Testing quantum gravity*

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The search for a theory of quantum gravity is the most fundamental problem in all of theoretical physics, but there are as yet no experimental results at all to guide this endeavor. What seems to be needed is a pragmatic way to test if gravitation really occurs between quantum objects or not. In this paper, we suggest such a potential way out of this deadlock, utilizing macroscopic quantum systems; superfluid helium, gaseous Bose–Einstein condensates and “macroscopic” molecules. It turns out that true quantum gravity effects — here defined as observable gravitational interactions between truly quantum objects — could and should be seen (if they occur in nature) using existing technology. A falsification of the low-energy limit in the accessible weak-field regime would also falsify the full theory of quantum gravity, making it enter the realm of testable, potentially falsifiable theories, i.e. becoming real physics after almost a century of theorizing. If weak-field gravity between quantum objects is shown to be absent (in the regime where the approximation should apply), we know that gravity then is a strictly classical phenomenon absent at the quantum level.

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The “holy grail” of fundamental theoretical physics is quantum gravity — the goal of somehow reconciling gravity with the requirement of formulating it as a quantum theory, i.e. “explaining” how gravity as we presently know it emerges from some more fundamental microscopic theory. The most serious obstacle — from the point of view that physics is supposed to be a natural science telling us something about the real world — is the total lack of experiments guiding us. Today there are as yet no detected observational or experimental signatures of any quantum gravitational effects. Naively, essentially from pure dimensional analysis arguments, quantum gravity experimentally seems to require an energy of

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roughly $E_P = \sqrt{\hbar c^3 G / \pi} \approx 10^{28}$ eV, the “Planck Energy” (or equivalently, the means for exploring length-scales of the order of the “Planck Length”, $l_P = \sqrt{\hbar G / c^3} \approx 10^{-35}$ m). Using existing technology, this would require a particle accelerator larger than our galaxy — so direct tests of quantum gravity seem, at first sight, impossible.

However, as quantum theory is supposed to be universal — no maximum length built into its domain of applicability — a low-energy, large length-scale, formulation of the theory should still apply. A falsification of the low-energy limit, in the experimentally accessible weak-field regime, would also falsify the full theory of quantized gravity, hence making it possible to test, and potentially rule out, quantum gravity with existing or near-future technologies. In fact, direct tests of the high-energy limit of general quantum gravity may never be possible. In that case, high-precision laboratory tests of weak-field quantum gravity will be the only possibility to make quantum gravity a physical (testable/falsifiable) theory instead of merely a mathematical one (as it has been until now).

But how can a quantum theory be applied to the fairly large bodies needed? The answer lies in macroscopic systems still obeying the rules and laws of quantum theory — in essence those described by macroscopic wavefunctions. For a free-falling, effectively two-body problem, it should then, in principle, be possible to measure, e.g. the resulting quantum gravitational excitation energies. A positive result would show that the gravitational field is quantized, just like the quantized energy levels resulting from the Schrödinger equation for hydrogen is implicit proof of the quantization of the electromagnetic field. We can immediately think of four such candidates (and combinations of them, and more fundamental electrically neutral particles like neutrons $\sim 10^{-27}$ kg):

(i) Superfluid helium-II.
(ii) Gaseous Bose–Einstein condensates ($\leq 10^9$ u $\sim 10^{-17}$ kg, presently).
(iii) Buckyballs or other “macroscopic” molecules known to still obey quantum mechanics ($\leq 10^4$ u $\sim 10^{-22}$ kg, presently).
(iv) Neutron stars, believed to contain a substantial portion of their mass as superfluid neutrons, which should give very significant quantum gravity effects, for instance potentially measurable as quantized (discrete) gravitational redshift, the normal component acting incoherently (where each neutron interacts individually with the test particle — adding probabilities not amplitudes), not screening the effect.

For superfluids, as the temperature decreases below the $\lambda$-transition the superfluid component rapidly approaches 100%. The helium atoms then condense into the same lowest energy quantum “groundstate” (losing their individual

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“Now if I consider only gravitostatics, I still have a problem. I still have a quantum theory of gravity.”, R. P. Feynman.

Previous work purporting to having seen quantum gravity effects have in reality only probed the “correspondence limit” of extremely high excitation, in the classical gravitational field of the whole earth, e.g.
identities), and it becomes the state of the macroscopic superfluid. Hence, the superfluid is described by a single quantum wavefunction, even though macroscopic in size and mass, and the same applies for gaseous Bose–Einstein condensates. It can then only behave in a completely ordered way in which the action of any atom is correlated with the action of all the others, and thus has extreme sensitivity to ultraweak forces (like gravity).

So, if superfluid systems, dominated by the superfluid state, interact solely/mainly through gravity with other quantum systems, we can obtain a test of low-energy quantum gravity. As the whole quantum “object” is described by a single wavefunction, quantum gravity affects, and is affected by, its whole mass.

We may consider several such possibilities:

A superfluid \((M)\) gravitationally binding a mass \((m)\) of either (a) a neutral quantum particle such as a neutron, (b) an atomic Bose–Einstein condensate or (c) a “macroscopic” quantum molecule. The system being in free-fall, inside a spherical Faraday cage, either in an evacuated drop-tower experiment on earth, in parabolic flight, or, ultimately, in permanent free-fall in a satellite experiment, e.g. at the International Space Station, or a dedicated satellite similar to the European Space Agency “STE-Quest” space mission proposal (Space-Time Explorer and QUantum Equivalence principle Space Test).

Also, a neutron star \((M)\) plus “test-particle” \((m)\) should exhibit substantial quantum gravity effects. Unfortunately, the formalism is strictly applicable only to weak fields where the static (potential) gravitational contribution overwhelms the dynamical.

However, just like Newtonian gravity is the weak-field/low-energy limit of general relativity, Newtonian quantum gravity must be the weak-field/low-energy limit of general (presently unknown) quantum gravity. The main advantage being that Newtonian quantum gravity is known and well-defined, and hence, in principle, testable today. If weak-field gravity between quantum objects is falsified (in the regime where it should apply), we know that general quantum gravity is falsified too, meaning that gravity is then a strictly classical phenomenon absent at the quantum level.

The gravitational energy levels between quantum systems \(m\) and \(M\) are

\[
E_n(\text{grav}) = -\frac{G^2 \mu m^2 M^2}{2\hbar^2} \frac{1}{n^2} = -E_g \frac{1}{n^2}, \quad (n = 1, 2, 3, \ldots), \tag{1}
\]

where

\[
\mu = \frac{mM}{m + M}, \tag{2}
\]

is the reduced mass, introduced to facilitate any combination of masses (\(\mu\) giving just \(m\) for \(m \ll M\), and \(\mu = m/2\) if \(m = M\)), and

\[
E_g = \frac{G^2 \mu m^2 M^2}{2\hbar^2}, \tag{3}
\]
is the quantum gravitational binding energy, i.e. the energy required to totally free
the mass \( m \) from \( M \) in analogy to the Hydrogen case, whereas the most probable
radial distance is

\[
\tilde{r}_{\text{grav}} \simeq \frac{n^2 \hbar^2}{G \mu m M}.
\]

(4)

All analytical solutions to the normal Schrödinger equation, the hydrogen wavefunc-
tions, carry over to the gravitational case with the simple substitution

\[
e^2/4\pi\epsilon_0 \rightarrow GmM,
\]

which is equivalent to replacing the reduced Bohr radius,

\[
a_0^* = \frac{\hbar}{\sqrt{\mu c}}
\]

(5)
in the wavefunctions

\[
\psi_{nlm} = R(r)\Theta(\theta)\Phi(\phi) = N_{nlm}R_{nl}Y_{lm}.
\]

(6)

Here, \( N_{nlm} \) is the normalization constant, \( R_{nl} \) the radial wavefunction, and \( Y_{lm} \) the spherical harmonics containing the angular parts of the wavefunction. The
gravitational Bohr radius, \( b_0^* \), also gives the distance where the probability density
of the ground state \( \psi_{100} \) peaks (and also the innermost allowed radius of orbits in
the old semiclassical Bohr model, equivalently, the radius where the circumference
\( 2\pi r \) equals exactly one de Broglie wavelength).

If we introduce the Planck mass

\[
m_P = \sqrt{\frac{\hbar c}{G}} \simeq 2.2 \times 10^{-8} \text{ kg},
\]

(7)

conventionally believed to be fundamental in quantum gravity, we can rewrite the
quantum gravitational binding energy and the reduced gravitational Bohr radius as

\[
E_g = \frac{\mu c^2}{2} \frac{m^2 M^2}{m^4_P},
\]

\[
b_0^* = \frac{\hbar}{\mu c} m M.
\]

(8)

(9)

where \( \hbar/\mu c \) in the last equation is just the reduced Compton wavelength for \( \mu \).
With \( m = M = m_P \), this yields \( E_g = E_P/4 \), i.e. 1/4th the Planck energy, and
\( b_0^* = 2l_P \), twice the Planck length, consistent with the naive expectation.

The quantum gravitational energy levels of the system are as quoted above.

For example, for a mass \( m = M = 8.6 \times 10^{-14} \text{ kg} \), the first few excited states above
the groundstate would require \( E_{1-2} = 2.2 \text{ eV}, E_{1-3} = 2.6 \text{ eV}, E_{1-4} = 2.8 \text{ eV} \).

One possibility (but by no means the only one) to investigate “quantum jumps”
between these gravitational quantum states, and hence potentially detect the quanti-
tization of the gravitational field, would be to use a laser calibrated to these energy
frequencies to experimentally detect and manipulate them. The system should not
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“jump” until the laser is in resonance with the possible quantum gravitational states of the system. It should be noted that the excitation of the states are then affected by electromagnetism, whereas the decay towards the ground state would be gravitational transitions with graviton emission. Even if the gravitational decay is incredibly slow/improbable (depending on the combinations of \( m \) and \( M \)), it is sufficient to observe photon absorption at the predicted resonance frequencies to verify the quantum gravity effect. (This being analogous to the fast production of e.g. strange particles, via the strong interaction, and their subsequent slow decay via the weak interaction.) An absorption spectrum will thus give the “fingerprint” of quantum gravity in the system under consideration. If the masses could be chosen to give well-separated energy states in the energy range of visible light (\( 1.7 \text{ eV} < E < 3.2 \text{ eV} \)), this would be completely analogous to optical absorption spectra in cold gases. As it is nowadays possible to identify single quanta with essentially 100% efficiency, having just one system (instead of billions of atoms in gases) should not be an impossible obstacle in principle. For ease of visualization and analogy with familiar physics, we have so far mentioned visible light. As seen in Table 1, and Figs. 1 and 2, maser energies hold more promise. Still, it turns out that it is rather hard to find an eutronic (“macroscopic” quantum molecule. These are all known and well-studied objects in their own right. More speculatively (and outside the weak-field limit), an electron (\( m \sim 10^{-30} \text{ kg} \)) gravitationally bound to a Preon Star with mass \( M \sim 10^{12} \text{ kg} \) tentatively gives \( [E_g \sim 1 \text{ eV}, b_0^* \sim 10^{-10} \text{ m}] \); a neutrino (\( m \sim 10^{-36} \text{ kg} \)) bound to a Preon Star of \( M \sim 10^{20} \text{ kg} \) gives \( [E_g \sim 10^{-2} \text{ eV}, b_0^* \sim 10^{-6} \text{ m}] \). The characteristic size of a Preon Star is comparable to its Schwarzschild radius: \( R_s(10^{12} \text{ kg}) \sim 10^{-15} \text{ m}, R_s(10^{20} \text{ kg}) \sim 10^{-7} \text{ m} \). The cosmic microwave background has an energy of \( 10^{-4} \text{ eV} \).

Table 1. Orders of magnitude for the quantum gravitational binding energy \( E_g \) in eV, and the “gravitational Bohr radius” \( b_0^* \) in meters, for a few potentially, physically and experimentally, interesting combinations of quantum masses \( m \) and \( M \), given in kg. SF = superfluid helium, BEC = gaseous Bose–Einstein condensate, BB = Buckyball (C\(_{60}\)) or similar “macroscopic” quantum molecule. These are all known and well-studied objects in their own right.

<table>
<thead>
<tr>
<th>( M ) (kg)</th>
<th>( m ) (kg) ( 10^{-20} ) BEC</th>
<th>( E_g ) (eV)</th>
<th>( b_0^* ) (m)</th>
<th>( m ) (kg) ( 10^{-23} ) BB</th>
<th>( E_g ) (eV)</th>
<th>( b_0^* ) (m)</th>
<th>( m ) (kg) ( 10^{-27} ) neutron</th>
<th>( E_g ) (eV)</th>
<th>( b_0^* ) (m)</th>
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<td>( 10^3 ) SF</td>
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<td>( 10^{-3} ) SF</td>
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<td>( 10^{-6} ) SF</td>
<td>( 10^{-6} )</td>
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Fig. 1. The quantum gravitational binding energy $E_{g}$ in eV, as a function of the quantum mechanical masses ("gravitational charges") $m$ and $M$, given in kg.

the energy of rotation is

$$E_{j} = j^{2} \frac{\hbar^{2}}{2mR^{2}},$$  \(10\)

where $j = (0, 1, 2, \ldots)$. For $m = m_{\text{He}} \simeq 6 \times 10^{-27}$ kg, and $R \simeq 10^{-3}$ m, $\hbar^{2}/2mR^{2} \simeq 5 \times 10^{-18}$ eV.

According to the equivalence principle, the backbone of general relativity, gravitation is equivalent to acceleration, which in this case is

$$a = j^{2} \frac{\hbar^{2}}{m^{2}R^{3}},$$  \(11\)

and as the acceleration is quantized, so is the equivalent gravitation. For the same parameter values as above $\hbar^{2}/m^{2}R^{3} \simeq 3 \times 10^{-7}$ m/s$^{2}$.

However, we immediately see that the groundstate ($j = 0$) does not accelerate at all, i.e. the equivalent quantum gravitational groundstate is unaffected and cannot “fall”, just like an electron cannot fall into the nucleus of an atom, which may resolve singularity problems arising in the classical theory. (Giving an innermost allowed gravitational “orbit” in the old interpretation of Bohr, its circumference being exactly one de Broglie wavelength, while $\hbar \rightarrow 0$ in Eqs. \(4\) and \(5\) gives
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In a simply connected vessel (no “hole”) the total angular momentum is still quantized, but there can no longer be any bulk rotation as the superfluid is irrotational (the hole in the torus being what allows this in such nonsimply connected vessels). Below the first critical angular velocity, the superfluid is stationary. As the circulation reaches $\kappa = h/m \simeq 10^{-7} \text{m}^2/\text{s}$, a first quantum vortex will form, at $2h/m$ a second one will appear, and so on. The resulting quantum vortices, $N$ individual ones all with $j = 1$ as higher $j$ are unfavorable energetically, should also be directly related to quantum gravity through the equivalence principle. As the core of the quantum vortex is of the order $R \sim 1\text{Å}$, the energy and acceleration for a single “fundamental” vortex is $E_1 \sim 10^{-4} \text{eV}$ and $a \sim 10^{14} \text{m/s}^2$. The nonrotating groundstate has no circulation, so no acceleration and again no equivalent effective gravity.

In conclusion, we have seen how quantum gravity in principle can be tested today, e.g. using the quantum gravitational behavior of combinations of macroscopic superfluids, large molecules, Bose–Einstein condensates and neutrons. Indirectly, the observed quantized rotation/acceleration of superfluids already hints at the
existence of quantum gravity. However, this assumes that the equivalence principle is still valid at the quantum level, which is far from proven.

References