Particle tracking in geometallurgical testing for Leveäniemi Iron ore, Sweden

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Abstract
In a particle based geometallurgical model, the behavior of the particles can be used for forecast the products and quantify the performance of the different ore types within a deposit. The particle tracking is an algorithm developed by Lamberg and Vianna 2007 whose aim is to balance the liberation data in a mineral processing circuit composed by several processing units. Currently, this tool is being developed for the HSC Chemistry software by Outotec.

The objective of this study is to understand and evaluate the particle tracking algorithm in a geometallurgical test for iron ore. To achieve this objective, the liberation data is balanced in a Davis tube test circuit. A total of 13 samples from Leveäniemi iron ore were process in a Davis tube circuit.

The magnetite is the main mineral in the Leveäniemi iron ore samples. Its high recovery in the Davis tube circuit along with the V, Ti and Mn suggest that these elements are present in the magnetite lattice. These penalty elements in the iron concentrates cannot be avoided at the stage of mineral concentrations.

The washing effect of the Davis tubes controlled by the rotational and longitudinal agitation of the tube perturb the particles agglomeration between the pole tips of the electromagnet. A higher agitation frequency and amplitude will wash away most of the gangue minerals and also fine grained magnetite.

In this work, the particle tracking is depicted and implemented in a magnetic separation circuit for high liberated material. The liberation data was balanced in a way that the particle classes can be followed through circuit and their recoveries can be calculated. Nevertheless, the algorithm requires further validation and analysis of its limitations in terms of resolution and reproducibility.

Keywords: Geometallurgy, Process mineralogy, Particle tracking, Leveäniemi, Mass balance, Iron ore, HSC
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# Table of Contents

Abstract .................................................................................................................................................. I
Acknowledgements ................................................................................................................................. II
List of Figures .......................................................................................................................................... V
List of Tables .......................................................................................................................................... VI
1 Introduction ......................................................................................................................................... 1
   1.1 Geometallurgical approach ............................................................................................................. 1
   1.2 Objective and hypothesis ............................................................................................................... 2
   1.3 Scope ............................................................................................................................................. 2
2 Literature review ................................................................................................................................. 2
   2.1 Magnetic separation ......................................................................................................................... 2
      2.1.1 Concepts ................................................................................................................................. 3
      2.1.2 Main parameters ....................................................................................................................... 5
      2.1.3 Davis tube magnetic separator ................................................................................................. 6
      2.1.4 Dynamic analysis of Davis tube test ......................................................................................... 7
   2.2 Particle tracking ............................................................................................................................... 14
      2.2.1 Element to mineral conversion ................................................................................................. 16
      2.2.2 Unsized and sized mass balance by mineral ............................................................................. 17
      2.2.3 Liberation mass balance ......................................................................................................... 18
      2.2.4 Practical overview of Particle tracking and scopes .................................................................. 20
   2.3 Leveäniemi Iron Ore ...................................................................................................................... 21
3 Materials and methods ......................................................................................................................... 22
   3.1 Samples and sample preparation for Davis tube test ..................................................................... 23
   3.2 Sampling error ............................................................................................................................... 24
   3.3 Davis tube test ............................................................................................................................... 26
   3.4 Element to mineral conversion: Set Up ......................................................................................... 27
3.5 Particle tracking algorithm ............................................................................................................... 29
   3.5.1 Adjusting particles ...................................................................................................................... 30
   3.5.2 Basic Binning ............................................................................................................................ 30
   3.5.3 Missing ..................................................................................................................................... 31
   3.5.4 Creating reference stream ........................................................................................................ 32
   3.5.5 Advanced Binning ..................................................................................................................... 33
   3.5.6 Smoothing .................................................................................................................................. 34
3.5.7 Particle mass balance ........................................................................................................... 34
3.5.8 Binary particles recoveries .................................................................................................. 37
3.6 Particle tracking strategy in Davis tube circuit ...................................................................... 38

4 Results .................................................................................................................................... 39
  4.1 Sized chemical mass balance ............................................................................................... 39
  4.2 Optical microscopy ................................................................................................................ 40
  4.3 Element to mineral conversion .............................................................................................. 43
  4.4 Bulk and sized mineral mass balance .................................................................................... 45
  4.5 Mineral Liberation Analysis .................................................................................................. 48
  4.6 Liberation mass balance ........................................................................................................ 52
    4.6.1 Liberated particles recoveries ......................................................................................... 54
    4.6.2 Binary particles recoveries ............................................................................................... 56
    4.6.3 Liberation mass balance methods for Davis tube circuit ............................................... 58

5 Discussions ............................................................................................................................. 59

6 Conclusion ................................................................................................................................ 61

7 References ................................................................................................................................ 62

8 Annexes .................................................................................................................................... 65
  8.1 Optical microscopy description of Leveäniemi products after Davis Tube test: PRE:L:8:G ... 65
    8.1.1 Concentrate 1 ...................................................................................................................... 65
    8.1.2 Concentrate 2 ...................................................................................................................... 67
    8.1.3 Concentrate 3 ...................................................................................................................... 69
    8.1.4 Final Tailings ....................................................................................................................... 72
  8.2 Sized mass balance results .................................................................................................... 76
    8.2.1 PRE:L:1 ................................................................................................................................ 76
    8.2.2 PRE:L:2 ................................................................................................................................ 78
    8.2.3 PRE:L:3 ................................................................................................................................ 80
    8.2.4 PRE:L:4 ................................................................................................................................ 82
    8.2.5 PRE:L:5 ................................................................................................................................ 84
    8.2.6 PRE:L:6 ................................................................................................................................ 86
    8.2.7 PRE:L:7 ................................................................................................................................ 88
    8.2.8 PRE:L:8 ................................................................................................................................ 90
    8.2.9 PRE:L:9 ................................................................................................................................ 92
    8.2.10 PRE:L:10 .............................................................................................................................. 94
List of Figures

Figure 1.1. Geometallurgical model based on particles (Koch 2017; P. Lamberg 2011) .................................. 1
Figure 2.1. Magnetic fields due to a magnetic moment (left) and a small circular current (right) (Yamauchi 2008) ................................................................. 3
Figure 2.2. Alignment of magnetic moments (Svoboda 2004) ........................................................................ 4
Figure 2.3. Magnetization curve (Svoboda 2004) .......................................................................................... 4
Figure 2.4. Davis tube experimental set-up at LTU laboratory ..................................................................... 7
Figure 2.5. Free-body diagram scheme: Single particle inside the inclined Davis tube (not at scale) .......... 8
Figure 2.6. 3D model of the DT (left) and values of the magnetic field in the central plane of the DT Gap (right) (Murariu and Svoboda 2003) ................................................................. 8
Figure 2.7. Dependence of coercive force and magnetic susceptibility of magnetite on the particle size (Murariu and Svoboda 2003) ................................................................. 9
Figure 2.8. Fl vs particle diameters for selected minerals (left) and magnetite (right) ................................. 10
Figure 2.9. Davis tube test configuration .................................................................................................. 11
Figure 2.10. Contribution of minerals to the total susceptibility of a mineral mixture (Svoboda 2004) ... 12
Figure 2.11. Magnetic susceptibility versus wt% of magnetite in particle ............................................. 14
Figure 2.12. Common data collection and mass balancing flow sheet for mineralogical circuit (P. Lamberg & Vianna, 2016) ................................................................. 15
Figure 2.13. Simple case of mass balance (taken from B. A. Wills & Finch, 2016) .................................... 17
Figure 2.14. Location of Leveäniemi ore (taken from Bremer, 2010) ...................................................... 21
Figure 2.15. Geological map of Leveäniemi ore. 1) Magnetite ore 2) Calcite-rich magnetite ore 3) Hematite-altered ore 4) Ore breccia 5) Leptite 6) Sericite schist 7) Metabasite 8) Lina granite 9) Skarn (Bremer 2010; Frietsch 1966) ................................................................. 22
Figure 3.1. Sample preparation flowsheet .................................................................................................. 24
Figure 3.2. Nomograms per samples for the sample preparation procedure ........................................... 26
Figure 3.3. Davis tube test flowsheet. ........................................................................................................ 26
Figure 3.4. Example of interpolation ........................................................................................................ 32
Figure 3.5. Ratio versus magnetite wt% in particle .................................................................................... 34
Figure 3.6. Simple 1-node mass balance circuit ......................................................................................... 35
Figure 3.7. First part of the constraints matrices ......................................................................................... 35
Figure 3.8. Second part of the constraints matrices ..................................................................................... 36
Figure 3.9. Third part of the constraints matrices ......................................................................................... 36
Figure 3.10. Fourth part of the constraints matrices .................................................................................... 37
Figure 3.11. Final constraints matrices ...................................................................................................... 37
Figure 3.12. Matrix defining the mass balance of the Davis tube circuit ................................................ 37
Figure 4.1. Fe cumulative recoveries for Leveäniemi samples by size ..................................................... 39
Figure 4.2. Selectivity curve for sample L8 ................................................................................................. 40
Figure 4.3. Selected photomicrographies for products of L8 (description in the text) ............................. 42
Figure 4.4. Pareto chart for chemical assays and back-calculated chemical proportions from SEM .......... 43
Figure 4.5 Pareto chart for mineral proportions from EMC and SEM ........................................... 44
Figure 4.6. Fe recovery versus Fe oxides in feed ........................................................................... 46
Figure 4.7. Selectivity curves for minerals in PRE:L:8 ................................................................. 47
Figure 4.8. Recovery-grade curve for Fe oxides. PRE:L:8 .............................................................. 48
Figure 4.9. Modal composition of products of DT test by size fraction and bulk ......................... 49
Figure 4.10. Mode of occurrence of Fe oxides of DT test by size fraction and bulk .................... 50
Figure 4.11. Cumulative liberation of Fe oxides ............................................................................. 51
Figure 4.12. Percentage of flowrate of balanced liberated and binary particles respect to total flowrate by stream from mass balance by size ................................................................. 54
Figure 4.13. Balanced and pre-balanced cumulative recoveries for mineral groups .................... 55
Figure 4.14. Balanced and back-calculated recoveries of binary particles as a function of wt% of magnetite in particle .................................................................................................................................. 57
Figure 4.15. Balanced and back-calculated recoveries of binary particles as a function of wt% of magnetite in particle .................................................................................................................................. 58

List of Tables

Table 2.1. Magnetic classification of the minerals ............................................................................. 5
Table 2.2. Davis Tube test parameters ............................................................................................. 7
Table 2.3. Parameters used in the calculation of the Force index vs particle diameter curve ............ 9
Table 2.4. Densities and mass magnetic susceptibilities of minerals used in calculations .............. 10
Table 2.5. Values of magnetic strength, its gradient and Force Index (Murariu and Svoboda 2003) .... 10
Table 2.6. Particle tracking algorithm inputs and outputs (modified from Pertti Lamberg & Vianna, 2007) .................................................................................................................................. 20
Table 3.1. Geological properties of the samples. (*) Pit samples from the south of eastern zone ........ 23
Table 3.2. Fundamental sampling error .......................................................................................... 25
Table 3.3. Mineral setup for EMC. (*) Stoichiometric mineral ........................................................ 28
Table 3.4. Rounds for the element to mineral conversion in HSC Chemistry 7 .................................. 29
Table 3.5. Stream properties .......................................................................................................... 29
Table 3.6. Particle properties ........................................................................................................ 30
Table 3.7. Particle class compositions .............................................................................................. 35
Table 3.8. Flowrates and composition of the mineral mass balance by size for size fraction 1 ....... 35
Table 3.9. Bin types mass proportions per stream ............................................................................. 36
Table 4.1.RMSD and absolute difference between SEM back-calculated chemical proportions and chemical assays (wt%) ........................................................................................................... 44
Table 4.2. RMSD and absolute difference between SEM and EMC mineral proportions (wt%) ....... 45
Table 4.3. Balanced iron oxide content in feed for Leveäniemi samples .......................................... 46
Table 4.4. Mineral mass balance by size for PRE:L:8 ................................................................. 52
Table 4.5. Residuals after imposing minimum NOP per particle class ............................................ 53
Table 4.6. Distribution of liberated and binary particles in the Feed of PRE:L:L8. Reconciled with mineral mass balance by size ........................................................................................................... 53
Table 4.7. Distribution of liberated and binary particles in the Feed of PRE:L:L8. No Reconciled with mineral mass balance by size ........................................................................................................... 53
Table 4.8. Incomplete beta function fitting results ................................................................. 56
Table 4.9. Residuals of mass balance of reconciled and no reconciled liberation data for all nodes
method. The RMSD between both results is shown ........................................................................ 59
Table 4.10. Residuals of mass balance of reconciled and no reconciled liberation data for node by node
method. The RMSD between both results is shown ........................................................................ 59

ABBREVIATIONS AND SYMBOLS

Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>EMC</td>
<td>Element to mineral conversion</td>
</tr>
<tr>
<td>Level 1</td>
<td>Unsized mineral mass balance</td>
</tr>
<tr>
<td>Level 2</td>
<td>Mineral mass balance by size</td>
</tr>
<tr>
<td>Level 3</td>
<td>Liberation mass balance</td>
</tr>
<tr>
<td>C1</td>
<td>Concentrate 1</td>
</tr>
<tr>
<td>C2</td>
<td>Concentrate 2</td>
</tr>
<tr>
<td>C3</td>
<td>Concentrate 3</td>
</tr>
<tr>
<td>T1</td>
<td>Tailing 1</td>
</tr>
<tr>
<td>T2</td>
<td>Tailing 2</td>
</tr>
<tr>
<td>T3</td>
<td>Tailing 3</td>
</tr>
<tr>
<td>DT</td>
<td>Davis tube</td>
</tr>
<tr>
<td>WLIMS</td>
<td>Wet low intensity magnetic separation</td>
</tr>
<tr>
<td>GCT</td>
<td>Geometallurgical comminution test</td>
</tr>
<tr>
<td>SEM</td>
<td>Scanning electron microscope</td>
</tr>
<tr>
<td>EDS</td>
<td>Energy dispersive spectroscopy</td>
</tr>
<tr>
<td>XRF</td>
<td>X-ray fluorescence</td>
</tr>
<tr>
<td>NOP</td>
<td>Number of particles</td>
</tr>
<tr>
<td>LS</td>
<td>Least square</td>
</tr>
<tr>
<td>NNLS</td>
<td>Non-negative least square</td>
</tr>
<tr>
<td>WLS</td>
<td>Weighted least squares</td>
</tr>
<tr>
<td>EWTLs</td>
<td>Element wise total least squares</td>
</tr>
<tr>
<td>RMSD</td>
<td>Root mean squared deviation</td>
</tr>
</tbody>
</table>
## Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\vec{H})</td>
<td>Magnetic field</td>
<td>A/m</td>
</tr>
<tr>
<td>(\vec{M})</td>
<td>Magnetization</td>
<td>A/m</td>
</tr>
<tr>
<td>(\vec{B})</td>
<td>Magnetic induction</td>
<td>T</td>
</tr>
<tr>
<td>(\mu_0)</td>
<td>Magnetic permeability of vacuum</td>
<td>(4\pi \times 10^{-7}) H/m</td>
</tr>
<tr>
<td>(\kappa_m)</td>
<td>Volume magnetic susceptibility</td>
<td>Dimensionless</td>
</tr>
<tr>
<td>(\chi_m)</td>
<td>Mass magnetic susceptibility</td>
<td>m³/kg</td>
</tr>
<tr>
<td>(\vec{F}_m)</td>
<td>Magnetic force</td>
<td>N</td>
</tr>
<tr>
<td>(\vec{W})</td>
<td>Weight</td>
<td>N</td>
</tr>
<tr>
<td>(\vec{F}_b)</td>
<td>Buoyancy</td>
<td>N</td>
</tr>
<tr>
<td>(\vec{F}_d)</td>
<td>Drag force</td>
<td>N</td>
</tr>
<tr>
<td>(\eta)</td>
<td>Dynamic viscosity</td>
<td>Pa s</td>
</tr>
<tr>
<td>(v_t)</td>
<td>Terminal velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>(\vec{g})</td>
<td>Gravitational acceleration</td>
<td>m/s²</td>
</tr>
<tr>
<td>(\rho)</td>
<td>Density</td>
<td>g/cm³</td>
</tr>
<tr>
<td>(d)</td>
<td>Diameter</td>
<td>m</td>
</tr>
<tr>
<td>(\Theta)</td>
<td>Inclination angle</td>
<td>radians</td>
</tr>
<tr>
<td>(Re)</td>
<td>Reynolds number</td>
<td>Dimensionless</td>
</tr>
</tbody>
</table>
1 Introduction

1.1 Geometallurgical approach

The concept of geometallurgy is not new in the mining industry, however, its popularity has increased within the last decade. Geometallurgy could be defined as an integration of the geological, mineralogical and metallurgical information in order to create spatially-based predictive models which provides relevant qualitative and quantitative information for the optimization of production planning and management. A geometallurgical program is the application of the concept of geometallurgy to the mining industry. It is an organized way to map the ore variability and forecast its metallurgical response (Lishchuk et al. 2015; P. Lamberg 2011; Lund, Lamberg, and Lindberg 2015; Parian 2017; Koch 2017).

The geometallurgical programs can be divide in three approaches (Lishchuk et al. 2015):

1. **Traditional**: the metallurgical responses are calculated from chemical assays
2. **Proxy**: geometallurgical tests for large amounts of samples are used to estimate the metallurgical response of the ore
3. **Mineralogical**: based on quantitative mineralogical data

The latter is favored because it considers the mineralogy rather than only the chemical composition as in the traditional approach. Lamberg (2011) proposed a particle-based approach where the particle (composed by minerals) is the key entity which links geology and metallurgy. This mineralogical approach based on particles and their properties uses the geological (spatial) model as input for a texture breakage model in order to generate particles and simulate its behavior in process models. The behavior of the particles can be used for forecast the products and quantify the performance of the different ore types within a deposit. The performance indicators are feedback to the geometallurgical model and used as a tool for mine planning and process design (Koch 2017; P. Lamberg 2011).

Within the process model, the particle behavior stands for the highest level of information regarding chemical composition, modal mineralogy and liberation by size fractions. Even though, the quantitative mineralogical data is highly valuable, especially when the liberation is an issue, it is rarely systematically introduce in a geometallurgical program due to its high economical costs (Parian 2017). The maximization of the potential of the liberation data in a process model set the basis for the understanding of the particle behavior and their further simulation.

*Figure 1.1. Geometallurgical model based on particles (Koch 2017; P. Lamberg 2011)*
The particle tracking is an algorithm developed by Lamberg and Vianna 2007 whose aim is to balance the liberation data in a mineral processing circuit composed by several processing units (P. Lamberg and Vianna 2016). Currently, this tool is being developed for the HSC Chemistry software by Outotec.

1.2 Objective and hypothesis
The objective of this study is to understand and evaluate the particle tracking algorithm in a geometallurgical test for iron ore. To achieve this objective, the liberation data is balanced in a Davis tube test circuit. The specific tasks in order to accomplish the objective of this work are the following:

1. Davis tube test of the Leveäniemi samples at Malmberget laboratory
2. Chemical, mineralogical and liberation analysis for products of the Davis tube tests

The hypothesis of this work is that the particle tracking algorithm will allow to balance the liberation data and to track the path of each particle class through the circuit.

The main motivation of this thesis work is based on the potential of the particle tracking for studying the behavior of particle in a relatively simple and inexpensive geometallurgical circuit. This evaluation will motivate further studies in other type of units and in more complex circuit at bigger scales and different ores. The outcomes of this thesis will be useful for the implementation of the mass balance of liberation in the Mass Balance Module of HSC Chemistry software. In terms of the geometallurgy, the particle tracking algorithm is the key component which will link the generated particles from the textural breakage model with the production forecast.

1.3 Scope
In this study, the mass balance by size of 13 samples from Leveäniemi iron ore will be performed for a Davis tube circuit. The liberation mass balance will be performed in one of the samples following the particle tracking algorithm proposed by (Pertti Lamberg and Vianna 2007). This algorithm will be programmed in Visual Basic .NET which would serve as a basis for further procedural implementation in HSC Chemistry 9.

2 Literature review
The literature review consists of three main topics which will be briefly discussed. For further understanding of the concepts, the reader is referred to the references. The topics to introduce are:

1. The magnetic separation: main concepts and most relevant variables in terms of particles and minerals
2. Particle tracking: stepwise algorithm from element to mineral conversion, bulk mass balance, mass balance by size and liberation mass balance. The algorithm proposed by Lamberg and Vianna (2007) is reviewed
3. Leveäniemi iron ore: the geology and main characteristics of the deposit in study are reviewed

2.1 Magnetic separation
The magnetic separation is a physical separation method which uses the difference of the magnetic properties of the material to concentrate valuable material (B. Wills and Finch 2016). An example of this is the concentration of magnetite from nonmagnetic minerals in Kiruna deposit, Sweden.
In this section, the main concepts of magnetism and magnetic properties of the minerals are summarized and applied in a dynamic analysis for the Davis tube test.

2.1.1 Concepts
An electrical current in a close loop wire produces a magnetic field such as the one generated by bar magnet. In the same way, the magnetic field due to a magnetic moment (or dipole) and an electron in rotational motion are comparable (Figure 2.1). In most of the materials, the electrons in their orbitals could be pictured as dipoles randomly oriented so no net field is produced. When a dipole is subjected to the influence of an external magnetic field, the dipole feels a torque which tends to align it with the magnetic field (Hughes 2005; Yamauchi 2008).

![Figure 2.1. Magnetic fields due to a magnetic moment (left) and a small circular current (right) (Yamauchi 2008)](image)

The properties that allow to understand the respond of a material to an external magnetic field \( \vec{H} \), are the magnetization \( \vec{M} \) and the magnetic induction \( \vec{B} \). The magnetization is defined as the total magnetic moment of dipoles per unit volume. In the vacuum, \( \vec{B} \) and \( \vec{H} \) are related by Eq. 2.1.

\[
\vec{B} = \mu_0 \vec{H} \tag{Eq. 2.1}
\]

Where \( \mu_0 \) (4π x 10\(^{-7}\) H/m) is the magnetic permeability of vacuum. In magnetic materials, the magnetic induction is equal to (Eq. 2.2)

\[
\vec{B} = \mu_0 (\vec{H} + \vec{M}) \tag{Eq. 2.2}
\]

In SI units, the magnetic induction is measured in T (tesla), while the magnetic field strength and the magnetization are measured in A/m.

According to the alignment of the magnetic moments due to an external magnetic field, the materials can be classified in five groups: diamagnetic, paramagnetic, ferromagnetic, antiferromagnetic and ferrimagnetic. In the diamagnetic materials, the resulting magnetic moment is in the opposite direction of the field therefore they are repelled from it. In the paramagnetic materials, the resultant magnetic moment causes the material to experience a magnetic force along the lines of the magnetic field. The ferromagnetic materials exchange coupling of the magnetic dipoles which allow their rapid alignment, resulting in a positive net magnetization. In the antiferromagnetic materials, the parallel and antiparallel moments are completely balance resulting in a zero net magnetization. In the ferrimagnetic materials,
some of the moments are aligned in an antiparallel sense so the net magnetization is lower than in a ferromagnetic material (Figure 2.2) (B. Wills and Finch 2016; Svoboda 2004).

![Diagram of magnetic moments alignment](image)

*Figure 2.2. Alignment of magnetic moments (Svoboda 2004)*

A measure of the magnetic response of a material to an external magnetic field is the volume magnetic susceptibility (dimensionless. In most of the materials, it can be calculated as the ratio between magnetization of the material and the magnetic field (Eq. 2.3).

$$κ_m = \frac{M}{H}$$  \hspace{1cm} Eq. 2.3

The magnetic susceptibility can be expressed with respect to the density of the material ρ. In SI, the mass magnetic susceptibility is measured in m³/kg and follows (Eq. 2.4)

$$χ_m = \frac{κ_m}{ρ}$$  \hspace{1cm} Eq. 2.4

Although the magnetization shows a linear function with the magnetic field for most of the materials, this behavior can vary as the values of H increase. The ferromagnetic materials show a nonlinear magnetization curve, meaning that once the external field $\vec{H}$ the magnetization remains (Figure 2.3).

![Diagram of magnetization curve](image)

*Figure 2.3. Magnetization curve (Svoboda 2004)*
The previous physical classification of the materials can be simplified to three groups in order to be utilized in a more practical approach. In the Table 2.1, the minerals from Leveäniemi iron deposit are classified within these three groups according to its properties and range of mass magnetic susceptibility. It is shown that biotite could be classified as weakly magnetic or non-magnetic depending on the iron content and the temperature. (Beausoleil et al. 1983; Svoboda 2004).

**Table 2.1. Magnetic classification of the minerals**

<table>
<thead>
<tr>
<th>CLASSIFICATION</th>
<th>TYPE OF MATERIAL</th>
<th>RANGE OF MASS MAGNETIC SUSCEPTIBILITY [m³/kg]</th>
<th>MAGNETIC SEPARATION PARAMETER RANGES</th>
<th>LEVEÄNIEMI MINERALS</th>
</tr>
</thead>
<tbody>
<tr>
<td>STRONGLY MAGNETIC</td>
<td>Ferromagnetic and ferrimagnetic material</td>
<td>&gt; 10⁴</td>
<td>Weak magnetic field (~0.15 T) and low field gradient (0.5 T/m)</td>
<td>Magnetite</td>
</tr>
<tr>
<td>WEAKLY MAGNETIC</td>
<td>Antiferromagnetic, paramagnetic and some ferrimagnetic minerals</td>
<td>10⁻⁷ to 5x10⁻⁶</td>
<td>Relatively high magnetic field (up to 1.0 T) and field gradient (50 - 500 T/m)</td>
<td>Hematite, Biotite</td>
</tr>
<tr>
<td>&quot;NON-MAGNETIC&quot;</td>
<td>very weakly paramagnetic, antiferromagnetic and diamagnetic materials</td>
<td>&lt;10⁻⁷ and also negative</td>
<td></td>
<td>Apatite, Tremolite-Actinolite, Biotite, Calcite, Scapolite, Albite, Muscovite</td>
</tr>
</tbody>
</table>

2.1.2 Main parameters

The magnetic separation is a physical separation method which uses the difference in the magnetic properties of the minerals in order to separate the magnetic particles from the non-magnetic ones. The separation is done by passing the particles through a non-homogeneous magnetic field which leads to the retention or deflection of the magnetizable particles. The magnetic separation can be operated in dry or wet mode and it is considered as an environmentally friendly technique (B. Wills and Finch 2016; Svoboda 2004).

The forces acting on an individual particle during the magnetic separation are depicted as follows according to Svoboda 2004. The subscripts $p$ and $f$ stand for particle and fluid, respectively.

- Magnetic force

  \[ \overline{F_m} = \frac{1}{\mu_0} (\chi_p - \chi_f) \rho_p V_p B \nabla B \]  
  \[ \text{Eq. 2.5} \]

- Weight and buoyancy

  \[ \overline{W} - \overline{F_b} = (\rho_p - \rho_f) V_p \ddot{g} \]  
  \[ \text{Eq. 2.6} \]

- Drag force for spherical particles

  \[ \overline{F_d} = 3 \pi \eta d v_t \]  
  \[ \text{Eq. 2.7} \]
where \( \eta \) is the dynamic viscosity of the fluid and \( v_t \) is the terminal velocity of the particle in the fluid. The terminal velocity according to Stoke’s law for particles with a Reynolds number \((Re_p) < 1\) are defined as:

\[
v_t = \frac{(\rho_p - \rho_f)gd^2}{18 \eta} \tag{Eq. 2.8}
\]

\[
Re_p = \frac{v_t \rho_fd}{\eta} \tag{Eq. 2.9}
\]

The Reynolds number can be seen as the ratio between the inertial and viscous forces acting on a control fluid. Therefore, a Reynolds number below 1 means that the viscous forces are predominant over the inertial ones and the flow can be considered as laminar. Otherwise, the flow is defined as turbulent (Smits 2017).

For a particle size below 100 µm, equations Eq. 2.5 to Eq. 2.9 can be considered valid. For bigger particles \((Re_p > 1000)\), the drag force can be calculated from other approaches.

Other forces, such as centrifugal forces, can be also accounted as forces that competes with the magnetic force. However, other forces will be neglected in this work for simplicity.

If only spherical particles were considered, the magnetic forces, weight and buoyancy will be proportional to \(d^3\) whereas the drag force is proportional to \(d\). For particles with a diameter >500 µm, the force of gravity is predominant, while the drag force is the main competing force for particles <50 µm (B. A. Wills and Finch 2016a).

The magnetic separation efficiency depends on the response of the particles to the magnetic, gravitational, hydrodynamic and other competing forces. Therefore, a successful separation requires that the magnetic force to be greater than the sum of the competing forces acting on the magnetic particles. However, the selectivity of the process will be compromised if the magnetic force is much greater than the competing forces. Thus, at the same time, the magnetic forces acting in the less or non-magnetic particles must be smaller than the competing forces (Svoboda 2004).

The efficiency of the magnetic separation is controlled by the properties of the particles and the technical parameters of the magnetic separator. Since gravitational, hydrodynamic and magnetic forces vary with the particle size, this property is considered to be the most important discriminating factor in a magnetic separation rather than only the magnetic susceptibility of the material.

### 2.1.3 Davis tube magnetic separator

The Davis tube (DT) magnetic separator is laboratory machine designed to separate small samples of strongly magnetic ore into strongly magnetic and weakly magnetic fractions. The DT is not designed specifically as a separator but it is commonly used for assessing the separability of an iron ore by low-intensity magnetic separation. In the DT, the material passes through a 25 mm diameter water filled inclined tube placed in between pole-tips of an electromagnet (Figure 2.4). Only the strongly magnetic fraction is retained in the tube (Niiranen and Fredriksson 2012; Svoboda 2004; Schulz 1964).
The parameters used in Davis tube previous tests in the LKAB’s mineral processing laboratory in Malmberget, Sweden, are shown in the Table 2.2. The most important technical parameter of the magnetic separator is the magnetic flux density and its gradient, which product is directly proportional to the magnetic force applied on a particle. In the Davis tube test, the current intensity is the parameter that controls the magnetic induction. All the other parameters remained constants (Niiranen and Fredriksson 2012; Svoboda 2004; Niiranen 2015; Farrell and Miller 2011).

Table 2.2. Davis Tube test parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Common values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of the sample</td>
<td>g</td>
<td>10 - 15</td>
</tr>
<tr>
<td>Water flow</td>
<td>liters/min</td>
<td>0.3</td>
</tr>
<tr>
<td>Angle of the tube</td>
<td>degrees</td>
<td>45</td>
</tr>
<tr>
<td>Frequency</td>
<td>Hz</td>
<td>1.33</td>
</tr>
<tr>
<td>Voltage</td>
<td>Volts</td>
<td>120</td>
</tr>
<tr>
<td>Current intensity</td>
<td>Amperes</td>
<td>0.1 – 0.2 – 0.5</td>
</tr>
<tr>
<td>Running time</td>
<td>min &amp; sec</td>
<td>2’10”</td>
</tr>
</tbody>
</table>

2.1.4 Dynamic analysis of Davis tube test

A simplified analysis of the forces acting in the Davis tube before starting the longitudinal and rotational motion of the pipe can be done. The main assumptions are that a particle is travelling along the inclined tube without touching it. The only effect of being in touch with the walls of the tube is that a Normal and a friction force will be added. The normal force is only a reaction force and the friction force will oppose to the movement to the particle. This two forces are neglected for simplicity. Furthermore, the effect of the interaction of the particles and the perturbations are not accounted in this analysis but they will be assess qualitatively later in this section. A free body diagram accounting for the main competing forces (Eq. 2.5 to Eq. 2.7) is shown in the Figure 2.5.
In this configuration, the magnetic force is the only one affected by the angle of the tube. The drag force could also be affected due to the path of the particle, but it is not considered for simplicity reasons. The magnetic force in the Davis tube shows its maximum value in the center of the gap of the between the pole tips and decreases exponentially with the distance (Murariu and Svoboda 2003).

These forces can be decomposed in $\hat{x}$ and $\hat{y}$ as follows.

$\hat{x}$: 
$$-F_m \cos \theta = m_p \ddot{x} \quad \text{Eq. 2.10}$$

$\hat{y}$: 
$$W - F_b - F_d - F_m \sin \theta = m_p \ddot{y} \quad \text{Eq. 2.11}$$

Considering spherical particles and replacing the expressions of the forces defined in the previous sections for the components in $\hat{y}$, the following formula is obtained.

$$\left(\frac{\rho_p - \rho_f}{6}\right) \pi d^3 g - 3\pi \eta d v_t - \left(\frac{\chi_p - \chi_f}{6\mu_0}\right) \rho_p \pi d^3 B V B \sin \theta = \frac{\rho_p \pi d^3}{6} \ddot{y} \quad \text{Eq. 2.12}$$
Imposing that the acceleration of the particle is zero, an expression for the minimum $B \nabla B$ required for a particle to be trapped by the magnetic force is obtained.

$$B \nabla B = \left( (\rho_p - \rho_f) g - \frac{\eta v_e}{2d^2} \right) \frac{\mu_0}{(\chi_p - \chi_f)\rho_p \sin \Theta}$$  \hspace{1cm} \text{Eq. 2.13}

The value $B \nabla B$ is known as the Force index (FI) and it is used for estimating the efficiency of the separation considering the main term of the magnetic force (Murariu and Svoboda 2003). Thus, considering the specific magnetic susceptibilities of some selected minerals and magnetite, the minimum force index for a particle to be trapped in the magnetic field as a function of the particle size can be obtained. It is fair enough to consider the magnetic susceptibilities of the diamagnetic and paramagnetic minerals as invariant with the magnetic field. However, in the case of magnetite, the magnetic susceptibility depends on the field and also on the particle diameter (Figure 2.7).

![Dependence of coercive force and magnetic susceptibility of magnetite on the particle size](image)

Figure 2.7. Dependence of coercive force and magnetic susceptibility of magnetite on the particle size (Murariu and Svoboda 2003)

The Force index curves are calculated for four minerals: magnetite, hematite, biotite and quartz. The parameters used are summarized in the Table 2.3 and Table 2.4. From the parameters is clear that the only changing curve is the one for magnetite. The main parameter that differentiates the curves is the magnetic susceptibility which can be several orders of magnitude different.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi_f$ water ($10^{-8}$ m³/kg)</td>
<td>-0.9</td>
</tr>
<tr>
<td>$\rho_f$ water (kg/m³)</td>
<td>1000</td>
</tr>
<tr>
<td>$g$ (m/s²)</td>
<td>9.8</td>
</tr>
<tr>
<td>$\eta$ (Ns/m²)</td>
<td>0.00089</td>
</tr>
<tr>
<td>$\mu_0$ (H/m)</td>
<td>$4\pi \times 10^{-7}$</td>
</tr>
<tr>
<td>$\Theta$ (rad)</td>
<td>$\pi/4$</td>
</tr>
</tbody>
</table>
Table 2.4. Densities and mass magnetic susceptibilities of minerals used in calculations

<table>
<thead>
<tr>
<th>Mineral</th>
<th>( \rho_p ) (kg/m³)</th>
<th>( \chi_p ) (10^{-8} ) m³/kg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnetite</td>
<td>5150</td>
<td>Variable</td>
</tr>
<tr>
<td>Hematite</td>
<td>5260</td>
<td>38.5</td>
</tr>
<tr>
<td>Biotite</td>
<td>3000</td>
<td>75</td>
</tr>
<tr>
<td>Quartz</td>
<td>2650</td>
<td>-0.6</td>
</tr>
</tbody>
</table>

The minimum Force index is considerably smaller for magnetite (FI<1) than for biotite and hematite (paramagnetics). For quartz, the minimum FI is more than 5 orders of magnitude higher than for magnetite. This means that the diamagnetic are absolutely not retain mean while the paramagnetic minerals have a chance to be retained along with magnetite if the FI is high enough (Figure 2.8).

Figure 2.8. FI vs particle diameters for selected minerals (left) and magnetite (right)

Values of the FI in the gap and 5 cm off are shown to be about 10 times different. In the center of the gap, even with the smallest B shown in Table 2.5, the magnetite particles will be retained regardless of their size. However, 5 cm off the center of the gap, the particles with a diameter below 20 µm will not be retained. In other words, the zone of retention of particles under the effect of the magnetic field of the Davis tube is mainly constrained to the center of the gap. Thus, if a particle is carried out of this zone, the gravitational forces will prevail over the magnetic forces and the particle will be washed. For all the other values of FI, the magnetite particles will be retained even 5 cm off the gap.

Table 2.5. Values of magnetic strength, its gradient and Force Index (Murariu and Svoboda 2003)

<table>
<thead>
<tr>
<th>Center of DT gap</th>
<th>5 cm off</th>
</tr>
</thead>
<tbody>
<tr>
<td>B (T)</td>
<td>( \nabla B ) (T/m)</td>
</tr>
<tr>
<td>0.1</td>
<td>3.40</td>
</tr>
<tr>
<td>0.2</td>
<td>5.70</td>
</tr>
<tr>
<td>0.4</td>
<td>10.00</td>
</tr>
<tr>
<td>0.5</td>
<td>11.80</td>
</tr>
<tr>
<td>0.6</td>
<td>12.85</td>
</tr>
</tbody>
</table>
Once the Davis tube test starts, the water starts to flow through the tube in movement. The movement agitates the water since the shear force from the wall of the tube are transferred to the fluid. These two effects combined, the flowing water and the motion of the tube, affect the drag force resulting in a washing effect of the particles retained within the magnetic field of the Davis Tube. Furthermore, since the magnetic particles tend to deviate the external magnetic field towards them, its presence will affect other particles even if there is no physical contact between them (Figure 2.9).

![Figure 2.9. Davis tube test configuration](image)

Using the dimensionless Reynolds number (Re), a runoff regime can be classified as laminar (Re<2000), turbulent (>4000) or transition laminar-turbulent. In pipes, the Reynolds number can be calculated as follows:

\[
Re = \frac{\bar{v}D}{\nu} = \frac{4Q}{\pi \nu D}
\]

*Eq. 2.14*

where D is the diameter of the pipe, Q is the flowrate, \( \nu \) is the viscosity of the fluid and \( \bar{v} \) is the average speed of the flow. Under the Davis tube conditions, the Reynolds number can be calculated as follows:

\[
Re = \frac{4Q}{\pi \nu D} = \frac{4 \times 5 \times 10^{-6} \left[ \frac{m^3}{s} \right]}{\pi \times 10^{-6} \left[ \frac{m^2}{s} \right] \times 0.025[m]} = 254.6
\]

In a steady state, the water flow inside the Davis tube should be classified as laminar. However, the tube is not static, it is perturbed in longitudinal and rotational direction. Furthermore, the material introduce during the tests also affects the runoff regime. Therefore, despite of the calculated Reynolds number, the perturbations and the material gives turbulent characteristics to the water. The main characteristic of a
turbulent flow is the random pattern presented by any variable of the flow in a defined period of time, such as the speed in any point of space.

The effect of entrapment of gangue minerals between the magnetic particles has been reported in other magnetic separations. In WLIMS, the entrapment chance of coarse gangue particles is higher than for fine particles during the process of drainage of the flocs (Parian 2017). In the case of Davis tube, the perturbation of the magnetic concentrate in the center of the gap facilitates the drainage of coarse particles due to the change in direction of the drag force.

2.1.4.1 Binary particles magnetic susceptibility

In a mixture of minerals containing ferromagnetic impurities, the magnetic susceptibility of the bulk will be determined by the concentration of the ferromagnetic. The contributions of the magnetic susceptibilities from each mineral to the bulk are not linear. In the case of magnetic ore, a mixture with more than 10 wt% of magnetite will have a volume magnetic susceptibility about 1 (Figure 2.10).

![Figure 2.10. Contribution of minerals to the total susceptibility of a mineral mixture (Svoboda 2004)](image)

As property of the material, the mass magnetic susceptibility of a binary particle can be estimated as the weighted average of the mass magnetic susceptibilities of the materials the compose it. Usually, the intensive properties such as density of a compound material are estimated by this method. However, the mass magnetic susceptibility of the magnetite depends on the size. From data in Figure 2.7, it is possible to calibrate an exponential equation Eq. 2.15 that relates \( X_{binary} \) \((\text{m}^3/\text{kg})\) and its diameter \( d \) (\(\mu\text{m}\)).

\[
X_{magnetite}(d) = C_0 \left(1 - e^{-\frac{d}{a_0}}\right)
\]  \( Eq. 2.15 \)

Where \( C_0 \) is the maximum value that reaches the magnetic susceptibility \( (17.5 \times 10^{-4} \text{ m}^3/\text{kg}) \) and \( a_0 \) (8.3 \( \mu\text{m} \)) is a calibration parameter obtained by minimizing the RMSD between the model and the data. The RMSD that fits the data and the model is \( 1.52 \times 10^{-5} \text{ m}^3/\text{kg} \).
The Eq. 2.15 can be used in the calculation of the mass magnetic susceptibility in binary particles as a function of the magnetite wt% in the particle, which could be described as follows (Leiβner 2016).

\[
\chi_{\text{mgt-gangue}} = \frac{\text{wt}\%_{\text{Mgt in particle}} \times \chi_{\text{mgt}}(d) + (100 - \text{wt}\%_{\text{Mgt in particle}}) \times \chi_{\text{gangue}}}{100}
\]  \hspace{1cm} \text{Eq. 2.16}

As observed in the Eq. 2.16, the magnetic susceptibility of a binary particles should behave as linear function between the magnetic susceptibility of magnetite (wt% of magnetite in particle = 100%) and the other mineral (wt% of magnetite in particle = 0%). However, by considering the variation of the magnetic susceptibility of the magnetite with size, the magnetic susceptibility of the binary particle can vary.

A first order approximation can be done if only spherical particles are considered. Since the percentage of magnetite in a particle is the main controlling variable for recoveries of particle classes, the equation Eq. 2.16 can be expressed in the following way.

Let two spherical particle with diameter D and d be considered (D>d). Consider the particle with diameter d be a particle of magnetite within the bigger one (D). Then, the mass of the magnetite and gangue particle can be expressed as:

\[
\text{wt}\%_{\text{Mgt in particle}} = \frac{\rho_{\text{mgt}} \pi d^3}{\rho_{\text{mgt}} \pi d^3 + \rho_{\text{gangue}} \pi (D^3 - d^3)}
\]  \hspace{1cm} \text{Eq. 2.17}

From Eq. 2.17, the diameter d can be calculated from the wt% of magnetite in the particle as:

\[
d = \left(\frac{\rho_{\text{gangue}} D^3 \text{wt}\%_{\text{Mgt in particle}}}{\rho_{\text{gangue}} + (1 - \text{wt}\%_{\text{Mgt in particle}}) \rho_{\text{Mgt}}}\right)^{1/3}
\]  \hspace{1cm} \text{Eq. 2.18}
Finally, using Eq. 2.15, Eq. 2.16 and Eq. 2.18, the mass magnetic susceptibility of a mixed particle can be estimated by the following equation.

\[ \chi_{mgt-gangue} = \frac{\text{wt\%}_{Mgt \text{ in particle}} C_0 \left( 1 - e^{-d/a_0} \right) + \left( 100 - \text{wt\%}_{Mgt \text{ in particle}} \right) \chi_{gangue}}{100} \quad \text{Eq. 2.19} \]

The magnetic susceptibility can be analyzed for the mass of a spherical magnetite particle in a particle with diameter D. If the geometric average of the size fractions are considered as the diameters D of the bigger particles, it is possible to estimate the magnetic susceptibilities of mixed particles. The geometric average for the size fractions 0-38, 38-75 and 75-106 µm are 8.72, 53.39 and 89.16 µm.

As it is expected from the increasing relation between the particle size and \( \chi_{mgt} \), the finer size fraction requires higher wt% of magnetite in particle than the coarser size fractions in order to achieve the same magnetic susceptibility. As the size increases, the magnetic susceptibility reaches a plateau and the relation between the mixed magnetic susceptibility and the wt% of magnetite in particle becomes linear. In the example, the binary association between magnetite and hematite is considered. Even though, the magnetic susceptibility of hematite is higher than the one for quartz or other minerals, the trend of the curves are maintained (Figure 2.11).

![Magnetic susceptibility average particle: Mgt - Hem](image)

*Figure 2.11. Magnetic susceptibility versus wt% of magnetite in particle*

This result can be used for comparison later on for comparison of the recovery of binary particles of magnetite and other minerals in the Davis tube test.

It is out of the scope of this simplified analysis to study the decrease in the magnetic susceptibility when a particle of magnetite is partially oxidized to hematite.

### 2.2 Particle tracking

Particle tracking is a technique developed by Lamberg and Vianna since 2007 used for mass balancing the liberation data in a mineral processing circuit. It considers all the mineral phases present in the particles.
at each size fraction. The objective of the particle tracking technique is to solve the mineral mass balance at particle level in order to be used as a reference in the modeling, simulation and optimization of processes. In the industry, the mass balance is commonly done by using the chemical components and the mineral by size mass balance is rarely performed. When liberation is one of the main issues in the performance of a process, the mass balance of particles delivers the highest level information for diagnose and optimization (P. Lamberg and Vianna 2016; Pertti Lamberg and Vianna 2007).

The algorithm is a particle mass balance consisting of four main steps performed sequentially: (1) Element to mineral conversion, (2) Bulk mineral mass balance (Level 1), (3) mineral-by-size mass balance (Level 2) and (4) liberation mass balance (Level 3). This stepwise approach uses the output of each step as a constraint for the next one in order to minimize error propagation.

A mineralogical study is the base for the particle mass balance. The aim of this study is to identify and determine the chemical composition of the most relevant minerals from the mineral processing and metallurgical point of view. Then, the mineral grades are calculated from the chemistry by performing an element-to-mineral conversion (EMC). Only after this, the mass balance level 1, 2 and 3 can be established (Pertti Lamberg et al. 1997; Pertti Lamberg and Vianna 2007; Whiten 2008; P. Lamberg and Vianna 2016).

In order to perform a mineralogical mass balancing such as particle tracking, several steps must be accomplished before for assuring the quality of the final results. These steps starts from the experimental design and sampling and pass through the level 1, 2 and 3 of mass balance. The steps 1 to 4 stand for the fact that the experiment design, sampling, sample preparation and assays must reach sufficiently high quality. The high quality involves collection of replicate samples and extra sample preparation in order to define the sampling, preparation and analytical errors. Furthermore, the data set must be complete, i.e. each size fraction must be analyzed for all the streams. The unsized chemical mass balance (step 5) is mandatory to assure the quality of the data for further analysis (Figure 2.12).

Figure 2.12. Common data collection and mass balancing flow sheet for mineralogical circuit (P. Lamberg & Vianna, 2016).

From the mathematical point of view, all the steps of the particle tracking algorithm are defined as a linear system of equations of the type $A\bar{x} = \bar{b}$, where the matrix $A \in \mathbb{R}^{mxn}$, $\bar{x} \in \mathbb{R}^{nx1}$ and $\bar{b} \in \mathbb{R}^{mx1}$. These
equations can be solved by different methods depending on the properties of the matrix $A$. If $m = n$, then the direct and most efficient method in terms of complexity and numerical precision to solve the system is the Gaussian elimination. However, when the system is overdetermined ($m > n$), meaning that there are more equations than unknowns, there is no solution but it is possible to find an approximate solution by minimizing a norm of the residual vector $\vec{r} \in \mathbb{R}^{mx1}$.

$$\text{Min } ||\vec{r}|| = ||\vec{b} - A\vec{x}||$$  \hspace{1cm} \text{Eq. 2.20}

The most used norm is the Euclidian norm or 2-norm due to its simplicity during calculations. This type of problems are known as Least Squares problems and from its initial development in 1795 by Carl Friedrich Gauss, there have been further advances in the efficiency of the algorithm as well as several variations of it. The Least Square Solution has an important property called the Gauss – Markoff Theorem which can be expressed as follows.

$$A\vec{x} = \vec{b} + \vec{\epsilon}$$  \hspace{1cm} \text{Eq. 2.21}

Where $\vec{\epsilon} \in \mathbb{R}^{mx1}$ is a vector of random error whose components $\epsilon_j$ are uncorrelated, with zero mean and with the same variance (Louveaux 2015; Gander, Gander, and Kwok 2014; Whiten 2008).

In the particle tracking algorithm implemented in the HSC software, different variations of the least squares method are used for achieving the required solutions. The weighted least squares (WLS), Non-negative least squares (NNLS) and the Element wise total least squares (EWTLS) methods are the main algorithms used for solving these overdetermined linear systems due to its constrains and efficiency. For more comprehensive understanding of this solutions to the least square problem refer to Lawson & Hanson (1974), Boyd and Vandenberghe (2009) and Markovsky, Luisa Rastello, Premoli, Kukush, & Van Huffel (2006).

The following sections explains the four steps of the particle tracking approach proposed by Lamberg & Vianna (2016) and Lamberg & Vianna (2007).

2.2.1 Element to mineral conversion

The element to mineral conversion is a linear algebra equation system where the chemical composition is used to calculate the mineral proportions in a sample (Whiten 2008; Pertti Lamberg et al. 1997). The problem $A\vec{x} = \vec{b}$ is characterized by the matrix $A$ containing the weight proportions of an element (columns) in the mineral phase (rows), $\vec{x}$ is the vector of the weight proportions of the minerals and $\vec{b}$ is the vector of weight proportions of elements in the sample. The $\vec{x}$ vector must fulfill two requirements: (1) the mineral grades are equal or greater than zero, and (2) the sum of the mineral proportions must be equal or smaller than 100%. In this case, the solution is commonly achieved by performing the least-squares or non-negative least-square algorithm in HSC Chemistry software (Pertti Lamberg 2016).

Due to the fact that the minerals are rarely stoichiometric in nature, the elemental composition of the minerals is required in order to get an accurate estimation of the mineral proportions (Pertti Lamberg and Vianna 2007; P. Lamberg and Vianna 2016). The elemental composition of a mineral can be obtained some advanced technique such as EPMA (Electron Probe Micro-Analyzer) for example. However, the
assumption that the composition of a mineral is the same (low variance) for the whole deposit must be sustained by an extensive analysis of particles.

2.2.2 Unsized and sized mass balance by mineral

Mass balancing is an application of the principle of the conservation of the mass used for assessing the performance of a plant and further optimization of a process. The mass balance accounts the material that enters and leaves a system. In the case of a circuit, the system could be subdivided in several process units. The simplest case of mass balancing is shown in the Figure 2.13, where a feed and two products are related to the process unit A.

![Figure 2.13. Simple case of mass balance (taken from B. A. Wills & Finch, 2016)](image)

If $W_F$, $W_C$ and $W_T$ are the mass flowrates of the feed, concentrate and tailings, respectively, then the mass balance equation can be written as follows.

$$ W_F = W_C + W_T $$

Eq. 2.22

Therefore, if $X_F$, $X_C$ and $X_T$ are the assays of an element of interest for the feed, concentrate and tailings, respectively, then the mass balance equation for this specific element can be written as follow.

$$ X_F W_F = X_C W_C + X_T W_T $$

Eq. 2.23

These equations can be generalized for the case of a defined mass flowrates enters and leaves a circuit defined by several process units. A network matrix $M \in \mathbb{R}^{m \times n}$, whose m rows represent different nodes (process units) and n columns, different streams can be defined for a circuit. The values of the matrix could be -1, +1 or 0 in case a stream (column) leaves a node (row), enters a node or neither option, respectively. A vector $\vec{W} \in \mathbb{R}^{m \times 1}$ contains the mass flowrates of each stream in the order defined in the matrix $M$. Finally, considering the diagonal matrix $\vec{W} \in \mathbb{R}^{m \times m}$ with the values of the vector $\vec{W}$, and the vector $\vec{X}_i \in \mathbb{R}^{m \times 1}$ as the chemical element assay i on each stream of the network, the mass balance equations can be written as follows (B. A. Wills and Finch 2016b).

$$ M \vec{W} = 0 $$

Eq. 2.24

$$ M \vec{W} \vec{X}_i = 0 $$

Eq. 2.25

These linear mass balance equations are solved in two steps in the HSC software. First, the total mass flowrates are balanced and then the assays. For solving the assays, different least square algorithms could
be used in order to obtain the optimum result. In HSC software, the chosen least square algorithm aims to find the solution by minimizing the weighted sum of squares (WSSQ), as follows.

\[ WSSQ = \sum_{j=1}^{k} \sum_{i=1}^{n} \frac{(a_{ij} - b_{ij})^2}{s_{ij}^2} \]

Eq. 2.26

Where \( j \) correspond to the stream, \( k \) is the number of streams, \( i \) refers to the components, \( n \) is the number of components, or analysis. The value \( a \), \( b \) and \( s \) are the measured value, balanced value and the standard deviation, respectively. A non-negative least square algorithm is chosen, then the values of \( a \) are restricted to be non-negatives. The solution method used in the software HSC is the element-wise total least squares (Outotec et al. 2016; Markovsky et al. 2006).

The sized mass balance by mineral, or mineral by size mass balance, has two constrains: (1) it should matches the unsized mass balance, and (2) the sum of the mineral proportions equals 100% (P. Lamberg and Vianna 2016).

2.2.3 Liberation mass balance

As the previous steps, the objective of the liberation mass balance is to adjust the liberation data in order to match the sized mass balance by mineral. This is achieved by balancing the weight proportions of particle classes for all the streams. The so-called level 3 mass balance consists in (P. Lamberg and Vianna 2016):

1. Adjust the particle grade (mass proportion) to match the mineral by size mass balance
2. Particle classification and binning
3. Extrapolating/interpolating to estimate the liberation data in the missing size fractions
4. Mass balance and reconciliation of each particle class

2.2.3.1 Particle grade adjustment to match the mineral by size mass balance

The mineral composition and mass proportion of each particle in a size fraction are required in this step. The minerals from the liberation analysis are grouped in order to match with the mineral by size mass balance. Then, the mass proportion of the particles are adjusted for every stream and size fraction. The mineral composition of the particles and the number of particles are not changed.

The method consists of the iterative calculation of a correction factor until the difference between the grade of the mineral from level 2 mass balance and the back-calculated mineral grade is close to zero. This routine is robust and the convergence is achieved usually after 25 rounds.

Considering a stream \( f \) and a size fraction \( s \), the algorithm can be described in 3 iterative steps, as follows.

1. Back-calculate the mass proportion (grade) of a mineral:

\[ M'_m = \sum_{i}^{n} P_i * X_{m,i} \]

Eq. 2.27

where \( n \) is the number of particles, \( P_i \) is the mass proportion (grade) of the particle \( i \) and \( X_{m,i} \) is the mass proportion of the mineral \( m \) in the particle \( i \).
2. The correction value \( Q_m \) is calculated for all minerals
\[ Q_m = \frac{M_m}{M'_m} \quad \text{Eq. 2.28} \]

where \( M_m \) is the mass proportion of the mineral \( m \) after the level 2 mass balance.

3. Update the mass proportions of each particle \( P_i \) using the previous value \( P'_i \).

\[ P_i = P'_i \cdot \sum_m X_{m,i} \cdot Q_m \quad \text{Eq. 2.29} \]

4. Repeat steps 1 to 3 until the difference \( \| M_m - M'_m \| \) is close to zero. In each step, the liberation degree of a mineral \( m \) can be calculated.

\[ \text{Liberation degree (m)} = \frac{P_{\text{fully liberated particles of mineral } m}}{M'_m} \quad \text{Eq. 2.30} \]

2.2.3.2 Particle classification and binning

For mass balancing the particles, the particle populations in each stream have to be identical. For this reason, the particles are binned in different classes. The binning of the particles consists of two stages: Basic and Advanced binning. In the first one, known as basic binning, four particle classes are created for each mineral: liberated particles, binary particles, ternary particles and more complex particles. It is common that after this stage, there are classes with a large amount of particles whereas others with a very small amount or even zero. In advanced binning, the basic particle classes are combined in order to achieve enough particles in each particle class for all the streams. The rules for the advanced binning are equal for all the streams, therefore, after the second stage, the particle classes are the same for all the streams in the flowsheet.

2.2.3.3 Interpolation and extrapolation

In case these size fractions are not measured, the missing liberation data can be extrapolated or interpolated from the measured size fractions. The method consists of a linear regression between the geometric mean sizes of the size fractions and the mass proportions of the particle classes in the respective size fractions of the stream.

2.2.3.4 Mass balancing and reconciliation of each particle class

In order to finally balance the particle classes in the circuit, the mass proportions of each particle classes need to be reconciled. Mathematically, this is achieved by minimizing the residual \( R \) for each size fraction and streams.

\[ R = W - \sum_b^k B_b \quad \text{Eq. 2.31} \]
where $W$ is the solid flowrate obtained in mass balance by size and $B_b$ are the flowrates of the binned particle class $b$.

### 2.2.4 Practical overview of Particle tracking and scopes

From a practical point of view, each step has inputs and outputs. In the case of element to mineral conversion, the inputs are the chemical composition of the minerals and the chemical analysis of the sample; and the outputs will be the mineral proportions. The Table 2.6 shows the analytical inputs and the balanced outputs matrices after each step in the particle tracking algorithm. The balanced outputs are also inputs for the next step.

#### Table 2.6. Particle tracking algorithm inputs and outputs (modified from Pertti Lamberg & Vianna, 2007)

<table>
<thead>
<tr>
<th>Step</th>
<th>Sub-steps</th>
<th>Analytical data input</th>
<th>Balanced output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Element to mineral conversion</td>
<td>-</td>
<td>Mineral setup &amp; Chemical assays</td>
<td>Mineral proportions</td>
</tr>
<tr>
<td>Level 1</td>
<td>Unsized mass balance by mineral</td>
<td>-</td>
<td>Total solids flowrates</td>
</tr>
<tr>
<td>Level 2</td>
<td>Sized mass balance by mineral</td>
<td>-</td>
<td>Chemical assays by size</td>
</tr>
<tr>
<td>Level 3</td>
<td>Liberation mass balance</td>
<td>Liberation data reconciliation</td>
<td>Particle data from liberation analysis</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Particle classification &amp; binning</td>
<td>Mineral composition of particles</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Smoothing data</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Mass balance of liberation data</td>
<td>-</td>
</tr>
</tbody>
</table>

The error in the mass proportions of the particles classes is inversely proportional to the square root of the number of particles (NOP) measured. The relative standard deviation of the mass proportions of the particles can be expressed as:

$$
\text{Relative standard deviation (\%)} = \frac{\sigma}{\mu} = \frac{100}{\sqrt{\text{NOP}}}
$$

where $\mu$ and $\sigma$ are the mean and the standard deviation of the mass proportions of the particles, respectively. Furthermore, the number of particles required to achieve enough confidence in the liberation analysis can be estimated by the following equation (P. Lamberg and Vianna 2016):

$$
\text{NOP to be measured} = \frac{10500}{\text{Lowest grade}}
$$

In the case of Leveäniemi, the grade of magnetite is relatively high (>50% wt) and the number of minerals is low (<10 minerals). Using the formula presented above, the minimum NOP required to measure is about 210. This characteristics make Leveäniemi a good candidate for the particle tracking testing.
2.3 Leveäniemi Iron Ore

Leveäniemi ore body is an apatite iron ore located in the Svappavaara ore field at approximately 2 kilometers southwest from the village of Svappavaara, Norrbotten area, Sweden (Figure 2.14). The Leveäniemi ore body account for 204 Mt of high-grade ore with around 64% Fe and 104 Mt of low-grade ore with 20% Fe. With a phosphorus content between 0.02% and 1.1%, Leveäniemi is considered to be the third largest apatite iron ore in Norrbotten (Martinsson et al. 2016; Grip and Frietsch 1973; Bremer 2010). When compared with other deposits in the Norrbotten area, Leveäniemi accounts for similar tonnage and grade to Malmberget ore deposit (Lund 2013).

The geology of Leveäniemi area consists of felsic to mafic metavolcanic and sedimentary rocks. The volcanic host rocks are characterized as trachyandesite with a mineralogy composed mainly by biotite, feldspar, amphiboles, quartz and plagioclase. A layer of metaconglomerates overlays the metavolcanic rocks. A banded section of pyrite, pyrrhotite, chalcopyrite and carbonated scapolite finishes the stratigraphic sequence (Gustafsson 2016).

![Figure 2.14. Location of Leveäniemi ore (taken from Bremer, 2010)](image)

The deposit is approximately 1500 m long and 600 m wide with a north-south orientation. The ore consists of several irregular bodies dipping towards north. A maximum depth of the ore of 500 m occurs in the central part of the deposit. Towards the south, the ore only reaches shallow depths. The main ore consists of massive magnetite and large ore breccia zones with a width up to 100 m. In the middle part of the
deposit, the magnetite is partially altered to hematite. The bedrock is characterized by volcanic and sedimentary rocks strongly metamorphosed and intruded by pegmatite and granite. The intermediate host rock is rich in biotite and scapolite with occasional muscovite (sericite), albite and tremolite-actinolite and possible calcite (Martinsson et al. 2016; Bergman, Kübler, and Martinsson 2001; Frietsch 1966; Bremer 2010).

Figure 2.15. Geological map of Leveäniemi ore. 1) Magnetite ore 2) Calcite-rich magnetite ore 3) Hematite-altered ore 4) Ore breccia 5) Leptite 6) Sericite schist 7) Metabasite 8) Lina granite 9) Skarn (Bremer 2010; Frietsch 1966)

3 Materials and methods
Samples from Leveäniemi iron ore were prepared for Davis tube test. The sample PRE:L:8 was chosen for further optical microscopy and SEM analysis because all the products presented enough mass for the polished sections preparation (>3 g). Furthermore, chemical analysis for all the size fractions of the products and the feed were also performed by ALS chemical.
The mass balance of particles is depicted in the following sections. The procedure consists of mass balance of chemical elements, element to mineral conversion, mass balance of minerals and mass balance of particles. All these steps are done as bulk and by size fraction.

3.1 Samples and sample preparation for Davis tube test

In total, 13 samples from Leveäniemi ore were prepared for the Davis tube test. These samples were taken in a previous sampling campaign. The samples 1 to 10 come from drill cores, and 11 to 13 are pit samples. The rock type, ore type and the mineralogy of alteration and mineralization of the samples are summarized in the Table 3.1.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Rock type</th>
<th>Ore type (MGT% CONTENT)</th>
<th>Alteration mineralogy</th>
<th>Mineralization</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Biotite schist</td>
<td>EMM &lt;50%</td>
<td>Scp, Kfel, Bt, Amph, Ser</td>
<td>Mgt</td>
</tr>
<tr>
<td>1</td>
<td>EMM &gt;60%</td>
<td>Amph, Scp</td>
<td></td>
<td>Mgt</td>
</tr>
<tr>
<td>1.1</td>
<td>Dyke</td>
<td>EMM &lt;50%</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>Dyke</td>
<td>EMM &gt;60%</td>
<td>Amph, Ser</td>
<td>Mgt ± Hem</td>
</tr>
<tr>
<td>3a</td>
<td>Dyke</td>
<td>H</td>
<td>-</td>
<td>Hem ± Mgt ± Mar &amp; V</td>
</tr>
<tr>
<td>3b</td>
<td>Dyke</td>
<td>H</td>
<td>-</td>
<td>Hem ± Mgt</td>
</tr>
<tr>
<td>4</td>
<td>Dyke</td>
<td>EMM &gt;60%</td>
<td>Bt, Amph</td>
<td>Mgt, Cp &amp; V</td>
</tr>
<tr>
<td>5</td>
<td>Dyke</td>
<td>EMM &gt;60%</td>
<td>Amph, Bt, Kfel, Cal</td>
<td>Mgt &amp; V</td>
</tr>
<tr>
<td>6</td>
<td>Trachyandesite</td>
<td>Shallow (MM) &amp; low grade disseminated mineralization</td>
<td>Kfel, Amph, Cal</td>
<td>Mgt</td>
</tr>
<tr>
<td>7</td>
<td>Dyke</td>
<td>Shallow (MM)</td>
<td>Amph, Cal</td>
<td>Mgt + Hem</td>
</tr>
<tr>
<td>8</td>
<td>Dyke</td>
<td>WMM &gt;50%</td>
<td>Amph, Bt, Cal</td>
<td>Mgt ± Py ± Cp</td>
</tr>
<tr>
<td>9</td>
<td>Dyke</td>
<td>WBM &lt;50%</td>
<td>Amph, Cal</td>
<td>Mgt ± Py ± Cp</td>
</tr>
<tr>
<td>10</td>
<td>Dyke</td>
<td>EMM &gt;60%</td>
<td>Amph, Scp, Cal</td>
<td>Mgt ± Py</td>
</tr>
<tr>
<td>11*</td>
<td>Dyke</td>
<td>EMM &gt;60%</td>
<td>Kfel, Amph, Cal</td>
<td>Mgt ± Py ± Cp &amp; V</td>
</tr>
<tr>
<td>12*</td>
<td>Dyke</td>
<td>EMM 50-60% (high grade breccia)</td>
<td>Kfel, Bt, Amph, Cal, Sil</td>
<td>Mgt ± Py ± Cp &amp; V</td>
</tr>
<tr>
<td>13*</td>
<td>Dyke</td>
<td>EMM &lt;50%</td>
<td>Kfel, Ep, Sil, Amph, Cal</td>
<td>Mgt ± Py ± Cp ± Chl &amp; V</td>
</tr>
</tbody>
</table>

The sample preparation procedure was carried out for the 13 samples in the laboratory of LTU (Figure 3.1). The sample preparation is based on a novel Geometallurgical Comminution Test (GCT) proposed by Mwanga, Lamberg, and Rosenkranz 2015. The procedure uses the GCT after size reduction (<3.36 mm).
and sampling (220 g) as preparation for further liberation analysis. The sample preparation procedure is depicted as follows.

1. Sample crushing below 3.36 mm. The product of the jaw crusher is sieved using a 3.36 mm sieve. The retained material is recirculated to the jaw crusher.
2. Splitting using the rotary splitter. 3 subsamples are obtained:
   - 10 kg for WLIMS
   - 1 – 2 kg for GCT
   - Back up
3. From the 1 – 2 kg for GCT, 5 subsamples of 220 g are obtained using the riffle splitter.
4. The 5 subsamples of 220 g are sent to GCT for 11 minutes
5. The GCT product is sieved during 10 minutes. Four fractions are obtained:
   - <38 µm
   - 38 – 75 µm
   - 75 – 106 µm
   - >106 µm
6. For the 3 finest fractions, 10 subsamples of 10 - 15 g are obtained.

\[ \sigma^2 (FSE) = \left( \frac{1}{M_s} - \frac{1}{M_{tot}} \right) f g \beta c d^3 \]  

\text{Eq. 3.1}

3.2 Sampling error
For the estimation of the sampling error, the theory of sampling proposed by Pierre Gy was used. Particularly, the simplified formula dedicated to ores allows to estimate a fundamental sampling error for each stage of the sample preparation.

Figure 3.1. Sample preparation flowsheet.
where $M_s$ and $M_{\text{lot}}$ are the mass of the sample and the mass of the initial lot, respectively. Since the mineral to study is magnetite (72.36 wt% of Fe) and it belongs to the isometric crystal system, meaning that the expected shapes of the crystals are related to the cubic system, the other parameters of Gy’s simplified formula are considered to be the following:

- The shape factor $f = \frac{\pi}{6} \approx 0.5$. Ratio between the volume and mean diameter of the particle. Magnetite belongs to the isometric crystal system, then its form can be approximated by an sphere.
- Particle size distribution factor $g = 0.25$. This factor stands for the degree of dispersion of the particle size. If all the grains have the same size, then $g = 1$. Higher dispersion means lower values of $g$.
- Liberation parameter $= \sqrt{\frac{d_l}{d_{95}}}$, where $d_l$ is the liberation size assumed to be 100 µm after optical microscopy analysis. In the coarsest size fraction (75-106 µm) the magnetite is mainly liberated, then the liberation size is at least 100 µm (See Optical microscopy description of Leveäniemi products after Davis Tube test: PRE:L:8:G).
- Size of the largest particle $d$: passing sieve size
- Mineralogical constitution parameter $c$: function of the proportion and density of the mineral

The constitution parameter $c$ is calculated for each sample separately depending on the grade of iron.

$$c = \frac{1-a}{a} \left[ (1-a) \rho_m + a \rho_g \right] \quad \text{Eq. 3.2}$$

where $\rho_m$ and $\rho_g$ are densities of magnetite (5.15 g/cm³) and gangue (2.8 g/cm³), respectively. The factor $a$ is the proportion of the mineral in the sample and it is equal to the ratio between iron content in the sample and the iron content in magnetite. This factor assumes that all the iron content is associated to magnetite which is not always true due to the presence of other iron oxides and Fe-bearing silicates in the gangue (B. A. Wills and Finch 2016b).

With these assumptions, it is possible to estimate a fundamental sampling error for each sample. The assumptions made in the parameters could lead to an overestimation of the sampling error (Table 3.2 & Figure 3.2).

The FSE has an inverse correlation with the grade, therefore, a relatively high grade of magnetite in the samples reduces the error.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Fe content (%)</th>
<th>Crush &lt;3.36 mm</th>
<th>Sample 220 g</th>
<th>GCT</th>
<th>Sample 10 g</th>
<th>FSE total</th>
<th>STD (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRE:L:1:M</td>
<td>28.29</td>
<td>3.1E-07</td>
<td>2.5E-05</td>
<td>1.2E-06</td>
<td>2.6E-05</td>
<td>5.2E-05</td>
<td>0.72</td>
</tr>
<tr>
<td>PRE:L:2:G</td>
<td>66.49</td>
<td>1.2E-08</td>
<td>9.8E-07</td>
<td>4.7E-08</td>
<td>1.0E-06</td>
<td>2.1E-06</td>
<td>0.14</td>
</tr>
<tr>
<td>PRE:L:3:G</td>
<td>64.57</td>
<td>1.9E-08</td>
<td>1.4E-06</td>
<td>6.6E-08</td>
<td>1.5E-06</td>
<td>2.9E-06</td>
<td>0.17</td>
</tr>
<tr>
<td>PRE:L:4:G</td>
<td>67.02</td>
<td>1.3E-08</td>
<td>8.8E-07</td>
<td>4.3E-08</td>
<td>9.4E-07</td>
<td>1.9E-06</td>
<td>0.14</td>
</tr>
<tr>
<td>PRE:L:5:G</td>
<td>60.93</td>
<td>3.3E-08</td>
<td>2.2E-06</td>
<td>1.1E-07</td>
<td>2.4E-06</td>
<td>4.7E-06</td>
<td>0.22</td>
</tr>
<tr>
<td>PRE:L:6:G</td>
<td>26.17</td>
<td>4.1E-07</td>
<td>2.8E-05</td>
<td>1.4E-06</td>
<td>3.0E-05</td>
<td>6.0E-05</td>
<td>0.77</td>
</tr>
<tr>
<td>PRE:L:7:G</td>
<td>69.33</td>
<td>5.6E-09</td>
<td>4.7E-07</td>
<td>2.3E-08</td>
<td>5.0E-07</td>
<td>1.0E-06</td>
<td>0.10</td>
</tr>
<tr>
<td>PRE:L:8:G</td>
<td>61.75</td>
<td>3.2E-08</td>
<td>2.0E-06</td>
<td>9.7E-08</td>
<td>2.1E-06</td>
<td>4.3E-06</td>
<td>0.21</td>
</tr>
<tr>
<td>PRE:L:9:G</td>
<td>35.02</td>
<td>2.2E-07</td>
<td>1.6E-05</td>
<td>7.7E-07</td>
<td>1.7E-05</td>
<td>3.4E-05</td>
<td>0.58</td>
</tr>
</tbody>
</table>
Table 1. Results for Davis tube tests.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Current (A)</th>
<th>Mass (g)</th>
<th>Fundamental Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRE:L:10:G</td>
<td>63.08</td>
<td>2.5E-08</td>
<td>1.7E-06</td>
</tr>
<tr>
<td>PRE:L:11:M</td>
<td>64.89</td>
<td>1.9E-08</td>
<td>1.3E-06</td>
</tr>
<tr>
<td>PRE:L:12:G</td>
<td>47.84</td>
<td>9.7E-08</td>
<td>6.9E-06</td>
</tr>
<tr>
<td>PRE:L:13:M</td>
<td>19.14</td>
<td>6.8E-07</td>
<td>4.7E-05</td>
</tr>
</tbody>
</table>

Figure 3.2. Nomograms per samples for the sample preparation procedure.

3.3 Davis tube test

The Davis tube tests were performed in the LKAB laboratory located in Malmberget, Sweden. The flowsheet of the procedure is shown in the Figure 3.3. The material was passed through the Davis tube with different currents. For each current, the products were dried and the tailings were fed to the next test. The only parameter that was controlled for the different tests was the current intensity.

Figure 3.3. Davis tube test flowsheet.
The Davis tube test was performed for the 3 size fractions: \(< 38 \, \mu m\), \(38 – 75 \, \mu m\) and \(75 – 106 \, \mu m\). For each size fraction, 5 rounds were carried out for the 10 to 15 g subsamples. Usually, about 20 g of samples are fed in each round of Davis tube test. Therefore, this range of masses were chosen in order to avoid saturation of the device (Personal communication at LKAB).

The tailings of each round were fed to the next Davis tube test with a higher current intensity. Finally, the products were collected, dried overnight, weighted, labeled and sent for chemical analysis to ALS Scandinavia AB in Piteå, Sweden (https://www.alsglobal.se/en).

The PRE:L:8 products were prepared as a polished section and carbon coated for further analysis in the scanning electron microscope (Merlin SEM – Zeiss Gemini) at Luleå University of Technology. The SEM-EDS combines the backscattered detector images and the EDS analysis in order to obtain the mineralogy of each particle in the polished section (Liipo, J. Lang, C. Burgess, S. Otterström, H. Person, H. Lamberg 2012; Parian 2017).

3.4 Element to mineral conversion: Set Up

A total of 30 minerals were considered for the mineral set up of the element to mineral conversion algorithm. This minerals were grouped 12 classes for further analysis in HSC Chemistry (Table 3.3). Since there is no satmagan analysis available for the products after the Davis tube test, magnetite and hematite are treated together. The mineral composition indicated in the Table 3.3 was obtained by Gustafsson (2015). Average stoichiometric minerals were added to the mineral groups.
Table 3.3. Mineral setup for EMC. (*) Stoichiometric mineral

<table>
<thead>
<tr>
<th>Mineral group</th>
<th>Mineral</th>
<th>SG</th>
<th>Si %</th>
<th>Ti %</th>
<th>Al %</th>
<th>V %</th>
<th>Fe %</th>
<th>Mn %</th>
<th>Mg %</th>
<th>Ca %</th>
<th>Sr %</th>
<th>Na %</th>
<th>K %</th>
<th>P %</th>
<th>C %</th>
<th>Cu %</th>
<th>S %</th>
<th>Ce %</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Fe oxides</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Magnetite</td>
<td>5.2</td>
<td>0.1</td>
<td>0.1</td>
<td>71.1</td>
<td>0.04</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>Hematite</td>
<td>5.2</td>
<td>0.29</td>
<td>0.1</td>
<td>67.9</td>
<td></td>
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<tr>
<td><strong>Amphiboles</strong></td>
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</tr>
<tr>
<td></td>
<td>Tremolite*</td>
<td>3.0</td>
<td>27.7</td>
<td>0.29</td>
<td>0.1</td>
<td>9.9</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>Actinolite</td>
<td>3.0</td>
<td>26.0</td>
<td>0.04</td>
<td>0.5</td>
<td>8.0</td>
<td>0.15</td>
<td>10.6</td>
<td>9.3</td>
<td>0.3</td>
<td>0.1</td>
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<tr>
<td></td>
<td>Hornblende</td>
<td>3.0</td>
<td>22.7</td>
<td>0.20</td>
<td>4.2</td>
<td>9.8</td>
<td>0.13</td>
<td>8.9</td>
<td>8.3</td>
<td>0.2</td>
<td>0.7</td>
<td>0.2</td>
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<td><strong>Micas</strong></td>
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</tr>
<tr>
<td></td>
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<td>36.8</td>
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</table>
The EMC system was solved in 3 rounds. The first one includes the non-silicates minerals which can be associated directly to a chemical element. The second round includes mainly the silicates and Ti oxides. The third round is only composed by the Fe oxides related to iron and vanadium (Table 3.4).

Table 3.4. Rounds for the element to mineral conversion in HSC Chemistry 7

<table>
<thead>
<tr>
<th>ROUNDS</th>
<th>MINERALS</th>
<th>ELEMENTS</th>
<th>ALGORITHM</th>
</tr>
</thead>
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<td>LS</td>
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<td>Cal</td>
<td>P</td>
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<td>Cu</td>
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<td>2</td>
<td>Amphiboles</td>
<td>Si</td>
<td>NNLS</td>
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<td>Micas</td>
<td>Al</td>
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<td></td>
<td>Plagioclases</td>
<td>Ca</td>
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<td></td>
<td>K feldspars</td>
<td>K</td>
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<td>Qtz</td>
<td>Na</td>
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<td>Pyroxenes</td>
<td>Mg</td>
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<tr>
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<td>Other Silicates</td>
<td>Ti</td>
<td></td>
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<tr>
<td></td>
<td>Ti oxides</td>
<td>Mn</td>
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<tr>
<td>3</td>
<td>Fe Oxides</td>
<td>Fe</td>
<td>NNLS</td>
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</table>

3.5 Particle tracking algorithm
In a mineral processing circuit, the material is transported between units as a mixture of solids, liquids and gases. These mixture is known as a streams and it has its own measurable properties. In HSC chemistry software, the streams are characterized by their solid, liquid and gas components. The solid components consist of particles. The properties of the streams and the particles that compose them shown in the following tables:

Table 3.5. Stream properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Identification</strong></td>
<td></td>
</tr>
<tr>
<td>ID</td>
<td>Feed, Concentrate, Tail,...</td>
</tr>
<tr>
<td><strong>Intensive</strong></td>
<td></td>
</tr>
<tr>
<td>Mineral composition by fraction</td>
<td>Mineral proportions obtained by mineral by size mass balance (level 2)</td>
</tr>
<tr>
<td>(wt%)</td>
<td></td>
</tr>
<tr>
<td>Wt% of bulk</td>
<td>Mass proportion of each size fraction of the total input. They add up to100%</td>
</tr>
<tr>
<td><strong>Extensive</strong></td>
<td></td>
</tr>
<tr>
<td>Total input (t/h)</td>
<td>Flowrate of a stream</td>
</tr>
</tbody>
</table>
**Table 3.6. Particle properties**

<table>
<thead>
<tr>
<th>Property</th>
<th>Definition or example</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Identification</strong></td>
<td></td>
</tr>
<tr>
<td>ID</td>
<td>M.P. &quot;N&quot; of minerals&quot;.&quot;B (if binary particle)&quot;.Min1-Min2.N° of Bins</td>
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<tr>
<td>Particle Type</td>
<td>Minerals that form the particle. For example, “Mgt-Hem” if the particle is composed by those 2 minerals only</td>
</tr>
<tr>
<td>N° of minerals</td>
<td>Number of particles that compose the particle</td>
</tr>
<tr>
<td>Min1,…, Min n</td>
<td>Mineral predominance. Each mineral in different column</td>
</tr>
<tr>
<td><strong>Intensive</strong></td>
<td></td>
</tr>
<tr>
<td>Specific gravity (g/cm³)</td>
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<tr>
<td>Mineral proportions (wt%, vol%, srf%)</td>
<td>Mineral proportions of the particle</td>
</tr>
<tr>
<td>Wt% in Bulk</td>
<td>Mass proportion of the particle respect to the total input. They add up to 100%</td>
</tr>
<tr>
<td>Wt% in Fraction</td>
<td>Mass proportion of the particles respect to the size fraction. They add up to 100% in each size fraction</td>
</tr>
<tr>
<td><strong>Extensive</strong></td>
<td></td>
</tr>
<tr>
<td>Flowrate (t/h)</td>
<td>Flowrate of particle class</td>
</tr>
<tr>
<td>NOP</td>
<td>Number of particles</td>
</tr>
</tbody>
</table>

Furthermore, these properties relate to each other by the following equations for a particle \(i\) in a stream.

\[
Flowrate_i = Total Input_{stream} \times Wt\% in Bulk_i
\]  \(\text{Eq. 3.3}\)

\[
Wt\% in Bulk_i = Wt\% in Fraction_i \times Wt\% of Bulk_{stream}
\]  \(\text{Eq. 3.4}\)

In the following steps of particle tracking, the properties of the particles need to be merged. The extensive properties of the particles are only added up, meanwhile the intensive properties are usually obtained by a weighted average of the properties using the flowrates (t/h) of each particle as weights.

### 3.5.1 Adjusting particles

From mass balance by size (level 2), the mass proportions of the minerals are balanced by size fractions. From the SEM, the wt% of each particle (“wt% in Fraction”) is obtained and are the values to be reconciled with the mass proportions from MB level 2 as input. Using the equations Eq. 3.3 and Eq. 3.4 mentioned above, the wt% in bulk and flowrates are obtained.

**Outputs:** wt% in fraction, wt% in bulk, flowrates

### 3.5.2 Basic Binning

For basic binning, there are 3 variables that need to be defined: threshold, tolerance and number of binary bins (NBB). A mineral proportion in a particle has to be equal or more than the threshold in order that particle to be considered as liberated (Recommended value is 100%). The tolerance is the value below
which the mineral proportions are not considered for the bin classification (Recommended value is 1%). The NBB is the number of bins that are considered in binary particles (Recommended value is 20).

The total number of basic bins is calculated as follows. If there are in total N minerals (from mineral set up) and 4 Bin types are generated, the number of bins can be calculated as follows:

- Liberated bins: N with a threshold value of 100 wt%
- Binary bins: if Number of Binary Bins (NBB) is 20 for each pair of minerals there are 20 bins. The 20 bins are: [95, 100]; [90, 95]; ...; [5, 10]and [0, 5]. The total number of binary bins can be calculated as \( \left( \frac{N}{2} \right) \times NBB \) With \( \left( \frac{N}{2} \right) = \frac{N!}{(N-2)!2!} \) which stands for all the possible pairs with no repetition with a N minerals
- Ternary bins: Only 6 bins are created for each triplet of minerals. The total ternary bins is calculated as \( \left( \frac{N}{3} \right) \times 6 \)
- Complex bins: only one

\[
\text{Total basic bins } = N + \left( \frac{N}{2} \right) \times NBB + \left( \frac{N}{3} \right) \times 6 + 1
\]

The basic binning process consists of 2 stages: Classification of the particles and merging the properties. The classification uses the previously defined parameters and each particle is classified as liberated, binary, ternary or more complex. In the case of binary and ternary, the mineral predominance is also given (Min 1, Min 2 and Min 3). The merging of the particles is done by weighted average of the intensive properties using the flowrates (t/h) as weights.

The basic binning only depends on the number of minerals, thus the number of bins will be the same for all the streams in the circuit after this step.

3.5.3 Missing

When there are missing liberation data for a size fraction, it can be estimated from the measured ones. For this estimation, it is required that the particle classes are identical for all the size fractions. This is achieved after basic binning or by imposing it in the advanced binning.

The missing “wt% in fraction” from one size fractions, as it is the case of the fraction 38-75 µm for C2 (Concentrate 2 after DT at 0.2 A), could be calculated using the other 2 size fractions, just by linear interpolation. The interpolation consists of calculating the equation of the curve:

\[
\text{wt\% in fraction}_i = m \times \text{Average size} \ (\mu m) + c
\]

Where m (%/µm) is the slope of the line and c (%) the constant when the average size is zero. Using the geometric average size of the missing size fraction allows to calculate the wt% in fraction for all the particle classes as the red point in the Figure 3.4. Then, the wt% in bulk and the flowrates (t/h) are calculated using the equations Eq. 3.3 and Eq. 3.4. The mineral proportions are calculated using the weighted average between the other size fractions. The NOP can also be calculated by linear interpolation.
3.5.4 Creating reference stream

A reference stream must be defined in order to calculate the distribution of the particle classes in each stream. The creation of a reference stream can be done in 2 ways: considering the feed as the reference, or creating an average reference stream as the sum of all streams. Once the streams are reconciled and the basic binning is done, a reference stream must be created in order to obtain the particle classes which will be balanced. Both methods are depicted for a simple 1-node circuit with feed, concentrate and tailings.

In case that the liberation data was only measured for the concentrate and tailing, then the back-calculated feed could be obtained as follows.

1. Reconstruct the extensive and intensive variables of the feed stream. The wt% of bulk for each fraction \( f \) are calculated by weighted average using the total inputs as weights.

\[
Total \ input_{Feed} \left[ \frac{t}{h} \right] = Total \ input_{Conc} \left[ \frac{t}{h} \right] + Total \ input_{Tail} \left[ \frac{t}{h} \right]
\]

\[
Wt\% \ of \ bulk_{f,Feed} = \frac{Total \ input_{Conc} \left[ \frac{t}{h} \right] * Wt\% \ of \ bulk_{f,Conc} + Total \ input_{Tail} \left[ \frac{t}{h} \right] * Wt\% \ of \ bulk_{f,Tail}}{Total \ input_{Feed} \left[ \frac{t}{h} \right]}
\]

2. Reconstruct extensive and intensive variables in the feed for each particle class \( i \) in each size fraction \( f \).

\[
Flowrate_{i,Feed} \left[ \frac{t}{h} \right] = Flowrate_{i,Conc} \left[ \frac{t}{h} \right] + Flowrate_{i,Tail} \left[ \frac{t}{h} \right]
\]

\[
NOP_{i,Feed} = NOP_{i,Conc} + NOP_{i,Tail}
\]

\[
Mineral \ wt\%_{i,Feed} = \frac{Flowrate_{i,Conc} \left[ \frac{t}{h} \right] * Mineral \ wt\%_{i,Conc} + Flowrate_{i,Tail} \left[ \frac{t}{h} \right] * Mineral \ wt\%_{i,Tail}}{Flowrate_{i,Conc} \left[ \frac{t}{h} \right] + Flowrate_{i,Tail} \left[ \frac{t}{h} \right]}
\]
\[
Wt\%\ in\ Bulk_{i,\text{Feed}} = \frac{Flowrate_{i,\text{Conc}} \cdot Wt\%\ in\ Bulk_{i,\text{Conc}} + Flowrate_{i,\text{Tail}} \cdot Wt\%\ in\ Bulk_{i,\text{Tail}}}{Flowrate_{i,\text{Conc}} + Flowrate_{i,\text{Tail}}} \quad \text{Eq. 3.12}
\]

3. Check wt% in Fraction for the feed

\[
Wt\%\ in\ Fraction_{i,f} = \frac{Wt\%\ in\ Bulk_i}{Wt\%\ of\ Bulk_f} \quad \text{Eq. 3.13}
\]

If the reference stream is considered as the summation of all the products and feed, it can be calculated as follows.

1. Calculate reference stream extensive properties for streams and particle classes. The number of particles is calculated as the arithmetic average of the 3 streams.

\[
Total\ input_{\text{Reference}} \left[\frac{t}{h}\right] = \sum_{\text{stream}} Total\ input_{\text{stream}} \left[\frac{t}{h}\right] \quad \text{Eq. 3.14}
\]

\[
Flowrate_{i,\text{Reference}} \left[\frac{t}{h}\right] = \sum_{\text{stream}} Flowrate_{i,\text{stream}} \left[\frac{t}{h}\right] \quad \text{Eq. 3.15}
\]

\[
NOP_{i,\text{Reference}} = \frac{1}{3} \sum_{\text{stream}} Flowrate_{i,\text{stream}} \quad \text{Eq. 3.16}
\]

2. Calculate Reference stream intensive properties using the flowrates of each particle class \(i\) for the bulk and by size fraction. The size, specific gravity and mineral proportions for each mineral \(m\) are obtained by weighted average with the flowrates of the particle classes \(i\) in each stream \(s\) as weights.

\[
Wt\%\ in\ Bulk_i = \frac{Flowrate_i \left[\frac{t}{h}\right]}{Total\ input_{\text{Reference}} \left[\frac{t}{h}\right]} \quad \text{Eq. 3.17}
\]

\[
Wt\%\ in\ Fraction_{i,f} = \frac{Wt\%\ in\ Bulk_i}{Wt\%\ of\ Bulk_f} \quad \text{Eq. 3.18}
\]

\[
Mineral\ wt\%_{m,f,\text{Reference}} = \frac{\sum_{i,f} Flowrate_{i,f,s} \left[\frac{t}{h}\right] \cdot Mineral\ wt\%_{m,f,s}}{\sum_{i,f} \phi_{i,f,s} \left[\frac{t}{h}\right]} \quad \text{Eq. 3.19}
\]

3.5.5 Advanced Binning

The objective is to adjust the NOP so the conclusions are statistically sound. Set a threshold of minimum NOP per bin (Recommended value is 25). The algorithm consists of merging the bins with the next one if the number of particles is below the minimum. This process is iterative until the minimum is reached. This applies for the binary and ternary bin types. For instance, if the values of minimum NOP per bin and NBB are set both to be 20, and there is 1 particle in each bin, then all the bins for that particle type will be
merged in a unique particle class. The final NOP will be 20 and the properties will be merged by the rules mentioned above.

3.5.6 Smoothing
The smoothing process consists of recalculating the wt% in bulk for every particle in order to decrease the dispersion of the data. The procedure is the following:

1. For each particle, calculate:
   \[
   \text{wt\% ratio} = \frac{\text{wt\% in bulk}_{\text{Stream}}}{\text{wt\% in bulk}_{\text{Reference}}}
   \]
   \[Eq. 3.20\]

2. For each binary and ternary bin types, the curve wt\% ratio versus wt\% of a mineral in particle is considered (Figure 3.5).

![Figure 3.5. Ratio versus magnetite wt\% in particle](image)

3. Do the best fitting of the curve by adjusting it to a selected library of continuous set of equations. The curve with the least sum of squares is selected as the best fit. The library is composed by polynomial equations and others such as incomplete beta distribution. The propagation error is calculated as the square-root of the inverse of the NOP.

4. Convert the ratios to wt\% and replace the originals values by the smoothed ones.

3.5.7 Particle mass balance
The final step is the mass balance of the particle classes' flowrates. For this purpose, the system of equations defined by \(Ax = \bar{b}\) must be solved. In this case, the matrix \(A\) and the vector \(\bar{b}\) contain all the constraints of the circuit, and the vector \(\bar{x}\) contains all the solutions for the particle classes' flowrates. This system is solved for each size fraction independently.

For illustration, a simple 1-node system (Figure 3.6) with 2 mineral groups, 3 streams and 4 particle classes for 1 size fraction is considered. Let \(n\) be the number of particle classes, \(m\) the number mineral groups, \(s\)
the number of streams and \( u \) the number of units or nodes. The constraints \( A \) and \( \vec{b} \) can be built in 4 parts. Each part of the constraints are described as follows.

![Diagram of a simple 1-node mass balance circuit](image)

\textit{Figure 3.6. Simple 1-node mass balance circuit}

1. Let \( A_1 \in \mathbb{R}^{(u \times n) \times (s \times n)} \) and \( \vec{b}_1 \in \mathbb{R}^{(u \times n) \times 1} \). These are the constraints for the mass balance of the flowrates per particle. The flowrate in the feed is equal to the summation of flowrates in the products (Figure 3.7).

\[
A_1 = \begin{bmatrix}
1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1
\end{bmatrix}, \quad \vec{b}_1 = \begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]

\textit{Figure 3.7. First part of the constraints matrices}

2. Let \( A_2 \in \mathbb{R}^{(m \times s) \times (s \times n)} \) and \( \vec{b}_2 \in \mathbb{R}^{(m \times s) \times 1} \). In these constraints, the composition of the mineral groups (Table 3.7) and the mineral mass proportions from the mass balance by size (Table 3.8) are considered.

<table>
<thead>
<tr>
<th>Particle class</th>
<th>Magnetite wt%</th>
<th>Quartz wt%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Particle class 1</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>Particle class 2</td>
<td>75</td>
<td>25</td>
</tr>
<tr>
<td>Particle class 3</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>Particle class 4</td>
<td>25</td>
<td>75</td>
</tr>
</tbody>
</table>

\textit{Table 3.7. Particle class compositions}

<table>
<thead>
<tr>
<th>Size fraction 1</th>
<th>Flowrate (t/h)</th>
<th>Magnetite wt%</th>
<th>Quartz wt%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feed</td>
<td>40</td>
<td>60</td>
<td>40</td>
</tr>
<tr>
<td>Concentrate</td>
<td>10</td>
<td>96</td>
<td>5</td>
</tr>
<tr>
<td>Tailing</td>
<td>30</td>
<td>48</td>
<td>52</td>
</tr>
</tbody>
</table>

\textit{Table 3.8. Flowrates and composition of the mineral mass balance by size for size fraction 1}
Then the matrix $A_2$ and vector $\overline{b_2}$ are defined as (Figure 3.8):

$$
A_2 = \begin{bmatrix}
100 & 0 & 0 & 75 & 0 & 0 & 50 & 0 & 0 & 25 & 0 & 0 \\
0 & 100 & 0 & 0 & 75 & 0 & 0 & 50 & 0 & 0 & 25 & 0 \\
0 & 0 & 100 & 0 & 0 & 75 & 0 & 0 & 50 & 0 & 0 & 25 \\
0 & 0 & 0 & 25 & 0 & 0 & 50 & 0 & 0 & 75 & 0 & 0 \\
0 & 0 & 0 & 0 & 25 & 0 & 0 & 50 & 0 & 0 & 75 & 0 \\
0 & 0 & 0 & 0 & 0 & 25 & 0 & 0 & 50 & 0 & 0 & 75 \\
\end{bmatrix}
\quad \overline{b_2} = \begin{bmatrix}
40 \times 60 \\
10 \times 96 \\
30 \times 48 \\
40 \times 40 \\
10 \times 5 \\
30 \times 52 \\
\end{bmatrix}
$$

$6 \times 12$  \quad $6 \times 1$

Figure 3.8. Second part of the constraints matrices

3. Let $A_3 \in \mathbb{R}^{s \times (s+n)}$ and $\overline{b_3} \in \mathbb{R}^{s \times 1}$. These constraints stand for the total flowrate from the mass balance by size. The matrix $A_3$ and vector $\overline{b_3}$ are defined as (Figure 3.9):

$$
A_3 = \begin{bmatrix}
1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\
\end{bmatrix}
\quad \overline{b_3} = \begin{bmatrix}
40 \\
10 \\
30 \\
\end{bmatrix}
$$

$3 \times 12$  \quad $3 \times 1$

Figure 3.9. Third part of the constraints matrices

4. Let $A_4 \in \mathbb{R}^{(#bintypes \times s) \times (s+n)}$ and $\overline{b_4} \in \mathbb{R}^{(#bintypes \times s) \times 1}$ where $#bintypes$ is the number of bin types present in the reference stream. This constrain aims to preserve the proportions of liberated, binaries, ternaries and more complex bin types obtained from the analytical method. In this case, there are only two bin types: liberated and binaries. Let the flowrate proportion of liberated and binaries per stream be the following.

Table 3.9. Bin types mass proportions per stream

<table>
<thead>
<tr>
<th>Flowrate (t/h)</th>
<th>Liberated (wt %)</th>
<th>Binaries (wt %)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Feed</strong></td>
<td>40</td>
<td>38.75</td>
</tr>
<tr>
<td><strong>Concentrate</strong></td>
<td>10</td>
<td>95</td>
</tr>
<tr>
<td><strong>Tailing</strong></td>
<td>30</td>
<td>20</td>
</tr>
</tbody>
</table>

Since there is only 1 particle class is fully liberated and 3 binaries, the matrix $A_4$ and vector $\overline{b_4}$ are defined as (Figure 3.10).
Finally, the adjacency matrix $A$ and the vector $\bar{b}$ are obtained by vertically putting the 4 matrices and vector as follows (Figure 3.11):

$$A = \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{bmatrix}, \quad \bar{b} = \begin{bmatrix} \bar{b}_1 \\ \bar{b}_2 \\ \bar{b}_3 \\ \bar{b}_4 \end{bmatrix}$$

This simple case of defining the matrices and vector for solving the systems can be used for any circuit in particle mass balance. However, depending on the number of streams, nodes, minerals and particle classes, the definition of the constraints can be more complex. In the example above, a 1-node system considered a matrix $M = \begin{bmatrix} 1 & -1 & -1 \end{bmatrix}$ was considered for describing the mass balance constraints for $A_1$ and $\bar{b}_1$. However, the mass balance for the Davis tube circuit can be described by a 3x7 adjacency matrix (Figure 3.12).

By using this matrix in the same example from above, the final matrix $A$ and the vector $\bar{b}$ will have a size of $(47 \times 28)$ and $(47 \times 1)$, respectively. Thus, circuit can be solved using different 2 strategies. The first one is to solve node-by-node independently and comparing the common streams errors. The second one is to solve the entire system using the $3 \times 7$ adjacency matrix. A comparison between these 2 methods will be done for the Davis tube test results in sample PRE:L:8.

3.5.8 Binary particles recoveries

In order to study the recovery of the binary particles as a function of the wt% of the mineral of interest in the particle, a model of binary particles recovery has to be taken into consideration. This type of models
adjust the recovery of the particles to a continuous function which can be used in further simulation of the behavior of the particles.

The cumulative recoveries of the binary particles can be modeled using an incomplete beta function (Abramowitz and Stegun 1964; Parian 2017) which is defined as follows:

\[ I_x(a, b) = \frac{1}{B(a, b)} \int_0^x t^{a-1} (1-t)^{b-1} dt \tag{Eq. 3.21} \]

Where the beta function \( B(a, b) \) is defined as:

\[ B(a, b) = \int_0^1 t^{a-1} (1-t)^{b-1} dt \tag{Eq. 3.22} \]

With \( a, b > 0 \) and \( 0 < x < 1 \). If for simplicity, the value of \( a \) is set to 1, the incomplete beta function becomes:

\[ R_x = I_x(1, b) = 1 - (1 - x)^b \tag{Eq. 3.23} \]

Then, the cumulative recoveries of the concentrates in Davis tube \( R_x \) can be modeled by fitting the parameter \( b \) in Eq. 3.23. The \( b \) values ranges from 0 to 30 and the one that minimizes the RMSD between the data (balanced or back calculated recoveries) and the model is kept.

### 3.6 Particle tracking strategy in Davis tube circuit

The particle tracking algorithm is the final step after solving the bulk and sized mass balance for elements and minerals. The liberation mass balance is based on the mineral mass balance by size. In the case of the Davis tube circuit, the liberation data of the products is reconciled with the mineral mass balance by size and the basic binning. After these steps, all the products will have the same particle classes for all the size fractions. Due to the lack of enough mass of the intermediate size fraction (38-75 µm) for 2nd concentrate, the liberation data is not available. However, the missing size fraction data can be interpolated from the other size fractions for the same stream.

Once the particle classes are the same for all the products, the intermediate streams (or relative feeds) are back calculated applying the principle of conservation of mass since no mass is retained in the circuit. Then, the feed is used as the reference stream for advanced binning. All the particle classes whose number of particles are not above a certain threshold are eliminated due to its low representativeness. Finally, the mass balance system is solved using the non-negative least squares algorithm (NNLS).

The smoothing is not performed before the mass balance since it is considered to modify the original liberation data. The liberation data can be modeled after it is balanced.
4 Results

Following a mineralogical approach to analyze the behavior of the iron ore in the Davis tube test, the results are presented in this order: sized chemical mass balance, mineralogical study (optical microscopy and element to mineral conversion), bulk and sized mineral mass balance, liberation analysis and liberation mass balance. In the following sections, only the highlights of the results are described. For a more extensive revision of the results for each sample, refer to the Annexes section.

4.1 Sized chemical mass balance

After the mass balance of the chemical elements by size, the focus of this sections are the iron recovery and grade of the Leveäniemi samples. Furthermore, the selectivity curves are also described in order to understand the behavior of the other elements in the Davis tube test as a function of the iron recovery.

The cumulative recoveries of iron show two trends. The first one is characterized by a linear correlation between the current (0.1, 0.2 and 0.5 A) and the recoveries. The second one is characterized by a high recovery (>80%) shown in the second concentrate (0.2 A). The latter results are shown in samples 1, 6, 7 and 10 which come from a previous campaign. This results can be biased due to the preparation error. In most of the cases, the trends are similar for 3 size fractions. The sample L3 presents a different trend due to its mineralogy composed mainly by hematite (Figure 4.1).

![Figure 4.1. Fe cumulative recoveries for Leveäniemi samples by size](image-url)
The selectivity curves are constructed for comparing the distribution of the chemical elements versus the recovery of iron. The identity line (\(y=x\)) in the graph represents the no selectivity of an element respect to iron, whereas the further to the right side of the graph the curves are means that the element is separated more efficiently from iron during the process. In the 3 currents Davis tube circuit, the elements that tend to behave along with iron are primarily V, followed by Mn and Ti. In some cases, Cu could be also considered to behave with low selectivity, meaning that some sulphide minerals are reported in the concentrates. The low selectivity of Al in some cases could be related to the presence of silicates in the concentrate, such as micas and amphiboles. In the case of the sample L8, the selectivity of the elements increases with the size fraction (Figure 4.2). Only V, Mn and Ti maintain their trend which could be explained due to its presence in the lattice of magnetite. These elements behave similarly to Fe and could substitute it due to present similar ionic potential (Bernal and Railsback 2008).

![Selectivity Curves](image)

**Figure 4.2. Selectivity curve for sample L8**

### 4.2 Optical microscopy

The polished sections of the products of the Davis tube circuit were described under the optical microscopy. The concentrates are mainly composed of highly liberated magnetite and some sulphides such as pyrite and chalcopyrite. The tailings are mainly composed of gangue minerals, sulphides, and hematite.
In the concentrates, the magnetite is present as clean irregular grains with occasional well-developed edges and cubic habits. It tends to occur more liberated as the size decreases. Starting from the second concentrate, some magnetite altered to hematite are reported. The pyrite in the concentrates present irregular edges and it can occur liberated or associated to magnetite. The gangue minerals are more present in the coarsest and in the third concentrate. They tend to present a prismatic and micaceous habits probably related to the amphiboles and micas, respectively.

In the tailings, the main mineral is hematite which occurs usually liberated but also associated to magnetite. Traces of pyrite and chalcopyrite are also reported.

In Figure 4.3, six selected photomicrographies are shown for the sample L8. In A, a general view of the C1 (75-106 µm) is characterized by the dominant presence of magnetite. In B, magnetite and pyrite can be differentiated in C3 (75-106 µm). In C, a particle of associated magnetite and hematite is shown for C2 (75-106 µm). In D, a general view of the tailings shows the predominance of gangue and some pyrite and hematite. In E, a liberated particle of chalcopyrite reported in the tailings is shown. Also some micaceous gangue is present. In F, a particle of magnetite being almost completely replaced by hematite is reported in the tailings (75-106 µm).
Figure 4.3. Selected photomicrographies for products of L8 (description in the text)
4.3 Element to mineral conversion

In order to validate the element to mineral conversion, the mineral proportions obtained by this method were compared to the SEM results for PRE:L:8. The chemistry and the mineralogy of this sample were compared for 13 size fractions. The chemical assay for C1 (0-38 µm) and the SEM results for C2 (38-75 µm) are missing.

For the comparison between results, the root mean square deviation (RMSD) was calculated as follows:

\[
\text{RMSD} = \sqrt{\frac{\sum_{i=1}^{n} (x_{\text{SEM}} - x_{\text{EMC}})^2}{n}}
\]

\[\text{Eq. 4.1}\]

where \(x\) are the mineral (or chemical) mass proportions obtained by SEM and EMC, and \(n\) is the number of pairs compared. Additionally, the average of the absolute differences are calculated. These values can be used to evaluate how dissimilar are the mineral and chemical mass proportions obtained by SEM and EMC.

In terms of chemical elements, RMSD and the average absolute difference are higher for Fe, Mg and Si. However, the general trends are maintained (Figure 4.4). The variations tend to be slightly higher in the 38-75 µm size fraction (Table 4.1).

![Pareto Chart of chemical elements](image)

**Figure 4.4. Pareto chart for chemical assays and back-calculated chemical proportions from SEM**

The high variation of Fe, Mg and Si mass proportions are expected since they are the most predominant elements in the sample. However, the highest variations are observed for the feed. If this stream were not considered for the calculation of the RMSD, this would decrease to 2.88 wt% instead of 5.14 wt%. Since the feed was analyzed in a previous campaign, the differences can be explained by sampling and preparation errors.
In terms of mineralogy, the general trends are maintained. However, they tend to be slightly off the identity line. The clear case are amphiboles and micas which are under and overestimated by EMC, respectively (Figure 4.5).
The mineral groups with higher RMSD are amphiboles, Fe oxides and Micas which are the predominant among the products. These mineral groups together have Fe, Mg and Si in their composition which can also explain the differences in chemical mass proportions. In general, the RMSD tends to increase with the size fractions (Table 4.2).

Table 4.2. RMSD and absolute difference between SEM and EMC mineral proportions (wt%)

<table>
<thead>
<tr>
<th>Mineral group</th>
<th>Bulk</th>
<th>0-38 um</th>
<th>38-75 um</th>
<th>75-106 um</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RMSD</td>
<td>Abs diff</td>
<td>RMSD</td>
<td>Abs diff</td>
</tr>
<tr>
<td>Amphiboles</td>
<td>3.76</td>
<td>2.22</td>
<td>3.27</td>
<td>2.21</td>
</tr>
<tr>
<td>Fe oxides</td>
<td>3.25</td>
<td>2.22</td>
<td>1.76</td>
<td>1.39</td>
</tr>
<tr>
<td>Micas</td>
<td>2.73</td>
<td>2.14</td>
<td>2.44</td>
<td>1.90</td>
</tr>
<tr>
<td>Plagioclases</td>
<td>2.51</td>
<td>1.60</td>
<td>2.53</td>
<td>1.47</td>
</tr>
<tr>
<td>Qtz</td>
<td>1.42</td>
<td>0.86</td>
<td>1.79</td>
<td>1.14</td>
</tr>
<tr>
<td>Pyroxenes</td>
<td>1.08</td>
<td>0.77</td>
<td>1.19</td>
<td>0.96</td>
</tr>
<tr>
<td>Kfs</td>
<td>0.85</td>
<td>0.47</td>
<td>0.81</td>
<td>0.49</td>
</tr>
<tr>
<td>Cal</td>
<td>0.69</td>
<td>0.39</td>
<td>1.02</td>
<td>0.48</td>
</tr>
<tr>
<td>Other silicates</td>
<td>0.22</td>
<td>0.16</td>
<td>0.42</td>
<td>0.33</td>
</tr>
<tr>
<td>Ti oxides</td>
<td>0.18</td>
<td>0.12</td>
<td>0.18</td>
<td>0.14</td>
</tr>
<tr>
<td>Phosphates</td>
<td>0.15</td>
<td>0.11</td>
<td>0.09</td>
<td>0.08</td>
</tr>
<tr>
<td>Sulphides</td>
<td>0.05</td>
<td>0.04</td>
<td>0.17</td>
<td>0.09</td>
</tr>
</tbody>
</table>

The element to mineral conversion method is highly dependent on the preselected chemical composition of the minerals and the assumption that this composition is invariant for all the samples might not be true. However, the EMC with a unique mineral composition would allow to compare samples and to have a first order characterization of the ore.

4.4 Bulk and sized mineral mass balance

Using the element to mineral conversion as a valid result for the bulk and sized mineralogy, the next step is to perform a mass balance of the mass proportions through the Davis tube circuit. Since there is no unit that alter the particle size distribution, the mass balance by size is straight forward. A brief description of the results for all the samples is depicted with more emphasis in the sample L8 which was selected for liberation analysis due to mass requirements.

When comparing the balanced mineralogy from the feed and the ore type of the samples obtained from the geological mapping, it turn out that they correspond to the macroscopic description. However, since the iron oxides are grouped, no difference between hematite and magnetite could be done at this stage, as it is the case of sample 3 which is mostly composed by hematite (Table 4.3).
### Table 4.3. Balanced iron oxide content in feed for Leveäniemi samples

<table>
<thead>
<tr>
<th>Sample</th>
<th>Rock type</th>
<th>Ore type (Magnetite% content)</th>
<th>Fe oxides % in Feed (bulk)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Biotite schist</td>
<td>EMM &lt;50%</td>
<td>42.74</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>EMM &gt;60%</td>
<td>89.04</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>H</td>
<td>88.24</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>EMM &gt;60%</td>
<td>90.81</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>EMM &gt;60%</td>
<td>83.28</td>
</tr>
<tr>
<td>6</td>
<td>Trachyandesite</td>
<td>Shallow (MM) &amp; low grade disseminated mineralization</td>
<td>36.39</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>Shallow (MM)</td>
<td>93.86</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>WMM &gt;50%</td>
<td>84.56</td>
</tr>
<tr>
<td>9</td>
<td></td>
<td>WBM &lt;50%</td>
<td>51.22</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>EMM &gt;60%</td>
<td>85.09</td>
</tr>
<tr>
<td>11</td>
<td></td>
<td>EMM &gt;60%</td>
<td>90.45</td>
</tr>
<tr>
<td>12</td>
<td></td>
<td>EMM 50-60% (high grade breccia)</td>
<td>62.74</td>
</tr>
<tr>
<td>13</td>
<td></td>
<td>EMM &lt;50%</td>
<td>24.71</td>
</tr>
</tbody>
</table>

**EMM**: Eastern massive magnetite zone  
**H**: Hematite zone  
**WMM**: Western massive magnetite zone  
**WBM**: Western breccia magnetite

In most of the samples, the cumulative recoveries for each size fractions reaches values around 90%. The recoveries of Fe tend to be higher as the Fe oxides content in the feed is higher. The most abrupt cases are the samples L1, L6, L9 and L13 whose Fe oxides content are about 50% or less and their recoveries reaches 80 to 90%. The exception to this trend is the sample L3 which belongs to the hematite zone. In this sample only the magnetic particles, for instance partially or totally oxidized magnetite, are recovered for currents 0.2 and 0.5 A. Almost no mass was obtained in the concentrate 1 for 0.1 A (Figure 4.6).

![Fe recovery versus Fe oxides in feed.](Figure 4.6. Fe recovery versus Fe oxides in feed.)
With the mass balanced mineralogy by size, the behavior of every mineral group can be characterize for the Davis tube circuit respect to the Fe oxides. As studied under the optical microscope, the Fe oxides for the sample L8 are mainly composed by magnetite and a small portion of partially altered magnetite to hematite. Therefore, the selectivity curves will reflect the degree of separation between magnetite and the non-magnetic minerals for the Davis tube test.

The selectivity of Ti oxides is the lowest one reaching about 70% recovery in the concentrates. The selectivity of this mineral group by size decreases with the size fraction. Other mineral groups report different distribution in the concentrates but they are still being recovered. This is the case of other silicates, micas and plagioclases whose distribution ranges from 10 to 30% in the total concentrate (Figure 4.7).
The recovery-grade curve for Fe oxides shows a trend in which the grade is almost constant for all the concentrates with an increasing recovery going from 10% in the first concentrate to more than 90% in the total concentrate (C1-C2-C3). In terms of the currents for the Davis tube test, the pattern shown by the cumulative recovery curve is strictly increasing with the current. The final concentrate has a recovery of 98% and a grade of 96 wt% of Fe oxides.

**Figure 4.8. Recovery-grade curve for Fe oxides. PRE:L:8**

4.5 Mineral Liberation Analysis

From the SEM analysis, the mineral composition, mode of occurrence and cumulative liberation by particles are obtained for the products of Davis Tube test: Concentrates (C1, C2 and C3) and Final tailing (T3) (Figure 4.9).

The concentrates present a fairly similar modal composition. The predominant minerals are the Fe oxides (magnetite and hematite) with about 96-97 wt%. The main gangue minerals are micas (~1.4 wt%) and amphiboles (<1 wt%). The main difference is the content of amphiboles in C3 (38-75 µm) where it reaches a value of 1.9 wt%.

The tailings present a content of Fe oxides <5 wt%. The predominant gangue are amphiboles, micas, plagioclases, calcite, quartz and phosphates. These minerals sum up more than 90 wt% of the tailings. Sulphides and Ti oxides are less than 1.5 wt%.
The mode of occurrence of the Fe oxides in the concentrates is mostly liberated ranging from 93.8% in C1 up to 96.9% in C3. The second most common occurrence is the binary association with micas ranging from 3.2 wt% in particle for C1 down to 1.7 wt% in particle for C3. These trends are shown in the fine and intermediate size fractions.

In the tailings, the 5 wt% of Fe oxides are mostly present as liberated particles. The binary associations with amphiboles show a considerable proportion (4.7 wt% in particle). This association also increases with the particle size. The binary association with micas is about 1% wt in particle, which is lower than the present in the concentrates. Thus, the binary particles with Fe oxides and amphiboles tend to be reported in the tailings meanwhile the ones with micas tend to be recovered preferably in the first stages of the Davis tube circuit.
The degree of liberation in the concentrates is above 94%, i.e. 94% of the particles of Fe oxides are fully liberated (100 wt% Fe oxides in particle). In the tailings, the degree of liberation is also above 90%. This fact indicates that although the current allows to reach a recovery of 98% of the magnetic fraction, there is still liberated particles being reported in the tailings. This could be explained by the washing effect due to the motion of the tube. As stated before, in the definition of the forces acting in a particle during the Davis tube test, the drag for is the main competing force for particles <50 µm. Thus, it is expected that only the most liberated particles are reported to the concentrates whereas the mixed particles are washed away towards the tailings (Figure 4.11).
The results of the mineral mass balance by size for the sample L8 are shown in the Table 4.4. The flowrates and composition by size are used as constrains for the liberation mass balance. In the end, the particles flowrates and total composition of the solids will add up to these values.
By doing this, the particle classes will be exactly the same for all the missing data for the size fraction 38 µm. The flowrates of the particle classes in the feed are the sum of the same particle classes in the other size fractions. Finally, the back-calculated feed contains all the particle classes and the total amount of particle by class which are distributed in the circuit.

### Table 4.4: Mineral mass balance by size for PREL:B

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Stream</th>
<th>Flowrate (t/h)</th>
<th>Fe oxides wt%</th>
<th>Amph wt%</th>
<th>Micas wt%</th>
<th>Phosphates wt%</th>
<th>Plg wt%</th>
<th>Qtz wt%</th>
<th>Other silicates wt%</th>
<th>Cal wt%</th>
<th>Sulphides wt%</th>
<th>Ti oxides wt%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bulk</td>
<td>Feed</td>
<td>181.33</td>
<td>84.56</td>
<td>5.04</td>
<td>4.80</td>
<td>0.54</td>
<td>2.02</td>
<td>0.72</td>
<td>0.74</td>
<td>1.24</td>
<td>0.12</td>
<td>0.21</td>
</tr>
<tr>
<td></td>
<td>C1</td>
<td>16.05</td>
<td>97.56</td>
<td>0.15</td>
<td>1.41</td>
<td>0.03</td>
<td>0.45</td>
<td>0.00</td>
<td>0.15</td>
<td>0.08</td>
<td>0.01</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>T1</td>
<td>165.28</td>
<td>83.30</td>
<td>5.52</td>
<td>5.13</td>
<td>0.59</td>
<td>2.17</td>
<td>0.79</td>
<td>0.80</td>
<td>1.35</td>
<td>0.13</td>
<td>0.21</td>
</tr>
<tr>
<td></td>
<td>C2</td>
<td>55.49</td>
<td>97.43</td>
<td>0.22</td>
<td>1.41</td>
<td>0.04</td>
<td>0.43</td>
<td>0.00</td>
<td>0.27</td>
<td>0.04</td>
<td>0.01</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>T2</td>
<td>109.79</td>
<td>76.16</td>
<td>8.19</td>
<td>7.01</td>
<td>0.87</td>
<td>3.05</td>
<td>1.19</td>
<td>1.07</td>
<td>2.01</td>
<td>0.20</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>C3</td>
<td>86.04</td>
<td>95.88</td>
<td>1.10</td>
<td>1.62</td>
<td>0.10</td>
<td>0.68</td>
<td>0.00</td>
<td>0.36</td>
<td>0.07</td>
<td>0.00</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>T3</td>
<td>23.75</td>
<td>4.70</td>
<td>33.89</td>
<td>26.52</td>
<td>3.67</td>
<td>11.64</td>
<td>5.51</td>
<td>3.62</td>
<td>9.05</td>
<td>0.92</td>
<td>0.47</td>
</tr>
<tr>
<td>&lt;38 µm</td>
<td>Feed</td>
<td>61.04</td>
<td>79.09</td>
<td>6.36</td>
<td>6.66</td>
<td>0.69</td>
<td>2.71</td>
<td>0.96</td>
<td>0.98</td>
<td>2.18</td>
<td>0.16</td>
<td>0.23</td>
</tr>
<tr>
<td></td>
<td>C1</td>
<td>5.15</td>
<td>97.76</td>
<td>0.01</td>
<td>1.42</td>
<td>0.03</td>
<td>0.37</td>
<td>0.00</td>
<td>0.18</td>
<td>0.08</td>
<td>0.01</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>T1</td>
<td>55.89</td>
<td>77.36</td>
<td>6.94</td>
<td>7.14</td>
<td>0.75</td>
<td>2.92</td>
<td>1.05</td>
<td>1.05</td>
<td>2.37</td>
<td>0.17</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>C2</td>
<td>20.65</td>
<td>97.71</td>
<td>0.00</td>
<td>1.42</td>
<td>0.04</td>
<td>0.22</td>
<td>0.00</td>
<td>0.41</td>
<td>0.04</td>
<td>0.01</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>T2</td>
<td>35.24</td>
<td>65.44</td>
<td>11.01</td>
<td>10.49</td>
<td>1.16</td>
<td>4.50</td>
<td>1.66</td>
<td>1.43</td>
<td>3.74</td>
<td>0.26</td>
<td>0.30</td>
</tr>
<tr>
<td></td>
<td>C3</td>
<td>23.07</td>
<td>97.34</td>
<td>0.15</td>
<td>1.54</td>
<td>0.04</td>
<td>0.35</td>
<td>0.00</td>
<td>0.34</td>
<td>0.09</td>
<td>0.00</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>T3</td>
<td>12.17</td>
<td>4.96</td>
<td>31.60</td>
<td>27.47</td>
<td>3.29</td>
<td>12.38</td>
<td>4.81</td>
<td>3.48</td>
<td>10.66</td>
<td>0.76</td>
<td>0.58</td>
</tr>
<tr>
<td>38-75 µm</td>
<td>Feed</td>
<td>63.41</td>
<td>87.94</td>
<td>4.30</td>
<td>3.62</td>
<td>0.45</td>
<td>1.54</td>
<td>0.55</td>
<td>0.67</td>
<td>0.66</td>
<td>0.11</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>C1</td>
<td>5.15</td>
<td>97.77</td>
<td>0.01</td>
<td>1.42</td>
<td>0.03</td>
<td>0.37</td>
<td>0.00</td>
<td>0.18</td>
<td>0.08</td>
<td>0.01</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>T1</td>
<td>58.26</td>
<td>87.07</td>
<td>4.68</td>
<td>3.82</td>
<td>0.49</td>
<td>1.64</td>
<td>0.59</td>
<td>0.71</td>
<td>0.71</td>
<td>0.11</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>C2</td>
<td>17.24</td>
<td>97.72</td>
<td>0.07</td>
<td>1.36</td>
<td>0.03</td>
<td>0.44</td>
<td>0.00</td>
<td>0.20</td>
<td>0.04</td>
<td>0.00</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>T2</td>
<td>41.02</td>
<td>82.60</td>
<td>6.62</td>
<td>4.85</td>
<td>0.69</td>
<td>2.15</td>
<td>0.85</td>
<td>0.92</td>
<td>0.99</td>
<td>0.16</td>
<td>0.18</td>
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<tr>
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<td>C3</td>
<td>35.58</td>
<td>94.51</td>
<td>1.88</td>
<td>1.79</td>
<td>0.18</td>
<td>0.86</td>
<td>0.00</td>
<td>0.52</td>
<td>0.08</td>
<td>0.00</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>T3</td>
<td>5.44</td>
<td>4.74</td>
<td>37.62</td>
<td>24.81</td>
<td>3.96</td>
<td>10.58</td>
<td>6.37</td>
<td>3.54</td>
<td>6.92</td>
<td>1.20</td>
<td>0.26</td>
</tr>
<tr>
<td>75-106 µm</td>
<td>Feed</td>
<td>56.88</td>
<td>86.67</td>
<td>4.45</td>
<td>4.12</td>
<td>0.49</td>
<td>1.82</td>
<td>0.66</td>
<td>0.58</td>
<td>0.88</td>
<td>0.11</td>
<td>0.23</td>
</tr>
<tr>
<td></td>
<td>C1</td>
<td>5.75</td>
<td>97.19</td>
<td>0.40</td>
<td>1.40</td>
<td>0.04</td>
<td>0.60</td>
<td>0.00</td>
<td>0.10</td>
<td>0.08</td>
<td>0.01</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td>T1</td>
<td>51.13</td>
<td>85.49</td>
<td>4.91</td>
<td>4.43</td>
<td>0.54</td>
<td>1.95</td>
<td>0.74</td>
<td>0.63</td>
<td>0.97</td>
<td>0.12</td>
<td>0.23</td>
</tr>
<tr>
<td></td>
<td>C2</td>
<td>17.60</td>
<td>96.82</td>
<td>0.62</td>
<td>1.44</td>
<td>0.04</td>
<td>0.67</td>
<td>0.00</td>
<td>0.17</td>
<td>0.04</td>
<td>0.01</td>
<td>0.19</td>
</tr>
<tr>
<td></td>
<td>T2</td>
<td>33.53</td>
<td>79.54</td>
<td>7.16</td>
<td>6.00</td>
<td>0.80</td>
<td>2.63</td>
<td>1.13</td>
<td>0.87</td>
<td>1.45</td>
<td>0.18</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>C3</td>
<td>27.39</td>
<td>96.44</td>
<td>0.89</td>
<td>1.48</td>
<td>0.04</td>
<td>0.73</td>
<td>0.00</td>
<td>0.17</td>
<td>0.04</td>
<td>0.00</td>
<td>0.21</td>
</tr>
<tr>
<td></td>
<td>T3</td>
<td>6.14</td>
<td>4.15</td>
<td>35.12</td>
<td>26.16</td>
<td>4.16</td>
<td>11.11</td>
<td>6.14</td>
<td>3.98</td>
<td>7.75</td>
<td>0.99</td>
<td>0.44</td>
</tr>
</tbody>
</table>

### 4.6 Liberation mass balance

The particle mass proportion reconciliation and basic binning are performed for the products of Davis tube: concentrate and tailings. By doing this, the particle classes will be exactly the same for all the streams. Then, the intermediate streams (T1 and T2) and the feed are back calculated in a way that the flowrates of the particle classes in the feed are the sum of the same particle classes in the products. A missing data for the size fraction 38-75 µm in the concentrate 2 was interpolated from the other size fractions. Finally, the back-calculated feed contains all the particle classes and the total amount of particle by class which are distributed in the circuit.
The coefficients used for the steps before the mass balance of particles are:

- Threshold for a particle to be considered as liberated = 100 wt% of mineral in particle
- Tolerance = 1 wt%
- NBB (# binary bins) = 20
- Minimum number of particles for Advanced binning = 25
- Minimum number of particles after Advanced binning for a particle class to be considered = 25.

After the basic binning, a total of 7959 particle classes are created. This number decreases to 956 after the advanced binning. Finally, after only considering the particle classes with more than 25 particles, the number of classes is reduced to 124. When eliminating the particle classes with less than 25 particles representing them in the feed, there is a residual flowrate for each stream that is removed.

Table 4.5. Residuals after imposing minimum NOP per particle class

<table>
<thead>
<tr>
<th>Residual flowrates (t/h)</th>
<th>Stream</th>
<th>Reconciled</th>
<th>No reconciled</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feed</td>
<td>1.19</td>
<td>1.84</td>
<td></td>
</tr>
<tr>
<td>C1</td>
<td>0.20</td>
<td>0.26</td>
<td></td>
</tr>
<tr>
<td>T1</td>
<td>0.99</td>
<td>1.59</td>
<td></td>
</tr>
<tr>
<td>C2</td>
<td>0.00</td>
<td>0.29</td>
<td></td>
</tr>
<tr>
<td>T2</td>
<td>0.98</td>
<td>1.29</td>
<td></td>
</tr>
<tr>
<td>C3</td>
<td>0.41</td>
<td>0.50</td>
<td></td>
</tr>
<tr>
<td>T3</td>
<td>0.57</td>
<td>0.79</td>
<td></td>
</tr>
</tbody>
</table>

Before the mass balance, the particle classes are classified according to their bin type. The results for the sample L8 shows that 95.4% of the particles are liberated and 4.6% are classified as binary particles. The percentage of binary particles increases with the size fraction, i.e. the liberation decreases with the size of the particles. The no reconciled liberation analysis and the reconciled data from the mineral mass balance by size show the same distribution of bin types. In both cases, the binary particles are represented by less than the 5% of the total number of particles (Table 4.6 & Table 4.7).

Table 4.6. Distribution of liberated and binary particles in the Feed of PRE:L:L8. Reconciled with mineral mass balance by size

<table>
<thead>
<tr>
<th>Bin type</th>
<th>Bulk</th>
<th>&lt;38 µm</th>
<th>38-75 µm</th>
<th>75-106 µm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liberated (%)</td>
<td>95.44</td>
<td>98.11</td>
<td>96.10</td>
<td>92.24</td>
</tr>
<tr>
<td>Binary (%)</td>
<td>4.56</td>
<td>1.89</td>
<td>3.90</td>
<td>7.76</td>
</tr>
<tr>
<td>Total NOP</td>
<td>71330</td>
<td>19229</td>
<td>29959</td>
<td>22142</td>
</tr>
</tbody>
</table>

Table 4.7. Distribution of liberated and binary particles in the Feed of PRE:L:L8. No Reconciled with mineral mass balance by size

<table>
<thead>
<tr>
<th>Bin type</th>
<th>Bulk</th>
<th>&lt;38 µm</th>
<th>38-75 µm</th>
<th>75-106 µm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liberated (%)</td>
<td>95.39</td>
<td>98.11</td>
<td>95.98</td>
<td>92.24</td>
</tr>
<tr>
<td>Binary (%)</td>
<td>4.61</td>
<td>1.89</td>
<td>4.02</td>
<td>7.76</td>
</tr>
<tr>
<td>Total NOP</td>
<td>71316</td>
<td>19223</td>
<td>29951</td>
<td>22142</td>
</tr>
</tbody>
</table>
The flowrate mass proportion of liberated minerals show a clear predominance in all the streams. The Fe oxides, micas and amphiboles are mostly liberated. The flowrate mass proportions of binary particles of Fe oxides and most of the minerals are usually lower than 1 wt%. The only exception are the binary particles of Fe oxides and micas whose values range from 1 wt% in the feed up to 3.5 wt% in the first concentrate (Figure 4.12).

4.6.1 Liberated particles recoveries

The balanced and pre-balanced cumulative recoveries of liberated particles are compared. The pre-balanced recoveries are reported to have different behavior for each mineral. Nevertheless, the balanced recoveries show that only two mineral groups maintain their previous trend: Fe oxides and micas. These mineral groups are the only ones whose flowrate % (Figure 4.12) are represented for more than 5% in most of the streams. Similar is the case of amphiboles even though its recoveries are low (<30%). After the mass balance of particles, the recoveries trends of the liberated particles of mineral groups with low representation in the streams are lost (Figure 4.13). This fact indicates that a mineral bin type (liberated in this case) requires to have a non-negligible flowrate in order to preserve its behavior through the process.
Figure 4.13. Balanced and pre-balanced cumulative recoveries for mineral groups
4.6.2 Binary particles recoveries

The recoveries of the binary particles modeled before and after the mass balance of particles. Before the mass balance, the back calculated feed and products had recoveries whose distribution show a smoother trend as a function of the wt% of magnetite in the particle. After the mass balance, the trend of the recoveries are sharper due to very low flowrates associated to the binary particle classes.

**Table 4.8. Incomplete beta function fitting results**

<table>
<thead>
<tr>
<th>Magnetite with</th>
<th>Size Fraction (µm)</th>
<th>b value</th>
<th>RMSD</th>
<th>Number points</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Balanced</td>
<td>Back Calculated</td>
<td>Balanced</td>
</tr>
<tr>
<td>Amphiboles</td>
<td>0-38</td>
<td>0.1</td>
<td>3.3</td>
<td>10.0</td>
</tr>
<tr>
<td></td>
<td>38-75</td>
<td>1.6</td>
<td>1.7</td>
<td>20.7</td>
</tr>
<tr>
<td></td>
<td>75-106</td>
<td>29.9</td>
<td>2.1</td>
<td>1.9</td>
</tr>
<tr>
<td>Micas</td>
<td>0-38</td>
<td>29.9</td>
<td>29.9</td>
<td>5.4</td>
</tr>
<tr>
<td></td>
<td>38-75</td>
<td>29.9</td>
<td>3.9</td>
<td>13.1</td>
</tr>
<tr>
<td></td>
<td>75-106</td>
<td>29.9</td>
<td>9.5</td>
<td>1.3</td>
</tr>
<tr>
<td>Quartz</td>
<td>0-38</td>
<td>0.1</td>
<td>1.4</td>
<td>3.9</td>
</tr>
<tr>
<td></td>
<td>38-75</td>
<td>0.1</td>
<td>0.9</td>
<td>14.9</td>
</tr>
<tr>
<td></td>
<td>75-106</td>
<td>0.5</td>
<td>1.2</td>
<td>26.0</td>
</tr>
<tr>
<td>Other silicates</td>
<td>0-38</td>
<td>29.9</td>
<td>4.6</td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td>38-75</td>
<td>7</td>
<td>3.1</td>
<td>4.9</td>
</tr>
<tr>
<td></td>
<td>75-106</td>
<td>0.5</td>
<td>29.9</td>
<td>15.9</td>
</tr>
</tbody>
</table>

For most of the binary particles, the back calculated recoveries show convex pattern with high increase of the recoveries at a relatively low wt% of magnetite in particle. Also, the modeled curves for different size fractions do not differ significantly from each other for the back calculated recoveries. Conversely, the balanced recoveries show significantly different patterns between size fractions for most of the binary particles. The only exception are the balanced recoveries of binary magnetite-micas particles which tend to show the same pre balanced and balanced pattern. Furthermore, for quartz and other silicates, the coarser size fractions tends to change the convexity before and after the mass balance (Figure 4.14 & Figure 4.15).
Figure 4.14. Balanced and back-calculated recoveries of binary particles as a function of wt% of magnetite in particle
These changes in the patterns can be associated to the low representativeness of the binary particle classes in the streams, i.e. low flowrate percentage. Intuitively, the residuals (section 4.6.3) between the liberation mass balance and the mass balance by size are the limit for the balanced flowrates. Any value balanced below the residuals can be considered as erratic and does not necessarily represents the behavior of the particles classes.

4.6.3 Liberation mass balance methods for Davis tube circuit

The previous results were obtained considering all the nodes and constrains for solving $A\hat{x} = \hat{b}$. In this section, two methods of solving the mass balance are compared. The first method considers all the nodes for the mass balance. The second method solves the mass balance node by node. These methods are used for solving the particle tracking in the liberation data reconciled with the mass balance by size and for the no reconciled liberation data (direct from SEM). The no reconciled liberation data and the mineral mass balance by size are used as references for comparing the results of the reconciled data.

The way to compare them is to calculate the RMSD (Eq. 4.1) between the solutions of reconciled and no reconciled and to calculate the residuals of the total flowrates by stream versus the mass balance by size.
A low RMSD will tell how comparable are the reconciled and no reconciled mass balance results, and low total residuals by stream will tell how comparable the results are with the mass balance by size.

When comparing the methods for all nodes and node by node, it is observed that the node by node reports lower residuals values, meaning that there is a better correlation between with the mass balance by size. However, the RMSD between the balanced reconciled and no reconciled liberation data is lower when all the nodes are considered at once, meaning that the reconciled and no reconciled liberation data are closer to each other (Table 4.9 & Table 4.10).

**Table 4.9. Residuals of mass balance of reconciled and no reconciled liberation data for all nodes method. The RMSD between both results is shown**

<table>
<thead>
<tr>
<th>Stream</th>
<th>Residuals</th>
<th>Reconciled</th>
<th>No reconciled</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RMSD</td>
<td>Bulk</td>
<td>0-35 µm</td>
</tr>
<tr>
<td>Feed</td>
<td>0.506</td>
<td>0.60</td>
<td>0.15</td>
</tr>
<tr>
<td>C1</td>
<td>0.038</td>
<td>0.10</td>
<td>0.03</td>
</tr>
<tr>
<td>T1</td>
<td>0.477</td>
<td>0.89</td>
<td>0.16</td>
</tr>
<tr>
<td>C2</td>
<td>0.352</td>
<td>0.19</td>
<td>0.08</td>
</tr>
<tr>
<td>T2</td>
<td>0.205</td>
<td>-0.04</td>
<td>0.00</td>
</tr>
<tr>
<td>C3</td>
<td>0.084</td>
<td>0.98</td>
<td>0.25</td>
</tr>
<tr>
<td>T3</td>
<td>0.151</td>
<td>-0.23</td>
<td>-0.09</td>
</tr>
</tbody>
</table>

According to the compared results of the two methods, the node by node solution shows lower residual values indicating that it is closer to the mineral mass balance by size. However, the common streams might require reconciliation between them which could vary the recovery results.

**Table 4.10. Residuals of mass balance of reconciled and no reconciled liberation data for node by node method. The RMSD between both results is shown**

<table>
<thead>
<tr>
<th>Node</th>
<th>Residuals</th>
<th>Stream</th>
<th>RMSD</th>
<th>Bulk</th>
<th>0-38 µm</th>
<th>38-75 µm</th>
<th>75-106 µm</th>
<th>Bulk</th>
<th>0-38 µm</th>
<th>38-75 µm</th>
<th>75-106 µm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Feed</td>
<td></td>
<td></td>
<td>0.516</td>
<td>-0.0013</td>
<td>-0.0009</td>
<td>-0.0002</td>
<td>-0.0002</td>
<td>0.0004</td>
<td>0.0003</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>C1</td>
<td></td>
<td></td>
<td>0.074</td>
<td>-0.0003</td>
<td>0.0001</td>
<td>-0.0002</td>
<td>-0.0002</td>
<td>0.0006</td>
<td>-0.0003</td>
<td>0.0009</td>
<td>0.0009</td>
</tr>
<tr>
<td>T1</td>
<td></td>
<td></td>
<td>0.499</td>
<td>0.0004</td>
<td>0.0005</td>
<td>0.0002</td>
<td>-0.0003</td>
<td>0.0005</td>
<td>0.0002</td>
<td>0.0003</td>
<td>0.0000</td>
</tr>
<tr>
<td>Node 1</td>
<td></td>
<td></td>
<td>T1</td>
<td>0.410</td>
<td>0.0005</td>
<td>0.0003</td>
<td>0.0003</td>
<td>-0.0001</td>
<td>0.0027</td>
<td>0.0010</td>
<td>0.0009</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C2</td>
<td>0.148</td>
<td>-0.0009</td>
<td>-0.0001</td>
<td>-0.0003</td>
<td>-0.0005</td>
<td>-0.0009</td>
<td>-0.0005</td>
<td>0.0002</td>
<td>-0.0006</td>
</tr>
<tr>
<td></td>
<td></td>
<td>T2</td>
<td>0.299</td>
<td>-0.0004</td>
<td>0.0002</td>
<td>-0.0002</td>
<td>-0.0004</td>
<td>-0.0012</td>
<td>-0.0006</td>
<td>0.0001</td>
<td>-0.0007</td>
</tr>
<tr>
<td>Node 2</td>
<td></td>
<td></td>
<td>C3</td>
<td>0.230</td>
<td>-0.0007</td>
<td>-0.0004</td>
<td>-0.0001</td>
<td>-0.0002</td>
<td>-0.0005</td>
<td>-0.0001</td>
<td>-0.0002</td>
</tr>
<tr>
<td></td>
<td></td>
<td>T3</td>
<td>0.186</td>
<td>0.0001</td>
<td>0.0001</td>
<td>0.0001</td>
<td>0.0001</td>
<td>-0.0002</td>
<td>-0.0003</td>
<td>0.0003</td>
<td>-0.0001</td>
</tr>
</tbody>
</table>

5  Discussions

The magnetite is the main mineral in the Leveäniemi iron ore samples. Its high recovery in the Davis tube circuit along with the V, Ti and Mn suggest that these elements are present in the magnetite lattice. Therefore, these penalty elements in the iron concentrates cannot be avoided at the stage of mineral concentrations. The quality of the iron concentrates needs to be improved downstream in the process.
The DT circuit results showed a high recovery and grade of Fe oxides (magnetite) in most of the cases. These results can be explained by the washing effect. This effect is controlled by the rotational and longitudinal agitation of the tube that perturb the particles agglomeration between the pole tips of the electromagnet. As showed before, the Force index decreases exponentially with the distance from the pole tips, then a higher agitation frequency and amplitude will wash away most of the gangue minerals and also fine grained magnetite. This could be stated as the main difference between the Davis tube and the Wet LIMS, since in DT there are almost no flocs formed and reported to the concentrates.

The estimation of the magnetic susceptibilities for binary magnetite particles show that the finer particles increase their magnetic susceptibility at a lower rate than the coarser ones as the wt% of magnetite increases in the particle. Conversely, the pre balanced recoveries of binary magnetite-micas particles show that the recoveries of finer particles is higher at a lower wt% of magnetite, whereas no difference is shown in the balanced recoveries. This results invalidates the simplified estimation of the magnetic susceptibility assuming spherical particles.

In terms of particle tracking, the representativeness of the particle classes in the feed is the key factor in order to get reliable representation of the behavior of the particles in the circuit. The predominant particles such as liberated magnetite, binary magnetite-micas and magnetite-amphiboles, show the same results for the pre balanced (back calculated) and balanced recoveries. All the other particle classes which are present only in a small mass proportion in the feed show erratic recoveries and the pre balanced and balanced recoveries do not match. Accordingly, the limitation of the particle tracking are related mostly to its resolution. The residuals can be used as an indicator of how reliable the mass balanced flowrates are. If the balanced flowrates are below the residuals, then the recoveries will be inconsistent with the liberation analysis.

In spite of the algorithm has its limitations, the definition of the set ups can influence as well. In this case, eleven mineral groups were considered and only a three were represented properly over the circuit. The other eight groups did not produced reliable results for their particle classes. Therefore, the definition of the mineral set up must be directed and limited to an amount of groups that allows to balance through the circuit.

The missing algorithm is a useful tool when the liberation data is not available for all the size fractions in a stream. However, it does not replaces the analytical data and it requires validation in order to obtain reliable results if required.

The smoothing algorithm was not performed in the DT circuit due to be considered as an unnecessary modification to the original liberation data. Nevertheless, a curve fitting for the balanced flowrates and recoveries is encouraged in further studies in order to be an input for simulation of particles behavior.

The node by node showed to be a better method for solving the liberation mass balance than the all nodes method. Nonetheless, it requires to deal with the difference between the common streams flowrates. In this case, the tailings of a DT unit is the feed of the next one. Both are balanced separately and a reconciliation between them is needed.

The particle tracking case presented in this study requires more validation. A different sample should be considered for future studies whose mass proportion of binary particle classes are relatively comparable to the liberated ones. Moreover, different ore types and units must be tested in order to study the limits
of the particle tracking. For instance, to study the recovery of binary particles of a low grade copper sulphide ore in a flotation circuit at laboratory scale.

In HSC chemistry software, particle tracking is only implemented for older versions. However, it is not being currently developed. Since particle tracking stands for the highest level of information in mass balance, it should be implemented in the mass balance module along with the bulk and sized mass balance. The software should allow to do the mass balances sequence proposed by Lamberg and Vianna 2007, going from chemical mass balance, passing through EMC, Bulk mineral, minerals by size and liberation mass balance. This will make easier to implement the results of particle tracking in the simulation of the behavior of the particles in a process.

6 Conclusion

In particle based geometallurgical modelling, the behavior of the particles in a process is the critical component for predictability. The particle tracking algorithm is a tool whose aim is to balance particle classes in a mineral processing circuit. Once the balance is done, the results serve as an input for modelling the particle behavior in the geometallurgical chain.

The Davis tube geometallurgical test is a tool that can be used for evaluating the response of magnetic material under the effect of an adjustable magnetic field. Nonetheless, due to the washing effect related to the agitation of the tube, the recoveries and grades obtained in the DT circuit are fairly high. If these results require to be compared with WLIMS in terms of modal mineralogy, the proposed circuit must be calibrated.

In this work, the particle tracking is depicted and implemented in a magnetic separation circuit for high liberated material. The liberation data was balanced in a way that the particle classes can be followed through circuit and their recoveries can be calculated. Nevertheless, the algorithm requires further validation and analysis of its limitations in terms of resolution and reproducibility.

In a process model, the particle tracking is the first step for understanding the particle behavior. The next step are the modelling of these behaviors for further implementation in simulation and production forecast.
7 References


8  Annexes

8.1  Optical microscopy description of Leveäniemi products after Davis Tube test: PRE:L:8:G

8.1.1  Concentrate 1

8.1.1.1  Size fraction: 75 – 106 µm

Total particles: 40%

Magnetite (37%): Mostly liberated. Rarely associated or included in gangue minerals. Irregular shape with some well-formed edges. Commonly the magnetite is present as clean crystals.

Gangue minerals (3%): Liberated or associated to magnetite. Irregular shapes usually granular.

Pyrite (<1%): Irregular grains associated to magnetite

8.1.1.2  Size fraction: 38 – 75 µm

Total particles: 50%

Magnetite (50%): Mostly liberated. Rarely associated to gangue minerals (<1%). Irregular shape with some well-formed edges. Commonly present as clean crystals.
Pyrite (<1%): Mostly liberated. Well-developed shape with irregular edges. It could present magnetic properties.

8.1.1.3 Size fraction <38 µm:
Total particles: 20%

Magnetite (20%): Liberated grains with irregular shape and edges. Occasionally, it present elongated irregular shapes. Rarely associated with gangue minerals (<1%).

Pyrite (<1%): Associated with magnetite, presenting irregular shape and edges.
8.1.2 Concentrate 2

8.1.2.1 Size fraction: 75 – 106 µm

Total particles: 30%

Magnetite (30%): Mostly liberated grains with irregular shapes and edges. Usually present as clean crystals. Occasionally, hematite (<1%) is present as an alteration of magnetite.

Pyrite (<1%): Mostly liberated, irregular shape and sharp edges.
8.1.2.2 Size fraction <38 µm

Total particles: 40%

Magnetite (40%): Liberated. Mostly present with granular shapes and irregular edges. Clean crystals commonly. Rarely associated with irregular crystals of pyrite (<1%)
8.1.3 Concentrate 3

8.1.3.1 Size fraction: 75 – 106 µm

Total particles: 40%

Magnetite (35%): Liberated. Irregular grains with occasional well developed edges with cubic habits. About 5% of the magnetite is associated with gangue minerals (b).

Gangue minerals (5%): Most of the gangue minerals present a prismatic and micaceous habits which could be related to the presence of actinolite and biotite, respectively.

Pyrite (<1%): Liberated and occasionally associated with gangue minerals. It shows well developed edges of the cubic system and also sharp ones. Occasionally associated with gangue minerals and as inclusions in them. The size is about 100 µm or smaller. The shape could vary depending on the breakage.
8.1.3.2  Size fraction: 38 – 75 μm
Total particles: 40%

Magnetite (35%): Liberated. Irregular grains with occasional well developed edges with cubic habits. About 5% of the magnetite is associated with gangue minerals (d).

Gangue minerals (5%): Most of the gangue minerals present a prismatic and micaceous habits.
8.1.3.3  **Size fraction <38 µm**

Total particles: 40%

Magnetite (35%): Liberated. Irregular grains with occasional well developed edges with cubic habits. About 5% of the magnetite is associated with gangue minerals (d).

Gangue minerals (5%): Most of the gangue minerals present a prismatic and micaceous habits. Granular grains also present.
8.1.4 Final Tailings

8.1.4.1 Size fraction: 75 – 106 µm

Total particles: 35%

Gangue minerals (30%): Most of the gangue minerals present a prismatic and micaceous habits which could be related to the presence of actinolite and biotite, respectively.

Pyrite (3%): Liberated. It shows well developed edges of the cubic system and also sharp ones. Occasionally associated with gangue minerals and as inclusions in them. The size is about 100 µm or smaller. The shape could vary depending on the breakage.

Hematite (2%): Liberated. It usually presents a tabular or elongated habit. Its dimensions goes around to 100 µm long and 50 µm width. Some crystals of magnetite (<1%) are partially altered to hematite and present a cubic habit.

Chalcopyrite (<1%): Liberated and fine grained.
8.1.4.2  Size fraction: 38 – 75 µm
Total particles: 35%

Gangue minerals (32%): Most of the gangue minerals present a prismatic and micaceous habits which could be related to the presence of actinolite and biotite, respectively.

Pyrite (1%): Liberated. Cubic forms and sharp edges presented depending on the breakage.

Hematite (2%): Liberated. Usually in irregular forms and occasionally tabular habit. Some crystals of magnetite (<1%) are partially altered to hematite and present a cubic habit (c).

Chalcopyrite (<1%): Liberated and fine grained (≈30 µm).
8.1.4.3  Size fraction <38 µm
Total particles: 40%

Gangue minerals (35%): Most of the gangue minerals present a prismatic and micaceous habits which could be related to the presence of actinolite and biotite, respectively.

Pyrite (2%): Liberated. Cubic forms and sharp edges presented depending on the breakage (d).

Hematite (3%): Liberated. Usually in irregular forms and occasionally tabular habit. Some crystals of magnetite (<1%) are partially altered to hematite and present a cubic habit (d).
8.2 Sized mass balance results

8.2.1 PRE:L1

8.2.1.1 Chemistry

8.2.1.2 Mineralogy
8.2.2 PRE:L2

8.2.2.1 Chemistry

![Graphs showing cumulative recovery and selectivity curves for iron (%).]

8.2.2.2 Mineralogy

![Graphs showing the balanced mineral composition and iron oxides recovery-grade (%).]
8.2.3 PRE:L3

8.2.3.1 Chemistry

8.2.3.2 Mineralogy
8.2.4 PRE:L4

8.2.4.1 Chemistry

8.2.4.2 Mineralogy
8.2.5 PRE:L:5

8.2.5.1 Chemistry

8.2.5.2 Mineralogy
8.2.6 PRE:L:6

8.2.6.1 Chemistry

8.2.6.2 Mineralogy
L6: Cumulative recovery Fe Oxides (%)

Selectivity curve (<38 μm)

Selectivity curve (38-75 μm)

Selectivity curve (75-106 μm)

Selectivity curve (Bulk)
8.2.7 PRE:L7

8.2.7.1 Chemistry

8.2.7.2 Mineralogy
8.2.8 PRE:L8

8.2.8.1 Chemistry

Cumulative recovery Fe (%)

Selectivity Curve (c<38 µm)

Selectivity Curve (38-75 µm)

Selectivity Curve (75-106 µm)

8.2.8.2 Mineralogy

Feed L8: Balanced Mineral Composition

L8: Fe oxides recovery-grade (%)
8.2.9  PRE:L:9

8.2.9.1  Chemistry

8.2.9.2  Mineralogy
8.2.10 PRE: L10

8.2.10.1 Chemistry

8.2.10.2 Mineralogy
8.2.11 PRE:L11

8.2.11.1 Chemistry

8.2.11.2 Mineralogy
8.2.12 PRE:L12

8.2.12.1 Chemistry

![Diagram of Cumulative recovery Fe (%)]

![Diagram of Selectivity Curve (<38 µm)]

![Diagram of Selectivity Curve (38-75 µm)]

![Diagram of Selectivity Curve (75-106 µm)]

8.2.12.2 Mineralogy

![Diagram of Feed L12: Balanced Mineral Composition]

![Diagram of L12: Fe oxides recovery-grade (%)]
8.2.13 PRE:L13

8.2.13.1 Chemistry

8.2.13.2 Mineralogy