic source as shown in figure. Fluctuating pressures were measured using three ½ inch condenser microphones thread mounted on the duct wall. The measurements were carried out using burst random signal (with 60% burst time) excitation and with different number of frequency domain averages (20, 50, 100 averages). The transfer function between the three microphone positions are measured and used to estimate the transfer matrix components.

![Diagram of One-Port Test Object](image)

**Figure 17. Layout of One-Port Test Object**

Three microphones were used to in order to cover the frequency range 100-2000 Hz while fulfilling the equation \[ 0.1\pi(1-M^2) < k < 0.8\pi(1-M^2) \]. The distance between the microphone 1 and 3 was 15cm giving approximately the frequency range 100-900Hz. The distance between microphone 2 and 3 was 5cm giving approximately the frequency range 350-2600Hz.
4.2 Two port Measurements with zero mean flow:

All two port measurements with zero mean flow were carried out in room temperature in the test setup at the Acoustic Competence Center (ACC). The test setup is as shown in Figure 18. The inlet and outlet pipes used during the measurements were made of standard steel with wall thickness 1.5 mm. The inlet and outlet pipe diameter was 51 mm and one loud speaker was used as an excitation source. Fluctuating pressures were measured using six ½ inch condenser microphones thread mounted on the duct walls. The microphone placement and the test setup are as shown in figure below. The measurements were carried out using Burst random (60% burst time) as the excitation signal and with different frequency domain averages. Since the measurements were done with zero flow, the number of averages did not really influence the quality of the result therefore 100 FDA were used during all these measurements. The two-port data was obtained using the two load technique by altering the loads at the termination as described in Section 3.3.2.

One of the four microphone signal is taken as the reference and the transfer function between the reference and the other three microphones are measure and used to estimate the transfer matrix components.
The same microphone separations described in the previous section for the one-port measurements were used, giving again the frequency range 100-900Hz for 15cm distance and 350-2600Hz for 5cm distance.

4.3 Two port Measurements with Mean flow:

The two-port measurements with mean flow were carried out at room temperature using the flow acoustic test facility at The Marcus Wallenberg laboratory for Sound and Vibration research at KTH. The muffler configurations were the same as used in ACC but only one inlet and outlet pipe diameter (67mm) was chosen to fit the test rig duct diameter. The loud speakers were divided equally between the upstream and downstream side. The microphone placement and the test setup are as shown in Figure 19. Fluctuating pressures were measured by six ¼ inch condenser microphones flush mounted on the duct wall. The measurements were carried out using stepped sine excitation in the frequency range 100 -1200 Hz using different frequency steps and different Frequency Domain averages. The two-port data was obtained using the source switching technique as described in section 3.3.1, where the measurements were made using the upstream loud speakers on and downstream loud speakers off and vice- versa.

Figure 19. Layout of Two-Port Test Object at MWL, KTH
The mean flow velocity was measured with a Pitot tube placed at the centre of the duct. It was assumed that flow the fully developed a boundary layer and the mean velocity = 0.8*Maximum Velocity (measured in the centre of the duct) [Ref-17 Schlichting (1968)]. Once the peak velocity was measured the Pitot tube was removed from the duct before taking the acoustic measurements as it might disturb the flow.
The flow velocity on the upstream side of the test object was measured separately before and after the acoustic measurements and average value was used.

As an additional data, the pressure drop across the test object was also measured for different flow speeds using the Pitot tube. The transfer function between the reference signal and the microphone signal was measured and used to estimate the transfer matrix components.

Two different microphone spacing were used to cover a wide frequency range 100-1200 Hz while fulfilling the equation \[0.1 \pi (1 - M^2) < ks < 0.8 \pi (1 - M^2)\]. The distance between microphone 1 and 2 was 10cm giving approximately the frequency range 170-1300 Hz and the distance between microphone 1 and 3 was 50cm giving approximately the frequency range 40-280 Hz.

4.4 Microphone calibration:

For ordinary sound pressure measurements, only amplitude calibration is enough but for two-microphone method we need both amplitude and phase calibration. The fluctuating pressures measured at each position have been corrected using the relative calibration between the microphone channels. Assuming that we have plane waves in a duct the sound pressure amplitude will be constant over the duct cross section and the sound pressure is measured by all microphones would give the same pressure amplitude with zero phase shifts. However there will in practice be a deviation from this ideal case due to the measuring chain, amplifiers, and cables etc., which introduce amplitude and phase shifts. Relative calibration of the microphone of the microphone measurement chain is therefore needed. In order to calculate the transfer-matrix equation, the transfer function between the microphones and the electrical loud-speaker signal, i.e. \(H_{r1}, H_{r2}, H_{r3}, H_{r4}, H_{r5}, \) and \(H_{r6}\) are needed. It is sufficient to measure the transfer function between these microphones and a reference microphone say microphone 1, \(H_{12}, H_{13}, H_{14}, H_{15}, \) and \(H_{16}\).
The calibration transfer functions, which will be used in the calculation of the transfer matrix, can then be obtained from

\[
H_{r1}^{\text{cal}} = H_{r1} \\
H_{r2}^{\text{cal}} = \frac{H_{r2}}{H_{12}} \\
H_{r3}^{\text{cal}} = \frac{H_{r3}}{H_{13}} \\
H_{r4}^{\text{cal}} = \frac{H_{r4}}{H_{14}} \\
H_{r5}^{\text{cal}} = \frac{H_{r5}}{H_{15}} \\
H_{r6}^{\text{cal}} = \frac{H_{r6}}{H_{16}}
\]  

(4.1)

**Figure 21. Calibration Tube**

A special calibration tube a shown in figure above was used to measure the transfer functions between the reference microphone and the other microphones. The calibration tube consists of a loudspeaker, a steel pipe, which has the same diameter as the test object and a microphone holder for six ¼ inch microphones. The holder is made-up of a plastic material to avoid possible grounding errors between the microphones. The length of the steel pipe is preferably short to minimize the number of resonance in the pipe.
4.5 Flow noise suppression:

An efficient way of suppressing turbulent pressure fluctuations is to use a reference signal, which is uncorrelated with the disturbing noise in the system and linearly related to the acoustic signal in the duct [Ref-9 Åbom (1989)]. A good choice for the reference signal is to use the electric signal driving the external sources as a reference. Deviation from a linear relation between the reference signal and the acoustic signal in the duct can for instance be caused by non-linearity of the loud speakers at high input amplitudes, temperature drift and non-linearity of the loudspeaker connections to the duct at high acoustic amplitudes. One possibility is to put an extra reference microphone close to a loud-speaker or even in the loud speaker box behind the membrane i.e., without contacting the flow. Otherwise one of the measurement microphones can be used as a reference.

The disadvantage of this technique is that one will get a minima’s at the reference microphone at certain frequencies or poor signal to noise ratio. To solve this problem one can use the microphone with the highest signal-to-noise ratio as the reference. In this work the electronic signals driving the loudspeakers were used as the reference.
5. Results and Discussion

5.1 Open End Reflection:

Figure 22. Test Set-up for reflection coefficient measurement at an open end

The reflection coefficient at the outlet cross section of an open pipe end without mean flow was measured as described in Section 4.1.

Figure 23 shows the absolute value of the measured reflection coefficient as compared with the BOOST SID theoretical model. There is very little information available about the acoustical lengths from the absolute value of the reflection coefficient therefore the real, imaginary and phase of the reflection coefficient which is very sensitive for these changes are shown compared with the BOOST SID theoretical results in Figure 24, Figure 25 and Figure 26.

We can see from these figures that the real and imaginary parts of the measured reflection coefficient have a very good agreement (there is no frequency shift therefore correct acoustic length) and the phase of the reflection coefficient agrees well with the theoretical models which show that the open out flow end corrections used in BOOST SID are correct.
Figure 23. Reflection Coefficient results for open end pipe

Figure 24. Reflection Coefficient results for open end pipe
Figure 25. Reflection Coefficient results for open end pipe

Figure 26 shows the predicted and measured phase in radians of the reflection coefficient $R$ at the open end of the pipe.

Figure 26. Reflection Coefficient results for open end pipe
By determining the complex value of the reflection coefficient $R$ at the opening, the phase of $R$ is converted to an end correction as described in Section 2.3.2. Davies gives the below expression for the reflection coefficient at a pipe opening at zero flow,

$$R_0 = 1 + 0.0133ka - 0.59079(ka)^2 + 0.33576(ka)^3 - 0.6432(ka)^4$$

in the useful range $0 < ka < 1.5$

*Where $R_0$ corresponds to the zero flow Mach number $M$.*

The and Davies suggests the following Open End corrections which is also used in BOOST SID,

$$\frac{l_0}{a} = 0.6133 - 0.1168(ka)^2 \quad ka < 0.5$$

$$\frac{l_0}{a} = 0.6393 - 0.1104ka \quad 0.5 < ka < 2$$

The figure below shows the measured end correction as described in, compared with the theoretical value given in the above equation at zero flow. The predicted end corrections agree well with the experimental value.

![Figure 27  End Correction Measurement](image-url)
5.2 Simple Expansion Chamber:

An Expansion chamber is the simplest form of reactive muffler using the property of sound reflection caused by the change in cross sectional area to cancel sound. A simple expansion chamber is made up of an inlet pipe, a Chamber and an Outlet pipe as shown in figure below.

An expansion chamber has a predictable transmission loss curve having maxima at

\[
f = \frac{nc}{4L}, \text{ where } n = 1,3,5,\ldots
\]  

\( c = \text{Speed of Sound} \)
\( L = \text{Length of Chamber} \)

The position of the maxima also depends on

\[
c = c_0 \sqrt{\frac{T}{T_0}}, \text{ where } T = \text{temperature in K}
\]  

(5.2)

Where \( c_0 \) is the speed of sound at \( T_0 \) degree K.

For example an expansion chamber of Length, L=500mm and Diameter, D=200mm will have a transmission loss curve as shown in the figure below at M=0 and T=293k where the speed of sound, c=343m/s. The first four maxima calculated from Equation 5.1 for n=1, 3, 5 & 7 matches the predicted curve at 172Hz, 515Hz, 856Hz and 1200Hz respectively.
Transmission Loss measurements were performed using the Two-load technique as discussed in the Section 4.2. The test objects used in this measurement are as in the table below,

<table>
<thead>
<tr>
<th>Muffler ID</th>
<th>Inlet Pipe Dimension in mm</th>
<th>Chamber Dimension in mm</th>
<th>outlet Pipe Dimension in mm</th>
<th>Expansion KARAL End correction in mm</th>
<th>Contraction KARAL End correction in mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dia 100</td>
<td>Dia 51, Len 96</td>
<td>Dia 100, Len 500</td>
<td>Dia 51, Len 96</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>Dia 200</td>
<td>Dia 51, Len 96</td>
<td>Dia 200, Len 500</td>
<td>Dia 51, Len 96</td>
<td>14.7</td>
<td>14.7</td>
</tr>
<tr>
<td>Dia 300</td>
<td>Dia 51, Len 96</td>
<td>Dia 300, Len 500</td>
<td>Dia 51, Len 96</td>
<td>17</td>
<td>17</td>
</tr>
</tbody>
</table>

Table 4. Dimensions of the simple expansion chambers

Geometric Model to Acoustic model:

Converting the geometric model of a muffler to an acoustic model is an important step in linear acoustic modelling. For example conversion of a diameter 200mm simple expansion chamber as in Table 4 to a BOOST SID model was made as shown below,
Geometric model of Diameter 200mm Expansion Chamber:

Suitable end corrections are applied to this geometric model. In our case, KARAL end corrections as shown in Table 2 & the values as in Table 4 are applied at the junctions where the inlet and outlet pipes are connected to the chamber and at the open end boundary.

Geometric model with end corrections applied:

As a result of these end corrections, the length of the inlet and outlet pipes are extended by a length \(l\), equal to the end correction and consequently, the length of the chamber is reduced as shown in figure below. The area between the extended pipe and the chamber forms quarter wave resonators on both ends as shown in figure.
**Acoustic Model:**

- Acoustic Inlet Pipe length = Geometric Inlet length + End correction.
- Acoustic Chamber length = Geometric chamber length – (2 x End correction).
- Acoustic Outlet Pipe length = Geometric Outlet length + End correction.

**Quarter wave Resonators:**
- Length = 14.7mm & Diameter = \((200^2 - 51^2)^{1/2}\)mm

**BOOST SID Model:**

- Acoustic modelling of this expansion chamber in BOOST SID can be done in three different ways,

1. **Modelling with Quarter Wave Elements:**
   - The end correction lengths are modeled as separate quarter wave resonators as shown in figure below as quarter wave resonators and Junctions as shown in the figure below,
These models are not useful when there is mean flow inside the muffler because the linear acoustic tool assumes that there is no flow inside the resonator which is not true if the resonator is inside the muffler.

2. Modelling with Restrictions:

The end correction lengths are modeled as extensions in the restriction elements as shown in figure below. Because of the flow limitation of the Quarter Wave model, this model with restrictions and extensions are used in this thesis.

3. Modelling with Plenum Chambers:

By modelling the expansion chamber with the plenum element, No end corrections have to be applied to these models since the higher order mode (Both Circumferential & Radial) effects which are excited above the first cut-on frequency are already accounted for.

There are two types of plenum chambers available in BOOST SID

- Extended Concentric-Accounts the Radial Mode effects.
- Flush Eccentric-Accounts the Circumferential Mode effects.
Figure 32 and Figure 33 compares the measured Transmission Loss results for a Diameter 100 and Diameter 200mm muffler with the predictions of the first two linear acoustic models from BOOST SİD without considering the higher order modes.

![Figure 32. Transmission Loss of Simple expansion Chamber](image)

![Figure 33. Transmission Loss of Simple expansion Chamber](image)
As described in Section 2.4, the cutoff frequency for circumferential and radial higher order modes in circular ducts are given by equation 2.6 and 2.7 respectively. The table below gives the cutoff frequency above which the first circumferential and first radial modes propagates in these three expansion chambers at 293k & M=0.

<table>
<thead>
<tr>
<th>Muffler Diameter in mm</th>
<th>Cutoff Frequency (Hz) 1st Circumferential Mode</th>
<th>Cutoff Frequency (Hz) 1st Radial Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>2008</td>
<td>4179</td>
</tr>
<tr>
<td>200</td>
<td>1004</td>
<td>2089</td>
</tr>
<tr>
<td>300</td>
<td>669</td>
<td>1392</td>
</tr>
</tbody>
</table>

Table 5. Cut off frequency of Expansion chambers of different diameter.

Our frequency range of interest is 0 to 2000 Hz so we are not concerned about the higher order modes propagating above 2000Hz. It can be seen from Table 5 that the first circumferential mode is cut on at 1004Hz and 669 Hz for a Diameter 200mm and 300mm expansion chambers respectively and from Figure 32 and Figure 33 we can see that there is no influence of these circumferential higher order modes. Whereas in Figure 34 we can see the effects of the first radial higher order mode in the diameter 300mm muffler starting from 1400Hz.

Figure 34  Transmission Loss for Simple Expansion Chamber
Therefore the diameter-300mm expansion chamber is modeled using a plenum considering the radial higher order modes and plotted in Figure 35. The reason for the amplitude difference in higher frequency region is not known.

![Graph showing transmission loss](image1)

**Figure 35. Transmission Loss for Simple Expansion Chamber**

Figure 36 shows the effect of varying chamber diameters on the Transmission Loss of simple expansion Chambers.

![Graph showing effect of diameter](image2)

**Figure 36. Effects of Diameter on Transmission Loss for Expansion Chambers**
Expansion Chamber with Mean flow:

Figure 37 and Figure 38 shows Transmission loss results of a Diameter 200mm expansion chamber compared with BOOST SID Theoretical models with mean flow (M=0.06 & M=0.1) as measured in the inlet pipe of the muffler.

Figure 37. Transmission Loss for Simple Expansion Chamber

Figure 38. Transmission Loss for Simple Expansion Chamber
Two microphone spacings were used to cover a wide frequency and this is the reason for two legends for the measured curves. As can be seen the agreement between the simulation and experiment results is good. When we take a close look at the zero and mean flow results of the diameter 200mm expansion chamber we can see that there is an amplitude difference between the two results, this is because of the difference in the inlet/outlet pipe diameters. For zero flow measurements, the inlet/outlet pipe diameter was 51mm and for all the mean flow measurements, the inlet/outlet pipe diameter was 67mm.

**Attenuation Measurements:**

As explained in chapter 2.2, the transmission Loss of a muffler is not influenced by the length of the inlet and outlet pipes therefore to study and validate the effects of the end corrections which is added as extra lengths to the inlet and outlet pipe, we measured the attenuation of the muffler which is dependent on the outlet pipe length.

The measured attenuation for Diameter 200mm and Diameter 300mm muffler is as shown in **Figure 39** and **Figure 40**.

![Figure 39](image)

**Figure 39**  Attenuation for a simple expansion chamber
Just like Transmission loss measurements, the Radial higher order modes effects are clearly visible above 1400Hz in the Attenuation curve for the 300mm diameter muffler in Figure 40.

![Attenuation of an Expansion Chamber of Diameter 300mm at , M=0 & T=293k](image)

**Figure 40** Attenuation for a simple expansion chamber

The KARAL’s end correction model as in Table 2 for an expansion and contraction was used in all these BOOST SID models. The good agreement between the measured and simulated attenuation results shows that the end corrections applied are correct which in turn shows that the acoustical length of the muffler is correct i.e. the end correction lengths added to the muffler outlet pipes are correct.

From these attenuation results we were able to create a correct acoustical model for a simple expansion chamber and the same technique was used to create acoustical models for other complex mufflers. Even though attenuation measurements were performed for other mufflers during the later stages of the thesis, they became less significant when the muffler were complex.
5.3 Expansion Chambers with Extensions:

The effect of inlet and outlet pipe extensions in a muffler is of great interest therefore the inlet and outlet pipes of the simple expansion chamber of Diameter 200mm discussed in the previous chapter are extended as below.

![Diagram of Expansion Chambers with Extensions](image)

**Figure 41 Possible Expansion Chambers with Extensions**

The acoustic model of the extended muffler is derived from the geometric model as shown in the set of figures in Section 5.2 the only change being the increase of the length of the QW Resonator corresponding to the extended length. The inlet and outlet extensions and the corresponding end corrections added to the inlet and outlet pipe act together as quarter wave resonators as shown in Section 5.2. The resonance frequency of quarter wave resonators is given by

\[ f_{QW} = \frac{n c}{4l}, \quad n = 1, 3, 5, \ldots \]  \hspace{1cm} (5.3)

\[ c = \text{Speed of Sound} \]

\[ l = \text{Effective length of extension} \]

Here the effective length, \( l \) is the extended length + the End Correction

The Transmission loss of the expansion chamber with various inlet and outlet extension combination is measured with and without mean flow and the results are compared with the BOOST SID simulation.
Figure 42 to Figure 46 shows the quarter wave resonator effects created by the extended inlet and outlet pipe on the TL of an expansion chamber.

Figure 42. Transmission Loss of Expansion Chamber with Extensions

Figure 43. Transmission Loss of Expansion Chamber with Extensions
Transmission Loss of an Expansion Chamber of Diameter 200mm at \( M=0 \) & \( T=293k \)

- MEASURED WITH INLET=50mm & OUTLET=150mm EXT
- BOOST SID WITH INLET=50mm & OUTLET=150mm EXT
- MEASURED WITHOUT EXTENSION
- BOOST SID WITHOUT EXTENSION

Figure 44. Transmission Loss of Expansion Chamber with Extensions

Transmission Loss of an Expansion Chamber of Diameter 200mm at \( M=0 \) & \( T=293k \)

- MEASURED WITH INLET=150mm & OUTLET=200mm EXT
- BOOST SID WITH INLET=150mm & OUTLET=200mm EXT
- MEASURED WITHOUT EXTENSION
- BOOST SID WITHOUT EXTENSION

Figure 45. Transmission Loss of Expansion Chamber with Extensions
Figure 46. Transmission Loss of Expansion Chamber with Extensions

The resonance peaks of the QW resonators can be clearly seen in these figures and they perfectly match the theory according to Equation 5.4.

The results in Figure 47 shows the Transmission loss for an expansion chamber of length 500mm with inlet extension 250mm and outlet extension 200mm, it is clear that the two open ends inside the chamber are too close to each other and the acoustical length is even closer.

Figure 47. Transmission Loss of Expansion Chamber with Extensions
This causes a shift in the measured Transmission Loss peaks towards the lower frequency which in turn means that there is an increase in the acoustical length of the quarter wave resonators.

More investigation about these near field effects are carried out in **Section 5.8**.

**Expansion chamber with extensions with mean flow:**

**Figure 48** and **Figure 49** shows the Transmission Loss of expansion chamber with inlet extension 150mm at \( M=0.06 \) and \( M=0.1 \) respectively. It can be seen that the amplitude of the resonance frequency of the quarter wave resonator formed by the inlet extension is damped with mean flow and has a good agreement with the BOOST SID predictions.

![Figure 48. Transmission Loss of Expansion Chamber with Extensions](image-url)
Transmission Loss of an Expansion Chamber of Diameter 200mm at, M=0, M=0.1 & T=293k
INLET EXTENSION 150mm

Figure 49. Transmission Loss of Expansion Chamber with Extensions

Figure 50 shows the effect of an outlet extension of 150mm under mean flow (M=0.06) and it can be seen that the outlet extension with flow has the same damping effect in the resonance peak of the quarter wave resonator but the theoretical predictions fail to have the same damping effect.

To support this point a muffler with both inlet and outlet extensions was built and the transmission loss was measured with (M=0.06) and without mean flow. The results compared with the BOOST SID predictions are shown in Figure 51.

The test chamber had an inlet extension of 150mm and outlet extension of 80mm. We can see from Figure 51 that the resonance peak of the quarter wave resonator formed by the outlet pipe extension has the same damping effect as the inlet pipe extension.
Transmission Loss of an Expansion Chamber of Diameter 200mm at \( M=0 \), \( M=0.06 \) & \( T=293 \)\( ^\circ \)F

**OUTLET EXTENSION 150mm**

- MEASURED \( M=0.06 \)
- MEASURED \( M=0.06 \)
- BOOST SID \( M=0.06 \)
- MEASURED \( M=0 \)
- BOOST SID \( M=0 \)

**Figure 50.** Transmission Loss of Expansion Chamber with Extensions

Transmission Loss of an Expansion Chamber of Diameter 200mm at \( M=0 \), \( M=0.06 \) & \( T=293 \)\( ^\circ \)F

**INLET EXTENSION=150mm & OUTLET EXTENSION 80mm**

- MEASURED \( M=0.06 \)
- BOOST SID \( M=0.06 \)
- MEASURED \( M=0 \)
- BOOST SID \( M=0 \)

**Figure 51.** Transmission Loss of Expansion Chamber with Extensions

A much clearer picture of the difference in the expansion and contraction models in BOOST SID can be obtained from **Figure 52**.
Figure 52 shows a clear picture of an expansion and contraction model with extensions used in BOOST SID which predicts that both these models behave in a different way with mean flow, whereas the measurement result contradicts this point. Therefore it will be of great interest to compare the measurement results with other expansion and contraction models available, one such model is the expansion and contraction model given by Davies [Ref-4 Davies (1988)].

The above result along with the Davies model of an expansion and contraction gives a transmission loss with mean flow (M=0.06 and M=0.1) as shown in Figure 53 and Figure 54 respectively.

We can see that the expansion models from BOOST SID & Davies have a good agreement but the contraction models are much different. But the contraction model of Davies has a very good agreement with the measurement results in both these flow conditions.
Figure 53  Transmission Loss of Expansion Chamber with Extensions

Figure 54  Transmission Loss of Expansion Chamber with Extensions
5.4 Expansion Chambers with Walls and Extensions:

The next configurations tested were mufflers with walls in the middle as shown in the figures below and these mufflers can be made more complex by having extensions in the inlet and outlet side as discussed in the previous chapter.

Figure 55 Possible Expansion chamber with walls and Extensions

Figure 56 shows the Transmission Loss of an expansion chamber of length 500mm and diameter 200mm with a wall thickness of 2mm at 257mm from the inlet side and a hole of diameter 67mm as shown in the figure below.

Figure 56. Transmission Loss of Expansion Chamber with Walls and Extensions
**Figure 57** shows the TL for the expansion chamber as shown below. The muffler had a wall with a hole of diameter 67mm at 257mm from inlet side. The inlet extension was 100mm and outlet extension was 50mm.

![Diagram of expansion chamber](image)

**Figure 58** shows the Transmission Loss of a muffler as shown in the figure below. The inlet and outlet extensions were 200mm each which means that the distance between the pipe openings and the wall was approximately 50mm on both sides. We can see that the open ends of the pipes are too close to the wall for the one-dimensional theory to be applicable.

![Diagram of muffler](image)
The peaks in the measured TL curve has a frequency shift towards the lower frequency because of these near field effects which is also seen in Figure 47. and further investigations are performed in Section 5.8.

Figure 58. Transmission Loss of Expansion Chamber with Walls and Extensions

Figure 59 Shows the Transmission Loss of an expansion chamber with a wall and a pipe as shown in figure at 257mm from inlet side

The pipe in the centre wall had a diameter of 65mm and a length of 200mm out of which 50mm was extending in the first chamber and the remaining 150mm was extending into the second chamber.
Transmission Loss of an Expansion Chamber of Diameter 200mm at \( M=0 \) & \( T=293k \)
WITH WALL AND A PIPE AT 257mm FROM THE INLET SIDE

Figure 59. Transmission Loss of Expansion Chamber with Walls and Extensions

Expansion chamber with walls and extension under mean flow:

Figure 60 and Figure 61 shows the measured Transmission Loss and the BOOST SID prediction of an expansion chamber with a wall with a hole of diameter 67mm at 375mm from the inlet side as shown in figure below with mean flow \((M=0 \text{ and } M=0.06)\) measured in the inlet pipe of diameter 67mm.

It can be seen that the agreement between theory and experimental results is good.
Transmission Loss of an Expansion Chamber of Diameter 200mm at M=0 & T=293k with wall at 375mm from the INLETSIDE

Figure 60. Transmission Loss of Expansion Chamber with Walls and Extensions

Transmission Loss of an Expansion Chamber of Diameter 200mm at M=0.06 & T=293k with wall at 375mm from the INLETSIDE

Figure 61. Transmission Loss of Expansion Chamber with Walls and Extensions
Figure 62 and Figure 63 show the Transmission Loss of an expansion chamber of length 500mm and diameter 200mm with two walls at 125mm and 375mm from the inlet with mean flow (M=0 and M=0.06).

The muffler had an outlet extension of 50mm and the first wall contained a pipe of diameter 65mm and length 100mm. Out of which 18 mm was inside the first chamber and the remaining 82mm was inside the second chamber as shown in figure below. The transmission loss of this muffler configuration is high at higher frequencies and the agreement between the theory and measured result is not good but interestingly the mean flow results in Figure 63 are better than the no flow results in Figure 62.

![Diagram of expansion chamber](image)

**Figure 62. Transmission Loss of Expansion Chamber with Walls and Extensions**
5.5 Mufflers with Flush Eccentric Inlet and Outlet Pipes:

Muffler with flush eccentric inlet pipe or outlet pipe or both is a common case in modern mufflers. In this section mufflers with the Inlet and outlet pipes of diameter 200mm expansion chamber are made flush eccentric to each other as shown in Figure 64 and the Transmission loss was measured.

![Figure 63. Transmission Loss of Expansion Chamber with Walls and Extensions](image)

![Figure 64. Possible Eccentric Inlet and Outlet pipe Muffler Configuration](image)
The BOOST SID model of this flush eccentric chamber was built with the plenum element with and without considering higher order modes as shown below,

![Diagram of Flush Eccentric chamber](image)

**Figure 65** shows the TL of the muffler with and without considering higher order modes in the plenum chamber. As discussed in the previous chapters, the first circumferential mode of a diameter 200mm expansion chamber is excited above 1004Hz which is visible in the measured Transmission Loss. Below this frequency, we have a very good agreement between the measured and BOOST SID predictions.

![Graph of Transmission Loss](image)

**Figure 65.** Transmission Loss of Mufflers with Flush Eccentric Inlet and Outlet
Figure 66 shows the Transmission Loss of the flush eccentric expansion chamber with an inlet extension of 100mm, as shown in figure below,

![Diagram of transmission loss setup](image)

Since the BOOST SID code does not have a model for flush eccentric mufflers with extended inlet/outlet pipes including higher order modes, only concentric models with extensions and considering higher order modes are compared with the measured Transmission loss results.

![Graph of transmission loss](image)

Figure 66. Transmission Loss of Mufflers with Flush Eccentric Inlet and Outlet
**Figure 67** shows the measured Transmission loss and the BOOST SID predictions of a flush eccentric Muffler with inlet and outlet extensions as shown below. The inlet extension was 250mm and outlet extension was 50mm, the same modelling technique used for the previous muffler was used here,

![Diagram of flush eccentric muffler](image)

**Transmission Loss of a Flush Eccentric Chamber of Diameter 200mm at , M=0 & T=293k**

**Inlet Extension 250mm & Outlet Extension 50mm**

From **Figure 66** & **Figure 67**, we can see that the first circumferential mode starts to propagate above the cut on frequency as given in **Table 5**.

The next configuration investigated was a muffler as shown in the figure below. Here both the eccentric inlet and outlet pipes have their openings in the same plane. And when the extension and contraction end corrections are applied, the acoustical openings cross each other. Therefore it is not possible to model this muffler using the concentric element in BOOST SID. Converting this geometric model to a valid BOOST SID model is a bit complicated and is done as shown in the following steps.
Geometric model:

In this BOOST SID model, the higher order mode effects caused by the eccentric pipes are not considered.

Acoustic model:

Pipe 1 --- QW 1 --- Pipe 2 --- QW 2 --- Pipe 3

BOOST SID Model:
Figure 68. Transmission Loss of Mufflers with Flush Eccentric Inlet and Outlet

The BOOST SID prediction compared well with the measured transmission loss as shown in Figure 68. Another muffler which is of interest is when those inlet and outlet pipe extensions are too long so that the openings cross each other as shown in Figure 69. The inlet extension was 350mm and outlet extension was 250mm and the chamber length being 500mm, the two pipe openings crossed each other. The same technique described for the previous muffler was used to build a BOOST SID model.

Figure 69. Flush Eccentric Mufflers
Figure 70 shows the Measured and Predicted Transmission loss of the muffler without considering higher order modes.

Figure 70. Transmission Loss of Mufflers with Flush Eccentric Inlet and Outlet

5.6 Mufflers with Flow Reversal:

Flow reversal is a common case encountered in modern mufflers and modelling these flow reversal chambers is of great interest. Three flow reversal chambers of length 500mm, 375mm and 250 were built as shown in figure and Transmission Loss was measured. Inlet/outlet extensions were made to further validate our reverse flow BOOST SID models.

Figure 71 Possible Reverse Flow Chambers
The BOOST SID models for the long and medium mufflers were created using two quarter wave resonators [Ref-13 Ih(1992)] (1st for the inlet extension / outlet extension / End Correction and 2nd for the remaining length of the chamber) as shown in the figure below,

![Diagram of muffler models](image)

**Figure 72** and **Figure 73** shows the transmission loss of the long and medium mufflers and the comparison looks good till 1000Hz after which the first circumferential mode is excited whereas the higher order modes are not included in the BOOST SID models.

![Transmission Loss Graph](image)

**Figure 72. Transmission Loss of Mufflers with Flow Reversal**
Figure 73. Transmission Loss of Mufflers with Flow Reversal

Figure 74 and Figure 75 shows the long and medium chambers with inlet extension of 257mm and 167mm respectively and the same modelling procedure is followed,

Figure 74. Transmission Loss of Mufflers with Flow Reversal
Double Flow Reversal:

The next configuration investigated was a muffler with double flow reversal as shown in figure below.

Figure 75. Transmission Loss of Mufflers with Flow Reversal

Figure 76 Muffler with Double Flow Reversal
As in reverse flow chambers the whole chamber where flow reversal takes place is modeled as a quarter wave resonator. Therefore the BOOST SID model of a double reversal chamber just looks like two independent flow reversal chambers in cascade. The acoustic model is as shown in figure

The BOOST SID model contains 2 QW resonators for the two chambers and 2 QW resonators for the expansion and contraction end corrections.

**Figure 77** Shows the Measured and BOOST SID predicted Transmission Loss for the muffler. As usual the higher order modes are not considered in this BOOST SID model therefore the comparison of results below 800Hz looks agreeable.
Figure 77. Transmission Loss of Mufflers with Flush Eccentric Inlet and Outlet

The short chamber with length 250mm gives the result as in Figure 78. And various possibilities of modelling a short reverse flow chamber is shown in the figure and it is better to consider it as a straight pipe instead of a quarter wave resonator if the frequency of interest is below the first cutoff frequency (800Hz).

Figure 78. Transmission Loss of Mufflers with Flow Reversal
5.7 Mufflers with Helmholtz Resonators:

Helmholtz resonator is a very important component in silencing systems. These resonators are very effective in the low frequency region.

The resonance frequency of a Helmholtz resonator is given by the equation

\[
f_{\text{Helm}} = \frac{c}{2\pi} \sqrt{\frac{S}{V l_{\text{eff}}}}
\]

5.4

- \( c \) = Speed of Sound
- \( S \) = Area of the Helmholtz neck
- \( V \) = Volume of the Helmholtz resonator
- \( l_{\text{eff}} \) = Effective length of the neck

The end corrections for the neck tube and neck cavity of the Helmholtz resonator are given as in Table 3. Four Helmholtz resonators of different neck lengths (2mm, 37mm, 100mm and 200mm) were created as shown in Figure 79 and the transmission Loss was measured.

Figure 79 Helmholtz Resonator
The above geometric model is converted into an acoustic model with the Helmholtz resonator as shown below:

![Helmholtz Resonator Diagram]

The corresponding BOOST SID model is as shown below:

![BOOST SID Model Diagram]

The Helmholtz resonator had an effective volume of 7 liters. Figures 80, Figure 81, Figure 82 and Figure 83 shows TL results for the respective neck lengths.

Apart from the neck lengths, the position of the neck in a Helmholtz resonator was also investigated. For example, for a muffler in Figure 79 we would expect a QW resonator to be formed because of the extended neck inside the Helmholtz resonator but no such effects is seen.

Whereas in Figure 82 the muffler investigated had the neck extension outside the Helmholtz resonator (as shown in the geometric model above) and the effect of the Quarter wave resonator can be seen around 1050Hz. What is more important is the end corrections added to the neck on both ends.
Transmission Loss of an Expansion Chamber of Diameter 200mm at M=0 & T=293k
With a Helmholtz Resonator of Volume 7 Litres and Neck Length 2mm

\[ \text{Measured} \quad \text{BOOST SID} \]

Figure 80. Transmission Loss of Muffler with Helmholtz Resonator

Transmission Loss of an Expansion Chamber of Diameter 200mm at M=0 & T=293k
With a Helmholtz Resonator of Volume 7 Litres and Neck Length 37mm

\[ \text{Measured} \quad \text{BOOST SID} \]

Figure 81. Transmission Loss of Muffler with Helmholtz Resonator
Transmission Loss of an Expansion Chamber of Diameter 200mm at $M=0$ & $T=293\,\text{k}$
With a Helmholtz Resonator of Volume 7 Litres and Neck Length 100mm

Figure 82. Transmission Loss of Muffler with Helmholtz Resonator

Transmission Loss of an Expansion Chamber of Diameter 200mm at $M=0$ & $T=293\,\text{k}$
With a Helmholtz Resonator of Volume 7 Litres and Neck Length 200mm

Figure 83. Transmission Loss of Muffler with Helmholtz Resonator
5.8 Mufflers with Bended Extensions:

Another frequent feature in modern day mufflers are the pipes with bends inside a muffler as shown in Figure 84,

![Bended Pipe in a Muffler](image)

The acoustic model is derived as shown in the figure below. We can see that the actual length of the bended pipe is greater than the length of the quarter wave it is creating.

Therefore two BOOST SID models with different quarter wave resonator lengths were created and the measurement results are compared as below,

Pipe length inside the muffler + EC = 200 + 14.7mm
Quarter wave length + EC = 200 + 14.7mm (Resonance freq = 400Hz)

**Figure 85** shows the BOOST SID model with a QW length equal to the bended pipe length and we can see that the prediction curve is shifted towards left indicating that the length of the QW resonator is too long.
Transmission Loss of an Expansion Chamber of Diameter 200mm at , M=0 & T=293k
Bended Outlet with Extension 200mm

---

**Figure 85** Transmission Loss of Mufflers with Bended Extensions

*Figure 86* shows the TL of the BOOST SID model with a shorter QW length as shown in the acoustic model and now the measured and the BOOST SID predictions have a good frequency match.

Quarter wave length + EC=169+14.7mm (Resonance freq=467Hz)

---

**Figure 86.** Transmission Loss of Mufflers with Bended Extensions
Then the effects of having this bended pipe close to the wall as shown in figure are studied.

![Bended Pipe in a Muffler (Close to the wall)](image)

Figure 87  Bended Pipe in a Muffler (Close to the wall)

We can see in Figure 88 that there is a small shift in the resonance frequency of the quarter wave resonator formed due to the extension, which shows that the acoustical length of the bended pipe changes when the open end is close to a wall whose effect is not accounted when modelling in BOOST SID.

![Transmission Loss of Mufflers with Bended Extensions](image)

Figure 88. Transmission Loss of Mufflers with Bended Extensions
Figure 89. Transmission Loss of Mufflers with Bended Extensions

We can see that the measured TL peak is shifted towards the lower frequency (measured=455Hz, simulation=467Hz) indicating that the acoustic length of the quarter wave is long when the opening is close to a wall. The pattern in which the shift occurs is not known which lead us to perform measurements to calculate the correct acoustic length (End corrections) when the pipe openings are close to a plate as shown in Figure 90.
End correction measurements were performed as described in Section 2.3.2 by placing a plate at various distances from the open end and the measured end corrections as a function of frequency are as shown in Figure 91.

![Figure 91: End Correction Measurement](image)

We can see that when the plate is moved closer to the open end, the end correction which has to be applied to the open end pipe increases which in turn increases the acoustical length, but BOOST SID uses an end correction which is not sufficient when the pipe is close to a wall thereby creating a frequency shift towards the higher frequency in the predicted models.

When the distance between the plate and the wall is less than 10mm, then the new end correction is a frequency dependent (decreasing with the increase in frequency) but when the distance is more than 10mm (which is more commonly found in mufflers because of the flow restrictions) the new end correction is not a frequency component anymore, therefore one possibility to account this near field effects is to add the extra length to the end corrections.
5.9 Expansion Chambers with Horn:

A muffler with a horn at one end was created and the Transmission Loss was measured. A horn is a pipe with a different start and end diameter as shown in the Figure 92.

The horn is modeled by splitting the length of the pipe into several segments and then considering each segment as a straight pipe. The number of segments is given as an input during acoustic modelling in BOOST SID.
The BOOST SID model for the horn is created using a pipe of a length equal to the horn length and its start and end diameters and the number of segments and the BOOST SID models is as shown in figure.

The predicted Transmission Loss from the BOOST SID model compared to the measured is as shown in Figure 93.

Figure 93  Transmission Loss of Expansion Chambers with Horn
Horn with Extensions:

The Final muffler configuration investigated in this thesis was a muffler with horn but with an extension as shown in figure below,

For a normal horn with an extension, a conical resonator is formed between the extended pipe and the horn wall. Whereas in our case the resonator formed is not a perfect cone or a perfect tube but a combination of both as shown in figure below,

Therefore the resonator formed in this case has a geometry as shown below

A normal tube resonator has a resonance frequency as
A conical resonator has a resonance frequency as

\[ f_{\text{conical}} = \frac{c}{2l} \]

Two BOOST SID model were created using these two resonators and the measured Transmission Loss is compared in Figure 94.

![Graph showing transmission loss comparison](image)

**Figure 94** Transmission Loss of Expansion Chambers with Horn and Extension

We can see that measured resonance frequency is in between the two resonance frequency of the resonators, a small adjustment is done in the geometry as shown in the figure.
A conical resonator of the length as given in the figure below gives the Transmission Loss prediction as in Figure 95 compared with the measured result.

\[ f_{\text{Conical}} = \frac{c}{2l} \]

Figure 95  Transmission Loss of Expansion Chambers with Horn and Extension
6. Conclusions:

A number of muffler configurations starting from simple expansion chambers to complex geometry have been investigated with and without the mean flow effects.

1. When validating the acoustic models created using a linear acoustic tool transmission loss measurement is not sufficient as it is independent of the inlet and outlet pipe lengths which make it difficult to study the effect of the end corrections. Sometimes other properties like attenuation, reflection coefficient has to be measured and compared with the predictions.

2. 1-D wave theory applies for Mufflers having concentric inlet and outlet pipes till the first radial higher order mode is excited as in equation 2.7.

For mufflers having eccentric Inlet and Outlet pipes, the circumferential higher order modes are excited above the first cut on frequency as in equation 2.6

3. Reverse Flow Mufflers: The above mentioned condition applies for flow reverse mufflers since the inlet or outlet or both the pipes are flush eccentric to the chamber and the predicted transmission loss result depends on the length of the reverse chamber.

4. Higher the Transmission Loss, the more difficult it becomes to measure. During this thesis some of the mufflers had a very high Transmission Loss (nearly 100dB) and the measurement results was not good.

5. The expansion model used in BOOST SID includes the flow related losses whereas the contraction model does not include these losses which can be seen from the mean flow measurements in Section 5.3.

6. BOOST SID does not include flow inside the Quarter Wave resonators. Since the mufflers investigated in the later part of the thesis is entirely constructed using QW resonators the BOOST SID predictions with and without flow will yield the same TL result which might not agree with the measurements. For example the influence of flow on the flow reversal and helmholtz muffler are inconclusive from our investigations.
7. When the acoustical openings inside a muffler are too close to each other or close to walls then acoustical length of the elements change therefore measurements were performed to find the end corrections when the open end is close to a wall.

8. Secondary or Internal sources cannot be used in BOOST SID. In a complex muffler when the flow velocities are high then the flow inside the muffler generates a considerable amount of noise which alters the quality of the measurement results.
7. References:

5. Z.TAO AND A.F.SEBERT, A Review of Current techniques for Measuring Muffler Transmission Loss, University of Kentucky, USA.
8. Appendix -

8.1 BOOST SID

8.1.1 Theory:

BOOST-SID linear acoustics is a frequency domain solver incorporated with AVL BOOST to determine the acoustic properties of a system. A BOOST SID model is made up of a source, a system and a termination. The term ‘system’ includes everything in between the source and the termination no matter how complex or how many elements are used in the modelling.

![Diagram of a linear acoustic system]

For application to engine exhaust or inlet systems the source is the engine, the system is the exhaust and the termination is the tailpipe ambient.

![Diagram of a basic linear acoustic system]

After the calculation of the transfer matrices, pressures and volume flows for each element in the model in the system the following main result are available:
• Transmission Loss
• Noise Reduction
• Insertion Loss (If a reference system is defined)
• Reflection Coefficient (at each element in the model)

8.1.2 Transfer Matrix Method:

The BOOST-SID code just like The KTH –SID code uses the transfer matrix method. This uses the relationship between the two pairs of state variables (vectors), \( x \) and \( y \), coupled by a two-dimensional matrix, \( T \):

\[
x = Ty
\]

This is the 4-pole method

\( x \) inlet side
\( y \) outlet side

In linear acoustics, the state variables are the temporal Fourier transforms of sound pressure \( p_1 \) and sound volume velocity \( q_1 \) at the inlet (=x) and the acoustic pressure \( p_2 \) and volume flow velocity \( q_2 \) at the outlet (=y) side and \( T \) is called the Transfer Matrix.

Provided certain assumptions, there exists a complex 2*2 matrix \( T \), which completely describes the sound transmission within a system at certain frequency \( f \):

\[
\begin{bmatrix}
  p_1 \\ q_1
\end{bmatrix} =
\begin{bmatrix}
  T_{11} & T_{12} \\ T_{21} & T_{22}
\end{bmatrix}
\begin{bmatrix}
  p_2 \\ q_2
\end{bmatrix}
\]

The sound field here is assumed to be linear, so the following variables can be separated into two parts:

\[
\begin{align*}
P &= p_o + p, & \quad n &= V_o + u, \\
\rho &= \rho_0 + \rho_{fl}, & \quad T &= T_o + \tau, \\
S &= S_0 + s_E,
\end{align*}
\]
Where the first term on the right hand side is the stationary mean value and the second term is the small disturbance. Only plane waves can propagate implies that the sound pressure is constant over the cross section.

Using the Transfer Matrix method an actual system can be modeled as shown below.

\[
\begin{align*}
Z_s & \quad q_1 \\
p_s & \quad P_1 \\
T_{11} & \quad T_{12} \\
T_{21} & \quad T_{22} \\
q_2 & \quad P_2 \\
\end{align*}
\]

**Active one-port**  **Two-port**  **Passive one-port**

**Linear Acoustic Model**

$Z_s$ and $p_s$ are the internal source impedance and source strength sound pressure $Z_T$ is the termination impedance. The system itself can be split up into basic linear acoustic elements and then solved.
8.1.3 Summary of Elements:

According to the linear acoustic theory [Ref 6-Glav (1994)], the transfer matrix of the basic elements is as given below,

8.1.3.1 Straight Ducts:

A straight duct is the simplest and the most common element encountered automobile exhaust systems, this element is use to connect all other elements in the system. They are usually straight or slightly curved with constant cross section and wall thickness of 1 to 2mm. For curved pipes, the influence of bend is neglected in the frequency range 30 to 2000 Hz.

For a hard walled pipe of length \( l \) the transfer matrix can be defined as

\[
t_{11} = \frac{1}{2} \{ \exp(ik_{+}l) + \exp(-ik_{-}l) \},
\]
\[
t_{12} = \frac{\rho_{0}c'}{2S} \{ \exp(ik_{+}l) - \exp(-ik_{-}l) \},
\]
\[
t_{21} = \frac{S}{2\rho_{0}c'} \{ \exp(ik_{+}l) - \exp(-ik_{-}l) \},
\]
\[
t_{22} = \frac{1}{2} \{ \exp(ik_{+}l) + \exp(-ik_{-}l) \},
\]

8.1.3.2 Area discontinuities:

The expansion and contraction models proposed by Glav [Ref 6 Glav (1994)], treats the extended lengths caused by the end correction at these area discontinuities as lumped parameters . The transfer matrix of the expansion and contraction elements are obtained by \( T=MxE \) and \( T=CxM \) respectively and the end corrections transfer matrix \( M \) in terms of lumped parameters is a given below,
\[ M = \begin{bmatrix}
1 & \frac{i\omega \rho \Delta_c}{\pi \cdot a^2} \\
0 & 1
\end{bmatrix} \]

### 8.1.3.3 Contraction:

For an area change as shown in the figure below, the transfer matrix of the contraction model taken from [Ref-6 Glav (1994)] is as follows:

\[ c_{11} = 1 - \frac{M_1 \cdot \rho \cdot c}{z \cdot S_1 \cdot (1 - M_1^2)} \]

\[ c_{12} = \rho \cdot c \cdot \left[ \frac{S_1}{S_2} - \frac{S_2}{S_1} \right] \cdot M_1 - \frac{\rho \cdot c \cdot M_1^2}{z \cdot S_2} \cdot \frac{1}{S_2 \cdot (1 - M_1^2)} \]

\[ c_{21} = \frac{1}{z \cdot (1 - M_1^2)} \]

\[ c_{22} = \left[ 1 - \left( M_1^2 + \frac{\rho \cdot c \cdot M_1}{z \cdot S_1} \cdot \left( \frac{S_1}{S_2} \right)^2 \right) \right] \cdot \frac{1}{(1 - M_1^2)} \]

And when the end correction is added in the form of lumped impedance, the transfer matrix \((T=CxM)\) of this area contraction element is given by
\[ t_{11} = 1 - \frac{M_1 \cdot \rho_0 \cdot c}{z \cdot S_1 \cdot (1 - M_1^2)} \]

\[ t_{12} = \frac{\rho_0 \cdot c}{S_2 \cdot (1 - M_1^2)} \left[ \left( \frac{S_1}{S_2} - \frac{S_2}{S_1} \right) \cdot M_1 - \frac{\rho_0 \cdot c \cdot M_1^2}{z \cdot S_2} \right] + m_{12} \cdot \left[ 1 - \frac{M_1 \cdot \rho_0 \cdot c}{z \cdot S_1 \cdot (1 - M_1^2)} \right] \]

\[ t_{21} = \frac{1}{z \cdot (1 - M_1^2)} \]

\[ t_{22} = \left[ 1 - \left( M_1^2 + \frac{\rho_0 \cdot c \cdot M_1}{z \cdot S_1} \right) \cdot \left( \frac{S_1}{S_2} \right)^2 \right] \cdot \frac{1}{(1 - M_1^2)} + m_{12} \cdot \left[ \frac{1}{z \cdot (1 - M_1^2)} \right] \]

8.1.3.4 Expansion:

For an area expansion as shown in figure below, the turbulent losses for expanding mean flow are considered and the expansion models is as given below.
\[ e_{12} = \frac{2 \cdot \rho_0 c M_1}{S_2} \cdot \left( \frac{S_1}{S_2} - 1 \right) \cdot \frac{1}{\det} \]

\[ e_{21} = \left[ 1 + \gamma \left( \frac{M_1 S_1}{S_2} \right)^2 \right] \cdot \frac{1}{z \cdot \det} \]

\[ e_{22} = \left\{ 1 + \left( \frac{S_1}{S_2} \right) \cdot \left[ 1 - 2 \cdot \frac{S_1}{S_2} \right] \cdot M_1^2 + \frac{2 \cdot \rho_0 c M_1}{z \cdot S_1} \cdot \left( \frac{S_1}{S_2} \right) \right\} \cdot \frac{1}{\det} \]

And when the end correction is added in the form of lumped impedance, the transfer matrix \( (T=Mx_E) \) of this area expansion element is given by

\[ t_{11} = \frac{1 + \left[ \lambda \cdot \left( \frac{S_1}{S_2} \right)^2 - 1 \right] \cdot M_1^2 + \frac{m_{12}}{z} \cdot \left[ 1 + \gamma \left( \frac{M_1 S_1}{S_2} \right)^2 \right]}{\det} \]

\[ t_{12} = \frac{2 \cdot \rho_0 c M_1 \cdot \left( \frac{S_1}{S_2} - 1 \right) + m_{12} \cdot \left[ 1 + \left( \frac{S_1}{S_2} \right) \cdot \left( 1 - \frac{2 \cdot S_1}{S_2} \right) \cdot M_1^2 + \frac{2 \cdot \rho_0 c M_1}{S_2^2 \cdot z} \cdot \left( \frac{S_1}{S_2} \right) \right]}{\det} \]

\[ t_{21} = \frac{1 + \gamma \left( \frac{M_1 S_1}{S_2} \right)^2}{z \cdot \det} \]

\[ t_{22} = \frac{1 + \left( \frac{S_1}{S_2} \right) \cdot \left[ 1 - 2 \cdot \frac{S_1}{S_2} \right] \cdot M_1^2 + \frac{2 \cdot \rho_0 c M_1}{z \cdot S_1} \cdot \left( \frac{S_1}{S_2} \right)}{\det} \]

Where determinant \( \det \) is given by
\[
\text{det} = 1 + M_1^2 \cdot \left[ \gamma - 1 + (\gamma - 1) \cdot \left( \frac{S_1}{S_2} \right)^2 + (1 - 2\gamma) \cdot \frac{S_1}{S_2} \right] + \frac{2 \cdot \rho_0 c M_1}{S_2 \cdot z}
\]

### 8.1.3.5 Resonators:

The transfer matrix of the resonators (Helmholtz, Quarter Wave, Conical) is given by [Ref-6 Glav (1994)] as

\[
t_{11} = 1
\]

\[
t_{12} = 0
\]

\[
t_{21} = \frac{S_r}{Z_r}
\]

\[
t_{22} = 1
\]

Where \( Z_r \) the impedance of the resonator as seen from the flow is duct and \( S_r \) is the area of the resonator mouth. The impedances \( Z_r \) for the three different resonators used in this thesis given by [Ref-6 Glav (1994)] are as follows

\[
Z_{hr} = R_{tu} + R_v + i \omega \rho_0 (l_n + \Delta_i + \Delta_o) - \frac{i \rho_0 c^2 S_n}{\omega V},
\]

\[
Z_{qr} = \rho_0 c \left( \frac{\exp(ik'l) + \Re \exp(-ik'l)}{\exp(ik'l) - \Re \exp(-ik'l)} \right),
\]

\[
Z_{cr} = \frac{i \rho_0 c}{\cot(kl) - (1/kl)},
\]
8.1.3.6 Flush Eccentric Chambers:

The transfer matrix of an expansion chamber with an offset inlet and outlet pipe is as follows. The higher order acoustic modes arising at the area discontinuities are included in the models,

\[
\begin{align*}
  t_{11} &= \exp\left[-\frac{jkMl}{(1-M^2)}\right] \cos\left[\frac{kl}{(1-M^2)}\right] \\
  t_{12} &= jZ \exp\left[-\frac{jkMl}{(1-M^2)}\right] \sin\left[\frac{kl}{(1-M^2)}\right] + M^2 \sin\left[\frac{kl}{(1-M^2)}\right] + \ldots \\
  t_{21} &= \frac{j}{Z} \exp\left[-\frac{jkMl}{(1-M^2)}\right] \sin\left[\frac{kl}{(1-M^2)}\right] \\
  t_{22} &= \exp\left[-\frac{jkMl}{(1-M^2)}\right] \cos\left[\frac{kl}{(1-M^2)}\right]
\end{align*}
\]

8.1.3.7 Concentric Chambers:

The transfer matrix of an expansion chamber with inlet and outlet extension including the higher order mode propagation is much more complex than the flush eccentric chambers. Therefore a detailed derivation and formulation of the transfer matrix for this element used in BOOST SID is given in Reference 25.