Determining Scattering Source Parameters from Radiation Detector Excitation Patterns

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Abstract

We deduce a simple equation that describes the irradiation pattern of radiation detectors along the beam pipe due to a localized source from which a beam is scattered.

Beam particles in charged particle accelerators sometimes scatter off of other material in their path due to Rutherford scattering on the atomic nuclei of the material. The source is often residual gas from imperfect vacuum conditions, an internal target for subatomic physics experiments, of other unwanted material that accidentally appears in the beam path, such as the Unidentified Falling Objects (UFOs) in LHC [1, 2]. In any case, it is desirable to determine the location and the thickness of the material that appears in the beam path. The longitudinal loss pattern of the scattered protons was already addressed in Figure 5.14 in ref. [3]. Here we calculate the corresponding distribution from a very simple model.

In this note we assume that some material that acts as a scattering source is localized at one distinct location and that it is sufficiently thick to warrant the approximation that the angular distribution of the scattered beam particles is Gaussian and given by

\[ \frac{dN}{d\phi} = \frac{1}{\sqrt{2\pi}\sigma} e^{-\phi^2/2\sigma^2} \]  \hspace{1cm} (1)

where \( \phi \) is the angle at which the particle is scattered and \( \sigma \) the root-mean squared angle of the distribution of scattering angles. It is related to the target thickness \( x \) by [4]

\[ \sigma = \frac{13.6 \text{MeV}z}{\beta pc} \sqrt{\frac{x}{X_0}} \]  \hspace{1cm} (2)

where \( z \) and \( p \) are the charge and momentum of the beam particle, respectively. \( X_0 \) is the radiation length of the source material. Moreover, we assume that there
Figure 1: The geometry with scattering source on the left from where a particles emanate with a Gaussian distribution that are recorded by detectors along the beam pipe.

is a number of detectors for ionization radiation placed along the beam pipe that has radius $R$, as is shown in Fig. 1.

The signals $S(z) = dN/dz$ of a detector at location $z$ downstream of the scattering source is given by the number of scattered beam particles $dN/dz$ hitting the detector per unit length. It is easily calculated by observing that the beam-pipe radius $R$, distance $s$ and the angle $\phi$ are related by

$$\tan \phi = R/z$$

which allows us to transform the scattering angles $\phi$ to the location $z$ where they impact the beam pipe, because there is a one-to-one relationship between the two. For the number of particles impacting at $z$ we have to collect all particles that scatter with angles given by Equation 3

$$\frac{dN}{dz} = \int \frac{dN}{d\phi} \delta(z - R/\tan \phi) d\phi$$

where the delta function takes care of the collecting. Next, we transform the argument of the delta function by using $\delta(g(x)) = \sum \delta(x - x_0)/|g'(x_0)|.$ Here $x_0$ are the zeros of $g(x)$ and $g'(x_0)$ are the derivatives of $g(x)$, evaluated at the zeros. For physical reasons, we constrain ourselves to the zeros between 0 and $\pi/2$, which lie in the upper half-plane of the forward direction of the beam-propagation. That is where we assume the detectors are located. Performing these manipulations on the delta function in Equation 4 leads to

$$\delta(z - R/\tan \phi) = \frac{\delta(\phi - \arctan(R/z))}{|d(R/\tan \phi)/d\phi|} = \frac{R}{\sin^2 \phi} \delta(\phi - \arctan(R/z))$$

and inserting into Equation 4 we obtain

$$\frac{dN}{dz} = \frac{1}{R} \int \frac{dN}{d\phi} \delta(\phi - \arctan(R/z)) \sin^2(\phi) d\phi .$$
After inserting $dN/d\phi$ from Equation 1 and performing the integral which is trivial, thanks to the delta function, we find
\[ \frac{dN}{dz} = \frac{1}{\sqrt{2\pi\sigma R}} e^{-\frac{\arctan(R/z)^2}{2\sigma^2}} \sin^2(\arctan(R/z)) \]
\[ \text{and, after some simplification the resulting equation is} \]
\[ \frac{dN}{dz} = \frac{1}{\sqrt{2\pi\sigma R^2 + z^2}} e^{-\frac{\arctan(R/z)^2}{2\sigma^2}}. \]

Normally, the scattering angles $\phi$ and their rms $\sigma$ are rather small, in the range of milliradians, which allows us to simplify by assuming $R \ll z$ and this results in the expression
\[ \frac{dN}{dz} \approx \frac{R}{\sqrt{2\pi\sigma z^2}} e^{-R^2/2\sigma^2 z^2}. \]

After introducing the abbreviation $y = \sqrt{2\sigma z}/R$ we can write the distribution of scattering products along the beam pipe as
\[ \frac{dN}{dz} \approx \left( \sqrt{\frac{2}{\pi R}} \right) \frac{e^{-1/y^2}}{y^2}. \]
Figure 3: The radiation pattern on detectors spaced by 5 m from a scattering center located at $z = 30$ m with a rms scattering width $\sigma = 10^{-3}$ radians. We also added 5% noise on the data points and the red curve shows a fit to the data points.

with one parameter describing the amplitude of the signal and the universal function $g(y) = e^{-1/y^2}/y^2$ describing the shape of the distribution of scattering products.

Experimentally determining the location of the scattering source and the rms scattering angle $\sigma$ from the profile we note that the curve in Figure 2 has its maximum at $y = 1$ which implies that the source point is at $\Delta y = 1$ before the peak. This leads us to the equation $\Delta y = 1 = \sqrt{2}\sigma \Delta z/R$ or

$$\Delta z = \frac{R}{\sqrt{2}\sigma}$$  \hspace{1cm} (11)

before the location of the peak, but we still need to determine $\sigma$ in order to be able to evaluate this. To this end we note that the universal curve $g(y)$ assumes the value 1/2 at $y_1 = 0.611$ and $y_2 = 2.076$ such that the full width at half maximum is given by $y_2 - y_1 = 1.465$ and we obtain $1.465 = \sqrt{2}\sigma (z_2 - z_1)/R$ or after solving for $\sigma$

$$\sigma = \frac{1.465R}{\sqrt{2}(z_2 - z_1)} \approx \frac{R}{z_2 - z_1}$$  \hspace{1cm} (12)
where $z_1$ and $z_2$ are the longitudinal positions where the signal assumes half its peak value.

The experimental determination of the source point and scattering width $\sigma$ are straightforward: First determine the longitudinal position of the peak and the two locations where the signal assumes half the value at the peak. Then calculate the scattering width $\sigma$ from Equation 12 and the position where the scattering center is located which is $\Delta z$ as given by Equation 11 before the peak. Of course, it is easy to improve this by a non-linear fit and using the approximate values as starting values for the fitting routine. Figure 3 shows an example where we have radiation detectors spaced at 5 m intervals and added 5% noise on the measurement values. First we calculated the fit parameters approximately and then added a nonlinear fit to refine the fit values. We see that the data are fitted quite well. In this case the location of the scattering source is $z = 30$ m, $\sigma = 10^{-3}$, and $R = 0.05$ m.

The method to determine target position and thickness presented in this note depends on a number of radiation detectors spaced along the beam pipe. The spacing need not be even. A varying radial distance of the detectors to the beam center can be accounted for by using $z_i/R_i$ for different detectors, labeled by index $i$, as independent variable in the analysis. A serious difficulty, however, are magnets between source point and detectors. The additional deflections need to be accounted for, provided they are large and complicate the analysis significantly, because tracking of the scattering produces is required.

References


