Suspensions of finite-size rigid particles in laminar and turbulent flows

by

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Cover: Suspension of finite-size rigid spheres in homogeneous isotropic turbulence.

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“Considerate la vostra semenza:
fatti non foste a viver come bruti,
ma per seguir virtute e canoscenza.”

Dante Alighieri, Divina Commedia, Inferno, Canto XXVI
Suspensions of finite-size rigid particles in laminar and turbulent flows

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Abstract
Dispersed multiphase flows occur in many biological, engineering and geophysical applications such as fluidized beds, soot particle dispersion and pyroclastic flows. Understanding the behavior of suspensions is a very difficult task. Indeed particles may differ in size, shape, density and stiffness, their concentration varies from one case to another, and the carrier fluid may be quiescent or turbulent. When turbulent flows are considered, the problem is further complicated by the interactions between particles and eddies of different size, ranging from the smallest dissipative scales up to the largest integral scales. Most of the investigations on this topic have dealt with heavy small particles (typically smaller than the dissipative scale) and in the dilute regime. Less is known regarding the behavior of suspensions of finite-size particles (particles that are larger than the smallest length scales of the fluid phase).

In the present work, we numerically study the behavior of suspensions of finite-size rigid particles in different flows. In particular, we perform direct numerical simulations using an immersed boundary method to account for the solid phase. Firstly, the sedimentation of spherical particles slightly smaller than the Taylor microscale in sustained homogeneous isotropic turbulence and quiescent fluid is investigated. The results show that the mean settling velocity is lower in an already turbulent flow than in a quiescent fluid. By estimating the mean drag acting on the particles, we find that non stationary effects explain the increased reduction in mean settling velocity in turbulent environments. Moreover, when the turbulence root-mean-square velocity is larger than the terminal speed of a particle, the overall drag is further enhanced due to the large particles cross-flow velocities.

We also investigate the settling in quiescent fluid of oblate particles. We find that at low volume fractions the mean settling speed of the suspension is substantially larger than the terminal speed of an isolated oblate. This is due to the formation of clusters that appear as columnar-like structures.

Suspensions of finite-size spheres are also studied in turbulent channel flow. We change the solid volume and mass fractions, and the solid-to-fluid density ratio in an idealized scenario where gravity is neglected. The aim is to independently understand the effects of these parameters on both fluid and solid phases statistics. It is found that the statistics are substantially altered by changes in volume fraction, while the main effect of increasing the density ratio is a shear-induced migration toward the centerline. However, at very high density ratios (~1000) the solid phase decouples from the fluid, and the particles
behave as a dense gas.
In this flow case, we also study the effects of polydispersity by considering Gaussian distributions of particle radii (with increasing standard deviation), at constant volume fraction. We find that fluid and particle statistics are almost unaltered with respect to the reference monodisperse suspension. These results confirm the importance of the solid volume fraction in determining the behavior of a suspension of spheres.
We then consider suspensions of solid spheres in turbulent duct flows. We see that particles accumulate mostly at the corners. However, at large volume fractions the particles concentrate mostly at the duct core. Secondary motions are enhanced by increasing the volume fraction, until excluded volume effects are so strong that the turbulence activity is reduced. The same is found for the mean friction Reynolds number.
The inertial migration of spheres in laminar square duct flows is also investigated. We consider dilute and semi-dilute suspensions at different bulk Reynolds numbers and duct-to-particle size ratios. The highest particle concentration is found in regions around the focusing points, except at very large volume fractions since particles distribute uniformly in the cross-section. Particles also induce secondary fluid motions that become more intense with the volume fraction, until a critical value of the latter quantity is reached.
Finally we study the rheology of confined dense suspensions of spheres in simple shear flow. We focus on the weakly inertial regime and show that the suspension effective viscosity varies non-monotonically with increasing confinement. The minima of the effective viscosity occur when the channel width is approximately an integer number of particle diameters. At these confinements, the particles self-organize into two-dimensional frozen layers that slide onto each other.

**Key words:** particle suspensions, sedimentation, homogeneous isotropic turbulence, turbulent channel flow, rheology, inertial migration, duct flow.
Numeriska studier av icke-sfäriska och sfäriska partiklar i laminära och turbulenta flöden

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Sammanfattning


I detta arbete studeras beteendet hos suspensioner bestående av stela sfärer (större än de minsta turbulenta virvlna) för olika strömningsfall med hjälp av direkt numerisk simulerings (DNS). I simuleringsarna hanteras partikelfasen (soliden) med hjälp av en såkallad immersed boundary metod. Först undersöker vi sedimentationen hos partiklar som är något större än Taylors mikroskala i homogen isotropisk turbulens samt i en stillstående fluid. Resultaten visar att den genomsnittliga sedimenteringshastigheten är lägre i en redan turbulent strömning jämfört den i en stillstående fluid. Genom att uppskatta det genomsnittliga motståndet på partiklarna, finner vi att icke-stationära effekter förklarar den ökade minskningen i genomsnittliga sedimenteringshastigheten som återfinns i turbulenta miljöer. När hastighetens standardavvikelsemedelvärde (RMS) är större än partiklens gränshastighet, ökar motståndskraften på grund av partiklarnas höga tvärströmshastighet. Sedimentering av oblata partiklar i stillstående fluid har också studerats. Resultaten visar att för små volymfraktioner är suspensionens medelhastighet betydligt större än gränshastigheten hos en enskild partikel, vilket är kopplat till formationen av kolumnliknande partikelstrukturer.

Turbulent kanalströmning innehållande sfäriska partiklar studeras också inom ramen för detta arbete. Partiklens volymfraktion varieras samt densitet förhållandet partikel och vätska (fluid) då strömningen är sådan att gravitationen kan försommnas. Målet är att förstå hur de oberoende effekterna av ovanstående


Slutligen studerar vi reologin hos avgränsade suspensioner med hög partikelkoncentrationen av sfärar i en enkel skjuvströmning. Vi fokuserar på det området som kännetecknas av svag tröghet och visar att suspensionens effektiva viskositet varierar icke-monotont med ökad avgränsninggrad. Den effektiva viskositeten uppvägs ett minsta värde då kanalens bredd är approximativt en multipel av partikeldiametern. Vid dessa avräknings där avståndet mellan två väggar minskas mer och mer så ordnar sig partiklarna i tvådimensionella lager som glider ovanpå varandra.

**Nyckelord:** partikelsuspensioner, sedimentering, homogen isotropisk turbulens, turbulent kanalströmning, reologi, tröghetsmigrering, duct flow.
Preface

This thesis deals with the study of the behavior of suspensions of finite-size particles in different flow cases. An introduction on the main ideas and objectives, as well as on the tools employed and the current knowledge on the topic is presented in the first part. The second part contains eight articles. The papers are adjusted to comply with the present thesis format for consistency, but their contents have not been altered as compared with their original counterparts.


November 2017, Stockholm

Walter Fornari
Division of work between authors

The main advisor for the project is Prof. Luca Brandt.

**Paper 1.** The simulation code for interface resolved simulations developed by Wim-Paul Breugem (WB) has been made triperiodic by Walter Fornari (WF), who also introduced a forcing to create a sustained homogeneous isotropic turbulent field. Simulations and data analysis have been performed by WF. The paper has been written by WF with feedback from LB and Prof. Francesco Picano (FP).

**Paper 2.** The point-particle and immersed boundary code have been modified by WF. Simulations and data analysis have been performed by WF. The paper has been written by WF with feedback from Gaetano Sardina (GS), LB and FP.

**Paper 3.** The code has been developed by MN. Simualtions and data analysis have been performed by WF. The paper has been written by WF with feedback from MN and LB.

**Paper 4.** The computations have been performed by WF and Cyan Umbert López (CL). Data analysis has been performed in part by CL and mostly by WF. The paper has been written by Prof. Dhrubaditya Mitra (DB), WF and FP with feedback from LB and Pinaki Chaudhuri (PC).

**Paper 5.** The code has been developed by WF and Mehdi Niazi Ardekani (MN). Simulations and data analysis have been performed by Hamid Tabaei Kazerooni (HT) and WF. The paper has been written by HT and WF with feedback from Jeanette Hussong (JH) and LB.

**Paper 6.** The computations have been performed by Alberto Formenti (AF) and WF. Data analysis has been performed by WF and AF. The paper has been written by WF with feedback from LB and FP.

**Paper 7.** The code has been developed by WF and Pedro Costa (PC) from TU Delft. Simulations and data analysis have been performed by WF. The paper has been written by WF with feedback from LB and FP.

**Paper 8.** The computations have been performed by WF. Data analysis has been performed by WF and HT. The paper has been written by WF with feedback from HT, JH and LB.

**Other publications**

The following paper, although related, is not included in the thesis.

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Part I

Overview and summary
Chapter 1

Introduction

Suspensions of solid particles and liquid droplets are commonly found in a wide range of natural and industrial applications. Environmental processes include the sediment transport in surface water flows, the formation and precipitation of rain droplets, dust storms and pyroclastic flows. Other examples include the transport of suspended micro-organisms in water (such as plankton), and various biological flows like blood. Typical engineering applications are instead food, oil chemical and pharmaceutical processes that involve, for example, particulate flows in fluidized beds, soot particle dispersion, and the pneumatic and slurry transport of particles. Due to the broad range of applications, it is hence important to understand and to be able to predict the bulk and microscopic behaviors of these multiphase flows. However, this is often a non-trivial task due to the complexity of the problem. Indeed, the presence of the dispersed phase alters the instantaneous and linear relationship between the applied strain and the resulting stress in the flow, typical of pure Newtonian fluids. A rich variety of complex rheological behaviors are hence observed, depending on the properties of the fluid and solid phases, on the flow regime, and on the system geometry. From a mathematical point of view, the difficulty in dealing with these complex fluids arises from the fact that it is necessary to couple the governing equations of the carrier fluid phase (typically considered as a continuum) – the Navier-Stokes equations – with those describing the dynamics of the dispersed phase. The former must also necessarily be complemented with suitable boundary conditions on the surface of each particle.

As mentioned, the suspension behavior depends on many different factors. First of all, the bulk flow regime may be laminar or turbulent. Secondly, particles may differ in density, size, shape and stiffness, and their solid mass and volume fractions ($\chi$ and $\phi$) are important parameters in defining the rheological properties of the suspension. In the so-called active fluids, micro-organisms may be capable of swimming, for example by sensing nutrient gradients (Lambert et al. 2013), and this should be accounted for in the formulation of the problem. Concerning particle dynamics, a very important parameter is the Reynolds number at the particle scale, defined as $Re_p = (2a)|U_r|/\nu$, where $a$ is the particle radius (or a characteristic length for non-spherical particles), $\nu$ is the fluid kinematic viscosity, and $|U_r|$ is the modulus of the relative (slip) velocity between fluid and particle (equal to the fluid velocity for fixed particles). This
nondimensional number quantifies the importance of inertia. Depending on the bulk flow conditions and on particle size, we can hence identify different regimes at the particle scale. For example, when particles are smaller than the smallest length scales of the flow, and fluid inertia at the particle scale is weak, the particle Reynolds number $Re_p$ tends to zero and the suspension is in the so-called Stokesian (or viscous) regime. In this scenario, the motion of an isolated particle is typically fully reversible and there is a linear relationship between its velocity and the drag force acting on it. It is interesting to note, however, that irreversible dynamics is observed in suspensions, due to the combined effects of non-hydrodynamic (e.g., roughness, collisions) and hydrodynamic interactions among particles (see for example the shear-induced diffusion and migration of particles in shear flows, Guazzelli & Morris 2011). This regime is typically found, for example, in microdevices.

The picture changes in the inertial regime (i.e., when the particle Reynolds number $Re_p$ is finite). The relation between drag and velocity becomes nonlinear. The unsteady forces related to the formation of the boundary layer close to the particle surface are altered due to the strong convection of vorticity. In addition, the symmetry of particle-pair trajectories is broken. The different dynamics is hence reflected at the macroscopic level of the suspension and new peculiar phenomena and rheological behaviors are observed. The situation is further complicated when the carrier fluid is turbulent, as particles interact with eddies of different size and lifetime. Inertial regimes are often found in environmental flows and in various engineering applications like slurry and pneumatic transport.

In the study of particle-laden flows, the system geometry is another important aspect that leads to different microscopic and macroscopic properties. Broadly speaking, we can distinguish among bounded flows (channel, duct or pipe flows) and unbounded flows. The latter scenario is common in numerical simulations of settling particles and droplets (or rising bubbles) in the atmosphere, suspended micro-organisms in oceanic waters, and planetesimals in accretion disks, to mention a few examples.

When dealing with bounded multiphase flows, it is of fundamental importance to study their rheological properties (Stickel & Powell 2005; Morris 2009). Indeed, the dispersion of few particles in a Newtonian fluid modifies the response to the local deformation rate and the mixture viscosity is no longer an intrinsic material property. Depending on the shear rate, these suspensions may either exhibit shear-thinning (a decrease in the suspension effective viscosity), shear-thickening (an increase of the effective viscosity), the appearance of normal stress differences (Brady & Bossis 1985), or other typical non-Newtonian behaviors (such as memory effects). In these flow cases, the particles microstructure and hydrodynamic interactions are generally responsible for the macroscopic behavior of the suspension (Brady & Bossis 1988). In turbulent flows, on the other hand, the presence of the solid phase may also lead to a strong modulation and modification of the turbulent field (Kulick et al. 1994; Zhao et al. 2010).
Depending on the type and size of the particles, turbulent velocity fluctuations may be increased or decreased as well as the overall drag.

The presence of walls also induces very interesting particle migrations. Particles are observed to undergo shear-induced migration in the viscous regime and inertial migration at finite $Re_p$; see for example the tubular pinch effect (Segre & Silberberg 1962). Inertial migration is also strongly related to the conduit shape and size (with respect to that of the particle). In pipe and duct flows of suspensions of spherical particles at same hydraulic diameters, particle radii, bulk and particle Reynolds numbers, particles are found to migrate towards distinct focusing positions. Another type of migration called turbophoresis (Reeks 1983) is instead observed for small heavy particles in wall-bounded turbulent flows.

Concerning open environments, one of the most common problems investigated is sedimentation. When particles are sufficiently small or when the fluid is highly viscous, the assumption of Stokes flow holds and the settling speed is obtained by the balance of Stokes drag and buoyancy (assuming the fluid to be quiescent). However, when inertial effects become important, the terminal speed can only be broadly estimated using empirical nonlinear drag corrections (Schiller & Naumann 1935). Since the terminal falling speed is typically unknown a priori, it is not convenient to use the particle Reynolds number to characterize the problem. Another non-dimensional number is instead used for this purpose. This is the so-called Galileo (or Archimedes) number, $Ga$, which quantifies the importance of the buoyancy forces acting on the particles with respect to viscous forces. Depending on the Galileo number and the solid-to-fluid density ratio, isolated particles exhibit different types of wakes and fall at different velocities along vertical, oblique, oscillating or chaotic paths (Uhlmann & Dušek 2014; Yin & Koch 2007).

When suspensions are considered, particle-particle and hydrodynamic interactions play an important role in the sedimentation process. The mean settling speed of the suspension strongly depends on the solid volume fraction $\phi$. In batch sedimentation systems, for example, the fixed bottom of the container forces the fluid to move in the opposite direction to gravity, so that the net flux of the mixture is zero. Hence, the mean settling speed is smaller than the terminal speed of an isolated spherical particle, and decreases with $\phi$ (Richardson & Zaki 1954; Yin & Koch 2007; Guazzelli & Morris 2011). However, as $Ga$ is increased, inertial effects become progressively more important and interesting fluid- and particle-particle interactions occur, further complicating the problem. An interesting interaction between two (or more) spherical particles is the so-called Drafting- Kissing-Tumbling (DKT). When a spherical particle has a sufficiently long wake and an oncoming particle is entrained by it, the latter will be strongly accelerated (drafted) towards the former. The particles will then kiss and the rear particle will tumble towards one side (Fortes et al. 1987). For oblate particles, however, DKT is modified and the tumbling phase is suppressed (Ardekani et al. 2017). Additionally, in a semi-dilute suspension of spherical
particles at large $Ga$ of about 180, Uhlmann & Doychev (2014) have observed the formation of particle clusters. These clusters fall substantially faster than an isolated particle, and as a result the mean settling speed increases above the terminal speed. Finally, when sedimentation occurs in an already turbulent field, the interactions among eddies of different sizes and particles alter the whole process. Particles may fall on average faster or slower than in quiescent fluid (Wang & Maxey 1993; Good et al. 2014; Byron 2015). The turbulent flow is also modulated due to the energy injection at the particle scale.

The aim of this summary is to give the reader a brief idea about the wide range of applications of particulate flows and especially about the complexity of the problem. Many experimentally and numerically observed behaviors are still far from clear; owing to the wide range of parameters involved there is still much to explore in each of the different flow regimes. In the present work, four main different scenarios have been studied. Concerning unbounded flows, the settling of finite-size spherical particles has been studied in both quiescent fluids and sustained homogeneous isotropic turbulence. Different volume fractions (between 0.5 and 1%), solid-to-fluid density ratios and Galileo numbers have been investigated. An effort has been made in order to understand the mechanisms leading to the different behaviors and settling speeds found in each case.

Up to date indeed, most of the works on sedimentation in turbulent environments have considered sub-Kolmogorov sized particles (where the Kolmogorov length and time scales are the scales of the smallest dissipative eddies) in the dilute regime (i.e. with very low volume fractions of the order of $10^{-5}$, Wang & Maxey 1993). When larger particles are considered (Lucci et al. 2010), the dynamics is strongly influenced by the ratios between the typical particle length and time scales (their diameter and the relaxation time) and those of the turbulent field (either the integral scales or the Kolmogorov, dissipative scales). Substantial amount of relative motion between particles and fluid is usually generated and predictions become almost prohibitive (Cisse et al. 2013). This has motivated us to study how the interaction with the background turbulence alters the settling process.

Less is known about the sedimentation of suspensions of finite-size nonspherical particles. Indeed, particle orientation plays an important role in the dynamics, and the sedimentation process is further complicated. The particle aspect ratio becomes an additional parameter of the problem. We have here considered the sedimentation of oblate particles of aspect ratio $1/3$, at fixed $Ga$ and different $\phi$. The objective is to understand how the mean settling speed changes as function of $\phi$, and how this is related to the suspension microstructure. Indeed, the DKT is modified for pairs of oblate particles (i.e., the tumbling phase is suppressed, as shown by Ardekani et al. 2017), and the collective behavior of the suspension is found to change in comparison to the case of spherical particles.
Several studies have also been performed concerning bounded particle-laden flows at different flow regimes. The first work is about the rheology of highly confined suspensions of rigid spheres in simple Couette flow at low Reynolds numbers. Over the years, indeed, much effort has been devoted to understand the effect of varying the imposed shear rate and volume fraction, mostly in the Stokesian regime (Einstein 1906; Morris 2009). However, not much is known about the weakly inertial regime and the effects of confinement. For example, will the rheological properties change monotonically with increasing confinement? In addition, understanding the behavior of confined suspensions at low Reynolds numbers is becoming more and more relevant due to its importance in microfluidic devices (Di Carlo 2009).

Still in the laminar regime, we have studied the behavior of semi-dilute suspensions of spherical particles in square duct flow. In particular, we have focused on the effects of changing the solid volume fraction (from 0.4% to 20%), the bulk Reynolds number (from 144 to 550) and the duct-to-particle size ratio on the rheology, the inertial migration of particles, and the secondary (cross-stream) fluid flows. In fact, up to date most studies had dealt only with isolated particles and dilute suspensions, and little was known about the dynamics of both phases at larger volume fractions.

The remaining works on bounded flows are all in the turbulent regime. As in sedimentation, most of the previous studies concerned either very small and heavy particles or finite-size particles at low volume fractions (Reeks 1983; Sardina et al. 2011; Shao et al. 2012). In turbulent channel flow, recent studies showed that as the volume fraction of neutrally buoyant finite-size spherical particles is increased from 0 to 20%, the overall drag is also increased. Interestingly, this is due to the growth of the particle induced stresses, while the turbulence activity is progressively reduced (Lashgari et al. 2014; Picano et al. 2015). However, the case of neutrally buoyant spherical particles is usually an idealized scenario (Prosperetti 2015), as these may differ in size, shape, stiffness and density. It is therefore crucial to understand how the results would change if more realistic suspensions were considered. Initially, we have considered monodispersed suspensions of spherical particles, and have studied the effects of varying independently the mass fraction and the solid-to-fluid density ratio (at constant volume fraction), in an idealized scenario where gravity is neglected. Indeed, while it has been shown how excluded volume effects strongly influence the dynamics of both phases, the importance of particle inertia has only been partially explored. Next, we have considered polydisperse suspensions of neutrally buoyant spheres. In particular, three normal distributions of particle radii and two volume fractions (2% and 10%) have been chosen. In the broader distribution, the bigger particles are four times larger than the smaller ones. Our interest was to understand if statistics obtained for semi-dilute monodisperse suspensions could be assumed to be valid also for suspensions of particles with different sizes (but with same global concentration).
1. Introduction

From both investigations it has emerged clearly that the suspension behavior is mostly governed by excluded volume effects, while small variations in density ratio and particle size do not alter significantly the results. Substantial changes in the results (at constant volume fraction) have only been observed for very large density ratios ($\geq 100$).

Finally, we have studied monodisperse suspensions of neutrally buoyant spheres in turbulent duct flows with volume fractions up to 20%. We have examined the behavior of both fluid and solid phases, paying particular attention on the secondary (cross-stream) flows, the mean streamwise vorticity, the mean particle concentration and the wall friction. Differences in the results with respect to two-dimensional channel flows have been highlighted.

Summarizing, the purpose of this work is to study the behavior of suspensions of finite-size particles in yet unexplored (or partially explored) flow cases. In particular, we have focused on the effects of changing the solid-to-fluid density ratio, the mass and volume fractions, particle size and shape, bulk flow regime, and the system confinement. In the following chapter, the governing equations describing the dynamics of the fluid and solid phases are discussed, as well as the immersed boundary method used for the direct numerical simulations (DNS). In chapters 3 and 4 the problems examined are more deeply discussed. Finally, in chapters 5 and 6 the main results are summarized and an outlook on future work is provided.
Chapter 2

Governing equations and numerical method

2.1. Navier-Stokes and Newton-Euler equations

When dealing with complex fluids it is necessary to describe the coupled dynamics of both fluid and solid phases.

Typically the fluid is treated as a continuum composed of an infinite number of fluid parcels. Each fluid parcel consists of a high number of atoms or molecules and is described by its averaged properties (such as velocity, temperature and density).

Many applications deal with either liquids or gases with flow speed significantly smaller than the speed of sound (less than 30%). Under this condition, the fluid can be further assumed to be incompressible (i.e., the total volume of each fluid parcel is always constant). The final set of equations describing the motion of these fluids is known as the incompressible Navier-Stokes equations and reads

$$\nabla \cdot \mathbf{u}_f = 0$$ (2.1)

$$\frac{\partial \mathbf{u}_f}{\partial t} + \mathbf{u}_f \cdot \nabla \mathbf{u}_f = -\frac{1}{\rho_f} \nabla p + \nu \nabla^2 \mathbf{u}_f$$ (2.2)

where $\mathbf{u}_f$, $\rho_f$, $p$ and $\nu = \mu/\rho_f$ are the fluid velocity, density, pressure and kinematic viscosity (while $\mu$ is the dynamic viscosity). For the mathematical problem to be well-posed, suitable initial and boundary conditions must be assigned. Typically, when these equations are written in non-dimensional form, the inverse of the Reynolds number $Re = UL/\nu$ appears in front of the diffusive (second) term on the right hand side of equation (2.2) (where $U$ and $L$ are a characteristic velocity and length scale of the system). The Reynolds number is a non-dimensional number that quantifies the importance of the inertial and viscous forces in the flow. The Navier-Stokes equations are second order nonlinear partial differential equations and analytic solutions exist only for a very limited set of problems. Therefore, either experimental or numerical investigations are commonly carried out. As already stated, in the present work all the results have been obtained by direct numerical simulations.

When a solid phase is dispersed in the fluid, the Navier-Stokes equations must be coupled with the equations of motion for the solid particles. Assuming the particles to be non-deformable and spherical, the rigid body dynamics is
described by a total of 6 degrees of freedom: translations in three directions and rotations around three axes. The particles centroid linear and angular velocities, \(u_p\) and \(\omega_p\) are then governed by the Newton-Euler Lagrangian equations,

\[
\rho_p V_p \frac{du_p}{dt} = \int_{\partial \mathcal{V}_p} \sigma \cdot n \, dS + (\rho_p - \rho_f) V_p g + F_c
\]  
(2.3)

\[
I_p \frac{d\omega_p}{dt} = \int_{\partial \mathcal{V}_p} \mathbf{r} \times \sigma \cdot n \, dS + T_c
\]  
(2.4)

where \(V_p = 4\pi a^3/3\) and \(I_p = 2\rho_p V_p a^2/5\) are the particle volume and moment of inertia, with \(a\) the particle radius; \(g\) is the gravitational acceleration; \(\sigma = -p\mathbf{I} + 2\mu\mathbf{E}\) is the fluid stress, with \(\mathbf{I}\) the identity matrix and \(\mathbf{E} = \left( \nabla \mathbf{u}_f + \nabla \mathbf{u}_f^T \right)/2\) the deformation tensor; \(\mathbf{r}\) is the distance vector from the center of the sphere while \(n\) is the unit vector normal to the particle surface \(\partial \mathcal{V}_p\). The second term in equation (2.3) is the buoyancy force, while \(F_c\) and \(T_c\) (from equation 2.4) represent additional forces and torques eventually acting on the particles (e.g., due to collisions). Note that in the case of non-spherical particles, the moment of inertia \(I_p\) changes with the particle orientation and it is therefore kept in the time derivative: \(d(I_p\omega_p)/dt = r.h.s..\).

Finally, to couple the motion of the fluid and the particles, Dirichlet boundary conditions for the fluid phase are enforced on the particle surfaces as \(\mathbf{u}_f |_{\partial \mathcal{V}_p} = \mathbf{u}_p + \omega_p \times \mathbf{r}\).

### 2.2. Numerical methods for particle-laden flows

From a numerical point of view, simulating suspensions of finite-size particles is a challenging task as the flow around each particle must be resolved accurately and, possibly, in a short amount of time. Efficient algorithms are hence needed, together with sufficient computational power. Over the years, several different approaches have been proposed to perform such interface-resolved direct numerical simulations (DNS).

Clearly, the most accurate way of dealing with such problems is to adopt unstructured body fitted meshes around the particles (see Zeng et al. 2005; Burton & Eaton 2005). However, this approach becomes computationally very expensive whenever particles are allowed to move and deform, as the mesh should be regenerated at each time step. Consequently, studying dense particle suspensions via this approach is infeasible. Instead, most of the numerical methods that are used nowadays adopt fixed and uniform meshes. The fluid phase is solved everywhere in the domain and the presence of particles is modelled by applying an additional force on the grid points located within the particle volumes. Among these approaches we recall the front-tracking method by Unverdi & Tryggvason (1992), the force-coupling method by Lomholt & Maxey (2003), different algorithms based on the lattice-Boltzmann solver (Ladd 1994a,b; Hill et al. 2001; Ten Cate et al. 2004), the Physalis method by Zhang & Prosperetti (2003, 2005), and the immersed boundary method (IBM) (Peskin
2.3. The immersed boundary method

In the immersed boundary method, the boundary condition at the solid surface (i.e., \( \mathbf{u}_f|\partial V_p = \mathbf{u}_p + \mathbf{\omega}_p \times \mathbf{r} \) for moving particles) is modelled by adding a force field \( \mathbf{f} \) to the right-hand side of the Navier-Stokes equations. The method was first developed by Peskin (1972), who used it to simulate blood flow patterns around heart valves, and has been widely modified and improved since then (Mittal & Iaccarino 2005).

Following Mittal & Iaccarino (2005), it is possible to distinguish between two main types of IBMs: the continuous forcing and discrete forcing approaches. In the first method, the IBM force is included directly into the continuous governing equation (2.2). The complete set of equations (2.1),(2.2) are then discretized on a Cartesian grid and solved in the entire domain. Therefore, this IBM force formulation does not depend on the specific numerical scheme adopted. On the contrary, in the discrete (or direct) forcing approach the IBM force is introduced after the discretization of the Navier-Stokes equations and, consequently, its formulation depends on the numerical scheme. The continuous forcing approach is particularly suitable to simulate elastic boundaries (see the seminal work of Peskin 1972). Instead, for the case of rigid boundaries the second approach is preferred, as it allows for a direct control over the numerical accuracy, stability, and conservation of the forces.

A computationally efficient direct forcing IBM to fully resolve particle-laden flows was originally proposed by Uhlmann (2005). The method was later modified by Breugem (2012), who introduced various improvements to make it second-order accurate in space. More specifically, the multidirect forcing scheme by Luo et al. (2007) and a slight retraction of the grid points on the particle surface towards the interior were used to better approximate the no-slip/no-penetration boundary conditions. Additionally, an improvement of the numerical stability for solid-to-fluid density ratios near unity was also achieved by directly accounting for the inertia of the fluid contained within the immersed (virtual) boundaries of the particles (Kempe & Fröhlich 2012).

The IBM version of Breugem (2012) is used in the present work to simulate particle suspensions. In particular, the fluid phase is evolved in the whole computational domain using a second-order finite difference scheme on a uniform staggered mesh (\( \Delta x = \Delta y = \Delta z \)). The time integration is performed by a third order Runge-Kutta scheme combined with a pressure-correction method at each sub-step. The same integration scheme is also used for the Lagrangian evolution of eqs. (2.3) and (2.4). The forces exchanged by the fluid and the particles are imposed on \( N_L \) Lagrangian points uniformly distributed on the particle
surface. The number of Lagrangian grid points, $N_L$, is chosen to guarantee that the volume of the Lagrangian grid cell, $\Delta V_l$, is as close as possible equal to that of the Eulerian grid cell, $\Delta x^3$. Specifically, the IBM force is computed in three steps: 1) the first prediction Eulerian velocity field is interpolated on the Lagrangian points; 2) the IBM force is then calculated based on the difference between the interpolated velocity and the local velocity at the particle surface (i.e., $\mathbf{u}_p + \mathbf{\omega}_p \times \mathbf{r}$); 3) the force is finally spreaded from the Lagrangian to the Eulerian grid points. Formally, the acceleration $\mathbf{F}_l$ acting on the $l$–th Lagrangian point is related to the Eulerian force field $\mathbf{f}$ (per unit density) by the expression $\mathbf{f}(\mathbf{x}) = \sum_{l=1}^{N_L} \mathbf{F}_l \delta_d(\mathbf{x} - \mathbf{X}_l) \Delta V_l$, where $\delta_d$ is the regularized Dirac delta (Roma et al. 1999). By using this approximated delta function, the sharp interface at the particle surface is replaced with a thin porous shell of width $3\Delta x$. The regularized Dirac delta function also guarantees that the total force and torque that fluid and particles exert onto each other are preserved in the interpolation and spreading operations. Moreover, to better impose the boundary conditions at the moving surfaces, the IBM forces are iteratively determined by employing the multidirect forcing scheme of Luo et al. (2007). The computed Eulerian force field $\mathbf{f}(\mathbf{x})$ is then used to obtain a second prediction velocity. Finally, the pressure-correction method is applied to update the fluid velocity and the pressure for the next time step. An illustration of the Eulerian and Lagrangian grids employed is shown in figure (2.1).
Using the IBM force field, equations (2.3) and (2.4) are rearranged as follows to maintain accuracy,

\[
\rho_p V_p \frac{d\mathbf{u}_p}{dt} = -\rho_f \sum_{l=1}^{N_l} \mathbf{F}_l \Delta V_l + \rho_f \frac{d}{dt} \int_{V_p} \mathbf{u}_f \, dV + (\rho_p - \rho_f) V_p \mathbf{g} + \mathbf{F}_c \quad (2.5)
\]

\[
I_p \frac{d\mathbf{\omega}_p}{dt} = -\rho_f \sum_{l=1}^{N_l} \mathbf{r}_l \times \mathbf{F}_l \Delta V_l + \rho_f \frac{d}{dt} \int_{V_p} \mathbf{r} \times \mathbf{u}_f \, dV + \mathbf{T}_c \quad (2.6)
\]

where \( \mathbf{r}_l \) is the distance from the center of a particle while the second terms on the right-hand sides are corrections to account for the inertia of the fictitious fluid contained within the particle volume. This helps the numerical scheme to be stable even for neutrally buoyant particles.

The force \( \mathbf{F}_c \) and the torque \( \mathbf{T}_c \) from equations (2.5) and (2.6) are used to account for particle-particle and particle-walls interactions. When the gap distance \( \epsilon_g \) between two particles is smaller than twice the mesh size, lubrication models based on the asymptotic solutions of Brenner (1961) and Jeffrey (1982) are used to correctly reproduce the interaction between the particles. The former symptotic solution is used to calculate the lubrication force of equally sized particles, while the latter is employed for particles of different radii. The problem with these solutions is that they diverge for \( \epsilon_g \to 0 \), whilst this is avoided in reality due to surface roughness. In the code, the effect of roughness is captured by fixing the lubrication force at very small gaps before the collision occurs.

When the gap distance between two particles reduces to zero, the lubrication model is switched off and the soft-sphere collision model is activated. Using this model, the particles are allowed to slightly overlap and the normal collision force is calculated as a function of the overlap between the particles and their relative velocity. An almost elastic rebound is ensured with a dry coefficient of restitution \( e_d = 0.97 \). The restitution coefficient, \( e \), is defined as the ratio between the relative velocity of the particles before and after the collision. The dry coefficient of restitution is the maximum value of \( e \) obtained for colliding spheres in a fluid of negligible resistance (Gondret et al. 2002). Another important input parameter for the model is the contact time defined as \( N_c \Delta t \), where \( N_c \) is the number of time steps. The collision time should not be too long nor too short in order to avoid extreme overlapping and to accurately resolve the collision in time. In our simulations, \( N_c = 8 \). The tangential force between colliding particles can be obtained in a similar way. When this is done, a Coulomb friction model is added to account for sliding motion (Costa et al. 2015).

The same models are used for the interaction between particles and walls. Walls are considered as spheres with infinite radius of curvature. More details and validations of the numerical code can be found in Breugem (2010, 2012), Lambert et al. (2013) and Costa et al. (2015).
2. Governing equations and numerical method

2.4. Stress IBM

The immersed boundary method can also be employed to create virtual walls in the computational domain. In this work we have used the stress IBM to create the walls of square ducts. By doing so, periodic boundary conditions can still be applied in all directions, together with a highly efficient pressure solver based on Fast Fourier Transforms (FFT).

The stress IBM is particularly suitable for rectangular-shaped obstacles immersed in a staggered rectangular grid, such that the fluid-solid interfaces coincide exactly with the faces of the grid cells. As a consequence, velocity nodes on the fluid-solid interfaces correspond to velocities directed normal to the interfaces, while parallel velocities are a half grid spacing away. Inside the solid phase and at the velocity nodes on the interfaces, the second prediction velocity is set to zero (to satisfy the boundary conditions). Instead, at the velocity nodes that are half a grid cell spacing away from the interfaces and in the fluid phase, the discretization of the advection and diffusion terms in the momentum equation are modified in order to perfectly satisfy the boundary conditions. For example, in two dimensions and in the absence of solid boundaries, the vertical advection and diffusion terms of streamwise momentum at node \((i, j)\) from figure (2.2) read

\[
- \frac{\partial u v}{\partial y} \bigg|_{(i,j)} = - \left[ \frac{u v(i, j + 1/2) - u v(i, j - 1/2)}{\Delta y} \right] \quad \text{(2.7)}
\]

\[
\nu \frac{\partial^2 u}{\partial y^2} \bigg|_{(i,j)} = \nu \left[ \frac{u(i, j + 1) - 2u(i, j) + u(i, j - 1)}{\Delta y^2} \right] \quad \text{(2.8)}
\]

where \(u v(i, j + 1/2)\) and \(u v(i, j - 1/2)\) are the values of \(u v\) at the locations of the blue circles. However, in the presence of an obstacle (i.e., colored square in figure 2.2) the discretization must be changed to account for the no-slip condition:

\[
- \frac{\partial u v}{\partial y} \bigg|_{(i,j)} = - \left[ \frac{u v(i, j + 1/2) - 0}{\Delta y} \right] \quad \text{(2.9)}
\]

\[
\nu \frac{\partial^2 u}{\partial y^2} \bigg|_{(i,j)} = \nu \left[ \frac{u(i, j + 1) - u(i, j)}{\Delta y} - \frac{u(i, j - 1) - 0}{\Delta y/2} \right] \quad \text{(2.10)}
\]

The difference between the two discretizations

\[
f \big|_{(i,j)} = - \left[ \frac{u v(i, j - 1/2)}{\Delta y} \right] - \nu \left[ \frac{u(i, j) + u(i, j - 1)}{\Delta y^2} \right] \quad \text{(2.11)}
\]

is hence added to the first prediction velocity \((u^{**} = u^* + \Delta tf)\) to enforce the proper boundary condition at the fluid-solid interface.

This so-called stress IBM was first developed by Breugem & Boersma (2005) and Pourquie et al. (2009), and is suitable for the simulation of both laminar and turbulent flows (Breugem et al. 2014). For immersed objects of irregular shape an alternative, a more suitable version of the IBM is that proposed by Fadlun et al. (2000).
In the following chapters, the problems studied in the context of this work are more thoroughly discussed. Up to date theoretical, numerical and experimental findings on the topics are reviewed, highlighting everytime the yet unknown facts and aspects.
Chapter 3

Sedimentation

3.1. Spherical particles in quiescent fluid

The problem of sedimentation has been extensively studied during the years due to its importance in a wide range of natural and engineering applications. One of the earliest investigations on the topic was Stokes' analysis of the sedimentation of a single rigid sphere through an unbounded quiescent viscous fluid at zero Reynolds number. Under this conditions, the motion of the particle can be assumed to be always steady, and the sedimentation (or terminal) velocity $V_s$ can be easily found by balancing the drag ($F_D = 6\pi\mu a V_s$) and the buoyancy forces acting on the particle (Guazzelli & Morris 2011). The sedimentation velocity can therefore be expressed as

$$V_s = \frac{2}{9} a^2 \frac{\rho_p - \rho_f}{\mu} g = \frac{2}{9} \frac{a^2}{\nu} (R - 1) g$$

(3.1)

where $R = \rho_p/\rho_f$ is the solid-to-fluid density ratio. In a viscous flow the sedimentation velocity $V_s$ of an isolated particle is directly proportional to the square of its radius $a$, to the density ratio $R$ and to the gravitational acceleration $g$, while it is indirectly proportional to the fluid viscosity $\nu$. However this result is limited to the case of a single particle in Stokes flow and corrections must be considered to account for the collective effects and inertia ($Re > 0$).

Under the assumption of very dilute suspensions and Stokes flow, Hasimoto (1959) and later Sangani & Acrivos (1982) obtained expressions for the drag force exerted by the fluid on three different cubic arrays of rigid spheres. These expressions relate the drag force only to the solid volume fraction $\phi$. For example, in the case of a simple cubic lattice the mean settling velocity $V$ of a very dilute suspension can be expressed as

$$\langle V \rangle_p = V_s \left[ 1 - 1.7601\phi^{1/3} + O(\phi) \right]$$

(3.2)

where $\phi$ is the solid volume fraction and $\langle \cdot \rangle_p$ denotes an average over the particles. For convenience, the settling velocity is assumed to be positive in the falling direction. A different approach was pursued by Batchelor & Green (1972), who found another expression for the mean settling velocity using conditional probability arguments:

$$\langle V \rangle_p = V_s \left[ 1 - 6.55\phi + O(\phi^2) \right]$$

(3.3)
3.1. Spherical particles in quiescent fluid

The mean settling velocity, $\langle V \rangle_p$, is hence a monotonically decreasing function of the solid volume fraction and is smaller than $V_s$ for all $\phi > 0$. However, these formulae are unable to properly predict $\langle V \rangle_p$ for semi-dilute and dense suspensions and empirical formulae are used instead. Among these, the most famous is that proposed by Richardson & Zaki (1954). This was obtained from experimental results in creeping flow and reads

$$\langle V \rangle_p = V_s [1 - \phi]^n$$

(3.4)

where $n$ is a positive exponent ($\approx 5$). Note that equation 3.4 is likely to be inaccurate when approaching the maximum packing fraction of the suspension ($\phi_{max} \sim 0.6$).

The reduction of the mean settling velocity $\langle V \rangle_p$ with $\phi$ is due to the hindrance effect (Climent & Maxey 2003; Guazzelli & Morris 2011). To better understand this effect, let’s consider a batch sedimentation system with a fixed bottom. The presence of the fixed bottom constrains the mean velocity of the mixture $\langle U_m \rangle$ to vanish:

$$\langle U_m \rangle = \phi \langle V \rangle_p + (1 - \phi) \langle U \rangle_f = 0$$

(3.5)

where $U$ is the fluid velocity and $\langle \cdot \rangle$, $\langle \cdot \rangle_p$ and $\langle \cdot \rangle_f$, denote averages over the entire suspension, and over the solid and fluid phases. Therefore, since particles fall towards the rigid bottom, the fluid is forced on average to move in the opposite direction hindering the settling. This dominant effect leads to the reduction of $\langle V \rangle_p$ with respect to $V_s$ and becomes more pronounced as $\phi$ increases.

The problem is further complicated when inertial effects become important. Indeed, at finite terminal Reynolds numbers, $Re_t = 2a|V|/\nu$, the assumption of Stokes flow is less acceptable (especially for $Re_t > 1$), the fore-aft symmetry of the fluid flow around the particles is broken and wakes form behind them. Solutions should be derived using the Navier-Stokes equations, but the nonlinearity of the convective term makes the analytical treatment of the problem extremely difficult. For this reason theoretical investigations have progressively given way to experimental and numerical approaches. Even if we limit our attention to the settling of an isolated sphere in quiescent fluid, it is usually not possible to accurately estimate the terminal velocity $V_t$ a-priori. As a result, the Reynolds number $Re_t$ cannot be used to characterize the problem. Using the Buckingham $\pi$ theorem, it can actually be shown that two parameters are necessary to describe the problem. These are the solid-to-fluid density ratio, $R$, and the Galileo (or Archimedes) number, $Ga$.

$$Ga = \frac{\sqrt{2g(2a)^3}}{\nu}$$

(3.6)

namely the ratio between the buoyancy and viscous forces acting on the particle. The Archimedes number is simply $Ar = Ga^2$. Particles with different density ratios $R$ and Galileo numbers $Ga$ fall at different speeds and along different paths, exhibiting various wake regimes (Jenny et al. 2004; Bouchet et al. 2006;
3. Sedimentation

Uhlmann & Dušek 2014). Changes in path and wake regimes depend mostly on $Ga$ and less on $R$. At low $Ga$, isolated particles fall along vertical paths with steady axi-symmetric wakes. Increasing $Ga$, the particle motion becomes (in sequence) oblique, time-periodic oscillating, zig-zagging, helical, and chaotic (for very large $Ga \geq 215$).

Broad estimates of the terminal velocity $V_t$ can be obtained by using empirical formulae. Typically, the drag coefficient on the particle, $C_D$, is related to the Reynolds number via nonlinear expressions of the form

$$C_D = \frac{24}{Re_t} \left( 1 + \alpha Re_t^\beta \right)$$

where $24/Re_t$ is the Stokes drag coefficient, while $\alpha$ and $\beta$ are coefficients that change with $Re_t$. In some formulations, also the coefficient $\beta$ can be a function of $Re_t$. Using equation (3.7) we can also derive a formula that relates the Reynolds number directly to the Galileo number. Indeed, the drag coefficient of a sphere settling at finite $Re_t$ is generally defined as

$$C_D = \frac{8D}{\rho_f V_t^2 \pi (2a)^2}$$

where, $D$ is the drag force. However, at steady state it can be assumed that $D$ is just balanced by the buoyancy force

$$D = \frac{\pi (2a)^3}{6} (\rho_p - \rho_f)g$$

and by introducing this definition in equation (3.8) we obtain

$$C_D = \frac{4Ga^2}{3Re_t^2}$$

Hence, from equations (3.7) and (3.10) we finally find

$$Ga^2 = 18Re_t \left( 1 + \alpha Re_t^\beta \right)$$

Note that in this formula, the dependence on the density ratio $R$ is completely neglected. Additionally, the $C_D$ estimated from the best empirical formulae proposed in the literature still vary by $\sim \pm 5\%$ with respect to measured and computed values (Schiller & Naumann 1935; Clift et al. 2005). As a result, a similar deviation is usually observed in the measured terminal velocity $V_t$. For example, from available numerical investigations (Yin & Koch 2007; Uhlmann & Doychev 2014; Fornari et al. 2016c,b), we find that the computed $V_t$ differs by approximately $\pm 6\%$ from that obtained via equation (3.11). Alternatively, in the limit of small $Re_t$, the empirical formulae (3.7) can be replaced by first and higher order analytic correlations. The well-know first order drag coefficient correlation is that proposed by Oseen (Lamb 1932):

$$C_D = \frac{24}{Re_t} \left( 1 + \frac{3}{16} Re_t \right)$$

For a review on these analytic formulae the reader is referred to John Veysey & Goldenfeld (2007).
When suspensions are considered, the scenario is further complicated due to particle-particle interactions dominated by interial effects related to their wakes. A most relevant effect occurs when a sphere is entrained in the wake of another particle of comparable size falling at finite $Re_t$. In the wake, the fluid motion is directed downwards leading to a lower drag force on the trailing particle. The latter will hence accelerate towards the leading particle. Eventually, the particles will touch (or kiss), and finally the trailing sphere will tumble laterally. This phenomenon is denoted as *drafting-kissing-tumbling* of a particle pair (Fortes et al. 1987), and during the draft phase the rear particle reaches speeds larger than the terminal velocity $V_t$. The extent of the increase of the trailing particle velocity with respect to $V_t$ depends on $Ga$. In semi-dilute suspensions ($\phi = 0.5\% - 1\%$) of spheres with density ratio $R = 1.02$ and $Ga = 145$ (Fornari et al. 2016c), we have found that these events are frequent and that involved particles reach falling speeds that are more than twice the mean $\langle V \rangle_p$. We also estimated that without these intermittent events, the mean settling velocity $\langle V \rangle_p$ would be smaller by about 3%.

In the inertial regime, the mean settling velocity $\langle V \rangle_p$ of a suspension of spheres has been recently shown to depend deeply on the Galileo number $Ga$. Below the critical $Ga \approx 155$, hindrance is still the dominant effect and the reduction of $\langle V \rangle_p/V_t$ with the volume fraction can be estimated sufficiently well with the modified Richardson & Zaki correlation:

$$\frac{\langle V \rangle_p}{V_t} = k [1 - \phi]^n$$

(3.13)

where $k$ is a correction coefficient for finite $Re_t$ that has been found to be in the range $0.8 - 0.92$ (Di Felice 1999; Yin & Koch 2007). The exponent $n$ has also been shown to be a nonlinear function of $Re_t$:

$$\frac{5.1 - n}{n - 2.7} = 0.1 Re_t^{0.9}$$

(3.14)

Note that equation (3.14) is also empirical. In figure (3.1) we show the typical mean settling speed of a suspension at finite Reynolds number, calculated using equations (3.4) and (3.13). Our results obtained via direct numerical simulations are also reported.

Above the critical $Ga$, however, equation (3.13) cannot be used to predict $\langle V \rangle_p$ as more complex particle-particle interactions occur. For example, for $R = 1.5$ and $Ga = 178$ Uhlmann & Doychev (2014) showed that clustering of particles occurs and, surprisingly, a suspension with $\phi = 0.5\%$ settles on average 12% faster than a single isolated particle. The formation of clusters is related to the steady oblique motion observed for isolated spheres with this combination of $R$ and $Ga$. Indeed, when $Ga < 155$ isolated spheres exhibit a steady vertical motion and no clustering is observed. These results were also confirmed numerically by Zaidi et al. (2014), and experimentally by Huisman et al. (2016), who also observed the formation of a columnar structure of falling spheres.
3.2. Spherical particles in turbulence

The problem becomes even more complex when the particles are suspended in a turbulent field. Indeed in a turbulent flow, many different spatial and temporal scales are active and the motion of a particle does not depend only on its dimensions and characteristic response time, but also on the ratios among these and the characteristic turbulent length and time scales. The turbulent quantities usually considered are the Kolmogorov length and time scales ($\eta = (\nu^3/\epsilon)^{1/4}$ and $t_{\eta} = (\nu/\epsilon)^{1/2}$ where $\epsilon$ is the energy dissipation) which are related to the smallest eddies. Alternatively, the integral length scale ($L_0 = k^{3/2}/\epsilon$ where $k$ is the turbulent kinetic energy) and the eddy turnover time ($T_e = k/\epsilon$) can also be used.

For the case of a small rigid sphere settling in a nonuniform flow, an equation of motion was derived already in the late 1940s and 1950s by Tchen (1947) and Corrsin & Lumley (1956). In the derivation, the authors assumed the particle Reynolds number to be very low, so that the unsteady Stokes equations could be solved. The added mass (the volume of surrounding fluid accelerated by the moving particle) and the Basset history forces were also included. The Basset force describes the temporal delay in the boundary layer development as the relative velocity (between fluid and particle) changes with time.

The equations were later corrected by Maxey & Riley (1983) and Gatignol (1983) to include the appropriate Faxén forces that appear in a nonuniform unsteady Stokes flow, due to the curvature of the velocity profile (Faxén 1922). The final form of this equation is often referred to as the Maxey-Riley equation.
and reads:
\[
\frac{4}{3} \pi a^3 \rho_p \frac{dV_p}{dt} = \frac{4}{3} \pi a^3 (\rho_p - \rho_f) g + \frac{4}{3} \pi a^3 \rho_f \frac{Du_f}{Dt} |_{x} - 6\pi a \mu [V_p - u_f(x, t)] - \frac{2}{3} \pi a^3 \rho_f \frac{d}{dt} [V_p - u_f(x, t)] - 6\pi a^2 \mu \int_0^t \left( \frac{d/\tau[V_p - u_f(x, t)]}{[\pi \nu(t - \tau)]^{1/2}} \right) d\tau
\]
(3.15)
where \( D/Dt|_{x} = \left( \frac{\partial}{\partial t} + u_f \cdot \nabla \right) \) is a time derivative following a fluid element, while \( d/dt = \left( \frac{\partial}{\partial t} + V_p \cdot \nabla \right) \) is a time derivative following the moving sphere. The terms on the right hand side of equation 3.15 are the buoyancy force, the stress-gradient force (related to the pressure gradient of the undisturbed flow), the viscous Stokes drag, the force due to the added mass and the Basset history force. For simplicity the Faxén corrections have been neglected in the last three terms.

The Maxey-Riley equation is usually extended to the case of low particle Reynolds numbers by using empirical nonlinear drag corrections (Clift et al. 2005) such as
\[
C_D = \frac{24}{Re_p} (1 + 0.15 Re_p^{0.687})
\]
(3.16)
and by changing the integration kernel of the history force, e.g., as proposed by Mei & Adrian (1992) and Mei (1994). Indeed, at small and finite Reynolds numbers, convective inertia changes fundamentally the temporal decay of the history force. It can be shown that at large times, the integration kernel decays (faster) as \( t^{-2} \), instead of \( t^{-1/2} \). In the limit \( 0 < Re_t \lesssim 1 \), Lovalenti & Brady (1993) used the reciprocal theorem to derive a new equation for particle motion that includes the effects of convective inertia. The difference with respect to equation (3.15) is in the history force that, however, tends to the Basset formulation for \( Re_t \to 0 \). However, the correction to the unsteady term is of order \( O(Re_t) \) and, as a consequence, the force is overestimated for \( Re_t \geq 1 \).

Recently, Loth & Dorgan (2009) tried to extend equation (3.15) to account for finite-size particles by means of spatial-averaging of the continuous flow properties. In fact, the main problem of the Maxey-Riley equation is that it is not strictly valid for particles that are substantially larger (in size) than the smaller scales of the flow. In such cases, a substantial amount of relative motion between particles and fluid is generated and the Reynolds number \( Re_t \) increases appreciably. As mentioned above, the behavior of the unsteady forces changes and the drag becomes nonlinear in \( Re_t \). Additionally, hydrodynamic interactions between particles (i.e., lubrication) are not captured.

From a numerical point of view, the Maxey-Riley equation is typically used to study the sedimentation of very dilute suspensions of small (either sub-Kolmogorov or Kolmogorov size) heavy particles, with low mass loadings. Indeed, at low mass and volume fractions (\( \phi \leq 10^{-6} \)), particles behave as if they were isolated and do not back-influence the carrier fluid. Since the fluid flow is
unaltered and there are no hydrodynamic- and particle-particle interactions, this is usually known as the one-way coupling regime (Balachandar & Eaton 2010). Instead, when the mass loading becomes important and the volume fraction increases \(10^{-6} \geq \phi \leq 10^{-3}\), the back-reaction due to the solid phase must be considered and we enter the so-called two-way coupling regime. In this regime, particle-particle interactions are still assumed to be negligible.

For dilute suspensions of small heavy particles it has been shown that turbulence can either enhance, reduce or inhibit the settling. Weakly inertial particles may be indefinitely trapped in a forced vortex (Tooby et al. 1977) but as particle density increases, their settling speeds may be increased due to the transient nature of turbulence. Indeed, it has been shown that highly inertial particles tend to be expelled from the vortex cores and accumulate in regions of low vorticity and high strain rate (Squires & Eaton 1991). Owing to this and to gravitational settling, particles are often swept into regions of downdrafts (the so called preferential sweeping). In doing so, the particles are also fast-tracked through the eddies and the mean settling velocity increases with respect to the terminal velocity in quiescent fluid. This was first observed by Maxey & Corrsin (1986); Maxey (1987) and Wang & Maxey (1993), who studied the settling of dilute suspensions of heavy point-particles (i.e. one-way coupled) in random flows and in homogeneous isotropic turbulence. These findings have later been confirmed by various experiments (Nielsen 1993; Aliseda et al. 2002; Yang & Shy 2003, 2005). In the two-way coupling regime, the mean settling velocity is further increased as shown by Bosse et al. (2006). In fact, where particles are preferentially concentrated (i.e., the regions of low vorticity), the surrounding fluid is accelerated due to particle drag and in turn this leads to an additional increase in \(\langle V \rangle_p\).

The mean settling velocity can also be reduced with respect to \(V_t\). Such behavior has been observed in both experiments (Murray 1970; Nielsen 1993; Yang & Shy 2003; Kawanishi & Shiozaki 2008) and simulations Wang & Maxey (1993); Good et al. (2014). However, in point-particle simulations the reduction of mean settling velocity can only be observed if nonlinear drag corrections are employed (i.e. for finite particle Reynolds numbers, \(Re_t\), as shown by Good et al. 2014). Nielsen (1993) suggested that fast-falling particles that bisect both downward- and upward-moving flow regions, need a longer time to cross the latter (a phenomenology usually referred to as loitering), especially if the particles settling speed is similar to the turbulence velocity fluctuations \(u'\). Good et al. (2014) performed a series of experiments and numerical simulations and found that reduction of the mean settling velocity occurs when the ratio \(\tau_p g / u'\) (where \(\tau_p = 2(\rho_p / \rho_f - 1)a^2 / (9\nu)\) is the inertial response time) is greater than one (i.e. when the particle terminal velocity is larger than the turbulent velocity fluctuations). Heuristically it can be said that when \(\tau_p g\) (the Stokes settling velocity) is sufficiently high, the particles fall along almost straight vertical paths, their horizontal velocity fluctuations are weak and hence they are unable to side step the turbulent eddies: fast-tracking is suppressed and
the mean settling velocity reduces due to the drag increase related to finite Reynolds number.

When finite-size particles are considered, we are typically in the four-way coupling regime since both mass loading and volume fraction are high ($\phi \geq 10^{-3}$), and interactions among particles cannot be neglected. Due to the difficulty of dealing with hydrodynamic and particle-particle interactions, there are up to date very few studies on the settling of such particles in turbulent flows. The experiments of Byron (2015) investigate the settling of Taylor-scale particles in turbulent aquatic environments using refractive-index-matched hydrogel particles and particle image velocimetry (PIV). These authors found that particles with quiescent settling velocities of the same order of the turbulent rms velocity $u'$, fall on average 40 – 60% more slowly than in quiescent fluid (depending on their density and shape). However, the reason behind the mean velocity reduction remains unclear. The overall drag acting on the particles could be increased either by stronger nonlinear effects due to the substantial amount of relative motion generated among phases (Stout et al. 1995), or by important unsteady history effects (Mordant & Pinton 2000; Sobral et al. 2007; Olivieri et al. 2014; Bergougnoux et al. 2014). Other interesting aspects that should be explored are, for example, one- and two-particle dispersions, particle velocity correlations and collision rates, turbulence eddies interactions with particle wakes and boundary layers, and more in general, the turbulence modulation at different Galileo numbers $Ga$, relative turbulence intensities $u'/V_t$, and solid volume fractions $\phi$.

The modulation of turbulence due to the addition of inertial particles is non-trivial and depends on a variety of factors such as surface effects, mass loading effects, inertial effects and time response effects. Surface effects become important when particles are larger than the dissipative Kolmogorov scale. The presence of such particles leads to a spatial filtering of the turbulent flow and to a consequent modification of the turbulence structures. Specifically, large non-sedimenting particles in homogeneous isotropic turbulence lead to an increase in the energy dissipation rate and a decrease in turbulence kinetic energy (Ten Cate et al. 2004; Yeo et al. 2010). The kinetic energy content decreases at large scales (except at the very large length scales where the energy is added to the system) while it increases at small scales, near the particle surfaces (Yeo et al. 2010). The pivoting length scale characterising the transition from damped to enhanced energy content has been shown to vary with the size of the particles, the ratio of particle size to flow scales and the density ratio $R$ (Gao et al. 2013). On the other hand, for Taylor-scale sized particles in decaying homogeneous isotropic turbulence Lucci et al. (2010) found that the kinetic energy is always reduced while the viscous dissipation is enhanced.

In all these studies, however, the effects of gravity were neglected. When particles are allowed to settle, it is observed that the energy dissipation rate is enhanced mostly in the front part of the particles. Additionally, turbulence kinetic energy can be increased due to the large velocity fluctuations induced by
the unsteady wakes of the falling particles. Anisotropy is therefore introduced in the system and the problem of turbulence modulation is further complicated. Ulterior investigations on the topic are certainly required.

3.3. Elliptic particles in quiescent fluid

When considering non-spherical particles, the sedimentation process is further complicated as the particle orientation plays a role in the dynamics.

Following Clift et al. (2005), we can classify particles based on their symmetry properties. This classification is useful to roughly predict how a certain particle move and behave. A particle is axisymmetric if its body is generated by rotating a closed curve around an axis. Within this category, spheroidal particles are of great interest as their shapes resemble those of drops, bubbles and some solid particles. Specifically, a spheroid is generated by rotating an ellipse about one of its principal axes. If this is the minor axis, the body is defined as oblate, otherwise the spheroid is prolate. Particles may also be orthotropic and/or spherically isotropic. Bodies that have three mutually perpendicular planes of symmetry are defined as orthotropic; if they are also isometric, then they are spherically isotropic. For example, cubes are spherically isotropic, spheroids are axysimmetric and orthotropic, and cones are just axysimmetric. Finally, particles may be irregular in shape without any axis of symmetry. This is the case for many particles in real applications. Such particles are then described using a variety of empirical factors: the volumetric shape factor, sphericity, circularity, operational sphericity and circularity, and the perimeter-equivalent factor. For the definition of these factors, the reader is referred to Clift et al. (2005).

The drag and torque that act on a translating and rotating particle of arbitrary shape in creeping flow, are determined by three second-order tensors which depend on the shape of the body and are proportional to the fluid viscosity \( \nu \). These are the symmetric translation tensor (that relates the force to the velocity), the symmetric rotation tensor (that relates torques and rotations), and the asymmetric coupling tensor (that relates torques to translational velocities, and forces to rotations). Due to the symmetry of the translation tensor, every particle has at least three mutually perpendicular axes. Hence, if a particle translates parallel to one of these axes without rotation, the total drag force is also parallel to that axis. Consequently, these axes are known as principal axes of translation. These axes are normal to the planes of symmetry for orthotropic particles, and parallel to the axis of symmetry for axysimmetric particles. Additionally, symmetric particles have a specific point about which the coupling tensor is zero and, as a result, a pure translation in creeping flow does not cause torques.

In this work we will only consider spheroids, and in particular oblate particles. The characteristic dimensions of a spheroidal particle are the equatorial and polar radii, \( b \) and \( c \), as depicted in figure (3.2). The aspect ratio is then defined as \( AR = c/b \). For oblate and prolate particles, \( AR < 1 \) and \( AR > 1 \);
the case of $AR = 1$ corresponds to a sphere. Another useful quantity is the equivalent diameter, $D_{eq} = 2(cb^2)^{1/3}$, i.e. the diameter of a sphere with the same volume of the spheroid. When studying the motion of a spheroid, its orientation must also be considered. The orientation vector is defined as the unit vector of the symmetry (polar) axis. For example, an oblate falling with its broad side perpendicular to gravity has the orientation vector $[O_x, O_y, O_z] = [0, 0, 1]$. The angle $\psi$ between the symmetry axis and the vertical direction is hence $\psi = acos(1) = 0^\circ$.

It is interesting to note that in the limit of vanishing Reynolds number, if a spheroid is released from rest and with a specific orientation (in an unbounded fluid), this will be maintained due to the reversibility of Stokes flow. Instead, at finite $Re$, the flow field exerts a torque that tends to turn the particle to the horizontal direction. This was shown, for example, by Feng et al. (1994) who performed two-dimensional numerical simulations of settling elliptic particles. The authors showed that in stable conditions an elliptic particle always falls with its long axis perpendicular to gravity. Three-dimensional oblates settling in steady motion at low $Re$ also display the symmetry axis in the gravity direction. However, increasing $R$ or $Ga$ the system becomes unstable and disc-like particles are observed to oscillate horizontally. For such particles, the Galileo number $Ga$ is typically defined by using the equivalent diameter $D_{eq}$. The settling regimes of disc-shaped cylinders and oblate particles with different aspect ratios $AR$, density ratios $R$, and Galileo numbers $Ga$ have been studied in detail by Chrust (2012). As for spheres, different falling paths and wakes are observed. At low $Ga$, an isolated particle settles along a straight vertical path with an axisymmetric wake. Increasing $Ga$, the particle first exhibits a vertical oscillatory motion that changes to oblique oscillatory and, finally, to chaotic motion (e.g., at large $Ga \geq 240$ for $AR = 1/3$ and $R = 1.14$). As explained by Magnaudet & Mougin
(2007); Ern et al. (2012), the path instability of spheroidal particles is closely related to their wake instability. Indeed, the release of vortices in the wake of a spheroid is modified as soon as the angle between the symmetry axis and the velocity direction is changed. Therefore, the ensuing wake instability is also strongly related to the particle aspect ratio $AR$.

The case of inertial prolates has been studied recently by Ardekani et al. (2017). It was shown that above a threshold $Ga$, while oblate particles perform a zigzagging motion, prolate particles rotate around the vertical axis with their broad side facing the falling direction. Additionally, the threshold $Ga$ is found to decrease as the aspect ratio $AR$ departs from 1. Particle-pair interactions were also studied and it was found that the drafting-kissing-tumbling is modified with respect to the case of settling spheres. In particular, for two oblate particles with $AR = 1/3$ and $Ga = 80$, the tumbling part is suppressed and the particles fall together with a mean speed that is substantially larger than $V_t$. Moreover, particles are attracted in the wake of a leading spheroidal particle from larger lateral distances than in the case of spheres. The absence of the tumbling phase was also found experimentally for pairs of falling disks with $AR = 1/6$ by Brosse & Ern (2011).

Much less is known about the sedimentation of suspensions of spheroidal particles. A pioneering study is that by Fonseca & Herrmann (2005), who studied numerically the settling of suspensions of oblates with $AR \simeq 0.27$ and $R = 4$ at the relatively low Reynolds numbers, $Re_t = 0.04$ and 7. Volume fractions up to $\phi \sim 20\%$ were considered. At the smallest $Re_t$, these authors found a local maximum of the mean settling speed $\langle V \rangle_p$ at $\phi = 5\%$. This value is however smaller than the terminal speed of an isolated oblate, $V_t$. On the other hand, at $Re_t = 7$ it was found that $\langle V \rangle_p \simeq 1.1V_t$ for $\phi \sim 1\%$. In any case, the behavior of suspensions of inertial oblate and prolate spheroids depends on four parameters, $AR$, $R$, $Ga$ and $\phi$, that up to date have been only scarcely explored.
Chapter 4

Particles in shear flows

4.1. Stokesian and laminar regimes

Understanding the rheological properties of suspensions in shear flows is not only a challenge from a theoretical point of view but has also an impact in many industrial applications. Even restricting the analysis to monodisperse rigid neutrally buoyant spheres in the viscous or laminar regime, the flow of these suspensions shows peculiar rheological properties such as shear thinning or thickening, normal stress differences and jamming at high volume fractions (Stickel & Powell 2005; Morris 2009). Indeed, the response to the local deformation rate is altered by the presence of the dispersed phase and, as a consequence, the effective viscosity of the suspension \( \mu_e \) differs from that of the pure fluid.

The earliest works in the field were those by Einstein (1905, 1906), who derived an expression to calculate the effective viscosity of a monodispersed suspension of neutrally buoyant spheres. The assumptions of Stokes flow and very low volume fractions were made (so that hydrodynamic interactions among particles are neglected). This expression, usually termed as Einstein viscosity, reads

\[
\mu_e = \mu \left(1 + \frac{5}{2} \phi\right)
\]

and shows that the normalized viscosity \( \mu_e/\mu \) grows linearly with the volume fraction \( \phi \). Due to the assumptions adopted in the derivation, equation (4.1) is valid only for very small \( \phi \). For higher volume fractions, equation (4.1) underpredicts the effective viscosity \( \mu_e \) since it does not account for particle interactions that would yield a viscosity contribution of \( O(\phi^2) \). The \( O(\phi^2) \) correction has been found for pure straining flow by Batchelor & Green (1972).

The mutual interactions between particles become increasingly critical when increasing the volume fraction. In the denser regime the effective viscosity increases by more than one order of magnitude until the system jams behaving as a glass or crystal (Sierou & Brady 2002). Moreover, as the system approaches the maximum packing limit, the effective viscosity of the suspension diverges (Boyer et al. 2011). The variation of \( \mu_e \) with \( \phi \) cannot be described anymore by the formulae of Einstein (1905, 1906) & Batchelor & Green (1972), and semi-empirical correlations are used instead (Stickel & Powell 2005). Among
Figure 4.1: The normalized viscosity $\mu_e/\mu$ as function of the volume fraction $\phi$. The Eilers fit, equation (4.3), is shown with a dashed line. Results from direct numerical simulations of a laminar duct flow laden with suspensions of rigid spheres are also reported. Note that in these simulations, the particle Reynolds number is approximately 2.

these we recall the fit by Krieger & Dougherty (1959)

$$\mu_e = \mu \left(1 - \frac{\phi}{\phi_m}\right)^{\frac{1}{2}\phi_m}$$

and the Eilers fit (Ferrini et al. 1979)

$$\mu_e = \mu \left(1 + \frac{\frac{1}{2}[\mu]\phi}{1 - \phi/\phi_m}\right)^2$$

where $[\mu]$ is the so-called intrinsic viscosity, and $\phi_m$ is the maximum packing fraction. Note that $[\mu] = 5/2$ in equation (4.1); typically $[\mu] = 2.5 - 3$. The maximum packing fraction, $\phi_m$, depends on particle size and shape, and on the imposed shear rate (Konijn et al. 2014). For suspensions of hard spheres, $\phi_m$ is chosen to be either 0.58 or 0.64 (Shewan & Stokes 2015). These values correspond respectively to the glass transition (typically observed in suspensions of colloidal particles\(^1\)), and to the random close packing (i.e. the most consolidated packing achievable by vibrating a container of particles). The normalized viscosity, $\mu_e/\mu$, obtained via the Eilers fit, equation (4.3), is shown in figure (4.1), together with our results from direct numerical simulations.

The intrinsic viscosity $[\mu]$ is directly related to the particle contribution to the average stress of the suspension, $\sigma^p$. For clarity, we will refer to these as

\(^1\)Colloids contain particles of size ranging from nanometers up to few micrometers. These particles usually undergo Brownian motion (the random motion resulting from collisions between particles and fast-moving atoms or molecules in the fluid).
In the limit of creeping flow, and for dilute monodispersed suspensions of spheres (Einstein 1905, 1906), we have:

\[ \sigma^p = 2[\mu] \phi \mu E^\infty = 5 \phi \mu E^\infty \]  

(4.4)

where \( E^\infty \) is the deformation tensor in the absence of any disturbance due to the particles. More generally, the particle-induced stress by rigid spheres in the absence of external torques reads (Batchelor 1970)

\[
\sigma_{ij}^p = \frac{1}{V} \sum A_p \int \frac{1}{2} \left( \sigma_{ik} x_j + \sigma_{jk} x_i \right) n_k \, dA - \frac{1}{V} \sum \frac{1}{2} \rho_f \left\{ f'_i x_j + f'_j x_i \right\} \, dV +
\]

\[- \frac{1}{V} \int V \rho_f u'_i u'_j \, dV + \sigma_{ij}^c \]  

(4.5)

where \( A_p \) and \( V_p \) are the particle surface area and volume, \( \sigma_{ij} \) is the fluid stress tensor at the particle surface, \( x_i \) is the material point, \( f'_i \) is the local acceleration of the particle relative to the mean acceleration in the averaging volume \( V = V_f + V_p \), and \( \Sigma \) represents a summation over all particles. The first term on the right hand side of equation (4.5) is the stresslet contribution to the bulk stress (obtained with the assumption of uniform stress state), which will be discussed later. The second term is the moment of the local material acceleration and is related to the stress inside the particle assuming the absence of all non-hydrodynamic forces (\( \nabla \cdot \sigma = D (\rho_f u_f) / Dt \), see Haddadi & Morris 2014). As pointed out by Marchioro et al. (1999), the physical meaning of the pressure and stress inside rigid particles is unclear. However, these terms appear in the typical two-phase flow averaged equations. The third term is a Reynolds stress related to the particle-induced fluctuating velocities. The last term, \( \sigma_{ij}^c \), is the interparticle stress that originates from near-field particle interactions (e.g. contact forces such as friction).

We have mentioned earlier that first term on the right hand side of equation (4.5) is due to the stresslet. The stresslet

\[ S_{ij} = \int \frac{1}{2} \left( \sigma_{ik} x_j + \sigma_{jk} x_i \right) n_k \, dA \]  

(4.6)

is the symmetric part of the first moment of the fluid stress tensor at the rigid particle surface. Note that the form is different for a deformable particle. Physically, the stresslet results from the resistance of the rigid particle to the straining motion. In Stokes flow, the stresslet of a rigid sphere is

\[ S_{ij} = \frac{20 \pi}{3} \mu a^3 E_{ij}^\infty \]  

(4.7)

and in the assumption of \( \phi \to 0 \), the particle-induced stress is given by \( \sigma_{ij}^p = n_0 \langle S_{ij} \rangle_p \), where \( n_0 \) is the number density of spheres. For equally-sized spheres, \( \langle S_{ij} \rangle_p \) is simply replaced by equation (4.7), resulting in equation (4.4). The effective viscosity \( \mu_e \) and the stresslet \( S_{ij} \) are hence strictly related\(^2\).

\(^2\)The bulk stress of the mixture is defined as: \( \sigma^m = -(p) \mathbf{I} + 2 \mu E^\infty + \sigma_0^p \). To obtain the Einstein viscosity we simply introduce the definition (4.4) finding: \( \sigma^m = -(p) \mathbf{I} + 2 \mu (1 + 2.5 \phi) E^\infty = -(p) \mathbf{I} + 2 \mu_e E^\infty \)
The suspension effective viscosity has also been shown to be a function of the shear rate $\dot{\gamma}$. Specifically, using again the Buckingham $\pi$ theorem and considering neutrally buoyant systems at steady state, it is found that $\mu_e/\mu = f(\phi, Re_\dot{\gamma}, Pe_\dot{\gamma})$, where

$$Re_\dot{\gamma} = \frac{a^2\dot{\gamma}}{\nu}, \quad Pe_\dot{\gamma} = \frac{6\pi \mu a^3 \dot{\gamma}}{kT}$$

(4.8)

(see Stickel & Powell 2005). The first dimensionless group is the particle Reynolds number defined with the shear rate $\dot{\gamma}$, while the second is the Péclet number (i.e. the ratio of shear-driven to Brownian motion), where $kT$ is the thermal energy ($k$ is the Boltzmann’s constant). In the limit of vanishing $Re_\dot{\gamma}$, $\mu_e/\mu = f(\phi, Pe_\dot{\gamma})$ and semi-empirical relations were provided by Krieger (1972). Instead, for non-Brownian particles $Pe_\dot{\gamma} \to \infty$ and we can assume that $\mu_e/\mu = f(\phi, Re_\dot{\gamma})$. Notice also that an additional dimensionless number can be introduced from the definitions of $Re_\dot{\gamma}$ and $Pe_\dot{\gamma}$. This is the Schmidt number (namely the ratio of viscous to thermal diffusion), defined as $Sc = Pe_\dot{\gamma}/Re_\dot{\gamma} = (6\pi \mu^2 a)/(\rho f kT)$.

The dependence of the effective viscosity on the shear rate is of particular interest. It is known that as the shear rate increases, the suspension shear thins from a zero-shear rate plateau viscosity until a minimum (or a second Newtonian plateau) is reached. For large $Pe_\dot{\gamma}$, further increasing the shear rate leads to shear-thickening of the suspension (Foss & Brady 2000; Stickel & Powell 2005).

For these complex fluids, the linear relationship between shear stress and shear rate is lost. The relation between these quantities is instead often described via a power law of the form

$$\sigma_{xy} = m(\dot{\gamma})^q$$

(4.9)

where $m$ and $1$ are the consistency and power law index. Consequently, the viscosity $\mu_e$ changes proportionally to $\dot{\gamma}^{q-1}$. The shear-thinning behavior is recovered with $q < 1$, while $q > 1$ describes shear-thickening. Shear-thinning is observed in many polymer melts ($q \approx 0.3 - 0.7$), and in fine particle suspensions like kaolin-in-water ($q \approx 0.1 - 0.15$). On the other hand, shear-thickening is observed in suspensions of corn flour in water (Chhabra 2010; Mewis & Wagner 2012). The problem with equation (4.9) is that it does not predict the Newtonian plateaus. Hence, more complicated empirical formulae are typically used, like the Cross-type viscosity model for shear-thinning suspensions.

Following Brown & Jaeger (2014), we can distinguish between continuous and discontinuous shear-thickening (DST). The former appears as a continuous increase in shear stress (or $\mu_e$) with $\dot{\gamma}$, and is usually absent for dilute suspensions. Instead, in DST both viscosity and shear stress appear to jump discontinuously (by orders of magnitude) as $\dot{\gamma}$ increases. Typically, DST follows the continuous shear-thickening for increasing packing fractions. Generally, shear-thickening is attributed to three different mechanisms: 1) the formation of hydroclusters at large shear, which in turn lead to increased lubrication drag forces (Brady & Bossis 1985); 2) the modification of particle layers from ordered to disordered
structures (the order-disorder transition) that results in drag increase (Hoffman 1974); 3) dilatancy, i.e. the increase under shear of the particulate packing volume, which results in additional stresses from solid-solid friction (Brown & Jaeger 2012).

The viscosity of suspensions may also exhibit time-dependent behavior. For example, when a steady shear is imposed on thixotropic materials, the viscosity decreases with time until it reaches an asymptotic value. However, after a period at rest, the suspension will recover its initial viscosity. When the shear rate is changed, these materials also show a time-dependent stress response. Thixotropy is believed to be related to a shear-induced change in the microstructure of the suspension (Guazzelli & Morris 2011). The opposite behavior to thixotropy is known as rheopexy (i.e. the viscosity increases under constant steady shear).

Changes in volume fraction and shear rate may also induce normal stress differences in dense suspensions of hard spheres, especially when $\phi > 0.25$ (Guazzelli & Morris 2011). In a Newtonian fluid, linear shear flow generates no normal stresses $\sigma_{ii}$ since there is no pressure response. However a suspension in shear flow exerts normal stresses that can be different in each direction. Therefore, the normal stress in a shear suspension loses isotropy and normal stress differences can be defined as $N_1 = \sigma_{xx} - \sigma_{yy}$ and $N_2 = \sigma_{yy} - \sigma_{zz}$ (in a frame of reference where $u_x = \dot{\gamma} y$).

Similar rheological behaviors can also be observed in the weakly inertial regime (Kulkarni & Morris 2008b; Picano et al. 2013; Zarraga et al. 2000). Indeed, when the particle Reynolds number $Re_\dot{\gamma}$ is finite, the symmetry of the particle pair trajectories is broken and the microstructure becomes anisotropic inducing shear thickening and normal stress differences (Kulkarni & Morris 2008a,b; Yeo et al. 2010). It is generally found that the effective viscosity, $\mu_e$, increases more than linearly with the Reynolds number, $Re_\dot{\gamma}$.

Recently Picano et al. (2013) showed that at finite inertia the microstructure anisotropy results in the formation of shadow regions with no relative flux of particles. Due to these shadow regions, the effective volume fraction increases and shear-thickening occurs. However, the understanding of such flows is still far from complete and research on the field is very active. For example, Davit & Peyla (2008) recently found that in the limit of $Re_\dot{\gamma} \to 0$, the effective viscosity $\mu_e$ changes with the gap width between the two parallel walls of the system. This finding motivated us to study the effect of confinement on the rheology of a dense suspension of neutrally buoyant hard spheres ($\phi = 0.3$) in the weakly inertial regime.

4.2. Shear-induced and inertial migration

Particle migration is another important feature observed in wall-bounded flows. Different types of migrations are observed, depending on the particle Reynolds number.
In the viscous regime ($Re_p \to 0$), particles are found to irreversibly migrate from high to low shear rate regions. As a result, in pressure-driven Poiseuille flow (either in tubes or channels), the particles migrate towards the centerline. This so-called *shear-induced* migration was first observed by Leighton & Acrivos (1987) in a Couette rheometer, and later by Koh *et al.* (1994) in a rectangular channel. To explain this phenomenon, Nott & Brady (1994) developed a suspension balance model that relates particle migration to the macroscopic pressure. In particular, the macroscopic pressure must remain constant across the direction of mean motion and as a consequence, particle migrations occur.

When inertial effects become important, a different kind of migration is observed, which is simply known as inertial migration. The inertial migration of neutrally buoyant rigid particles in Poiseuille flow has been the object of several studies since the work by Segre & Silberberg (1962). These authors studied experimentally the flow of a dilute suspension of spherical particles in a tube at very low bulk Reynolds number ($Re_b = O(1)$). They found that particles migrate radially towards a focusing (or equilibrium) position located at a distance of approximately $0.6R$ from the centerline of the tube, being $R$ the pipe radius. Since particles concentrate on this annulus at $0.6R$, this migration has been termed *tubular pinch*.

While shear-induced migration is driven by particle-particle interactions, inertial migration results from fluid-particle interactions within the conduit. Indeed, the focusing position is determined by the balance between the wall repulsive lubrication force and the shear-induced lift force on the particle due to the curvature of the velocity profile (Matas *et al.* 2004b; Zeng *et al.* 2005). As a consequence, the focusing position changes with $Re_b$. For a dilute suspension with $\phi < 1\%$, the effects of $Re_b$ and of the pipe-to-particle size ratio, $R/a$, have been studied by Matas *et al.* (2004c). It was found that as $Re_b$ increases, the focusing position approaches the wall. However, at large $Re_b$ and depending on the pipe-to-particle size ratio, the particles are also found to accumulate on an inner annulus in the cross section. In particular, the formation of the inner annulus was observed for particles with $R/a = 10$, and was related by Matas *et al.* (2004b) to the finite-size of the particles. This speculation aroused from the fact that theoretical predictions using the point-particle approximation, can only predict the progressive shift of the focusing position towards the wall (Matas *et al.* 2004b). More recently, Morita *et al.* (2017) suggested that the occurrence of the inner annulus is a transient phenomenon that would disappear for long enough pipes with $Re_b < 1000$.

In dilute suspensions, spherical particles have also been observed to align into extended and long-lived *trains* (Matas *et al.* 2004b). Additionally, within these trains the axial particle separation has been found to decrease as the particle Reynolds number, $Re_p \sim (a/R)^2 Re_b$, grows.

In the past few years, inertial migration has been used as a passive method for the separation and sorting of cells and particles in microfluidic devices (Di Carlo 2009; Amini *et al.* 2014). In such applications, however, channels with
rectangular cross sections are mostly used and, due to the loss of cylindrical symmetry, the particle behavior is modified with respect to pipe flows.

If we limit our attention to the case of an isolated rigid sphere in a square duct, we see that the particle undergoes an inertial migration away from the core. Differently from pipe flows, however, at least two possible equilibrium positions exist, at the duct corners and at the wall centers (Chun & Ladd 2006). Depending on $Re_b$ and $h/a$ (the duct-to-particle size ratio), one of the two equilibrium positions is stable, while the other is unstable. Typically, for low bulk Reynolds numbers the duct corners are unstable equilibrium positions as shown by Di Carlo et al. (2009). Indeed, the duct wall centers are the only points where the wall lubrication and shear-induced lift forces balance each other and are hence stable equilibrium position. These results have also been confirmed recently by the theoretical work of Hood et al. (2015), in which an asymptotic model was used to predict the lateral forces on a particle, and to determine its stable equilibrium position.

The equilibrium position at the duct corner is unstable until the bulk Reynolds number $Re_b$ exceeds a critical value of about 260 (Nakagawa et al. 2015). Around this critical $Re_b$, additional equilibrium positions are shown to appear on the heteroclinic orbits that join the channel faces and the corner equilibrium positions. As $Re_b$ increases, the wall center equilibrium positions are first shifted towards the wall and later away from it ($Re_b \gtrsim 200$). The corner equilibrium positions are instead monotonically shifted towards the corners.

Interesting behaviors have also been observed for dilute suspensions of spherical particles. For example, Choi et al. (2011) found that for $Re_b = 12$ and relatively high duct-to-particle size ratio ($h/a = 6.25$), a ring of particles parallel to the duct walls is formed, at a distance of around 0.6$h$ from the centerline. However, as $Re_b$ increases to 120, the particle ring breaks and four focusing points are observed at the duct wall centers, see figure (4.2). Similar results have been obtained by Abbas et al. (2014). As for isolated particles, the corner equilibrium positions are observed only at relatively high bulk Reynolds numbers, $Re_b > 250$ (Chun & Ladd 2006; Miura et al. 2014).

In the present work, we further study the inertial migration of spherical particles in a square duct at semi-dilute conditions. In particular, we investigate the effects of varying $Re_b$, $h/a$ and $Re_p$, and we consider different volume fractions ranging from 0.4% to 20%. Indeed, the suspension behavior and inertial migration had not yet been studied at relatively large $\phi$.

4.3. Bagnoldian dynamics

We have seen in chapter 4.1 that the shear stress and effective viscosity of a suspension change as inertial effects become increasingly important.

A seminal work on the behavior of inertial suspensions under shear is that by Bagnold (1954). The author performed a set of experiments on a suspension of neutrally buoyant solid spheres, sheared with increasing $\dot{\gamma}$ in the annular space between two concentric drums. It was shown that the shearing of grains leads to
the generation of a shear stress and a radial dispersive pressure exerted between the grains. The shear stress and the pressure were found to be proportional, and their ratio was found to be approximately constant in both the viscous and inertial regimes (with different values in each regime). In the macro-visous regime, observed at low shear rates, the shear stress was shown to increase linearly with the shear rate. Indeed, in this viscosity dominated regime, the shear stress can be estimated from the fluid viscosity modified by the presence of particles. On the other hand, at high shear rates the dynamics is dominated by inertial effects. Consequently, the grain shear stress can be regarded as an additive contribution to any residual fluid shear stress, and was shown by Bagnold (1954) to be proportional to the square of the shear rate. Note that the quadratic relationship between shear stress and shear rate resembles the dynamics of dry granular-material flows. In these flows, the effects due to the interstitial fluid are negligible and the dynamics is solely dominated by particle-particle interactions (i.e. collisions). As a result, these flows are usually considered as dispersed single-phase rather multiphase flows (Campbell 1990). Since $\mu_e$ scales as $\dot{\gamma}^{q-1}$ and $q = 2$ (see equation 4.9 and its discussion), in this grain-inertial regime the effective viscosity $\mu_e$ is found to increase linearly with the shear rate $\dot{\gamma}$.

In addition, Bagnold (1954) introduced a new dimensionless number (later called the Bagnold number) to distinguish between the different regimes (macro-visous and grain-inertial). The Bagnold number is defined as the ratio between
4.4. Suspensions in turbulent wall-bounded flows

Typically, as the Reynolds number increases, inertial forces become more and more important until the flow undergoes a transition to the turbulent regime. This is generally the case for unladen flows, while the transition may be anticipated, delayed, or even inhibited by the presence of a dispersed solid phase.

For pipe flows laden with spherical particles, the critical Reynolds number (i.e. the value of $Re_b$ above which the transition occurs) has been found to depend on the pipe-to-particle size ratio, $R/a$, and the solid volume fraction, $\phi$ (Matas et al. 2003; Yu et al. 2013). Different transition scenarios are observed depending on particle size. For small neutrally buoyant particles ($R/a > 65$), the critical Reynolds number increases monotonically with the volume fraction $\phi$, due to the raise in effective viscosity. On the other hand, for large particles ($R/a < 65$) transition shows a non-monotonic behavior which cannot be solely explained in terms of an increase of the effective viscosity $\mu_e$.

The critical Reynolds number has been found to decrease in plane channel flows laden with dilute suspensions of finite-size spheres (Lashgari et al. 2015; Loisel et al. 2013). Additionally, at fixed Reynolds number and volume fraction, the initial arrangement of particles was also found to be important to trigger the transition.

Once the flow is turbulent, the dynamics of the suspension is deeply altered due to the interaction between particles and vortical structures of different size. Yet, before looking in detail at the main findings regarding the behavior of both phases, we shall recall some basic knowledge on wall-bounded turbulence.

In wall-bounded turbulent flows it is generally possible to identify an outer length scale (typically the boundary layer thickness or the channel half-width $h$) and an inner length scale typical of a region in which viscous effects are significant (Pope 2000). The inner length scale is expressed as $\delta^* = \nu / U^*_b$ where $U^*_b = \sqrt{\tau_w / \rho_f}$ is the friction velocity, while $\tau_w$ is the wall-shear stress (due to the no-slip boundary condition at the wall). For turbulent channel flow, using these we can define the friction Reynolds number, $Re_f = U^*_s h / \nu$, and the skin-friction coefficient, $C_f = 2 \tau_w / (\rho_f U_b^2) = 2 (U_s / U_b)^2$ (where $U_b$ is the bulk velocity). Note also that in turbulent channel flow, the mean axial pressure

$$Ba = \frac{4 \lambda^{1/2} \rho^{2 \lambda \gamma}}{\nu} = 4 Re_s \lambda^{1/2}$$

(4.10)

where $\lambda = \frac{1}{(0.74 / \phi)^{1/3} - 1}$ is the linear concentration given as the ratio between grain diameter and mean free dispersion distance. For $Ba \leq 40$ the suspension is in the macro-viscous regime, while for $Ba \geq 450$ the suspension is in the grain-inertial regime. In the latter regime, Bagnold also predicted that the residual fluid stress due to turbulence should be progressively replaced by the grain shear stress. However, an overall shear stress increase was observed.

Note that none of the simulations performed in this study are in the grain-inertial regime.
gradient is uniform in the wall-normal direction, and that \(-dp_w/dx = \tau_w/h\) (i.e. the constant mean pressure gradient that drives the flow is balanced by the shear stress gradient). Consequently, the shear stress profile is linear in the wall-normal direction \((\tau(y) = \rho_f \nu dU_{f,x}/dy - \rho_f \langle u_{f,x}' u_{f,y}' \rangle = \tau_w(1 - y/h),\) where \(U_{f,x}(y)\) is the mean streamwise velocity).

In the near-wall flow, different layers can be identified based on \(y^+ = y/\delta_\star\). The layer located at \(y^+ < 50\) is called the viscous wall region. Here, there is a direct effect of viscosity on the shear stress. Instead, the region found at \(y^+ > 50\) is dominated by inertia and is called the outer layer. The overlap region links the two layers. Within the viscous wall region, we can then define the viscous sublayer, the buffer layer and the well-known log-layer. In the viscous sublayer (located at \(y^+ < 5\)), the Reynolds shear stress is negligible compared with the viscous stress, and \(u^+\) increases approximately linearly with \(y^+\). In the buffer layer (found at \(5 < y^+ < 30\)), the transition between the viscosity-dominated and the turbulence-dominated parts of the flow occurs. Finally, in the log-layer (\(y^+ > 30\)) the mean streamwise velocity grows as the natural logarithm of the wall-distance scaled in inner units \((y^+ = y/\delta_\star)\)

\[
U_{f,x}^+ = \frac{U_{f,x}}{\bar{U}_*} = \frac{1}{\kappa} \ln y^+ + B
\]  

where \(U_{f,x}\) is the mean streamwise velocity, \(\kappa\) is the von Kármán constant, and \(B\) is an additive coefficient. This self-similar solution is known as the log law, and it is the specific form at large values of \(y^+\), of the more generic law of the wall (Prandtl 1925). From experiments and direct numerical simulations of turbulent channel flows, the values of these constants are found to be within 5\% of \(\kappa = 0.41\) and \(B = 5.2\).

The addition of polymers or particles to the flow typically results in a modification of the turbulent flow field. As a consequence, the mean streamwise fluid velocity profile is modified with respect to that of the single-phase flow (and so are the profiles of all other relevant mean and fluctuating quantities). Typically, the log law still holds but with different coefficients. Hence, changes in the log law coefficients are related to changes in the overall drag. Typically, a decrease in \(\kappa\) denotes drag reduction while small or negative \(B\) is related to drag increase (Virk 1975).

For suspensions of rigid particles, turbulence modulation is observed when both mass and volume fractions are sufficiently high. On the contrary (i.e. for very dilute suspensions), the particles are transported by the turbulent flow exhibiting interesting dynamics, while the turbulence is unaltered. Many studies have been performed in this one-way coupling regime. It is known that heavy rigid particles smaller than the dissipative scales tend to migrate from regions of high to low turbulence intensities, i.e. towards the wall (Caporaloni et al. 1975; Reeks 1983). This phenomenon is known as turbophoresis, is due to particle inertia, and the turbophoretic velocity is proportional to the gradient of the local turbulence velocity fluctuations. The strength of the turbophoresis effect is related to the ratio between the particle inertial time scale and the
4.4. Suspensions in turbulent wall-bounded flows

The turbulent near-wall characteristic time (Soldati & Marchioli 2009). This non-dimensional number is an important parameter in the study of particle dynamics in turbulence, and it is called the Stokes number \( St = \tau_p / T_f \), where \( \tau_p \) is the particle relaxation time, and \( T_f = \nu / U^2 \) is the viscous time scale of the flow. More recent numerical results have also shown that small-scale clustering occurs and together with turbophoresis this leads to the formation of streaky particle patterns (Sardina et al. 2011, 2012).

When the mass loading of the particles becomes sufficiently high, the back-reaction of the dispersed phase on the fluid must be considered (i.e. we now are in the two-way coupling regime). In turbulent channel flow, sub-Kolmogorov spherical particles at high mass loadings reduce the turbulent near wall fluctuations in the spanwise and wall-normal directions, as well as the Reynolds stress. On the contrary, the streamwise mean and fluctuating velocities are enhanced. Therefore drag reduction is achieved in a fashion similar to that obtained by using polymeric or fiber additives (Zhao et al. 2010). However, when particles larger than the dissipative length scale are considered, both turbulence intensities and Reynolds stress \( \langle u_x u_y \rangle \) are increased (Pan & Banerjee 1996). Indeed, the presence of particles directly affects the ejection-sweep cycle that is connected to turbulence production. In particular, sweeps (i.e. the inrush of high-momentum fluid towards the wall) are suppressed by spheres that are smaller than the dissipative scale, while they are enhanced by larger particles. In the first case, the Reynolds stress and turbulence production decrease; in the second they increase.

When the volume fraction \( \phi \) of these finite-size particles is sufficiently high, all the possible interactions among particles must be considered in DNS. Experiments become instead complicated, since at large concentrations it is difficult to distinguish between the various particles and to estimate many statistics. Concerning turbulent channel flows, Shao et al. (2012) showed numerically that a semi-dilute suspension of neutrally buoyant spheres \( (\phi \approx 7\%) \) attenuates the large-scale streamwise vortices and reduces the fluid streamwise velocity fluctuations (except in regions very close to the walls or around the centerline). On the other hand, the particles increase the spanwise and wall-normal velocity fluctuations in the near-wall region by inducing small scale vortices.

More recently, Picano et al. (2015) observed that as the volume fraction \( \phi \) of a suspension of rigid neutrally buoyant spherical particles is increased from 0% to 20%, also the overall drag is increased. Interestingly, the drag is higher at the highest volume fraction \( (\phi = 20\%) \) although the turbulence intensities, the Reynolds shear stresses, and the von Kármán constant are strongly reduced. Analyzing the mean momentum balance of the mixture (Marchioro et al. 1999; Zhang & Prosperetti 2010), the authors derived the following equation for the total shear stress across the channel, \( \tau(y) \):

\[
\frac{\tau(y)}{\rho_f} = -\langle u'_c u'_c \rangle + \nu (1 - \phi) \frac{dU_f x}{dy} + \frac{\phi}{\rho_f} \langle \sigma_p \rangle = \nu \frac{dU_f x}{dy} \left( 1 - \frac{y}{h} \right) \tag{4.12}
\]
where \( \nu \frac{dU_f}{dy} \bigg|_w \) is the stress at the wall \( \tau_w \). Equation 4.12 shows that the total stress is given by three contributions: the viscous stress, \( \tau_V / \rho_f = \nu(1-\phi) \frac{dU_f}{dy} \); the turbulent Reynolds shear stress \( \tau_T / \rho_f = -\langle u'_{c,x} u'_{c,y} \rangle = -(1-\phi) \langle u'_{f,x} u'_{f,y} \rangle - \phi \langle u'_{p,x} u'_{p,y} \rangle \); and the particle-induced stress \( \tau_P / \rho_f = \phi \langle \sigma_{xy} \rangle / \rho_f \). In Figure (4.3) we show the typical shear stress budget for a monodispersed suspension of spheres with volume fraction \( \phi = 10\% \). Using this stress budget equation, it was found that at the highest volume fraction there is a sharp increase of the particle-induced stress. As a consequence, although the Reynolds stress is reduced, the overall drag increases. The increase in drag is therefore not related to an enhancement of the turbulence activity, but to an increase of the effective viscosity of the suspension.

Based on different volume fractions and bulk Reynolds numbers, Lashgari et al. (2014) further identified three different regimes in which the flow is dominated by one of the components of the total stress. These are the laminar, turbulent and inertial shear-thickening regimes, in which the predominant stress components are the viscous stress, the Reynolds stress, and the particle-induced stress.

As noted by Prosperetti (2015), however, results obtained for rigid neutrally buoyant spherical particles, cannot be easily extrapolated to other cases. Additionally, monodispersed suspensions are hardly found in nature and industrial applications. Therefore, the importance of other aspects such as polydispersity, particle density, shape and stiffness should be studied thoroughly.
In the present work, we consider a turbulent channel flow laden with finite-size rigid spheres of size $a/h = 1/18$, that corresponds to $y^+ \approx 10$. In an idealized scenario where gravity is neglected, we study the effects of varying independently the density ratio $R$ at constant $\phi$, or both $R$ and $\phi$ at constant mass fraction $\chi$. The main scope is to understand independently the importance of excluded volume effects (i.e. of $\phi$) and particle inertia ($R_{cp}$ and $R$) on the statistical observables of both fluid and solid phases. In the same set-up, we also study the effects of polydispersity on the turbulence modulation and on the solid phase behavior. In particular, we consider three different Gaussian distributions of particle radii, centered on the reference radius of the monodispersed suspension, $a/h = 1/18$.

4.5. Turbulent duct flow

Particle-laden turbulent flows are often studied in canonical geometries (i.e. in channels or in boundary layers). However, internal flows relevant to many industrial applications typically involve more complex geometries in which secondary flows, flow separation and other non-trivial phenomena are observed. It is hence important to understand the behavior of particle suspensions in more complex and realistic geometries. In this work we focus on turbulent flows in square ducts, where gradients of the Reynolds stresses induce the generation of mean streamwise vortices. These are known as Prandtl’s secondary motions of the second kind (Prandtl 1953).

To better understand the production of secondary flows, we shall write the Reynolds-averaged streamwise vorticity equation:

\[
0 = U_{f,y} \frac{\partial \Omega_f}{\partial y} + U_{f,z} \frac{\partial \Omega_f}{\partial z} - \nu \left( \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \Omega_f + \frac{\partial^2 \left( \langle u_{f,z}^2 \rangle - \langle u_{f,y}^2 \rangle \right)}{\partial y \partial z} + \left( \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2} \right) \langle u_{f,y}^2 \rangle \langle u_{f,z}^2 \rangle
\]

(4.13)

where the mean vorticity is defined as

\[
\Omega_f = \frac{\partial U_{f,z}}{\partial y} - \frac{\partial U_{f,y}}{\partial z}.
\]

(4.14)

The first two terms on the right hand side of equation (4.13) represent the convection of mean vorticity by the cross-stream flow, and have been shown to be negligible. The third term represents the viscous diffusion of vorticity. The remaining terms are the most interesting ones, being responsible for the generation of mean streamwise vorticity and, consequently, of secondary flows ($U_{f,y}$ and $U_{f,z}$). Specifically, the forth term that involves the gradients in the anisotropy of the cross-stream normal stresses is a source of vorticity in the viscous sublayer. As discussed by Gavrilakis (1992), the production of vorticity within the viscous sublayer is the main responsible for the presence of vorticity in the bulk of the flow. The last term involving the secondary Reynolds stress acts instead as source or sink of vorticity (Gessner 1973; Gavrilakis 1992). In the near-wall region, it typically acts to dissipate the vorticity, in contrast to
the term that involves the normal stress difference. Secondary motions of the second kind are hence entirely due to turbulence and indeed they are absent in laminar duct flows. Note also that differently from channel flows, there are now two inhomogeneous directions and therefore all components of the Reynolds stress tensor play a role in the dynamics. This secondary motions consist of pairs of counter-rotating vortices in each corner, with the flow directed towards the corners along the bisectors. As a result, they convect mean velocity from regions of large shear along the walls towards regions of low shear along the corner bisectors. Typically, their mean (cross-stream) velocity is approximately 2% – 3% of the bulk velocity.

When sub-Kolmogorov heavy particles are released in a turbulent duct flow, these are found to accumulate in regions of high compressional strain and low swirling strength. Hence, these particles are mostly found in the near-wall and vortex center regions (Winkler et al. 2004). Sharma & Phares (2006) showed that there is a clear difference in the motion of low- and high-inertia particles. The former, as well as passive tracers, stay within the secondary swirling flows, circulating between the duct core and the boundaries. On the contrary, high inertia particles accumulate close to the walls, mixing more efficiently in the streamwise direction. In particular, these particles tend to deposit at the duct corners. More recently, Noorani et al. (2016) considered duct flows at higher Re instead than in the work of Sharma & Phares (2006) and studied the effect of varying the duct aspect ratio on the particle transport. It was found that in square ducts, particle concentration in the viscous sublayer is maximum at the centerplane. However, increasing the aspect ratio, the location of maximum concentration moves towards the corners as also the kinetic energy of the secondary flows increases closer to the corners.

Up to date, there are instead very few studies on the dynamics of large particles in turbulent duct flows. Recently, Lin et al. (2017) used a direct-forcing fictitious method to study turbulent duct flows laden with a dilute suspension of finite-size spheres heavier than the carrier fluid. Spheres with radius \(a = 0.1h\) were considered at a solid volume fraction \(\phi = 2.36\%\). These authors show that particles sedimentation breaks the up-down symmetry of the mean secondary vortices. This results in a stronger circulation that transports the fluid downward in the bulk center region and upward along the side walls. Additionally, the mean secondary motions at the bottom wall are enhanced. In turn, this leads to a preferential accumulation of particles at the face center of that wall.

In this work, we study the turbulence modulation and particle dynamics in turbulent square-duct flows laden with dense particle suspensions. In particular, we consider neutrally buoyant finite-size spheres with radius \(a = h/18\), and increase the volume fraction up to \(\phi = 0.2\). Special attention is given to changes in mean streamwise vorticity and secondary motions, friction, particle concentration (in the cross section), and Reynolds stresses.
In the next chapter, the main findings of the works on sedimentation, channel and duct flow, and confined rheology are summarized.
Chapter 5

Summary of the papers

Paper 1

_Sedimentation of finite-size spheres in quiescent and turbulent environments._

Particle sedimentation is encountered in a wide number of applications and environmental flows. It is a process that usually involves a large number of particles settling in different environments. The suspending fluid can either be quiescent or turbulent while particles may differ in size, shape, density and deformability. Owing to the range of spatial and temporal scales generally involved, the interaction between the fluid and solid phases is highly complex and the global properties of these suspensions can be substantially altered from one case to another. Although sedimentation has always been an active field of research, yet little is known about the settling dynamics of suspensions of finite-size particles in homogeneous isotropic turbulence (HIT).

In this work we perform direct numerical simulations of sedimentation in quiescent and turbulent environments using an immersed boundary method to account for the solid phase. We consider a suspension of rigid spheres with diameter of about 12 Kolmogorov length scales $\eta$ and solid-to-fluid density ratio $R = 1.02$. Based on these values, the Galileo number $Ga$ of the particles is about 145. Two solid volume fractions are investigated ($\phi = 0.5\%$ and 1%). An unbounded computational domain with triperiodic boundary conditions is used, ensuring at each time step a zero total volume flux. For the turbulent cases, an homogeneous isotropic turbulent field is generated and sustained using a $\delta$-correlated in time forcing of fixed amplitude. The achieved Reynolds number based on the Taylor microscale $Re_\lambda = \lambda u'/\nu$ (where $\lambda$ is the Taylor microscale and $u'$ is the turbulence root-mean-square velocity) is approximately 90.

Comparing the results obtained in quiescent fluid and homogeneous isotropic turbulence we find the interesting result that finite-size particles settle more slowly in the turbulent environments. The mean settling velocity is reduced by about 8.5% respect to the quiescent cases for both volume fractions. The reduction with respect to the isolated particle in quiescent fluid is about 12 and 14% for $\phi = 0.5\%$ and 1%. The probability density functions (p.d.f.s) of the particle velocities in the directions parallel and perpendicular to gravity are also computed. In the direction of gravity, the p.d.f.s are found to be
almost Gaussian in the turbulent cases while large positive tails are found in the quiescent cases. In the latter, the \textit{p.d.f.}s are positively skewed and the flatness is substantially larger than 3. The positive tails of the \textit{p.d.f.}s are due to the intermittent fast sedimentation of particle pairs in drafting-kissing-tumbling motions (DKT). The DKT is highly reduced in the turbulent cases since the particle wakes are quickly disrupted by the turbulent eddies.

Particle velocity autocorrelations and single particle dispersions are also examined. It is found that particle velocity fluctuations decorrelate faster in homogeneous isotropic turbulence. Particle lateral dispersion is found to be higher in the turbulent cases while the vertical one is found to be of comparable magnitude for all cases examined. However, in the quiescent case at lowest volume fraction ($\phi = 0.5\%$), longer times are needed before the diffusive behavior is reached.

Finally, we estimate the mean relative velocity and the mean drag coefficient of the falling spheres, and we show that non-stationary effects explain the increased reduction in mean settling speed in turbulent environments.

\textbf{Paper 2}

\textit{Reduced particle settling speed in turbulence.}

In this work we further investigate the settling of finite-size rigid spheres in sustained homogeneous isotropic turbulence (HIT). In particular, we study semi-dilute suspensions at constant solid volume fraction ($\phi = 0.5\%$) and at different Galileo numbers $Ga$. The Galileo number is here varied via the solid-to-fluid density ratio $R$, and the focus is on particles that are slightly heavier than the fluid. The homogeneous isotropic turbulent field is generated and sustained using a forcing of fixed amplitude that is $\delta$-correlated in time. The achieved Reynolds number based on the Taylor microscale ($Re_\lambda = \lambda u'/\nu$) is approximately 90 for all cases. On the other hand, varying $Ga$ leads to different terminal falling speeds, $V_t$, and therefore we study the importance of the relative turbulence intensity, $u'/V_t$, on the behavior of the settling suspension.

We find that in HIT, the mean settling speed is less than that in quiescent fluid. In particular, the reduction of the mean settling speed with respect to the terminal velocity of the same isolated sphere in quiescent fluid increases from 6\% to 60\%, as the relative turbulence intensity $u'/V_t$ increases, see figure (5.1). Analysing the fluid-particle relative motion, we find that the mean settling speed is progressively reduced while reducing $R$, due to the increase of the vertical drag induced by the particle cross-flow velocity. Indeed, lighter particles are subjected to horizontal velocity fluctuations one order of magnitude larger than their mean settling speeds. Consequently, the variance of the settling angle (i.e. the angle between the mean particle velocity and the vertical axis) strongly increases above $u'/V_t \sim 1$ (at low $Ga$). Unsteady effects are instead found to contribute by approximately $6\% - 10\%$ to the mean overall drag.
We then compute the probability density functions, p.d.f.s, of particle velocities and accelerations. Concerning the p.d.f.s of velocities we find that the variance is similar in all directions for all $Ga$. It is just slightly smaller in the vertical direction at large $Ga$ (i.e. small $u'/V_t$). The p.d.f.s of accelerations are instead similar for all $Ga$ indicating that particle dynamics is strongly influenced by the features of the turbulent flow field. This is also supported by the results on the particle mean-squared displacement in the settling direction. Indeed, this is found to be similar for all $Ga$ provided that time is scaled by $(2a)/u'$ (where $2a$ is the particle diameter).

**Paper 3**

*Clustering and increased settling speed of oblate particles at finite Reynolds number.*

Most numerical studies on sedimentation at finite Reynolds number deal with spherical particles. However, little is known about non-spherical particles. For such particles, the orientation plays an important role in the dynamics and the settling process is further complicated. Here, we study the settling of rigid oblates in quiescent fluid at finite Reynolds number. We consider semi-dilute suspensions of oblate particles with aspect ratio $AR = 1/3$, solid-to-fluid density ratio $R = 1.5$, Galileo number (based on the diameter of a sphere with equivalent volume) $Ga = 60$, and solid volume fractions $\phi = 0.5\% - 10\%$. With this choice of parameters an isolated oblate falls vertically with a steady wake with its broad side perpendicular to the gravity direction.
Figure 5.2: The mean settling speed of the suspension of oblate particles as function of the volume fraction $\phi$. The empirical fit proposed by Richardson & Zaki for spherical particles of equal $Ga$ is also shown (dashed line).

At this $Ga$, the mean settling speed of spheres is a decreasing function of the volume $\phi$ and is always smaller than the terminal velocity of the isolated particle, $V_t$. On the contrary, we show here that the mean settling speed of oblate particles increases with $\phi$ in dilute conditions and is 33% larger than $V_t$. At higher concentrations, the mean settling speed decreases becoming smaller than the terminal velocity $V_t$ between $\phi = 5\%$ and 10%, see figure (5.2).

We show that the increase of the mean settling speed at low $\phi$ is due to the formation of particle clusters that appear as columnar-like structures. Within these structures, the fluid is strongly dragged by the particles and in the vertical direction it reaches speeds almost of the order of $V_t$. From the pair-distribution function we observe that it is most probable to find particle-pairs almost vertically aligned. However, the pair-distribution function is non-negligible all around the reference particle indicating that there is a substantial amount of clustering at radial distances between 2 and 6$c$ (with $c$ the polar radius of the oblate). Above $\phi = 5\%$ hindrance becomes the dominant effect and the mean settling speed decreases below $V_t$.

As the volume fraction increases, the mean particle orientation changes and the mean pitch angle (i.e. the angle between the horizontal plane and the plane defined by the particle equatorial radius $b$) increases from $23^\circ$ to $47^\circ$. Finally, from the joint $p.d.f.s$ of settling speeds and falling orientation, we find that particles that settle with larger speeds than the mean, fall on average with larger pitch angles.

Paper 4
Figure 5.3: Surface plot of the normalized effective viscosity $\mu_e/\mu$ as a function of $\xi$ and $Re$, for $\phi = 30\%$. The inset shows $\mu_e/\mu$ versus $\xi$ for $\phi = 30\%$ and $Re = 1, 5$ and 10.

Rheology of confined non-Brownian suspensions.

Suspensions of solid particles in simple shear flows exhibit different rheological behaviors depending on their size, shape, volume fraction $\phi$, solid-to-fluid density ratio $R$ and imposed shear rate $\dot{\gamma}$. Typical rheological properties include finite normal stress differences, shear-thinning or thickening, thixotropy and jamming at high volume fractions.

It has been shown that in the weakly interal regime, the symmetry of the particle pair trajectories is broken inducing an anisotropic microstructure. This in turn leads to shear-thickening in dense suspensions of rigid spheres. Recently, intriguing confinement effects have been discovered for suspensions of spheres in the Stokesian regime. Here we therefore investigate the effect of confinement at low but finite particle Reynolds numbers.

In this work we study the rheology of highly confined suspensions of rigid spherical (non-Brownian) particles by performing direct numerical simulations. We considered a plane-Couette flow seeded with neutrally buoyant spheres. The suspension volume fraction is fixed at $\phi = 30\%$ and the confinement is studied by changing the dimensionless ratio $\xi = L_z/(2a)$, where $L_z$ is the channel width. In particular, the channel width is decreased from 6 to 1.5 particle diameters. The simulations are performed at three different particle Reynolds numbers $Re = \rho_f \dot{\gamma} a^2/\mu = 1, 5$ and 10.

The most striking result is that the effective viscosity of the suspension does not show a monotonic behavior with decreasing $\xi$, but rather a series of maxima and minima as shown in figure (5.3). Interestingly the minima are found when the channel width is approximately an integer number of particle diameters.
At these $\xi$ indeed, particle layering occurs and wall-normal migrations are drastically reduced. When layering occurs, the p.d.f. of the particle wall-normal displacement is shown to possess exponential tails indicating a non-diffusive behavior. The typical diffusive behavior is instead recovered at intermediate $\xi$ (i.e. when there is no layering). This is also reflected in the particle mean-squared wall-normal displacement. When there is no layering, the mean-squared displacement grows linearly in time; however for integer values of $\xi$, it follows a power-law of the form $\sim t^\beta$ with $\beta<1$.

Finally, we observe that the two-dimensional particle layers are structurally liquidlike, but their dynamics is frozen in time.

**Paper 5**

*Inertial migration in dilute and semi-dilute suspensions of rigid particles in laminar square duct flow.*

Understanding the inertial migration of particles is of fundamental importance in the context of many industrial and microfluidic applications. While most studies on the topic have focused on very dilute particle suspensions, little is known about denser cases (i.e. at larger volume fractions). In this work, we investigate the inertial migration of finite-size neutrally buoyant spherical particles in dilute and semi-dilute suspensions in laminar square duct flow. In particular, we investigate the effects of the bulk Reynolds number $Re_b$, particle Reynolds number $Re_p$ and duct-to-particle size ratio $h/a$, at different solid volume fractions $\phi$. The volume fraction is increased from 0.4% to 20%. The stress immersed boundary method is used to create four virtual walls.

At low volume fraction ($\phi = 0.4\%$), low bulk Reynolds number ($Re_b = 144$), and $h/a = 9$, we find that particles accumulate at the center of the duct walls. As $Re_b$ is increased, the focusing position moves progressively towards the corners of the duct. At different $h/a$ and same $Re_b = 550$, the focusing positions are always located around (or at) the corners, showing the importance of the bulk Reynolds number. At higher volume fractions, $\phi = 5\%, 10\%$ and 20%, and in wider ducts ($h/a = 18$) with $Re_b = 550$, particles are found to migrate away from the duct core toward the walls. In particular, for $\phi = 5\%$ and 10%, particles accumulate preferentially at the corners. At the highest volume fraction considered, $\phi = 20\%$, particles sample all the volume of the duct, with a lower concentration at the duct core. The mean particle concentration in the duct cross-section is shown in figure (5.4) for $\phi = 5\% - 20\%$, $Re_b = 550$, and $h/a = 18$.

For both $h/a = 9$ and $h/a = 18$, and volume fraction $\phi = 5\%$, the highest particle concentration is found around the focusing points observed in dilute conditions (i.e. at the corners). Instead, for lower bulk Reynolds number $Re_b = 144$, $h/a = 9$, and $\phi = 5\%$ the particle concentration is large all along the walls, with higher values at the locations of the focusing points (i.e. at the wall-bisector). These observations reveal that the mean particle distribution in
Figure 5.4: Contours of mean particle concentration $\Phi$ for $Re_b = 550$, $h/a = 18$. Results for $\phi = 5\%$ are shown in the left half, $z/h \in [-1; 0)$, while the results for $\phi = 20\%$ are shown in the right half, $z/h \in (0 : 1]$.

semi-dilute suspensions at low bulk Reynolds number depends also on the solid volume fraction and the duct-to-particle size ratio.

The presence of particles induces secondary cross-stream motions in the duct cross section, for all $\phi$. The intensity of these secondary flows depends strongly on the particle rotation rate, on the maximum concentration of particles in focusing positions, and on the solid volume fraction. We find that the secondary flow intensity increases with the volume fraction up to $\phi = 5\%$. However, beyond $\phi = 5\%$ excluded volume effects lead to a strong reduction of the cross-stream velocities.

**Paper 6**

*The effect of particle density in turbulent channel flow laden with finite size particles in semi-dilute conditions.*

Suspensions of finite-size particles are also found in many applications that involve wall-bounded turbulent flows. Concerning turbulent channel flows, it has been shown that the presence of a dispersed phase may alter the near-wall turbulence intensities and the Reynolds stress. Therefore, streamwise coherent structures are modified and drag is either enhanced or reduced. For dense suspensions ($\phi \sim 20\%$) of large neutrally buoyant spheres ($a \sim 10^+$), it has been shown that while turbulent stresses are reduced, particle-induced stresses are enhanced. As a consequence, the overall drag is also increased.

Generally, the addition of the particle parameters (size, number, density, shape, stiffness) to those of the fluid generates a vast parameter space. Up
to date, however, only very few of these parameters have been explored. In this work, we study the importance of particle inertia, mass loading and solid volume fraction.

We perform direct numerical simulations of a turbulent channel flow laden with finite-size rigid spheres. The imposed bulk Reynolds number, $Re_b$, of the reference unladen case is chosen to be 5600, giving a friction Reynolds number $Re_\lambda$ of about 180. The ratio between the particle radius and the channel half-width is fixed to $a/h = 1/18$ (i.e. the radius $a$ is about $10^+$). Two sets of simulations are initially performed. First the mass fraction $\chi$ is kept constant while changing both the volume fraction $\phi$ (from 0.2% to 20%) and the density ratio $R$ (from 1 to 100). Then, the volume fraction $\phi$ is fixed at 5% while the density ratio is increased from $R = 1$ to 10. In this idealized study, we neglect gravity to understand the importance of excluded volume effects ($\phi$) and particle inertia ($R$) on the behavior of the suspension.

We find that both fluid and solid phase statistics are substantially altered by changes in volume fraction $\phi$, while up to $R = 10$ the effect of the density ratio is small. Increasing the volume fraction drastically changes the mean fluid velocity profiles and the fluid velocity fluctuations. Respect to the unladen case, the mean streamwise velocity is found to decrease close to the walls and increase around the centerline. Fluid velocity fluctuations are found to increase very close to the walls and to substantially decrease in the log-layer.

In the cases at constant $\phi$, the results at higher $R$ are found to be similar to those of the neutrally buoyant case. The main result found at constant $\phi$ is a shear-induced migration of the particles towards the centerline (absent for $R = 1$), as can be seen from the mean particle concentration profiles depicted in figure (5.5). This effect is shown to be more important at the highest density ratio ($R = 10$) and it is therefore a purely inertial effect.

Finally, we fix the volume fraction at $\phi = 5\%$ and increase the density ratio to $R = 1000$. We observe that under these conditions, the solid phase decouples from the fluid phase. In particular, the solid phase behaves as a dense gas and moves with an uniform streamwise velocity across the channel. Both particle and fluid velocity fluctuations are drastically reduced. Furthermore, the $p.d.f.$ of the modulus of the particle velocity fluctuations closely resembles a Maxwell-Boltzmann distribution typical of gaseous systems. In this regime, we also find that the collision rate is high and governed by the normal relative velocity among particles.

**Paper 7**

*The effect of polydispersity in a turbulent channel flow laden with finite-size particles.*

In this work, we aim to understand the importance of polydispersity in turbulent channel flows laden with finite-size particles. In particular, we study
monodisperse and polydisperse suspensions of neutrally buoyant rigid spheres. Suspensions with 3 different Gaussian distributions of particle radii are considered (i.e. 3 different standard deviations). The distributions are centered on the reference particle radius of the monodisperse suspension. In the most extreme case, the radius of the largest particles is 4 times that of the smaller particles. We consider two different solid volume fractions, 2% and 10%.

We find that for all polydisperse cases, both fluid and particles statistics are not substantially altered with respect to those of the monodisperse case. Mean streamwise fluid and particle velocity profiles are almost perfectly overlapping. Slightly larger differences are found for particle velocity fluctuations. These increase close to the wall and decrease towards the centerline as the standard deviation of the distribution is increased. Hence, the behavior of the suspension is mostly governed by excluded volume effects regardless of the particle size distribution (at least for the radii here studied). Interestingly, we also find that for all polydisperse cases, the mean Stokes number is always approximately equal to that of the monodispersed case. Due to turbulent mixing, particles are uniformly distributed across the channel. However, smaller particles can penetrate more into the inner-wall layer and velocity fluctuations are therein altered.

Non trivial results are presented for particle-pair statistics. In particular, collision rates between particles of different sizes are dominated by the behavior of smaller particles. Additionally, by calculating the impact Stokes number we show that film drainage occurs between particle-pairs, leading to an enduring contact.

Paper 8
Suspensions of finite-size neutrally buoyant spheres in turbulent duct flow.

Particle suspensions are typically studied in canonical geometries (i.e. channel and simple shear flows). However, internal flows relevant to many industrial applications typically involve more complex, non-canonical geometries in which secondary flows, flow separation and other non-trivial phenomena are observed. Here we investigate turbulent duct flows of dense suspensions of finite-size neutrally buoyant particles. We consider ducts of squared cross-section at bulk Reynolds number (based on the hydraulic diameter) $Re_b = 5600$. The four walls are generated by means of the stress IBM. Suspensions consist of rigid spherical particles of size $h/a = 18$, and the solid volume fraction is increased from $\phi = 5\%$ to $20\%$. In turbulent duct flows, due to the additional inhomogeneous direction fluid statistics vary in the cross-section and secondary motions of the second kind appear. Our aim is to investigate how fluid phase statistics change with the presence of solid particles, and how the latter behave in this complex geometry.

We observe that for $\phi = 5\%$ and $10\%$, particles preferentially accumulate on the corner bisectors, close to the duct corners as also observed for laminar duct flows of same duct-to-particle size ratios. At the highest volume fraction, there is still a large particle concentration at the corners. However, particles are here found to preferentially accumulate in the core region. On the contrary, for channel flows particles are found to be uniformly distributed across the channel. The peculiar particle distribution is therefore due to the additional lateral confinement that prevents particles to diffuse away from the core.

We then find that by increasing the volume fraction up to $\phi = 10\%$, the intensity of the cross-stream secondary flows grows (with respect to the unladen case). In agreement, the mean fluid streamwise vorticity is also found to increase. Therefore, the presence of particles introduces a new production mechanism of streamwise vorticity. Instead, for $\phi = 20\%$ the turbulence activity is strongly reduced and the intensity of the secondary flows reduces below that of the unladen case, as can be seen from figure (5.6).

The friction Reynolds number is initially found to increase with $\phi$, as observed for channel flows. However, at the highest $\phi$, the mean friction Reynolds number decreases below the value for $\phi = 10\%$. 


Figure 5.6: Contours and vector fields of the mean fluid cross-stream velocity for the unladen case (top-left quadrant), and for the laden cases with $\phi = 5\%$, $10\%$ and $20\%$ (respectively top-right, bottom-right and bottom-left quadrants).
Chapter 6

Conclusions and outlook

In the present work we have investigated suspensions of finite-size rigid particles in different flow cases. In particular, we have performed direct numerical simulations using an immersed boundary method to account for the dispersed solid phase. We have considered both unbounded, triperiodic computational domains, as well as wall-bounded geometries, at different flow regimes.

6.1. Main results

Concerning unbounded flows, we have studied the settling of semi-dilute suspensions of finite-size spheres in quiescent fluid and in homogeneous isotropic turbulence. The focus has been on particles that are slightly heavier than the fluid. We have explored two solid volume fractions, different Galileo numbers (changed via the density ratio) and, consequently, different relative turbulence intensities (defined as the ratio between the turbulence velocity r.m.s. and the terminal velocity). We have found that the mean settling speed of the suspension is always smaller in homogeneous isotropic turbulence than in quiescent fluid. At large Galileo numbers (and small relative turbulence intensities), this is due to the quick disruption of the wakes that results in less frequent drafting-kissing-tumbling events, and to the appearance of important unsteady effects that increase the overall mean drag. Instead, at low Galileo numbers and large relative turbulence intensities, the reduction of the mean settling speed is substantially larger due to the increase of the nonlinear drag. In this case, turbulent eddies induce strong lateral/cross-flow motions on the settling particles and, consequently, the magnitude of the fluid-particle relative velocity and the particle Reynolds number increase significantly in comparison to the quiescent cases, leading to drag enhancement.

We have also investigated the effects of particle shape on the settling behavior. Indeed, for non-spherical particles the orientation plays a role in the dynamics and the sedimentation process is further complicated. Specifically, we have considered the sedimentation in quiescent fluid of oblate particles with aspect ratio $AR = 1/3$, Galileo number $Ga = 60$ (based on the equivalent diameter), density ratio $R = 1.5$, and solid volume fractions in the range $\phi = 0.5\% - 10\%$. At low volume fractions we have found that the mean settling speed is substantially larger than the terminal velocity of an isolated oblate, $V_t$. 

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This is due to the formation of clusters that appear as columnar-like structures of particles. Note that this behavior is not observed for suspensions of spheres at similar $Ga$ and $\phi$. At larger volume fractions instead, hindrance becomes the dominant effect and the mean settling speed decreases below $V_t$.

We have then focused on wall-bounded flows. One study has been devoted to the understanding of the rheology of highly confined suspensions in simple shear flow. We have considered a plane-Couette flow seeded with neutrally buoyant spheres in the weakly inertial regime. We have found that as confinement is increased, the effective viscosity of the suspension does not change monotonically. Instead, it shows minima when the channel width is approximately an integer number of particle diameters. At these channel widths, layering of particles occurs and wall-normal migrations are drastically reduced. The particle layers have also been found to be structurally liquidlike with a dynamic that is frozen in time.

We have also studied the inertial migration of neutrally buoyant spheres in laminar square duct flow. The focus has been on semi-dilute suspensions at different bulk Reynolds number and duct-to-particle size ratios. At low volume fractions ($\phi = 0.4\%$) and bulk Reynolds numbers we have observed that particles accumulate at the center of the duct walls. Increasing the bulk Reynolds number the location of maximum concentration moves towards the duct corners. At larger volume fractions ($\phi \leq 10\%$), particles have also been found to migrate towards the walls, with a higher concentration around the focusing positions identified at lower $\phi$. The focusing is instead lost when the volume fraction is further increased. We have also discussed the secondary motions induced by the particles. We have shown that their intensity increases with the volume fraction up to $\phi = 5\%$.

Finally, we have investigated suspensions of finite-size spheres in turbulent channel and duct flows. Concerning channel flows, we have first studied the importance of the mass and volume fractions, and the solid-to-fluid density ratio on the statistics of both fluid and solid phases. To better isolate the effects of the different parameters, we have considered an idealized scenario where gravity is neglected. We have found that fluid and particle statistics are mostly altered by changes in volume fraction. On the other hand, at constant volume fraction and increasing density ratio, fluid statistics are only slightly modified. However, buoyant particles undergo an inertial shear-induced migration towards the centerline of the channel. The centerline concentration increases with the density ratio. At very high density ratios ($R \sim 1000$), the solid phase decouples from the fluid and behaves as a dense gas.

In the same set up we have also studied the effect of polydispersity. We have considered suspensions of neutrally buoyant spheres with 3 different Gaussian distributions of particle radii (i.e. 3 different standard deviations), and two volume fractions ($\phi = 2\%$ and $10\%$). We have observed that for all polydisperse cases, fluid statistics are not substantially altered with respect to those of the monodisperse case. Slightly larger differences are found for particle velocity
fluctuations. These increase close to the wall and decrease towards the centerline as the standard deviation of the distribution is increased. These results indicate that the behavior of the suspension is mostly governed by excluded volume effects regardless of the particle size distribution.

To conclude, we have considered suspensions of neutrally buoyant spheres in turbulent duct flows, in the range of volume fractions $\phi = 5\% - 20\%$. We have found that for $\phi \leq 10\%$ the mean particle concentration is largest close to the corners. Instead, for $\phi = 20\%$ the largest concentration is found in the core of the duct, feature that is absent in turbulent channel flows of same $h/a$. We have also found that the secondary cross-stream velocity grows with the volume fraction up to $\phi \approx 10\%$. For larger $\phi$, the turbulence activity is reduced and the cross-stream velocity decreases below the value of the unladen case. Concerning the mean friction Reynolds number we have seen that it initially increases with the volume fraction up to $\phi = 10\%$. Differently from channel flows, however, this quantity is then found to decrease for larger $\phi$.

6.2. Future work

As noted by Prosperetti (2015), the combination of fluid and particle parameters generates a huge parameter space and consequently the present work can be extended in plenty of directions. For example, concerning the settling of finite-size spheres in homogeneous isotropic turbulence, we have considered two different volume fractions of 0.5% and 1%. These results should certainly be extended by considering larger volume fractions and ellipsoidal particles. We have already performed simulations at high volume fractions ($\phi \sim 10\%$), and we observe that there is a clear background fluid flow modification due to strong excluded volume effects. As a consequence, the mean settling speed is almost equal to that of the suspension settling in quiescent fluid. Concerning non-spherical particles, we have seen how the microstructure and the dynamics are deeply influenced by the particle orientation. It is hence interesting to understand how this parameter (and in general particle shape) influence the settling process in homogeneous isotropic turbulence. In these studies we have also started to investigate the turbulence modulation due to the solid phase. While several works on the topic have been performed for non-sedimenting particles, the case of falling particles has not been deeply investigated. Due to the presence of settling particles, there are additional mechanisms that produce and dissipate turbulent kinetic energy. The aim is to understand these mechanisms, and to isolate them from those of the background turbulence.

Regarding the sedimentation in quiescent fluid of oblate particles, we have investigated only one Galileo number and one aspect ratio. It is known, however, that the settling of such particles strongly depends on these two parameters. Therefore, it will be interesting to explore more in detail the parameter space defined by the Galileo number, the density ratio, the aspect ratio and the solid volume fraction. Considering prolate particles is challenging from a numerical
6. Conclusions and outlook

point of view (due to the large resolution required to resolve the high curvature), but it is certainly of fundamental and practical importance.

In the context of wall-bounded flows at low Reynolds numbers, we can extend our works on rheology and particle migration to consider polydisperse suspensions, as well as non-spherical particles and different density ratios. Additionally, it would be relevant to study time-dependent rheological behaviors such as thixotropy and rheopexy. For example, such behaviors could be modelled by applying attracting and repulsive potentials between particles. Such potentials should then depend on the relative distance between particles (eventually by also defining a specific cut-off distance) and the local or average shear rate.

Concerning turbulent channel flows laden with buoyant finite-size particles, we can extend the results of this work by allowing the particles to settle. We could then examine the importance of different parameters such as the bulk Reynolds number, the Stokes number, the Shields number (i.e. the ratio of the shear stress over the net gravity) and the volume fraction, on the behavior of both phases. The same could also be studied in turbulent duct flows. In the latter geometry it would also be interesting to investigate the flow of suspensions of non-spherical, possibly buoyant, particles.
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