Orientation of fibres in suspensions flowing over a solid surface

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Abstract

The orientation of fibres suspended in a viscous fluid, flowing over a solid surface, has been studied experimentally. A shear layer was generated, by letting the suspension flow down an inclined plate. Far upstream from the measuring section the suspension was accelerated to obtain an initial orientation of the fibres aligned with the flow direction. A CCD-camera was used to visualise the fibres. The velocity profile of the fibres coincided with the theoretical expression for fully developed flow of Newtonian liquid down an inclined wall.

The orientation of the fibres was analysed in planes parallel to the solid surface. At distances from the wall larger than one fibre length the fibres performed a tumbling motion in the flow-gradient plane in what appeared to be Jeffery-like orbits. Closer to the wall a difference was found between fibres of aspect ratio $r_p = 10$ and 40. The longer fibres of $r_p = 40$ kept their orientation, aligned with the flow, also in the near wall region. For the shorter fibres the orientation shifted gradually, to orientations closer to the vorticity axis, when the distance from the wall was decreased. In the very proximity to the wall the fibres were aligned with the vorticity, perpendicular to the direction of the flow. Another distinction, most likely related to the fibre orientation, was seen in the wall normal concentration profile. Due to sedimentation effects fibres accumulated in the near wall region. For fibres of $r_p = 10$ a peak in concentration was found at the wall, while for $r_p = 40$ the maximum concentration was found approximately half a fibre length from the wall. It is previously known that a fibre can interact with the wall in what is referred to as a "pole vaulting" motion away from the wall. It is suggested, as a likely explanation to the location of the maximum concentration, that fibres of $r_p = 40$ perform this motion, while fibres of $r_p = 10$ do not.

In another experiment the surface of the wall was modified with ridges. For fibres of $r_p = 10$ there were no longer any fibres oriented perpendicular to the flow direction in the near wall region.

The main application in mind throughout this work is papermaking. The study is considered to be of fundamental character and is not applicable in a direct sense. The difference between the flow situation in the experiments and the paper machine is discussed further.

Descriptors: fluid mechanics, fibre orientation, shear flow, fibre suspension, papermaking
Preface

This licentiate thesis in fluid mechanics deals with fibre orientation in shear flows. Particular emphasis is put on flows where the suspended fibres are influenced by a solid surface. The primary application in mind is manufacturing of paper, where shear layers are generated along all the solid surfaces present in the headbox. The thesis is divided into two parts. Part I provides a brief introduction to papermaking as well as an overview of relevant work performed in the area of fibre orientation. Part II consists of three papers that, for consistency, have been adjusted to the format of the thesis.

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Part I

Overview and summary
CHAPTER 1

Introduction

This thesis deals with the orientation of fibres suspended in shear flows, near a solid surface. Rod-like particles in flowing suspensions are present in various applications. The primary application in mind in this work is paper production. Experiments have been performed on fibres suspended in a viscous fluid. Although not applicable, in a direct sense, it is the author’s belief that the thesis may provide insights that can be useful for papermaking. For readers, who are not familiar with papermaking and the relevance of fibre orientation, a brief background is given. The primary sources for the text in this chapter is Fellers & Norman (1998) and Gavelin (1990).

1.1. Paper manufacturing

The ability to produce paper has its origins in China, where paper was manufactured roughly 2000 years ago. Originally all paper sheets were made by hand in a slow process, whereas today there are machines that produce more than 100,000 tons/year. Although paper as a product has been around for several centuries, all of the physical mechanisms present in the manufacturing process are not fully understood. Thus, the prospects of improving the process are still promising.

Paper consists of a network of fibres, where the most commonly used fibres in manufacturing are cellulose fibres from wood. To produce paper a fibre suspension, a mixture of water and cellulose fibres, with a mass fraction of fibres less than 1% enters a part of the paper machine often referred to as a headbox. The main assignment of the headbox is to transform a pipe flow, with a diameter of about 800 mm, to a uniform jet around 10 mm thick and up to 10 m wide. The jet leaving the headbox impinges on one or two permeable bands called wires. This is followed by large machinery consisting of a press section and a drying section. The primary aim of the press and drying section is to remove the water from the suspension. A schematic of the initial part of a paper machine is shown in figure 1.1.

1.2. Fibre orientation

A parameter of relevance for the mechanical properties of a paper sheet is the fibre orientation. A lot of work has been performed concerning fibre orientation
in papermaking. Odell & Pakarinen (2001) made a recent overview on fibre orientation related defects on different scales and their effect on the paper sheet. The majority of the fibres, in the final product, are oriented in the machine direction. This leads to a stronger tensile strength in the machine direction, than in the cross direction. The orientation of the fibres in the paper sheet is determined early in the paper machine. Shortly after the impingement of the jet the fibres create a network and can no longer move in relation to each other.

Several studies have shown that the flow conditions in the headbox are significant for the fibre orientation in the final paper sheet. Due to the streamwise acceleration of the suspension in the headbox the fibres tend to align with the flow direction. Jansson (1998) investigated the effect, on the fibre orientation, of the contraction ratio of the headbox nozzle at minimum shear conditions during dewatering. The anisotropy of the fibre orientation distribution was low at the surfaces, even though the anisotropy around the sheet centre was high. It could be argued that this effect is related to the impingement of the jet, where the initial rate of dewatering is high. Another possible explanation was proposed by Asplund & Norman (2003). Measurements of the fibre orientation were made over the thickness of the jet leaving the headbox. It was shown that, already at this stage of the process, the anisotropy was clearly lower near the surfaces of the jet. This suggests that the low anisotropy, seen at the surfaces

\[ \text{Figure 1.1. Schematic of a headbox.} \]
of the paper sheet, are related to the shear layers generated along the solid surfaces in the headbox nozzle.

Another property of interest is the formation, i.e. the local mass distribution of fibres, of the paper. One often desires a paper to be as smooth as possible. In an empty headbox nozzle, which can be regarded as a convergent channel, large-scale fluctuations may occur in the flow. This can in turn lead to a poor formation. In order to reduce the problem a set of flexible flow dividers, here referred to as lamellas, are usually implemented in the headbox. At the interface of the suspension and solid surfaces the velocity of the fluid is zero. This leads to a formation of thin layers of shear along the surfaces of the lamellas, where the velocity gradually increases. The hydromechanical term for these shear layers is boundary layers. How fibres behave in shear flows when a solid surface is present is not obvious and is the main theme of this thesis.
1. INTRODUCTION
CHAPTER 2

Rod-like particles in shear flow

In this chapter a review is made of the progress in the field of orientable particles. Extensive work has been done in the field and related areas. In order not to depart too far from the theme of this work some restrictions are introduced.

The fluid in which the particles are suspended is assumed to be Newtonian, i.e. the shear stress of the fluid is linearly proportional to the rate of strain. This assumption holds in paper production, where water is the main ingredient. It should be mentioned that when particles are immersed in a Newtonian liquid the properties of the mixture as a whole could show non-Newtonian properties. The study of non-Newtonian fluids is termed rheology. The rheological properties of a suspension generally depend on the orientation of the particles. However, studies concerning the rheology of suspensions are not regarded here, at least not in a rheological sense.

Another restriction is that the effect of Brownian diffusion is considered to be negligible. Brownian motion is particularly significant for suspensions with very small particles. It is convenient to introduce a rotary Peclet number $Pe = \dot{\gamma}/D$ to characterise the influence of Brownian effects, see for instance Chen & Jiang (1999). In the expression $\dot{\gamma}$ is the shear rate of the fluid and $D$ is a rotary diffusivity coefficient dependent on the temperature, viscosity and particle parameters. In the shear layers in a headbox $Pe \approx 10^{11}$ and thus molecular diffusion is negligible.

2.1. Fluid motion

The motion of an incompressible Newtonian fluid is described by Navier-Stokes equations

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

(2.1)

$$\nabla \cdot \mathbf{u} = 0,$$  \hspace{1cm} (2.2)

where $\mathbf{u}$ is the fluid velocity, $t$ is the time, $p$ is the pressure and $\mathbf{f}$ is a body force term. The fluid properties, density and kinematic viscosity, is denoted by $\rho$ and $\nu$, respectively. In order to get an indication of the characteristics of a flow a non-dimensional number, referred to as the Reynolds number $Re = UL/\nu$, is often introduced. The parameters $U$ and $L$ correspond to $a$, for the particular
flow, characteristic velocity and length scale, respectively. For steady flows where the inertial effects are negligible as compared to effects of viscosity, i.e. \( Re << 1 \), equation (2.1) is approximately reduced to
\[
\frac{1}{\rho} \nabla p = \nu \nabla^2 u + f. \tag{2.3}
\]
Flows that are described by equation (2.3) are generally called Stokes flows.

### 2.2. Unbounded shear flow

#### 2.2.1. Single particles

The motion of a solid ellipsoid, with a surface defined by \( \frac{x'^2}{a^2} + \frac{y'^2}{b^2} + \frac{z'^2}{c^2} = 1 \), suspended in a simple shear flow, was computed analytically by Jeffery (1922). For the case where \( b = c \), i.e. as a spheroid, the solutions are
\[
\dot{\phi} = -\frac{\dot{\gamma}}{r_e^2 + 1}(r_e^2 \sin^2 \phi + \cos^2 \phi) \tag{2.4}
\]
\[
\dot{\theta} = \left(\frac{r_e^2 - 1}{r_e^2 + 1}\right) \frac{\dot{\gamma}}{4} \sin 2\phi \sin 2\theta, \tag{2.5}
\]
where \( \dot{\gamma} \) is the shear rate and \( r_e = a/b \) is the ellipsoidal aspect ratio. Jeffery’s equations are valid for both prolate spheroids \( (r_e > 1) \) and oblate spheroids \( (r_e < 1) \), but since the main focus of this study concerns rod-like particles only the motion of prolate spheroids will be considered. For a simple shear, with a velocity defined by \( u = \dot{\gamma}y e_x \), the angle \( \phi \) is taken from the flow direction \( x \) to the projection of the \( x' \)-axis in the \( xy \)-plane and \( \theta \) is the angle from the vorticity axis \( z \) to \( x' \). The coordinate system is illustrated in figure 2.1.

Equations (2.4) and (2.5) are valid for conditions in which the particle Reynolds number \( Re_L = (\dot{\gamma} L^2)/\nu << 1 \), where \( L \) is typically \( a \) or \( b \). Directly from the equations it is seen that the angular velocities \( \dot{\phi} \) and \( \dot{\theta} \) depend linearly with the shear rate. The spheroid will rotate in closed orbits with a period
\[
T = \frac{2\pi}{\dot{\gamma} \left( \frac{r_e^2 + 1}{r_e} \right)}. \tag{2.6}
\]
If equations (2.4) and (2.5) are integrated with respect to time they may be rewritten to
\[
\cot \phi = -r_e \cot \left( \frac{2\pi t}{T} + \phi_0 \right) \tag{2.7}
\]
\[
\tan \theta = \frac{C r_e}{(r_e^2 \sin^2 \phi + \cos^2 \phi)^{1/2}}, \tag{2.8}
\]
where \( C \) is the orbit constant and \( \phi_0 \) is the initial value of \( \phi \). A number of possible orbits are illustrated in figure 2.2, for \( r_e = 40 \). For \( C = 0 \) the spheroid is oriented with its major axis aligned with the vorticity axis and it rotates around this axis with a constant angular velocity \( \dot{\gamma}/2 \). As \( C \) approaches infinity the major axis of the spheroid will be located in the flow-gradient plane,
aligned with the flow for most of the time to occasionally, with a period of $T/2$, flip around the vorticity axis. In the remainder of this text, when discussing the orientation of a spheroid, or another elongated particle, the major axis is implied.

Bretherton (1962) extended Jeffery’s analysis to be valid for almost any body of revolution, with a fore-aft symmetry. As a result equations (2.4) - (2.8) are valid also for particles of cylindrical shape, provided that an equivalent ellipsoidal axis ratio is found. For certain very long bodies, the orbits will however be different from those of Jeffery.

The orbit that a spheroid will undergo is defined by $C$ and the spheroid will, according to the equations, follow this orbit for an indefinite time. Jeffery also calculated the average rate of dissipation of energy during the periodic motion and came up with a "minimum energy" hypothesis. Accordingly, the spheroid will tend to perform the motion that results in the minimum average dissipation of energy. For prolate spheroids this motion is given by $C = 0$.

In an attempt to verify Jeffery’s minimum energy hypothesis Taylor (1923) was the first to perform an experimental study on spheroids, in a flow between two concentric cylinders. Prolate spheroids of $r_e < 3$ aligned with the axis.
Figure 2.2. The path of one of the end points of a cylindrical particle, of aspect ratio $r_e = 40$, rotating in Jeffery orbits. The path is projected on the $xy$-plane for various values of $C$.

of vorticity as suggested by Jeffery. The process was gradual and before the final state was reached Taylor observed the oscillating motion described by Jeffery, although no measurements were made to analyse the orbits. A study on a similar experimental setup was performed by Binder (1939) on cylindrical particles of varying aspect ratios $r_p = l/d$, where $l$ and $d$ is the length and diameter of the cylinder, respectively. For shorter particles than $r_p \approx 15$ the particles finally reached a state where orbits corresponding to $C = 0$ were observed. For longer particles orbits of large $C$ were observed, thus not in consistency with Jeffery’s minimum energy hypothesis. Binder suggested that the reason for the discrepancy might lie in the neglect of inertia in Jeffery’s analysis.

First to verify Jeffery’s equations experimentally were Trevelyan & Mason (1951). The experiments were performed in a Couette apparatus on cylindrical particles with $r_p$ in the range from 20 to 120. An equivalent aspect ratio was determined by measuring the period of rotation and extract $r_e$ from (2.6).

A number of analytical studies, under flow conditions in which inertia can be neglected, has been done concerning the force and torque acting on long slender bodies. Cox (1970, 1971) and Tillett (1970) considered the force and torque on bodies of circular cross-section, where Cox also allowed a curvature of the body. A similar work was done by Batchelor (1970) for straight bodies.

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1 Although Taylor (1923) performed initial experiments on spheroids, the first measurements on the periodicity of spheroids is not reported until several years later by Anzurowski & Mason (1968). This is probably related to the difficulties of manufacturing spheroids.
with an arbitrary non-circular cross-section. Some years later Keller & Rubinow (1976) investigated the force per unit length on a slender body of circular cross-section. In addition to earlier studies the body was also permitted to twist and dilate. Yet another theoretical work was presented by Geer (1976), where there were no restrictions on the flow other than the fulfillment of Stokes equations. Chwang & Wu (1975) presented a number of exact solutions to Stokes flow problems, including flows past prolate spheroids. For a spheroid suspended in a paraboloidal flow governed by \( u = K(y^2 + z^2)e_x \), where \( K \) is a constant of unit \([1/\text{ms}]\), Chwang (1975) concluded that the motion of a spheroid could be described by Jeffery’s equations with the shear rate evaluated at the centre of the spheroid. As a consequence of this result, sufficiently small spheroids will move in Jeffery orbits also in Poiseuille flow.

Goldsmith & Mason (1962) performed an experimental study with single rods of cylindrical shape in a flow satisfying Stokes equations. The particles were suspended in a circular Poiseuille flow, i.e. with a velocity governed by

\[
    u(r) = \frac{2Q}{\pi R^3}(R^2 - r^2),
\]

where \( Q \) is the volume rate of flow, \( r \) is the radial distance from the center of the tube and \( R \) is the radius of the tube. The motion of the rods was in good agreement with Jeffery’s solutions, provided that an equivalent aspect ratio was found from equation (2.6), with \( \dot{\gamma} \) taken at the \( r \)-position where the centre of the rod was located.

As already mentioned work has been conducted in order to find the equivalent ellipsoidal aspect ratio for cylindrical particles. This has been studied by Trevelyan & Mason (1951) and Goldsmith & Mason (1962), but also by Anczurowski & Mason (1968). Anczurowski & Mason (1968) measured the periodical orbits of spheroids in a Couette flow. The results were in good agreement to Jeffery’s equations. In the same study experiments were conducted on cylindrical particles mainly in order to determine the point of transition from discs \((r_e < 1)\) to rods \((r_e > 1)\), corresponding to orbits of oblate and prolate spheroids, respectively. The transition was found to take place at a particle aspect ratio of \( r_p = 0.86 \). A second result from the study was that \( r_p = r_e \) when \( r_p = 1.68 \). Experiments were also carried out on particles of \( r_p \) up to 100 and the equivalent \( r_e \) was calculated. One of the key results found by Cox (1971) was an expression relating \( r_p \) of cylindrical bodies to an equivalent \( r_e \)

\[
    r_e = \left( \frac{8\pi}{3L} \right)^{1/2} r_p (\ln r_p)^{-1/2},
\]

where \( L \) is a constant dependent on the shape of the blunt ends of the body. Cox compared equation (2.10) to the experiments conducted on cylindrical particles by Anczurowski & Mason and concluded that \( L = 5.45 \) resulted in the best fit. Equation (2.10) is derived for long bodies and is valid only for large \( r_p \). Another expression relating \( r_p \) of cylindrical particles to an equivalent \( r_e \)
2. ROD-LIKE PARTICLES IN SHEAR FLOW

Figure 2.3. Equivalent ellipsoidal aspect ratio $r_e$ of cylindrical particles with aspect ratio $r_p$.

was deduced by Harris & Pittman (1975). Experiments were made in a plane Couette flow, which resulted in

$$r_e = 1.14 r_p^{0.844}.$$  \hspace{1cm} (2.11)

The expression was reported to agree with the measurements of Trevelyan & Mason, within 5%, down to $r_p = 1$. In figure 2.3 equation (2.11) by Harris & Pittman is shown and compared with equation (2.10) by Cox.

2.2.2. Particle interactions

In suspensions of rod-like particles, the particles can interact through fluid stresses or direct mechanical contact. In this text $n l^3$, the number of fibres within a volume $l^3$, is used to denote the concentration of the suspension. A common procedure to get an indication of how frequent particle interactions are is to consider different regimes of concentration. For concentrations of $n l^3 << 1$, where particle interactions can be neglected, the suspension is called dilute. The regime $n l^3 >> 1$ and $n l^2 d << 1$ is called semi-dilute and is dominated by hydrodynamic interactions. Mechanical contact becomes relevant in the semi-concentrated regime where $n l^2 d = O(1)$.

In the dilute regime the results discussed in the previous section are valid. However, it turns out that Jeffery’s equation provide a good approximation also for higher concentrations. Koch & Shaqfeh (1990) derived a correction to the rate of rotation, due to hydrodynamic interaction in a semi-dilute fibre suspension. For a Jeffery rotation rate of $O(\dot{\gamma})$ in a dilute suspension the
correction was shown to be \( O(\dot{\gamma}/\ln(1/c_v)) \) in the semi-dilute regime, where \( c_v \) is the volume fraction of fibres.

Mason & Manley (1956) studied the motion of cylindrical particles in a shear flow. Experiments were performed on low concentration suspensions \((nl^3 < 1)\) with \( r_p \) in the range between 20 and 120. A drift towards a preferential orientation in the flow direction was seen, for all initially isotropic suspensions. The drift was stronger for larger \( r_p \). Similar experiments were performed by Anczurowski & Mason (1967). The orbit distribution of rods of \( r_p = 18.4 \) was investigated for concentrations in the range \( nl^3 = 0.016 \) to 0.52. For \( nl^3 < 0.1 \) the distribution of orbits was independent of concentration. About 50% of the fibres rotated in orbits with \( C < 0.2 \). Note that this does not necessarily contradict a preferential direction near the flow direction. When moving in Jeffery orbits, the fibres spend most of their time nearly aligned with the \( xz \)-plane. This means that, in this phase of the orbit, for \( C = 0.2 \) and \( r_e \approx 13 \), the fibres are oriented only about 20 degrees from the flow direction. Thus, in the experiments by Anczurowski & Mason (1967), more than 50% of the fibres will spend most of their time oriented less than 20 degrees from the flow direction. Although only low concentrations were under study a small shift in the direction of orbits corresponding to higher values of \( C \) was seen when the concentration was increased.

Stover, Koch & Cohen (1992) performed experiments on index of refraction matched suspensions in order to visualise suspensions in the semi-dilute regime. The experiments were done in a cylindrical Couette apparatus on suspensions with \( r_p = 16.9 \) and 31.9 and concentrations between \( nl^3 = 1 \) and 45. The particles were reported to rotate around the vorticity axis in the manner described by Jeffery, also for the highest concentration. At small concentrations lower values of \( C \) was favoured, but with an increase of concentration a more uniform \( C \)-distribution was found. Recalling the results of Anczurowski & Mason (1967) this indicates that the shift towards a more preferential orientation in the flow direction, with increasing concentration, continues also in the semi-dilute regime.

At higher concentrations, in the semi-concentrated regime, Sundararajakumar & Koch (2005) performed dynamic simulations. In the study hydrodynamic interactions were neglected and only interactions due to direct mechanical contact were included. It was concluded that collision between fibres caused them to flip more frequently. Experiments on fibre suspensions of \( r_p \geq 50 \), with concentrations between \( nl^2d = 0.2 \) and 3, were conducted by Petrich, Koch & Cohen (2000). In consistency with Sundararajakumar & Koch the period of rotation was shorter than the period predicted by Jeffery. At \( nl^2d = 0.2 \) the period was about 20% smaller than the period given by equation (2.6). However, when the concentration was increased the period returned to values close to the dilute result. The fibre orientation was also considered. With an increasing
concentration the distribution of orbits shifted slightly towards smaller values of $C$.

### 2.3. Inertial effects

It has been previously mentioned that the result of Binder (1939), where fibres of large $r_p$ were found non-consistent with Jeffery’s minimum energy hypothesis, could be due to inertial effects. Otherwise, up to now the flows under study have been assumed to be free from inertia, i.e. the particle Reynolds number $Re_l$ has been small. In many applications the term $(\mathbf{u} \cdot \nabla)\mathbf{u}$ is not negligible in equation (2.1). Equation (2.3) will thus not be valid. For instance, in the shear layers along the lamellas in the headbox of a paper machine, $Re_l$ is typically larger than 250.

An analytical study concerning the inertial effects on long slender bodies was performed by Khayat & Cox (1989). The hydrodynamic force and torque on the body, held fixed in a uniform flow, were derived. For finite Reynolds numbers an orientation dependent non-zero torque was found. As a result a body with fore-aft symmetry, when sedimenting in the vertical direction, rotates to an equilibrium horizontal orientation.

The effect of inertia on a cylinder aligned with the vorticity axis was studied numerically by Ding & Aidun (2000). For low $Re_{dd} = \gamma d^2/\nu$ the motion of the cylinder was in good agreement with Jeffery’s analysis. For higher $Re_{dd}$ the period of rotation was shown to increase and eventually become infinitely large at a sufficiently large $Re_{dd}$.

In a recent theoretical study Subramanian & Koch (2005) explored the influence of inertia on the motion of fibres in a simple shear flow. One of the conclusions made in the study is that, for a small but finite $Re_l$, fibres drift towards orbits of higher $C$, i.e $C \to \infty$ as $t \to \infty$. For $Re_l$ larger than a critical Reynolds number

$$Re_{l,c} = \frac{15 \ln(r_p)}{\pi r_e} \sin^{-2}(\theta),$$

(2.12)

the fibres cease to rotate and drift monotonically to the flow-gradient plane. Subramanian & Koch also examined a case where both shear and sedimentation were accounted for. When the force of gravitation align with the gradient direction the evolution of $C$ is given by

$$\frac{dC}{dt} = \left( \frac{2\pi Re_l}{15 \ln(r_p)} \sin^2(\theta) \cos^2(\phi) - \frac{5}{16 \ln(r_p)} \frac{Re_{sed}^2}{Re_l} \right) C,$$

(2.13)

where $Re_{sed} = U_{sed}/\nu$ is a Reynolds number based on the sedimentation velocity $U_{sed}$. Hence, for sufficiently large $Re_{sed}$, as compared to $Re_l$, the orbit constant will drift towards zero and finally align with the vorticity axis.

Qi & Luo (2003) numerically investigated the motion of spheroidal particles of $r_e = 2$, suspended in simple shear flows for $0 < Re_l < 467$, with $l = 2a$. 
where \( a \) is the half length of the spheroid. Two transitions in the final rotation state were observed within the studied range of \( \text{Re}_l \). For \( 0 < \text{Re}_l < \text{Re}_{l,1} \approx 205 \) there was a drift of \( C \) towards larger values and the spheroid finally rotated in the flow-gradient plane. In an intermediate region \( \text{Re}_{l,1} < \text{Re}_l < \text{Re}_{l,2} \approx 345 \), the mean value of \( \theta \) decreased monotonically with increasing \( \text{Re}_l \). For \( \text{Re}_{l,2} < \text{Re}_l < 467 \) an equilibrium state was found for \( \theta = 0 \), i.e. when the spheroid was aligned with the vorticity axis, rotating around its major axis. Notable from this study is that the spheroid never ceased to rotate, although the period increased with \( \text{Re}_l \).


2.4. Wall-bounded shear flow

The analysis by Jeffery were made under the assumption that Stokes equations are valid. Apart from this assumption it was also assumed that there were neither particle interactions nor any wall effects. A few studies have been made concerning the motion of elongated particles in the presence of solid boundaries.

Dabros (1985) calculated numerically the motion of a prolate spheroid, with an aspect ratio \( r_e = 2 \), close to a solid boundary. The spheroid was located in the flow-gradient plane, i.e. far away from the wall the motion would be described by Jeffery’s equations with \( C \) approaching infinity. At large distances from the wall the angular velocity \( \dot{\phi} \) of the spheroid coincided with the solution of Jeffery. Near the wall, at a distance of \( y/a = 1.05 \), where \( y \) is the distance from the wall to the particle centre and \( a \) is the half length of the spheroid, the angular velocity of the spheroid was smaller. This was in particular seen in the phase of the orbit when the spheroid was oriented parallel to the wall, i.e. \( \phi = 0 \).

Hsu & Ganatos (1989, 1994) calculated the hydrodynamic force and torque on an arbitrary body of revolution, suspended in a shear flow, with a varying orientation angle relative to a solid wall. The solution was used in order to compute the motion of a prolate spheroid, at distances from the wall down to \( y/a = 1.1 \). As in the study by Dabros the spheroid was fixed in the flow-gradient plane. The spheroid underwent a periodic tumbling motion similar to the motion described by Jeffery, but also oscillated periodically in the wall normal direction. A similar study with similar results was done by Gavze & Shapiro (1997, 1998). Also here a periodic oscillation was found toward and away from the wall at \( y/a = 1 \). It was also concluded that the tumbling motion could be described by Jeffery’s equations, but with a larger period closer to the wall. A first-order approximation to account for small particle inertia was introduced. Similar to the case of no inertia, the inertial spheroids performed an oscillatory motion, in the wall normal direction, but simultaneously drifted towards the wall.

Pozrikidis (2005) made a numerical analysis on the motion of a spheroid near a solid wall. This study, was however not restricted to motions where the
2. ROD-LIKE PARTICLES IN SHEAR FLOW

Figure 2.4. Projection of the particle centre, of a spheroid of \( r_e = 4 \), in the \( xy \)- and \( xz \)-planes for \( y/a = 1.25 \). The various lines correspond to \( \Phi_0/\pi = 0 \) (dotted line), \( \Phi_0/\pi = 0.125 \) (dashed line), \( \Phi_0/\pi = 0.25 \) (solid line), \( \Phi_0/\pi = 0.375 \) (thick solid line) and \( \Phi_0/\pi = 0.46875 \) (bold solid line). From Pozrikidis (2005).

particle was fixed in the flow-gradient plane. The initial inclination angle \( \Phi_0 \) of the spheroid, to the flow-gradient plane, was varied in the computations. The motion of the particle centre is shown in figure 2.4. Also in this study the particle centre moved periodically in the wall normal direction, when located near the wall. A periodical motion parallel to the vorticity axis was also found when the spheroid was not initially located in the flow-gradient plane nor directed parallel to the vorticity axis. For all initial conditions under study a longer period was found near the wall than far from the wall, where Jeffery’s equations were verified. For a particle of \( r_e = 4 \) the period increased with approximately 10%, at a distance from the wall of \( y/a = 1.25 \), for any initial angle to the flow-gradient plane.

An experimental study in pressure-driven flow between two solid walls was performed by Stover & Cohen (1990). A special device made it possible to suspend single fibres of \( r_p = 12 \) and adjust them to a desired position and orientation. For distances from the wall larger than one fibre length Jeffery’s equations were verified for small and high values of \( C \). Closer to the wall than one fibre length the motion was still periodic, but with a longer period than would be predicted by the shear rate. This trend was seen independent of the value of \( C \). Furthermore, an interaction with the wall was found for fibres with a large \( C \), located closer to the wall than half a fibre length. In what was referred to as a ”pole vaulting” motion, the fibre moved away from the wall to a point where the centre of mass of the fibre was located at approximately \( y = l/2 \).
2.5. SUMMARY

Another experimental study on the wall effect was conducted by Moses, Advani & Reinhardt (2001). The experiments were made in a simple shear flow. The fibres were suspended one at a time, oriented perpendicular to the flow direction, in the flow-gradient plane. For distances at greater distances from the wall than one fibre length Jeffery's equations were verified. Closer to the wall the motion could still be described by Jeffery’s equations if an increased effective shear rate was used. Notable is that according to equation (2.6) this would result in a shorter period, thus inconsistent with the other reviewed studies.

The orientation of fibres in a shear flow, near a solid wall, was studied by Holm (2005). A fibre suspension was flowing down an inclined wall. Far upstream from the position where the measurements were made the suspension was accelerated, in the flow direction, to align the fibres with the flow. Experiments were performed on suspensions with aspect ratios between $r_p = 10$ and 40 and the orientation $\beta$ in a plane parallel to the wall was analysed. For small $r_p$ a significant amount of the fibres were oriented perpendicular to the flow direction.

2.5. Summary

Several studies have been done in the field of fibre orientation. In particular for unbounded shear flows where the particle motion given by Jeffery’s equations have been verified both numerically and experimentally. Accordingly an elongated particle, with a for-aft symmetry, will move in closed orbits, referred to as Jeffery orbits. In Jeffery’s analysis no account is taken for wall effects or particle interactions. Furthermore, Stokes equations are fulfilled, thus removing inertia from the problem.

Experimental studies on suspensions, where particle interactions can not be neglected, has yielded a drift towards higher values of the orbit constant $C$. This has been seen for concentrations in the semi-dilute regime. However, for even higher concentrations, in the semi-concentrated regime, Petrich, Koch & Cohen (2000) reported a drift towards lower values of $C$. Notable from all these studies are that the particles have been rotating around the vorticity axis in a motion similar to that described by Jeffery.

For flows where inertia is not negligible, relatively little work has been done, on freely suspended particles. For a small but finite $Re_l$ Subramanian & Koch (2005) and Qi & Luo (2003) reported a drift towards higher values of $C$. Subramanian & Koch predicted that for sufficiently large $Re_l$ the fibre would stop rotating, whereas according to Qi & Luo the fibres never cease to rotate, although the period of the motion do increase for larger $Re_l$. Furthermore, Qi & Luo found a second transition towards a preferential orientation. For $Re_l > 345$ the fibres drifted towards lower values of $C$. The only experimental study under review, suggesting that inertia might be of significance, is the
work of Binder (1939). Binder varied the aspect ratio and reported a final state, corresponding to a large value of $C$, for sufficiently long fibres.

From numerical studies on the wall effect it has been concluded that the mass centre of the particle will oscillate in the wall normal direction as well as in the direction parallel to the vorticity axis. A longer period, than what would be expected from the shear rate, has been found close to the wall. This was also found experimentally by Stover & Cohen (1990), although a contradicting result was reported by Moses et al. (2001). In none of the reviewed studies has there been any discussion concerning a preferential value of $C$ due to the presence of the wall.
Fibre orientation in a flow over an inclined plane

In this chapter the author’s own contribution to the field of fibre orientation is summarised. A brief overview of the experimental setup and the essential results is reviewed. For a more detailed description the reader is referred to Part II, the paper section of this thesis.

3.1. Experimental setup

On essentially the same experimental setup as Holm (2005), measurements were done by Carlsson, Lundell & Söderberg (2006a,b). Fibre suspensions consisting of cellulose acetate fibres suspended in a viscous fluid were studied. A schematic of the setup is shown in figure 3.1. A film of the suspension, with a thickness of \( h \approx 17 \text{ mm} \), was flowing down an inclined solid surface, thus generating a shear layer. To visualise the fibres in a plane parallel to the wall a CCD-camera was placed underneath the wall.

Two different aspect ratios (\( r_p = 10 \) and \( r_p = 40 \)) and two different wall structures were studied. The first wall structure was a smooth surface and the other a structured surface with ridges, oriented 30 degrees to the flow direction.

Image analysis was used in order to determine the orientation and velocity of individual fibres. By placing the camera to capture images in the flow-gradient plane, it was concluded that the velocity of the fibres correlated well with the distance from the solid surface. This together with the velocity, made it possible to determine the distance from the wall of the fibres also in the images captured from underneath the wall. The studied orientation \( \beta \) of the fibres is defined as the angle, in a plane parallel to the wall, taken clockwise from the flow direction.

3.2. Results

For distances further away from the wall than one fibre length, i.e \( y > l \), the majority of the fibres kept their orientation aligned with the flow \( (\beta = 0) \), as initially aligned. For fibres of \( r_p = 40 \) this was also the dominating orientation of the fibres, in the near wall region. For fibres of \( r_p = 10 \) most fibres were oriented in the flow direction at \( y = l \), but gradually shifted to an orientation close to perpendicular to the flow direction \( (\beta = 90) \) in the very proximity to
3. FIBRE ORIENTATION IN A FLOW OVER AN INCLINED PLANE

the wall. In figure 3.2 $F(\beta) = P(B \leq \beta)$, i.e. the probability that a fibre is oriented between $\beta = 0$ and $\beta$, is shown.

Carlsson et al. also found a difference in the wall normal concentration distribution between the two aspect ratios under study. Due to gravitational effects fibres accumulated in the near wall region. For $r_p = 10$ the maximum concentration of fibres was found very close to the wall, while for fibres of $r_p = 40$ the largest concentration was found at $y \approx l/2$. It was suggested that fibres of $r_p = 40$ were performing the pole vaulting motion described by Stover & Cohen (1990), while fibres of $r_p = 10$ were not.

The reason for the difference found in the near wall region, between longer and shorter fibres, in the work by Carlsson et al. is not perfectly clear. Recalling the results of Subramanian & Koch (2005) it is reasonable to believe that inertial effects could be contributing to the observed discrepancy. The particle Reynolds number in the experiments was $Re_l \approx 0.01$ and 0.20 for $r_p = 10$ and $r_p = 40$, respectively. Putting $\theta = 90$ in equation (2.12), thus restricted to the flow-gradient plane, results in the corresponding $Re_{l,c} = 1.38$ and 0.69. Thus, $Re_l < Re_{l,c}$ for both $r_p$. Note that $Re_{l,c}$ is a lower limit defining where fibres will cease to rotate altogether, according to Subramanian & Koch. A drift towards higher values of $C$ was reported also for values below $Re_{l,c}$. With this in mind, inertial effects could be of significance, for $r_p = 40$ in particular. Another result of Subramanian & Koch, that could be of relevance, is equation (2.13). In the study by Carlsson et al. $Re_{sed} \ll Re_l$ and although $C$ will decrease for finite periods, related to certain orientations, $C \to \infty$ as $t \to \infty$.

If the mechanism, due to inertia, is present, the fibres will tend to drift toward higher values of the orbit constant. However, if the presence of the
3.2. RESULTS

Figure 3.2. $F(\beta)$ for different wall normal distances: (a) $r_p = 10$ and $nl^3 = 0.48$, (b) $r_p = 40$ and $nl^3 = 0.48$. The concentration is here expressed as $nl^3$ which is the number of fibres within a volume of $l^3$. From Carlsson et al. (2006b).

Another interesting result from the experiments was found for the case when the solid surface was modified with ridges. For all cases, including fibres of $r_p = 10$, the majority of the fibres in the near wall region were orientated in the flow direction.
3. FIBRE ORIENTATION IN A FLOW OVER AN INCLINED PLANE
Relevance for paper manufacturing

In Chapter 3 and Part II results from performing experiments, on the orientation of fibres in a wall bounded shear flow, are reported. In this chapter no emphasis is laid on the results of the study, but rather on a brief comparison between the flow and suspension properties in the experiments and in the paper machine.

4.1. Flow properties

The flow in the experiments is a laminar film flow with a film thickness of $h \approx 17$ mm. The Reynolds number based on $h$ is $Re = U_s h / \nu \approx 8$, where $U_s$ is the free surface velocity. In the headbox the situation is quite different. The flow can be modeled by a 2D convergent channel, illustrated in figure 4.1. Due to the flow rates the flow in a headbox is to a large extent turbulent. Parsheh (2001) performed experimental investigations of the flow in a 2D convergent channel, with application to headboxes. It was found that towards the end of the contraction the mean velocity profile of an initially turbulent boundary layer approached a self-similar laminar state. The corresponding similarity solution can be found in Schlichting (1979), where the velocity is defined by

$$\frac{u}{U_e} = 3 \tanh^2 \left( \frac{\eta}{\sqrt{2}} + 1.146 \right) - 2,$$

where $U_e$ is the velocity of the fluid outside the boundary layer and $\eta$ is defined as

$$\eta = y' \sqrt{\frac{U_e}{-(x' - x_0') \nu}},$$

with the coordinates defined in figure 4.1. The occurrence, that a turbulent boundary layer approach a laminar state, is often termed re-laminarisation and requires a sufficiently strong acceleration. Thus, although the external flow is turbulent, the boundary layers are expected to show a laminar-like behaviour near the exit of the headbox. Note that a self-similar mean flow profile is not sufficient to conclude that the flow is truly laminar in the boundary layer. Studies have shown that parts of the turbulent structures can remain in a re-laminarised boundary layer, see for instance Warnack & Fernholz (1998) and Talamelli, Fornaciari, Westin & Alfredsson (2002). The remaining turbulent
structures may lead to a rapid re-transition to turbulence once there is no longer any acceleration acting on the flow. Acceleration present in the headbox is also expected to affect the fibre orientation. One consequence of the acceleration is that the boundary layer thickness decreases in the flow direction. The order of magnitude of the boundary layer thickness is roughly 1 mm near the end of the lamellas.

4.2. Suspension properties

As already indicated inertial effects play a significant role on the fibre dynamics. For a fibre of length $l = 0.5$ mm and diameter $d = 50$ µm the particle Reynolds number based on $l$ and $d$ are given by $Re_l = \dot{\gamma}l^2/\nu$ and $Re_d = \dot{\gamma}ld/\nu$, respectively. In the experiments ($Re_l \approx 0.01$, $Re_d \approx 0.001$) and in the headbox ($Re_l \approx 250$, $Re_d \approx 25$) under typical shear rates in respective case. The Reynolds numbers suggest that the fibres will rotate in Jeffery-like orbits in the experimental case, although drifts in the orbit constant may occur due to wall effects, effects of inertia and particle interactions. In the headbox inertial effects are significant and it is unclear whether or not the fibres will rotate in orbits similar to those of Jeffery.

In the experiments the concentration range between $n/l^3 = 0.01$ and 3.82, whereas typical values of $n/l^3$ in the headbox are between 0.5 and 20. Apart from fibres, there are also fillers, fines and various chemical substances present in the paper production. The precise composition depends on the pulp and the specific application, but these additives are likely to have an impact on the fibre motion.

Yet another parameter of significance is the fibre flexibility. In the experiments the fibres are considered to be rigid. However, this is generally not the case in the headbox. Wet pulp fibres are flexible and can bend when suspended in the headbox. The fibre flexibility is different for different pulps. Tam Doo & Kerekes (1981, 1982) developed a method to measure the flexibility of wet fibres. Tests showed that chemical pulps were up to 30 times more flexible than mechanical pulps from the same wood. However, although fibres in mechanical pulps are stiffer than in chemical pulps, they can not be assumed to be rigid.
Concluding remarks

Based on the comparison made in Chapter 4 it is clear that a lot of work remains to be done in order to fully understand the situation in a headbox. A first step, before introducing turbulence, is to understand the laminar case thoroughly. It is believed that proceeding in this manner will facilitate understanding of the more complex situation present in the paper machine.

From the review made in Chapter 2 it is concluded that there are not many studies concerning the wall effect on the fibre dynamics. In basically all studies with a solid boundary, inertia has been totally neglected. To the author’s knowledge the only exception is the numerical study by Gavze & Shapiro (1998), where a small but finite inertia was included. In fact, there seems to be essentially no experimental studies where effects of inertia are discussed. This is the case also for unbounded shear flows. Experimental studies in the region where the effect of inertia becomes significant ($Re_l < 1$) are thus needed. The experiments would preferably be done in a setup where it is possible to follow the fibres for a long period of time, to be able to detect small drifts in the orbit constant.

In order to understand the fibre motion in the headbox boundary layers more thoroughly two suggestions of topics that need to be addressed are stated.

- The only study found in the region $Re_l > 1$ is the numerical work by Qi & Luo (2003). Complementary experimental studies are required in this region, recalling that typically $Re_l > 250$ in the headbox.
- Pulp fibres are generally flexible. Experimental studies concerning the influence of the fibre elasticity are thus required.

More points could obviously be added. For instance, as mentioned in Chapter 4 there are various interactions with different kinds of fillers and chemical substances, depending on the specific pulp. Nevertheless, it is believed that further insights on the two points mentioned are essential if the situation in the headbox is to be understood.
5. CONCLUDING REMARKS
CHAPTER 6

Papers and authors contributions

Paper 1

Fibre orientation control related to papermaking

A. Carlsson (AC), F. Lundell (FL) & L. D. Söderberg (DS).

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The wall effect on the orientation of fibres immersed in a laminar shear flow is investigated. Experiments are performed on two different structures of the solid surface, one smooth and another modified with ridges. The wall is shown to have an effect on the orientation. For fibres of aspect ratio $r_p = 10$ the fibres are oriented close to perpendicular to the flow direction near the wall, for the smooth surface. When the surface is modified with ridges, this effect is seemingly absent. AC performed the experiments and analysis under supervision of FL and DS. AC, FL and DS wrote the paper jointly. Parts of these results have been presented at (i) ASME Joint U.S.-European Fluids Engineering Summer Meeting, Miami, FL, USA 2006, (ii) Euromech Fluids Mechanics Conference 6, Royal Institute of Technology, Stockholm 2006 and (iii) Nordic Rheology Conference, Royal Institute of Technology, Stockholm 2006.

Paper 2

Orientation of fibres in a flowing suspension near a plane wall

A. Carlsson, F. Lundell & L. D. Söderberg.

The orientation of fibres flowing over a solid surface is studied experimentally. Measurements are performed on two different aspect ratios, $r_p = 10$ and 40. For fibres of $r_p = 40$ the majority of the fibres are oriented in the flow direction, as initially aligned, independent of the wall-normal position. For $r_p = 10$ the orientation shifts gradually, from being nearly aligned with the flow for distances from the wall larger than one fibre length, to being oriented close to perpendicular to the flow direction at the wall. AC performed experiments and analysis under supervision of FL and DS. AC, FL and DS wrote the paper jointly.
Paper 3

Evaluation of steerable filters for detection of rod-like particles in flowing suspensions
A. Carlsson, F. Lundell & L. D. Söderberg.

A filter within the class of steerable filters is evaluated for suitability of detecting fibres in flowing suspensions. The concept of steerable filters is concluded to be an efficient method of finding the position and orientation of fibres. Experiments, analysis and writing were performed by AC under supervision of FL and DS.
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Prof. Bo Norman and Krister Åkesson are greatly acknowledged for improving my initially and still lacking knowledge of paper manufacturing. On this topic I would again like to include the already mentioned Dr. Söderberg.

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