Beam-to-Beam Contact and Its Application to Micromechanical Simulation of Fiber Networks

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Abstract

This doctoral thesis covers the topic of modeling the three-dimensional fiber networks with the finite element method. It contains the part addressing the numerical aspects of the modeling, namely, the contact formulation and application of the developed methods to the fundamental questions such as the effect of randomness in fiber properties and effect of fines and hygroexpansion.

In the approached used in the work, the fibers were meshed with beam elements and the bond between fibers is modeled using point-wise beam-to-beam contact. Contact between beam elements is a specific category of contact problems, which was introduced by Wriggers and Zavarise in 1997 for normal contact [1] and later extended by Zavarise and Wriggers to include tangential and frictional contact [2]. These formulations encompass a large number of derivations and provide the consistent tangent matrix. We showed, however, the resulting numerical implementations based on these consistent formulations are not sufficiently robust in modeling random fiber networks with a large number of contacts. In the first papers, we proposed a simpler non-consistent formulation, which turned out to be superior in terms of convergence stability with respect to the load step size for a wide range of loading cases. Having these advantages, it remained equally accurate as the original formulation. The first paper covered the formulation of normal and tangential contact, and the second paper contains two formulations with both the consistent and non-consistent linearizations for in-plane rotational contact of beams.

We use the developed formulations to address fundamental problems within the area of fiber networks, which cannot be solved purely with experimental tools. In the third article, we investigated the effect of fiber and bond strength variations on the tensile stiffness and strength of fiber networks and concluded that in cases of skewed distribution, using mean values for fiber and bond properties instead of the distributions is not always adequate to assess the changes these properties have on the average mechanical characteristics of the entire network.

In the fourth paper, the mechanisms behind the improvement of stiffness and strength after PFI refining in the papermaking process is investigated. The PFI refiner is very popular for studying the effect of refining in the lab scale. By using a combination of experimental and numerical tools, we found that density, which is often mentioned as the main reason behind the improvement of mechanical properties after PFI refining, cannot solely explain the degree of the change observed experimentally. We concluded the remaining part of the improvement is caused by the fibrillar fines,
in particular, by the fines that cannot be detected with modern automated fiber characterization tools due to the limited resolution of such tools. Finally, in the fifth paper, we suggested a multi-scale model to study hygroexpansion/shrinkage properties of paper. Due to the anisotropy of the fibers, the stress transfer at the bonded sites has a dominant role in the behavior of paper when exposed to moisture change. While we modeled the bonds between fibers using point-wise contact elements, such stress transfer requires a finite contact area. To solve this limitation and yet preserve the advantages for using beams for modeling fiber networks, we developed a concurrent multi-scale approach. In this approach, the bond model is resolved for every bond in the network, and the exchange between the network and bond model is maintained through the current configuration of the fibers being passed to the bond scale, and the inelastic strains being transferred back to the network scale. We demonstrated the effectiveness of such approach by comparing it with a full-scale continuum model. Using this approach, we were able to complete the existing experimental observation with key insights using the advantage of having unlimited access to the details of the network at each stage of the deformation.


Sammanfattning

Denna doktorsavhandling behandlar modellering av tredimensionella fibernätverk med finita element metod. Dels studeras numeriska aspekter av modelleringen, framförallt formuleringen av kontakten mellan fibrer, och dels tillämpningar av de utvecklade metoderna på fundamentala frågor som effekten av spridning i fiberegenskaper, inverkan av finmaterial och hygroexpansion.


Vi använde de utvecklade formuleringarna för att studera fundamentala problem inom fibernätverk som inte kan lösas med rent experimentella metoder. I den tredje artikeln undersöks effekten av variationer i fiber- och fogstyrka på dragstyvhet och nätverksstyvhet och slutsatsen kan dras att för skeva fogstyrkfördelning leder användandet av medelvärdet för fibrers och fogars styrkeegenskaper istället för hela fördelningen inte till en representativ beskrivning av nätverket.

I den fjärde artikeln undersöks mekanismerna bakom förbättringar i styvhet och styrka efter PFI-malning, vilket är en vanlig metod för att studera effekterna av malning på laboratorieskala. Genom användning av en kombination av experimentella och numeriska metoder fanns att en ökning i densitet, som ofta beskrivs som en av de huvudsakliga anledningarna bakom förbättringar av de mekaniska egenskaper efter PFI-malning, inte ensamt kan förklara de förändringar som observeras experimentellt. Slutsatsen är därför att den återstående delen av förbättringen orsakas av


Preface

The presented work has been carried out in the Department of Solid Mechanics, School of Engineering Sciences, Royal Institute of Technology (KTH), Stockholm Sweden, between June 2012 and March 2018 and was funded by the following organizations:

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Finally, I would like to express my sincerest gratitude to my beloved wife, Sahar, for her endless support. It would not be possible without you. Thank you for being a great friend and fantastic partner. I love you and want you to know that you and Adrian are everything to me.

List of appended papers

**Paper A**: On the use of consistent and non-consistent beam-to-beam contact formulations in implicit finite element analyses with large deformations  
Hamid Reza Motamedian and Artem Kulachnko  
*Report TRITA-SCI-RAP 2018:003, KTH Engineering Sciences, Royal Institute of Technology, Stockholm, Sweden*  
Submitted for publication

**Paper B**: In-plane Rotational Contact of Beams in 3D Space  
Hamid Reza Motamedian and Artem Kulachnko  
*Report 590, Department of Solid Mechanics, KTH Engineering Sciences, Royal Institute of Technology, Stockholm, Sweden*  
Submitted for publication

**Paper C**: Effect of Fiber and Bond Strength Variations in the Tensile Stiffness and Strength of Fiber Networks  
Svetlana Borodulina, Hamid Reza Motamedian and Artem Kulachnko  
*International Journal of Solids and Structures, 2016*

**Paper D**: Mechanisms of Strength and Stiffness Improvement of Paper after PFI Refining with a Focus on the Effect of Fines  
Hamid Reza Motamedian, Armin Halilovic and Artem Kulachnko  
*Report TRITA-SCI-RAP 2018:004, KTH Engineering Sciences, Royal Institute of Technology, Stockholm, Sweden*  
Submitted for publication

**Paper E**: Simulating the Hygroexpansion of Paper Using a 3D Beam Network Model and Concurrent Multiscale Approach  
Hamid Reza Motamedian and Artem Kulachnko  
*Report TRITA-SCI-RAP 2018:005, KTH Engineering Sciences, Royal Institute of Technology, Stockholm, Sweden*  
Submitted for publication
Authors’ Contribution

The author’s contributions to the appended papers are as follows:

Paper A: Principal author. Performed all mathematical derivations, implementations and getting results for the provided examples. Artem Kulachenko reviewed the article.

Paper B: Principal author. Performed all mathematical derivations, implementations and getting results for the provided examples. Artem Kulachenko helped with results for the last example.

Paper C: Contributed to the text by writing the parts related to network generation, beam-to-beam contact. Wrote the code to create the geometry. Extracted data and graphs from geometry. Also part of discussions of the results. Svetlana Borodulina was responsible for writing the article. She also gathered simulation outputs and interpreted them with help of other authors and together with Artem Kulachenko was in charge of simulations. Artem Kulachenko reviewed the article.

Paper D: Principal author. Performed all simulations, gathered and interpreted results. Armin Halilovic performed experiments and extracted data micro tomography images. Artem Kulachenko planned and reviewed the article.

Paper E: Principal author. Performed all mathematical derivations, implementations, simulations, and gathered and interpreted results. Artem Kulachenko reviewed the article.
Other Contributions

In addition to the appended papers, the work has resulted in the following publications and presentations:

**Network Generation for Fiber-Based Paper Network Simulation**
Hamid Reza Motamedian and Artem Kulachnko
Presented at *European Forest Product Doctoral Symposium*, Helsinki, Finland, 2013

**Beam-to-Beam Contact with Rotational Friction/Adhesion**
Hamid Reza Motamedian and Artem Kulachnko
Presented at *11th World Congress on Computational Mechanics (WCCM XI)*, Barcelona, Spain, 2014

**Beam-to-Beam Normal, Tangential and Rotational Contact**
Hamid Reza Motamedian and Artem Kulachnko

**A Robust Algorithm for Normal and Tangential Contact of Beams in 3D Space**
Hamid Reza Motamedian and Artem Kulachnko
Presented at *Svenska Mekanikdagar 2015 (SMD 2015)*, Linköping, Sweden 2015

**In-plane Rotational Contact of Beams in 3D Space**
Hamid Reza Motamedian and Artem Kulachnko
Presented at *12th World Congress on Computational Mechanics (WCCM XII)*, Seoul, 2016

**Effects of Variation in Morphological Properties of Fibers and Bond Strength on the Tensile Stiffness and Strength of Fiber Networks**
Svetlana Vorodulina, Hamid Reza Motamedian and Artem Kulachnko

**Modeling the Hygroexpansion of Paper Using a 3D Network Model**
Hamid Reza Motamedian and Artem Kulachnko
Presented at *Euromech Colloquium 592 – Deformation and Damage Mechanism of Wood-Fibre Network Materials and Structures*, Stockholm, Sweden, 2017
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**Paper C**
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Introduction

With computers gaining more processing power, it is possible now to analyze complex and outstanding scientific problems using advanced numerical tools. Among these complex problems in material mechanics, there are many that involve contact between two or more bodies. Contact problems get more challenging when large deformations or friction is a part of the problem. The focus of the thesis is micromechanics of fiber networks in which we use beam finite elements in describing individual fibers in contact. Handling contact between beam elements undergoing large deformations in 3D space is a relatively new subject in the field of finite element analysis and although well-established algorithms exist for surface-to-surface contact problems, beam-to-beam contact formulation still presents room for improvements and new formulations are developed to overcome shortcomings in accordance with the intended usage of the formulation.

Some examples of usage of beam-to-beam contact formulation include analyses of space structures, robotic arms in motion, springs, ropes consisting of multiple strings, cables, knots, woven fabrics, foam ligaments undergoing crushing, and random networks such as paper.

In the first part of the work, we review the existing formulations for normal, tangential and in-plane rotational contact of beams and present a simple yet more robust approach for these cases.

In the second part of the thesis, we used the developed formulation to address practical problems, namely, assessing the impact of fiber and bond variability, effect of fines and hygroexpansion.
Motivation

We used beam contact algorithms in studying the micromechanics of large three-dimensional networks of fibers and solving problems related to paper and packaging materials. Paper has been produced for more than 2000 years and has been used for different purposes e.g. writing, printing, decorations, packaging, cleaning, building, etc. Today paper can be produced at very high speeds of over 100 km/h and optimization of the production process from points of view of raw material usage and energy consumption are of main concerns, especially due to constantly growing environmental demands. Despite the long history of existence and usage of paper, our knowledge about the micromechanical processes in paper are still limited to certain extent. These limitations are mainly because of the experimental difficulties in operating and observing the paper constituents on the micro-scale and below. In addition, the effect of some properties cannot be investigated independently as it is not easy/possible to modify some properties without affecting others.

Among the nagging problems that require the information from the microscale is the effect of variability of different characteristics of raw materials, which is tightly connected to the problems of furnish optimization. Furthermore, the mechanisms behind processes like refining that improve the mechanical properties of produced paper are not completely clear. Another example of a problem is the accumulation of dried-in strains in paper during production, which are released upon moisture cycles and cause stability problems. Finally, probably the most difficult of all is the problem of paper breaks, which was shown to be connected to disordered nature of paper on the micro-scale.

Using simulations on the microlevel, enables us to gain more knowledge about the contribution of factors such as bond properties, fiber properties, fiber alignments etc, to the performance of paper product in the end-use. This motivation brought us to the problem of resolving many contacts between the fibers represented as a series of beam elements. By using numerical modeling, we can conduct controlled numerical experiments, study size effects, probe things which are inaccessible by the conventional experimental tools or continuum modeling. A reconstructed micro-CT scan of paper and numerically generated fiber network are shown in Figure (1a).

The use of commercial software to do the simulations is practically impossible due to a number of potential exceptions in a system with so many randomly oriented fibers having millions of contacts. Finding these exceptions and ensuring the stability during solution required robust pre-processing algorithms and solutions procedures.
Figure 1: (a) Micro-CT scan of a paper network, and (b) a top view of a numerically generated random network and (c) the side view of the same network.

The majority of existing contact formulations presented in the literature were rather complex for implementation and their robustness for modeling complex networks of interconnected fibers was never proven. Furthermore, some formulations were only available for describing normal and tangential contact between beams, ignoring the rotational contact constraints. Overcoming the complexity of formulation and making it robust have been one of the primary focus in this work. Figure (2), shows as an example of what can be achieved with the numerical tools; the displacement field mapped on the deformed fibers in the network subjected to tensile loading.

Figure 2: the displacement field mapped on the deformed fibers in the network subjected to tensile loading.
Previous Work on Beam-to-Beam Contact

Beam-to-beam contact is a relatively new branch of computational contact mechanics. A formulation for point-wise normal contact of beams elements with circular cross-sections was first suggested in 1997 by Wriggers and Zavarise [1]. In 2000, Zavarise and Wriggers presented a formulation for tangential contact of beams including friction [2]. Later Litewka and Wriggers applied the formulations to beams with rectangular cross-sections [3] by changing the contact search algorithm. Afterwards, Litewka implemented the formulations using penalty stiffness and Lagrange methods [4] and suggested smoothing techniques for a frictional contact when the contact point moves along the contacting beams from one element to another [5, 6, 7]. Recently, this formulation was extended by allowing the beams to have multiple contact points [8].

In all the above-mentioned formulations, the contact is assumed to happen point-wise and rotational constraints and moments at the contact point are ignored. Also to derive equations for the tangent stiffness matrices of normal and tangential contacts, linearization techniques are utilized to relate the the incremental change of the strain energy of the system and its variation to the incremental change of the gap function and its variation and subsequently to the incremental change of the nodal displacement vector and its variation. Due to complexity of derivations, some limitations have to be assumed for shape functions of the beam elements, e.g. to derive the tangential contact equations, linear shape functions are assumed for the geometry and deformations of beam elements.

Applications

After developing the formulation for beam-to-beam contact, it was used in studying different phenomena related to paper. We approach paper from the micromechanical stand-point, resolving the individual fibers in 3D and interaction between them. The geometry of the network plays an essential role in defining the mechanical properties. We reconstruct the geometry using a fiber deposition technique. In this approach, single fibers are generated based on the data gathered from the characterization of wet pulp and corrected using micro-tomography technique. Single fibers are deposited one by one first on a base plane and then over previously laid fibers until the desired grammage of the network is reached. When a fiber is put in the network, its intersections with previously deposited ones are found and the geometry of fiber
is modified so that it passes over the previous fibers in a smooth manner. More
details on this procedure can be found in paper E.

After generation of the network geometry, it must be discretized, and we do so
with help of 3-node Reissner-Timoshenko beam elements. The bond between fibers
are modeled using the presented point-wise beam-to-beam contact elements.

Three different phenomena are studied using the developed tool. In the first
work presented in Paper C, we studied the effect of variability in the fiber and bond
strength properties and how it affects the properties of the network. Afterwards,
we investigated the effect of refining on the strength and stiffness of paper. The
study consisted of experimental and simulation parts. We captured some changes
in the pulp after refining and observed improvements in the mechanical proper-
ties of the handsheets made from refined pulp. Using numerical simulations, we
looked at potential reasons and mechanism for these improvements and concluded
that the formation of fines is a major factor. Finally, in the last paper, we studied
the hygroexpansion in paper. The change of moisture causes anisotropic expan-
sion/shrinkage in the longitudinal and transverse direction of fibers and as a result,
the stress transfer between fibers at the bonded sites becomes a critical component.
As we modeled the inter-fiber bonds with point-wise contacts, the model was not
readily capable of resolving the bond-level transverse shrinkage. To overcome this
problem, we suggested a concurrent multi-scale simulation scheme based on the orig-
inal fiber network model and enhanced with a detailed model of individual bonds,
which are solved concurrently with the network-level deformations with appropriate
data exchange. A short summary of the papers can be found in the next section.

Summary of Appended Papers

Paper A: On the Use of Consistent and Non-consistent Beam-to-
Beam Contact Formulations in Implicit Finite Element Analyses
with Large Deformations

In the first paper, we presented a simple and robust non-consistent formation for the
beam-to-beam contact and compared its performance against the existing formul-
ations. It is significantly simpler however limited to the penalty method, which is, on
the other hand, a very popular method for handling contact problems. We demon-
strated the advantages over the previously presented formulations in a number of
benchmarks, where apart from the simplicity and ability to include higher order
shape functions, we showed the better convergence and stability of the formulation for both normal and tangential contact. The basic difference between the presented method and the previous ones is that the location of the contact point and the directions of the contact normal and tangents are assumed to remain constant at a given iteration. On this assumption, there is no need to linearize the orthogonality conditions and to predict the change in contact location. We demonstrated that this assumption is not critical for the accuracy of the analysis. A sample load case studied in this paper is shown in Figure (3a), and the performance of consistent and non-consistent formulations in handling this example for a bonded contact can be seen in Figure (3b).

The main advantages of the proposed non-consistent algorithm in comparison with the similar consistent ones is as follows:

- Shorter solution time, specially with larger load steps
- Possibility to use higher order shape functions
- Possibility to use larger values for penalty stiffness
- Smoothness and predictability of change in the number of iterations upon increased load step size

The disadvantage is mainly slower convergence with either small frictional sliding or very tight convergence tolerance.

**Paper B: In-plane Rotational Contact of Beams in 3D Space**

In the second paper, we presented the weak formulation for both the Lagrange multiplier and penalty stiffness methods. The Lagrange multiplier and penalty stiffness are two standard methods, successfully used with FEM for contact analysis. Next, we suggested two alternative formulations for the in-plane rotational contact of beams. In the first method, with the assumption of linear shape functions for the beams, an in-plane rotational gap was defined and using the variational methods and by linearizing the nonlinear equations, the tangent stiffness matrix was derived for both Lagrange multiplier and penalty stiffness methods. Afterwards, in the second method, using a different approach and assuming that the contact normal is constant (independent of deformations) at each iteration, while it is updated between iterations, we presented an alternative, simpler formulation with the penalty
Figure 3: (a) A sample initial geometry, boundary conditions and loading from paper A, and (b) comparison of number of iterations needed for convergence for consistent and inconsistent formulations.

stiffness method. We showed the efficiency of the formulations by solving several examples and demonstrated the effect of including the in-plane rotational contact into simulation of a network of randomly oriented, interconnected fibers (Figure 4).
### Paper C: Effect of fiber and bond strength variations on the tensile stiffness and strength of fiber networks

It is generally accepted that the tensile strength of paper can be described by mean properties of its constituents, namely fibers and bonds \[?\]. In the next paper, we addressed the question related to the effect of variability of the fiber and bond properties on the average strength and stiffness of fiber networks. To do so, we generated...
a set of reference networks using a deposition technique and verified the consistency of the geometry using micro-tomography data from physical samples. Afterwards, we modified the distribution of certain fiber and bond parameters while keeping the other network generation parameters unchanged. These parameters include fiber length, diameter, wall thickness, shape factor (curvature), and bond strength. We found that while using the length-weighted average instead of the distribution data can be sufficient for fiber diameter and wall-thickness when evaluating their contribution to mechanical properties of the sheets, it is not the case for fiber length and shape factor, see Figure (5a). When it came to bond strength, we concluded that using an average value suffices only if the distribution is symmetric (Figure(5b)).

Figure 5: Effect of (a) the distribution of fiber properties, and (b) bond strength distribution on the average network properties.

**Paper D: Mechanisms of Strength and Stiffness Improvement of Paper after PFI Refining with a Focus on the Effect of Fines**

In the fourth paper, we looked at refining (also known as beating) in papermaking process which is used to improve the mechanical properties of the final product. We conducted measurements and experiments on isotropic handsheet from pulp refined at different levels. We observed that refining results in higher density, more stiff and stronger networks. In fact, in literature density is suggested to be the main reason for increased strength and stiffness [9] on the macro-scale. On micro-scale the most important effects of refining are internal fibrillation [10, 11, 12, 13], external fibrillation [12, 14, 15, 16, 17, 18] and formation of fines [19, 11, 20, 21, 22] where the last two are closely related [19, 23]. All these factors are known to
increase bonding properties which in turn leads to increased density after refining. We studied the effect of density on mechanical properties of refined paper using numerical simulations and found out that although increased density is a major factor for mechanical improvements, it cannot be responsible for whole modifications. In Figure (6) the effect of increased density captured through numerical simulations is compared to the mechanical improvements seen in the experiments.

![Stress-Strain Curve](image.png)

**Figure 6:** Effect of density on simulated stress-strain curves in comparison with experimental results. Bars show the maximum and minimum of the strength results from 10 experimental or 5 numerical samples for each density and the curves represent the averages.

Then we studied two other factors suggested in the literature, namely bond strength and creation of fines as possible mechanisms. We concluded that stronger bonds lead to stronger paper but do not affect the stiffness. We also noticed that the mere existence of fines, even fines with low strength and stiffness, has a large contribution to both stiffness and strength. And the effect of fines with smaller diameters which are not detectable by characterization devices has the utmost importance. Figure (7) shows the effect of fine diameter (for constant mass percentage of fines) on the strength and stiffness of a sample network.

Finally we demonstrated that the combined effect of increased density and fines can explain the improvements due to refining as shown in Figure (8).

**Paper E: Simulating the Hygroexpansion of Paper Using a 3D Beam Network Model and Concurrent Multiscale Approach**

In the last paper, we simulated the effect of hygroexpansion/shrinkage in paper. A number of dimensional stability problems experienced by paper products, such
as curl, fluting or cockling are attributed to hygroexpansion [24]. Fibers in paper have anisotropic response to a change of moisture, with a much larger coefficient of hygroexpansion in the transverse direction, compared to the longitudinal one. As a result, an important part of hygroexpansion/shrinkage phenomena in paper is attributed to the stress transfer between longitudinal and transverse directions of fibers at the bonded sites.

Some numerical models have been used for relating the hygroexpansion of single wood fibers with the properties of fiber’s constituents [25, 26, 27]. The contribution of inter-fiber bonds to hygroexpansion and dimensional stability of paper has also been investigated using the combination of analytical and numerical tools on the continuum level [28, 29]. Some analytical approaches were also used for connecting the hygroexpansion properties of paper to the properties of fibers and bonds combined [30, 25, 31]. Only a few attempts are done in order to use numerical simu-
lations accounting for more of the randomness nature of the sheet in the calculations [32, 33, 34, 35, 36]. In these simulations still many simplifying assumptions such as ignoring the 3D nature of the networks and using 2D models are made.

In our simulations of fiber networks, the inter-fiber bonds are modeled with the beam-to-beam contact presented in the first two papers and is treated as a contact point. In order to account for the stress transfer in the bonds which requires a contact area instead of a point, we used a concurrent multi-scale model. We suggested 1D and 2D models for a single bond. This model is resolved for every bond in the network by transferring required geometrical and mechanical data from the network scale to this bond scale model. The results are then transferred back to the network in terms of inelastic strains. A sample of results that can be extracted from our simulations is shown in Figure (9). These results correspond to drying of a network under uniaxial constraints.

![Figure 9](image-url)  
**Figure 9:** Computed (a) total deformation, (b) out-of-plane deformation of the network, (c) equivalent stress, and (d) plastic strains in the fibers of a sample network
Using a robust 2D bond model and 3D fiber network we were able to capture experimental results previously reported. For example, simulation results shown in Figure (10) correspond to experimental results on the deformation of bonded and free segments of fibers in a network during drying under various boundary conditions [37]. Another example shown in Figure(11) is a comparison of experimental and numerical results on how the hygroexpansion coefficient of a handsheet varies with its stiffness if the stiffness is modified either through increased pressing pressure or using debonding agents during production [38].

**Figure 10:** Average shrinkage of sheet, fibers, and bonded and free segments of fibers during drying under different constraints.

**Figure 11:** Variation of hygroexpansion coefficient versus specific elastic modulus of an isotropic handsheet

The set of presented methodologies paves the way to modeling various phenomena related to mechanics of fiber networks. The use of beam elements in combina-
tion with beam-to-beam contact is a computationally effective approach which gives possibilities to include all the relevant mechanisms on the network scale such as plasticity, frictional interactions, debonding and hygroexpansion. We demonstrated that accuracy of the method for calculating mesoscale properties of reasonably large and dense networks. We proposed a method for overcoming the inability to resolve the areal contact using beam elements, which is required for modeling hygroexpansion. It combines the beam model with a subscale model for representing the bond between the fibers in a concurrent multiscale approach. Along with fiber-level characterization tools, the presented computational framework can be used in the material development and solving the outstanding problems in paper mechanics by providing the insights inaccessible by conventional experimental and analytical approaches.

**Future Work**

There are various possible extensions of the work which can be undertaken from this point. They include considering the applicability of the model for compressive strength studies, looking at the effect of the network parameters on the strength distribution and long-term failures, studying the dimensional stability of paper products associated with moisture changes and spatial variability of the network characteristics.
References


Appended Papers