

# AC cables strengthening railway low frequency AC power supply systems – a deepened study

Lars Abrahamsson  
Electric Power Engineering  
Luleå University of Technology  
931 87 Skellefteå, Sweden  
Email: lars.abrahamsson@ltu.se

**Abstract**—In railway power supply systems using AC frequencies lower than the public grids of 50/60 Hz, high voltage AC overhead transmission lines in railway grid frequency are used as one measure of strengthening the systems. An increased resistance to overhead high voltage AC transmission lines, may motivate cables for future railway power systems.

With the frequency of 50/60 Hz, reactive power produced in lowly utilized cables imposes an obstacle. For low frequency AC, this issue is less significant. Moreover, in converter-fed railways, no reactive power will leak into the feeding public grid.

This paper studies AC cables in low-frequency AC railway. Two reinforcement cable solutions are compared with no reinforcement. A simplified load model of trains, with thyristor bridges and DC motors, is used.

## I. INTRODUCTION

For a more sustainable transport sector, the share and amount of railway traffic needs to increase, since rail is the most energy-efficient land-based means of transportation [1], and electrical motors are more energy efficient than combustion engines [1]. Thus, one can expect railway power grid strengthening to become inevitable world-wide. There are many ways of strengthening a converter-fed railway power system – most of them briefly mentioned in Section IV-D. The interest for converter feeding of railways also using public grid frequencies has increased lately [2], [3], some benefits of doing so are treated in [4]. This paper studies AC cables as a means of strengthening the system.

AC cables are in use in the Tokyo sections of the Tohoku and Tokaido Shinkansen [5]. They are expensive but do on the other hand require little land and space use. In Sweden on some places 15 kV cables are used, and to a smaller extent (9 km) also 132 kV cables [6].

The excess of reactive power production in lowly utilized AC cables is less significant with a lower AC frequency, which some railway systems use. Converters feeding (such) railways can absorb this reactive surplus, leaving the feeding public grid unaffected.

In relation to a previous study [7], a more robust and reliable computational approach (c.f. Section II-C) has been used, facilitating the determination of loadability limits and more studied cases. Moreover, a thorough comparison of resulting voltages to voltage level standards [8], [9] has been made in this paper.

TABLE I: Line, cable, and transformer data

	$R, \left[\frac{\Omega}{\text{km}}\right]$	$X, \left[\frac{\Omega}{\text{km}}\right]$	$C, \left[\frac{\mu F}{\text{km}}\right]$
BT catenary (120 mm <sup>2</sup> , 2A)	0.2	0.2 j	0
15 kV cable	0.12	0.054 j	0.16
132 kV cable	0.1009	0.026 j	0.16
	$R, [\Omega]$	$X, [\Omega]$	—
16 MVA transformer	0.065	0.85 j	
25 MVA transformer	0.037	0.54 j	

## II. PROBLEM SETUPS, MODEL, AND SOLUTIONS

### A. Problem setup

The distance between the pair of converter stations involved in the study is set to 100 km.

The reinforcement cable is connected to the catenary at three equidistant points, as illustrated in Figs. 2 and 3. Three intermediate transformers is representative for the Swedish national railway power system [7].

Both converter stations have four activated rotary converters of the Q48/Q49 type [10], shown in Figs. 1 to 3.

### B. Models

The system studied is a typical low-frequency AC railway system fed by rotary converters and BT (booster transformer) [11] catenaries. The line, transformer, and cable data are listed in Table I. Line and cable data are taken from [6]. Transformer data are representative, and in line with [12].

In the numerical simulations, p.u. has been used with base voltage  $U_b^{15 \text{ kV}}$  set to 16.5 kV, base voltage  $U_b^{132 \text{ kV}}$  set to 132 kV, and base power  $S_b$  set to 5 MVA. Train loads  $\lambda$  are equal and set to  $k \cdot 0.85 + |k| \cdot 0.15j$  p.u. to in a simplified manner represent thyristor-based trains with DC motors [13, Chapter 6-3],[10], [14]. The scalar  $k$  denotes a load multiplication factor.

The rotary converter models and the load flow equations for low frequency AC railways used are the same as in [7].

In this study, over and under voltage protection in trains, converters, or transformers are not considered.

For simplicity, and conservative results, the electrical line models in this study all represent single-track railway.

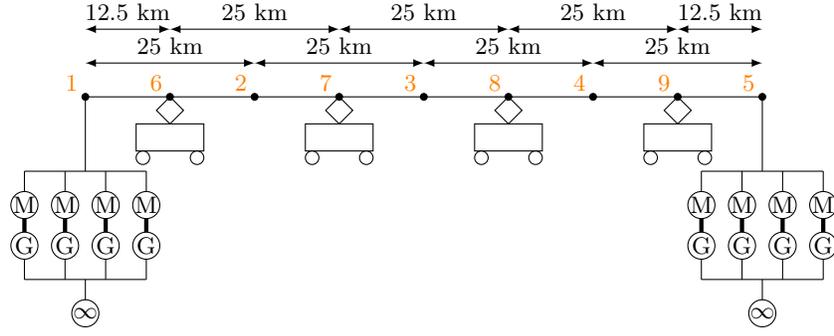


Fig. 1: Illustration of the system studied for the pure BT case. Node numbers in orange.

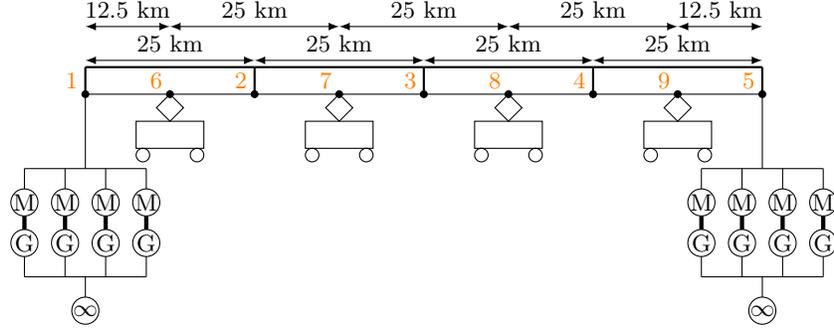


Fig. 2: Illustration of the system studied for the 15 kV cable case. Node numbers in orange. Thick lines denote cables.

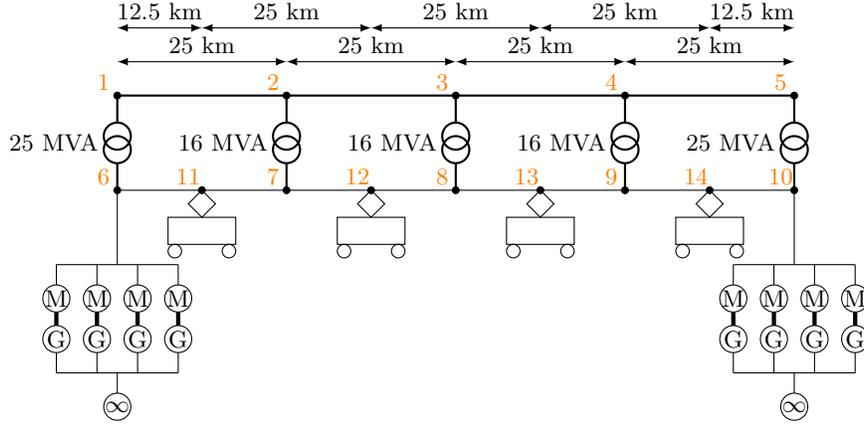


Fig. 3: Illustration of the system studied for the 132 kV cable case. Node numbers in orange. Thick lines denote cables.

### C. Solution approaches

The loadability limits in terms of the extreme values of the load factor  $k$ ,  $k_{\max}$  and  $k_{\min}$ , are determined by

$$\max_{x,k} k, \quad \text{subject to} \quad f(x, k \cdot \lambda) = 0, \quad (1)$$

$$\min_{x,k} k, \quad \text{subject to} \quad f(x, k \cdot \lambda) = 0, \quad (2)$$

where  $f$  represents the power flow and frequency converter equations, and  $x$  the state variables.

Thereafter a selection of cases

$$\min_{x_j, s_i^+, s_i^-} \sum_i s_i^+ + s_i^-, \quad \begin{cases} f_i(x_j, k \cdot \lambda) + s_i^+ - s_i^- = 0 \\ s_i^+, s_i^- \geq 0 \end{cases} \quad (3)$$

where  $s^+$  and  $s^-$  represent positive and negative slack respectively, and where  $k \in \{\mathbb{Z} | k_{\min} < k < k_{\max}\}$ , are solved. The solver LINDO [15] is used to solve both Eqs. (1) and (2) as well as Eq. (3) as NLPs [15]. In [7],  $f(x, k \cdot \lambda) = 0$  was solved by IPOPT [15] as an CNS [15].

### III. RESULTS

#### A. General remarks

1) *Voltages*: All the numerical results from [7] except for the case with a pure BT catenary system and load factor  $k = 2$  have been verified. The  $k_{\max}$  for that system turned, using Eq. (1), out to be 1.35, confer Figs. 4 and 5.

In the standard [8], two under voltage levels are defined for 15 kV AC railways:  $U_{\min,1}$  at 12 kV, which is allowed during less than 2 minutes; and  $U_{\min,2}$  at 11 kV. In addition, three over voltage levels are defined:  $U_{\max,1}$  at 17.25 kV, not to be exceeded more than 5 minutes;  $U_{\max,2}$  at 18 kV, except in Sweden where it is 17.5 kV, here denoted  $U_{\max,2,SE}$ , which can be exceeded up to 1 s; and  $U_{\max,3}$  at 24.3 kV that can be violated less than 20 ms.

Another standard, [9], stipulates that the train tractive effort should be limited linearly down to zero from  $a \cdot U_n = 0.95 \cdot 15 \text{ kV} = 14.25 \text{ kV}$  down to  $U_{\min,2}$ .

Loadability is not necessarily the same as transferability: the largest regenerative braking is possible in the 15 kV cable of Fig. 2. The transfer losses are so high that the converters will need to regenerate less for a given regenerative braking, than for the 132 kV cable of Fig. 3. The largest converter regeneration however takes place in the latter.

Common for all cases is that the voltage levels in the system increases until they reach a point where they start to decrease for increased levels of regeneration until the system eventually collapses. The change in behaviour occurs in different nodes at different load factors  $k$ .

2) *Powers*: In the legends of Figs. 5, 7 and 10, the denotations  $P_G$ ,  $Q_G$ ,  $S_G$ ,  $P_L$ ,  $P_D$ ,  $Q_D$ ,  $Q_{G,cab}$  are used to represent the total active power injected to the railway by the converters, the total reactive power injected, the total converter apparent power, the power transfer losses, the total active power loads, the total reactive loads, and the total reactive power produced in the cables.

#### B. The base case, BT (booster transformer) catenaries

1) *Voltages*: This system is a bit too weak for the load sizes and densities of this study. This can be seen in Fig. 4. Already for  $k = 1$ , full tractive effort will not be allowed for the middle train pair [9]. For  $k = k_{\max} \approx 1.35$  the middle trains should have stopped obeying [9].

Similarly, for  $k = -1$ , the mid trains exceed  $U_{\max,2}$ , whereas the outer trains only exceed  $U_{\max,1}$ . For  $k = -2$  the outer trains' voltages are slightly below  $U_{\max,2}$ . For  $k = -3$  all trains exceed  $U_{\max,2}$ . For load factor  $k = -6$  the outer trains are below  $U_{\max,2,SE}$  and for  $k = k_{\min}$  their voltage levels look fine, with the system close to collapse.

The highest obtained inner train voltages are for  $k = -5$ , the highest outer train voltages are for  $k = -4$ .

2) *Powers*: Powers are shown in Fig. 5. The maximal apparent power for  $k < 0$  is 20.7 p.u., and 7.7 p.u. for  $k > 0$ . The minimal active power generation for regenerating trains (of the studied cases of  $k$ ) is -10.2 p.u. (for  $k = -6$ ),

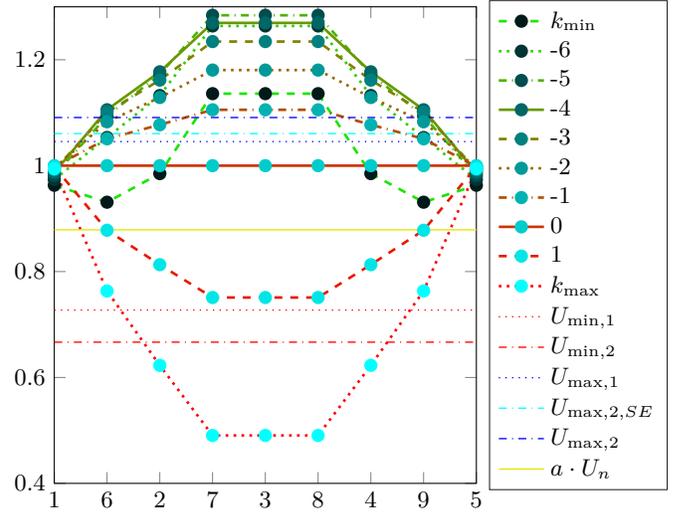


Fig. 4: Voltages [p.u.] plotted against node numbers together with voltage limits, in the pure BT system.

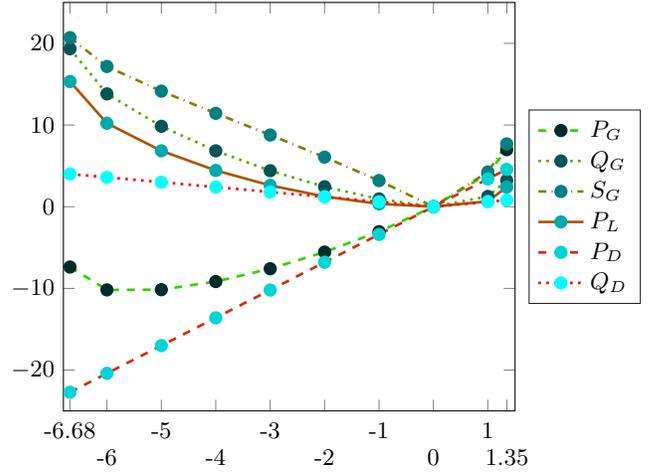


Fig. 5: Various powers [p.u.] plotted against load scaling factors, for the pure BT system.

whereas the amount is -7.4 p.u. for  $k = k_{\min}$ . The maximal active power generation is 7.0 p.u.

The apparent power follows  $|k|$  almost linearly until the last few feasible steps of increasing  $|k|$ .

#### C. The compromise case, 15 kV feeder cable

1) *Voltages*: In this system, Fig. 6, no voltage limit is violated for  $k = \{0, 1\}$ . For  $k = 2$ , the inner trains are affected by the  $a \cdot U_n$  limit, and for  $k = \{3, k_{\max}\}$  the outer trains are affected by the  $a \cdot U_n$  limit, whereas the inner trains have voltages far below  $U_{\min,2}$ .

For  $k = -1$ , the inner pair of trains are above  $U_{\max,1}$ , but the outer trains are fine. For  $k = -2$ , the inner trains have voltages above  $U_{\max,2}$  and the outer ones lie more or less on  $U_{\max,2,SE}$ . For  $-4 \leq k \leq -18$  all train voltages lie above  $U_{\max,2}$ , and for  $k = k_{\min}$  the outer trains lie between  $U_{\max,2}$  and  $U_{\max,2,SE}$ , but the system is unstable.

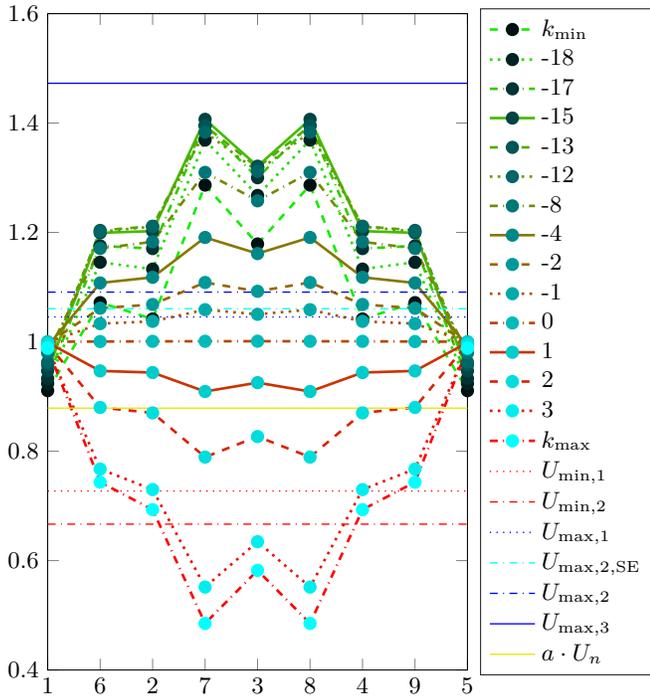


Fig. 6: Voltages [p.u.] plotted against node numbers together with voltage limits, in the 15 kV cable system.

The voltages for the middle trains are the highest at  $k = -15$ , and for the outer trains at  $k = -13$ .

2) *Powers*: Powers are shown in Fig. 7. The maximal apparent converter power for  $k < 0$  is 49.4 p.u., and 17.7 p.u. for  $k > 0$ . The smallest simulated active power converted for regenerating trains is -25.1 p.u., for  $k = -15$ , whereas the amount is -15.0 p.u. for  $k = k_{\min}$ . The maximal active power generation is 16.5 p.u.

Comparatively low cable voltage, and the low AC frequency, makes the impact of  $Q_{G,\text{cab}}$  negligible. One can notice a slight tendency of concavity for "moderate" levels of  $k < 0$  for  $S_G$  in Fig. 7.

#### D. The proposed case, 132 kV feeder cable

1) *15 kV subsystem voltages*: The 132 kV cable offers, as expected, and as can be seen in Fig. 8, a quite strong system. No voltage thresholds are overridden for  $k = \{0, \dots, 3\}$ . For  $k = 5$  all trains are in the zone of reduced tractive effort. For  $k = 6$  the inner trains are below  $U_{\min,1}$ . Finally, for  $k = k_{\max}$  the outer trains are below  $U_{\min,1}$  and the inner ones below  $U_{\min,2}$ .

For  $k = -1$ ,  $U_{\max,1}$  is exceeded for the pair of inner trains. For  $k = \{-3, -6, -8\}$  the voltages lie between  $U_{\max,2}$  and  $U_{\max,2,\text{SE}}$  for all the trains. For  $k = -11$  the outer train voltages are above  $U_{\max,2,\text{SE}}$  but only above  $U_{\max,1}$  for the inner trains – an indication of a stressed system. For  $k = -13$  only the outer pair of trains still exceed  $U_{\max,1}$ . At the 16 MVA transformer connection points, confer Fig. 3, the voltages are lower than  $a \cdot U_n$ .

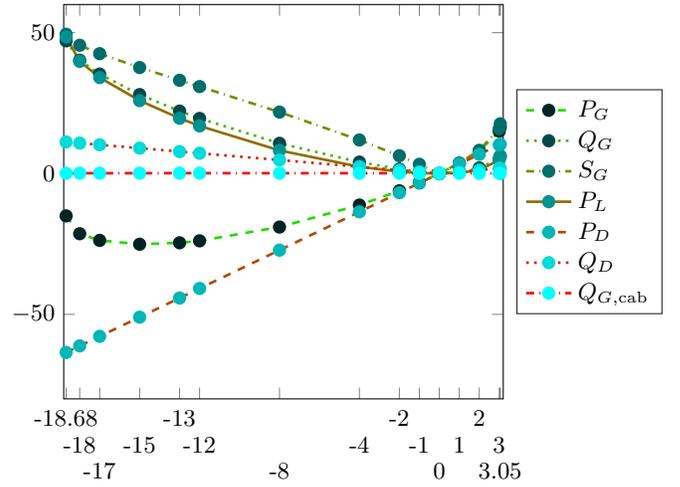


Fig. 7: Various powers [p.u.] plotted against load scaling factors, for the BT system with attached 15 kV cables.

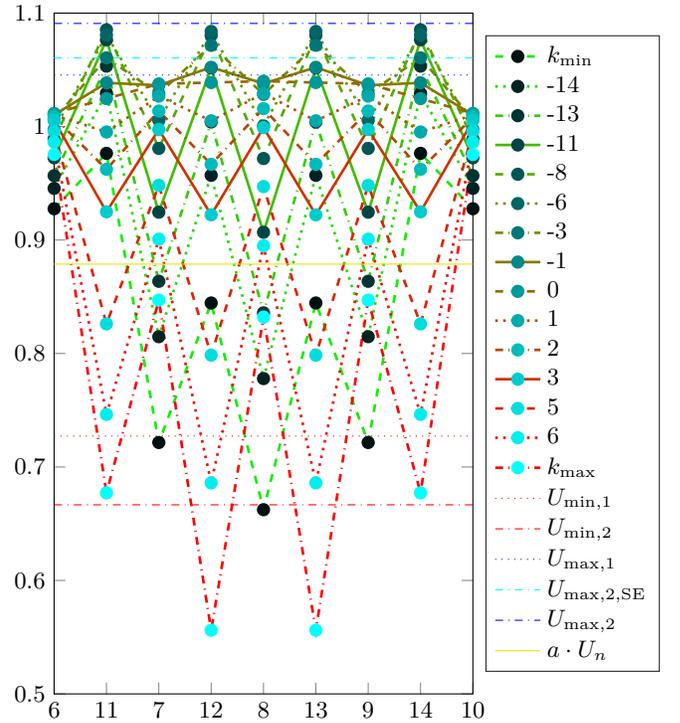


Fig. 8: Voltages [p.u.] in the 15 kV part of the grid plotted against node numbers together with voltage limits stipulated in standards, in the system with 132 kV cables.

The  $a \cdot U_n$  threshold does however not make any practical sense for regenerating trains. These are signs of an even more stressed system. For  $k \leq -14$  the trend worsens until reaching the loadability limit.

The pair of inner trains maximize their voltages for  $k = -6$ , whereas the outer pair does it for  $k = -8$ .

2) *132 kV subsystem voltages*: The voltages on the high-voltage part of the grid are presented in Fig. 9. Voltages

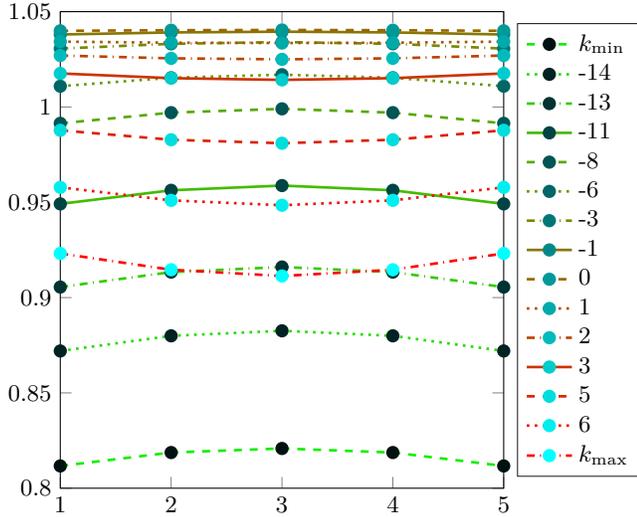


Fig. 9: Voltages [p.u.] in the 132 kV part of the grid plotted against node numbers, in the system with 132 kV cables.

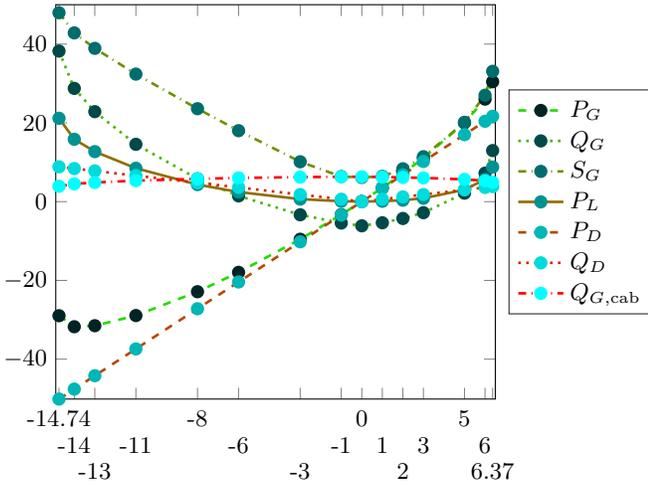


Fig. 10: Various powers [p.u.] plotted against load scaling factors, for the BT system with attached 132 kV cables.

are maximal at  $k = 0$  because of the impact of  $Q_{G,cab}$  shown in Fig. 10. For  $k < 0$ , the highest voltage is in the middle, and, for  $k > 0$  the lowest voltage is in the middle – reflecting the power flow direction. The voltages are above nominal for  $-6 \leq k \leq 3$  and below nominal for other  $k$ .

3) *Powers*: Powers are shown in Fig. 10. The maximal apparent power for  $k < 0$  is 48.0 p.u., and 33.1 p.u. for  $k > 0$ . The smallest simulated active power converted is -31.8 p.u., for  $k = -14$ , -29.0 p.u. for  $k = k_{min}$ . The maximal active power converted is 30.4 p.u.

The impact of  $Q_{G,cab}$  can no longer be neglected. Still at  $k = 2$  and  $k = -1$ ,  $Q_{G,cab}$  is the largest contributor to  $S_G$ , the apparent power of the converters.

The apparent power curves assumes a more convex shape in Fig. 10 than in Figs. 5 and 7.

#### A. Summary

In this extended conceptual study, the loadability limits for motoring as well as regenerating trains have been identified and many intermediate loading levels have been evaluated.

For the proposed solution, in the no-load case of  $k = 0$ , about 15 MVAR has to be consumed in each converter station, meaning that the converters are utilized at 37.5% of their rated capacity – for no traffic! Decreasing the number of committed converters in a station will result in more reactive power per station, and consequently an even higher share of the rated capacity per unit.

At least two converters per station needs to be committed with the proposed solution.

For a medium-level-utilized railway, the intermediate 15 kV solution might be attractive. The 15 kV cable of Table I is designed for 132 kV usage [6] making the system prepared for a possible future upgrade.

Converter losses for a railway with 132 kV cable transmission, could be reduced by switching to the 15 kV cable operation mode for load factors  $-0.5 \lesssim k \lesssim 1.5$ .

#### B. Future Work

Since significant levels of reactive converter power regeneration is needed at low load, converter losses needs to be considered in future deepened studies.

In future, deepened case studies, unsymmetrical systems in regards to spatiality, load factors, converter station configurations, etc. would be of relevance to study.

Further down the line, actual moving load simulations are necessary, as well as analyses of resonances, etc.

#### C. Case Study Result Analysis

The converters in the study are rated to 10 MVA continuously and to 14 MVA for 6 minutes [16], so the total amount of rated power in the system is 16 p.u. continuously, and 22.4 p.u. for 6 minutes.

This means that for the 132 kV cable solution,  $k = 3$  is the highest studied  $k$  for which the level of loading is continually feasible and that  $k = 5$  is the highest  $k$  which is feasible for up to 6 minutes. For motoring trains, the converters cannot be overloaded in the pure BT case study, whereas for the 15 kV cable case,  $k = 3$  is the highest studied  $k$  which is continually feasible.

For regeneration,  $k = -5$  is the lowest studied continuously feasible  $k$  in the BT system. Corresponding figures for the 15 kV and the 132 kV cable systems are  $k = -4$  and  $k = -3$ ; and  $k = -8$  and  $k = -6$ , up to 6 minutes.

#### D. Cost Analysis

This is not the time and place to make a deep quantitative cost analysis. The main reasons for using AC cables as a means of strengthening a railway power supply system are when other options turn out less attractive:

- 1) If increasing the transfer capacity of the railway power system (by the different means discussed below) is cheaper than distributing the converter stations denser along the contact line system.
- 2) If overhead transmission lines are legally impossible or the time and money needed for getting a permission to build such are unacceptable.
- 3) If a strengthening of the contact line will ...
  - ... not give the needed power transfer capacity,
  - ... imply reinforcement/replacement of the poles holding up the contact line system leading to overall higher costs than transmission cables,
  - ... be costlier than transmission cables.
- 4) And, finally, if the grid can be strengthened with AC cables instead of DC cables.

The first option is rarely desired. It will not only increase the number of subscriptions to the feeding public grid (if it is locally strong enough). It will also probably result in larger, but less utilized, levels of installed converter capacity. World-wide high voltage transmission solutions for converter-fed railway are reviewed in [7].

The second option provides low losses and no uncontrolled reactive production.

The third option includes dual-voltage systems of AT (auto transformer) [17], [18] type, and/or extra/thicker reinforcement line(s) and/or return conductor(s). This is a classical approach. Transfer losses are likely to be higher than for transmission lines/cables, but offers a less complicated solution.

Transformers are cheaper than DC-AC converters, and the control and protection is simpler and thus cheaper with an AC solution. Technically, the fourth option of VSC-HVDC transmission [12] is preferable to AC cables, in terms of control options.

## V. CONCLUSIONS

In this paper, the lion's share of the results of [7] have been verified and confirmed, confer Section III-A1. That prior study was extended with maximizing and minimizing the load factors, and studying more cases. Both these contributions were possible due to a slight modification of the models, and the usage of another solver – c.f. Section II-C. The focus of [7] was to argue for AC cables, whereas focus in this paper is to gain a deeper understanding of the behaviour of such systems.

## REFERENCES

- [1] S. Östlund, *Electric Railway Traction*. KTH, Stockholm, 2012.
- [2] S. Leonard, R. Ollerenshaw, A. Wrightson, and R. Hargrave, "First static converter project in the United Kingdom at Doncaster," *Elektrische Bahnen*, vol. 113, no. 6-7, pp. 332–335, 2015.
- [3] I. Perin, S. Matthews-Frederick, P. F. Nussey, and G. R. Walker, "Static frequency converters – world's first application for 50 Hz/50 Hz," *Elektrische Bahnen*, vol. 113, no. 8, pp. 392–399, 2015.
- [4] L. Abrahamsson, T. Schütte, and S. Östlund, "Advocating for the use of converters for feeding of AC railways for all frequencies," *Elsevier Energy for Sustainable Development*, vol. XX, no. 3, p. XX, 2012.
- [5] Y. Oura, Y. Mochinaga, and H. Nagasawa, "Railway Electric Power Feeding System," *Japan Railway & Transport Review*, vol. 16, pp. 48–58, 1998.
- [6] E. Friman, "Impedances for contact line and 132 kV, 30 kV and 15 kV feeder lines (original title in Swedish: Impedanser för KTL och 132 kV, 30 kV och 15 kV ML)," Swedish Railway Administration (Banverket), Tech. Rep. BKE 02/28. Rev. F., Jul. 2006.
- [7] L. Abrahamsson, D. S. Jimenez, J. Laury, and M. Bollen, "AC Cables Strengthening Railway Low Frequency AC Power Supply Systems," in *2017 Joint Rail Conference*, Philadelphia, PA, USA, Apr. 2017, paper No. JRC2017-2258.
- [8] CENELEC - European Committee for Electrotechnical Standardization, *Railway applications – Supply voltages of traction systems*, Std. EN-50 163-2004, Nov. 2004,
- [9] —, *Railway Applications - Power supply and rolling stock*, Std. EN 50 388:2012, Mar. 2012.
- [10] M. Olofsson, "Power Flow Analysis of the Swedish Railway Electrical System," Royal Institute of Technology (KTH), Stockholm, Sweden, Tech. Rep., 1993, Licentiate Thesis.
- [11] L. Abrahamsson, "Railway Power Supply Models and Methods for Long-term Investment Analysis," Royal Institute of Technology (KTH), Stockholm, Sweden, Tech. Rep., 2008, Licentiate Thesis.
- [12] J. Laury, "OPF for an HVDC Feeder Solution for AC Railways," Master's thesis, Royal Institute of Technology (KTH), 2012.
- [13] N. Mohan, T. M. Undeland, and W. P. Robbins, *Power Electronics: Converters, Applications, and Design, 3rd Edition*. Wiley, 2003.
- [14] T. Kulworawanichpong and C. J. Goodman, "Optimal area control of ac railway systems via pwm traction drives," *IEEE Proceedings - Electric Power Applications*, vol. 152, no. 1, pp. 33–40, 1 2005.
- [15] GAMS Development Corporation, "GAMS — The Solver Manuals," Washington, DC, USA, Tech. Rep., 2017.
- [16] M. Olofsson, "Optimal Operation of the Swedish Railway Electrical System - An Application of Optimal Power Flow," Ph.D. dissertation, KTH, Stockholm, Sweden, 1996.
- [17] A. Bülund, P. Deutschmann, and B. Lindahl, "Circuit design of the Swedish railway Banverket in catenary network (original title in German)," *Elektrische Bahnen*, vol. 102, no. 4, pp. 184–194, 2004.
- [18] F. Martinsen, M. Nordgård, and T. Schütte, "A new type of autotransformer system for the railway in Norway," *Elektrische Bahnen*, vol. 108, no. 8–9, pp. 334–344, 2010.