Currency Basis Swap Valuation -
Theory & Practise
Master’s thesis

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June 14, 2017

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Department of Mathematics and Mathematical Statistics

MSc, Industrial Engineering and Management
Master’s thesis, 30 ECTS
Spring 2017

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Abstract

Banks finance their operations in several ways, by shareholders equity, receiving deposits from customers and by borrowing from investors and other financial institutions. One widely used approach is to issue a bond. Bonds issued on the foreign capital markets is a way to increase the financing options and mitigate risk exposure. When a bank converts foreign capital to domestic capital, there is a degree of currency risk involved. One commonly used instrument for converting capital from one currency to another is a cross currency swap.

Since the Global Financial Crisis 2007-2009 regulations imposed by regulators have increased. Banks are required to have sound risk management practises where risk exposure is estimated. In response to recent regulations banks have several departments which assess and follow up risks taken in the operations. As a result, at least two systems are used when valuing financial instruments, one where all trades are conducted, the front office system, and one where risk exposure is estimated, the risk system.

The aim of this project is to investigate why there is a discrepancy between the two systems. We will also analyse how this discrepancy affects risk measures.

By replicating the two systems’ valuation it is possible to distinguish why there is a discrepancy between the systems, regarding the valuation of cross currency basis swaps. When the replication is in place, risk measure calculations are conducted to enable analysis of the impact on risk measures. There are two main differences found between the two systems and how they value a cross currency basis swap: (i) how the underlying risk factors are used; and (ii) how an upcoming cash flow is settled. The effect of these discrepancies are that the risk system overestimate the risk exposure compared with the front office system.


Syftet med detta projekt är att undersöka eventuella skillnader i värderingen av valutabasis-swappar och vidare analysera hur detta påverkar olika riskmått.

Genom att replikera de två systemens värdering är det möjligt att urskilja varför det finns en diskrepans. Replikering av de två systemen låg till grund för beräkningen av riskmått samt analysen av hur skillnaderna påverkar dessa. De huvudsakliga skillnaderna mellan de två systemen avsesade värderingen av valutabasis-swappar är: (i) hur de underliggande riskfaktorerna används, och (ii) hur nästkommande kassaflöde (kupong) bestäms. Effekten av dessa skillnader är att systemet där riskexponering estimeras övervärderar risken jämfört med om risken skulle estimerats i systemet där all handel utförs.
Acknowledgements

This thesis is the concluding project to receive my MSc, Industrial Engineering and Management at Umeå university, specialised in Risk Management. The thesis covers 30 ECTS and is conducted at a bank in Stockholm during the spring of 2017.

Initially I want to thank my supervisor at the bank, Mert Camlibel. He has guided me in an extraordinary way throughout the project with his knowledge about financial instruments, risk management, and programming expertise. Further, I want to thank the rest of the Market and liquidity risk group, for all their support and help.

I also want to thank my supervisor Lisa Hed at Umeå University. She has aided me in an outstanding way with her dedication. She has contributed with new angles and been keen on to keep it simple.

Josef Larsson

Stockholm, June 14, 2017
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1 Introduction

1.1 Preliminary

The traditional role of a bank is to give out loans and administer deposits received by customers [1]. When a bank offers loans in any form, they must finance their lending either by shareholders equity, receiving deposits from customers or by borrowing from investors and other financial institutions. Normally banks have a mix of shareholders equity, deposits and borrowings. Borrowing capital can be done in several ways, one widely used approach is to issue a bond. Bonds have countless of different features and are issued on both domestic and foreign capital markets. Foreign capital markets are a way to increase financing options and mitigate risk exposure. When a bank converts foreign capital to domestic capital (e.g. EUR to SEK), there is a degree of currency risk involved [2].

Two commonly used instruments for converting capital from one currency to another, known as foreign exchange (FX), are FX swaps and cross currency swaps. The FX swap is a short term derivative (usually three months) where the parties swap currencies at the issue date, using the spot exchange rate [3], and at maturity the parties swap back according to the forward exchange rate [2, 4, 5], which was agreed up on at the issue date [2]. The cross currency swap is a similar derivative used for long term (> 3 months) investments where both principals and interest rates are exchanged (see Section 2.5). The interest rates are determined by either a fixed or floating rate.

In recent years regulations imposed by authorities and regulators have led to that banks are required to monitor and manage risks, e.g. credit, currency, and interest rate risks, more thoroughly. In Sweden, supervision of financial institutes is done by Finansinspektionen (FI). FI receives directions from the government and the European Banking Authority (EBA), where the main purpose of supervision and regulations is to ensure that banks follow current regulations and hold enough capital (capital requirements) for the risks they are exposed to [6]. Following the Global Financial Crisis 2007-2009, banks had to increase their capital for market and credit risk [6]. Market risk refers to risk of losses related to unfavourable movements in the market price, e.g. interests rates, foreign exchange rates, commodities and equities [7]. Financial institutes are required to have suitable systems for risk management, where regulations postulate a benchmark model for risk control and management responsibilities in the organisation. The benchmark model, known as three lines of defence, was developed to coordinate control responsibilities in an effective manner throughout the bank [8].

The revenue-generating business units are the first line of defence and form the basis. These units perform a rough control based on the daily operations, e.g. trading and treasury activities. This enables a direct management and implementations of necessary procedures to minimise risks on a daily basis. Units which are part of the first line are treasury and asset management [6].

The second line of defence contains various risk management and compliance functions, e.g. risk control, and compliance, which are responsible for monitoring the first line of defence. The second line must be independent of the first line, and work under a clear risk assessment criteria. Further, the second line assess and report risk exposure on an ongoing basis (e.g. daily or monthly). These independent control functions define and monitor control requirements which the first line must comply with [8].
Internal audit is the third line of defence which provides an independent assurance to senior management and the board of directors. Controls are based on effective risk assessment methodologies. In practice the third line conducts a comprehensive risk assessment at least yearly. Furthermore it covers areas which are not considered by the two first lines of defence [8].

This study will focus on the valuation of cross currency swaps in the first line and the second line of defence. Furthermore, different valuations affect risk measures and the value of a portfolio containing currency swaps. This study has been conducted at a bank, with their head quarter in Stockholm. The layout of this report is such that a reader with modest prerequisite knowledge in the subject can read and acquaint oneself with the problem. Whereas, for a more advanced reader some sections might be ignored.

1.2 Background

In this section the reader is introduced to the first two lines of defence and how these operate independently. Furthermore the basic mechanics regarding funding on the foreign capital market and related risks are explained.

1.2.1 Internal risk control

In response to recent regulations banks have established several departments where risks are assessed, quantified, analysed, and reported on a continuous basis. The market and liquidity risk unit at a bank is responsible for quantification, and analysis of risks to ensure that no limits constituted by senior management are breached. This unit is a part of the independent risk control (the second line of defence) which monitors the daily activities carried out by the first line. All trading activities executed by the first line (see Section 1.1) are conducted in a front office system where some basic risks are assessed.

The second line use a risk system where the activities carried out by the first line are monitored and more advanced risk measures are calculated. The two main differences between the front office system and the risk system are; the front office system is designed for trading and profit and loss (P&L) analysis; while the risk system is configured to estimate risk exposure and support advanced risk calculations, e.g. various types of simulations. In Figure 1 a schematic picture of the first two lines of defence and their respective system.

![Figure 1: Illustration of the first and the second line and how they operate independently of each other. The risk system monitors the activities conducted in the front office system, hence basic information generated in the front office system is sent to the risk control which uses the risk system.](image)
The risk system (RS) works independently from the front office system (FOS), nonetheless the RS uses data generated by the FOS, such as basic information about each trade, which include type, size, maturity, etc. With this information the risk system revalues each position to ensure that the calculations in the front office system are correct. Beyond these calculations the risk system performs additional risk measure calculations, e.g., Value at Risk (VaR). Furthermore this system is used to analyze the impact on risk measures when different scenarios are applied to underlying risk factors, e.g., foreign exchange rates and interest rates.

Since the RS is used to estimate risk based on information from the FOS, valuation of positions in the portfolio should be coherent, if not, risk measure calculations could be misleading. According to regulations a small deviation between the two systems is allowed. At the bank where this project is conducted they have found that there is a significant discrepancy between the two systems when cross currency basis swaps are valued.

1.2.2 Risk related to cross currency basis swaps

A cross currency basis swap is a swap where both legs are determined by a floating rate. As mentioned in Section 1.1 a bank can finance their lending through several channels. In Figure 2 a schematic picture of how the lending can be funded through deposits and a bond issued on the foreign capital market. To convert the funds received in the foreign currency a cross currency basis swap is used. The dashed and dotted arrows represent cash outflow and the solid arrows symbolize cash inflows.

![Figure 2: A schematic picture of how a bank can fund their lending through deposits and by issuing a bond on the foreign capital market together with a cross currency basis swap. The red dashed arrows represent cash outflows, the black arrows illustrate cash inflows, whereas the dotted red arrow represent lending to customers.](image)

Below follows an explanation of how the funding in Figure 2 is conducted:
• The bank issues a 5 year bond with fixed coupons in EUR. The lending is short term (3 months) with floating rate in SEK.
• To hedge the gap between funding and lending the bank enters in to an interest rate swap where the outstanding fixed rate $r_{EUR}$ is transformed to a floating rate $r_{EURIBOR}$.
• By entering a cross currency basis swap the floating rate $r_{EURIBOR}$ is exchanged to floating rate in SEK $r_{STIBOR}$ plus a basis spread $r_{SEK/EUR}$ basis spread.
• The floating rate which is paid in the cross currency basis swap is covered by the floating rate paid by customers.
• The premium $r_{lend}$ paid by customers covers the deposit rate $r_{depo}$ and other operational costs, i.e this is the banks main source of income.

Therefore, the only cash flow not covered in Figure 2 is the SEK/EUR basis spread which is a liquidity premium charged by the market for receiving SEK. Hence the basis swap spread is an inherent risk which the bank is exposed to when funding on the foreign capital market [9].

1.3 Purpose and goal

The purpose of this thesis is to examine why there is discrepancy between the front office system and the risk system when valuing cross currency basis swaps. Further the intention is to analyse how this discrepancy affect risk measures.

The main goal of this project is to develop a program which replicate the portfolio, containing solely cross currency basis swaps, using the two systems valuation methods to enable comparison between the two valuation methods. When this foundation is in place the affect on risk measures such as Value at Risk (VaR) and Expected Shortfall (ES) can be studied.

1.4 Scope and outline

As mentioned this project is conducted at a bank, since the bank’s reporting currency is SEK and the EUR market is one market where the bank issues bonds only cross currency basis swaps denominated in EUR and SEK will be considered.

This paper is built on a theoretical framework found in Section 2. The framework covers financial instruments whose price depend on the present value of future interest rates. Furthermore a methodology for valuation of these instruments is given, the methodology is concluded with theory regarding currency swaps in Section 2.5. In Section 2.6 risk measures are introduced.

From Section 3 the reader will obtain sufficient knowledge about the methods and procedure used to replicate the portfolio containing cross currency basis swaps and how risk measures are calculated. Section 4 display the result which is followed by a discussion found in Section 5. The discussion address the reasons why there is a discrepancy between the front office system and the risk system, and further the affect on risk measures are analysed. Conclusions are summarised in Section 6.
2 Theory

To understand the financial market and the process of pricing (i.e., valuing) financial instruments there are several concepts to comprehend. In this section, basic concepts and underlying factors will be reviewed, commencing with basic concepts such as interest rates in Section 2.1, bonds (see Section 2.2), followed by more complex instruments, e.g., interest rate swaps in Section 2.4. When the foundation is set currency swaps will be studied (see Section 2.5) to finally examine the relation between risk measures and financial instruments in Section 2.6. For a more advanced reader Section 2.1 to 2.4 can be overlooked.

2.1 Interest rates and the risk-free rate

Interest rates are one of the fundamental elements of financial services. It is used to measure expected return, payments on loans etc. When forecasting future values of cash flows interest rates play a central role [10]. In the process of evaluating financial instruments the risk-free rate is widely used, where the risk-free rate is the rate of interest which can be earned without assuming any risk [11]. The risk-free rate is also known as the default free rate, since the risk related to interest rates often involve the risk that a counterparty can not fulfil their obligation, i.e. they default. All currencies have a risk-free rate, where historically the risk-free rate was the rate implied by government bills and bonds, since the government is unlikely to default on their obligations [10, 11].

Up to the Global Financial Crisis 2007-2009 financial institutions used the LIBOR (London Interbank Offered Rate) rate as the risk-free rate. The LIBOR rate is a reference rate which reflects the interest rate at which large banks borrow from other banks [11]. LIBOR is quoted on maturities from 1 week to 12 months in all major currencies (e.g. STIBOR Stockholm Interbank Offered Rate) [6]. After the Global Financial Crisis the risk-free rate is a combination of deposit rates, swap rates, forward rate agreement rates and LIBOR rates.

2.2 Bonds

One foundational concept in modern finance is bonds. A bond is a debt instrument where the issuer is in debt to the holder of the instrument. There are various types of bonds, e.g. zero coupon bonds, fixed coupon bonds, and floating coupon bonds, all with different features. Bonds are also known as fixed income instruments since they provide a deterministic stream of income to the holder [3, 10].

2.2.1 Zero coupon bonds

As the name explains a zero coupon bond (zcb) is a bond where no coupons are paid during the life of the bond. Denote the maturity date $T$ (i.e., the time horizon) which is the date when the issuer of the bond repays the debt by paying the notional value (equivalently, face value, principal value, par value). The zcb interest rate is the rate of interest earned on the interval $[t, T]$, where the time horizon can be one week, two months etc. [11]. When valuing financial instruments the term discount factor (DF) is widely used. The discount factor is the factor used to calculate the present value (PV) of a future cash flow (CF). In the case of a zcb, the future
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CF is the notional. Let \( P(t; T) \) be the price of a zcb which matures at \( T \) valued at \( t \), where \( t < T \).

For simplicity let the zero coupon bond pay one unit at maturity (if the bond pays more, the nominal amount is easily scaled up). Further the default risk is not included in the calculations. The price of a zcb (i.e. the bond value) at time \( t \) with maturity at time \( T \) is defined as \[10\]

\[
P(t; T) = \text{present value of notional amount}.
\]

In mathematical terms

\[
P(t; T) = CF_T \cdot DF(t; T),
\]

where \( CF_T \) is the cash flow at maturity and \( DF(t; T) \) is the discount factor. The discount factor using continuous compounding is defined as \[11\]

\[
DF(t; T) = e^{-r(T-t)}, \tag{2.2.1}
\]

where \( T \) is the maturity date, \( t \) is the valuation date, and \( r \) is the interest rate. \( T \) and \( t \) are expressed in years. The relation \( DF(t; t) = 1 \) must hold for all \( t \) to avoid arbitrage \[3\].

Further it is possible to construct the forward rate prevailing at time \( t \) for the period \([t_{i-1}, t_i] \), where \( t \leq t_{i-1} < t_i \). The non arbitrage relation for the forward rate is \[3, 12, 13\]

\[
F(t; t_{i-1}, t_i) = \frac{DF(t; t_{i-1}) - DF(t; t_i)}{\delta_i \cdot DF(t; t_i)}, \tag{2.2.2}
\]

where \( F(t; t_{i-1}, t_i) \) is the simply compounded forward rate prevailing at time \( t \) concerning the interval \([t_{i-1}, t_i] \), and \( \delta_i \) is the year fraction between \( t_{i-1} \) and \( t_i \). A formal definition of the forward rate and the simply compounded rate is found in Appendix A.

When observing the market price of a zcb one know the time horizon (\( T \)) and the price, whereas the interest rate is unknown. Hence one must solve for the implied interest rate, known as the yield to maturity \[10\]. Solving equation (2.2.1) for \( r \) for prices observed on the market one can construct a relationship between time to maturity and the yield to maturity, denote the yield \( y_T \). Let \( y_T \) be a function of time to maturity \( T \), this relation is called the yield curve. From the yield curve one can derive the term structure of interest rates \[10\]. The yield can be viewed as the return on the investment.

2.2.2 Coupon bonds

A coupon bond is a bond where coupons are paid regularly to the holder. Coupons can be paid with different tenor (time), e.g. quarterly, semi-annual or annual. Similarly to a zero coupon bond the price, \( P(t; c; T) \), of a coupon bond with fixed coupons, \( c \), paid on fixed dates \( t_i \), \( i \in \{1, \ldots, n\} \) where \( t_0 = t \) and \( t_n = T \) is \[10\]

\[
P(t, c; T) = \text{present value of future cash flows + present value of notional amount}.
\]
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The coupon us usually expressed as a percentage of the notional amount. The price of the coupon bond with coupon \( c \) (and \( N = 1 \)) is \([3, 11]\)

\[
P(t, c; T) = \sum_{i=1}^{n} CF_i \cdot DF(t; t_i) + DF(t; T),
\]

where \( CF_i \) is the coupon \( c \) paid at \( t_i \).

2.3 Interest rate curve issues

2.3.1 Interpolation

Assume that we have a set of distinct time steps \( t_i \), with corresponding interest rates \( r_i \), observed on the market, for any \( i \in \{1, \ldots, n\} \). These distinct interest rates are often referred to as nodes or tenors on the interest rate curve. Further we have a future cash flow which takes place at time \( t^* \), where \( t_0 < t^* < t_n \) and \( t^* \neq t_i \). Hence the prevailing interest rate \( r^* \) at time \( t^* \) is unknown. In order to calculate the present value of the cash flow at \( t^* \) we need to determine the interest rate \( r^* \). Initially we find \( i \) such that \( t_{i-1} < t^* < t_i \), then linear interpolation is used to determine the interest rate \( r^* \). It is possible to use other interpolation techniques, e.g. raw, cubic splines, natural cubic splines, Bessel (Hermite) cubic splines, etc. \([14]\). When performing a linear interpolation on spot rates Equation (2.3.1) is used \([14, 15]\)

\[
r^* = \frac{(t^* - t_{i-1}) \cdot r_i + (t_i - t^*) \cdot r_{i-1}}{t_i - t_{i-1}}.
\] (2.3.1)

Linear interpolation is commonly used on interest rates since no assumptions about the forward rate curve has to be done. While other methods will require some property of the forward rate function, for example, piecewise constant \([14]\).

2.3.2 Day count

Interest rates are usually quoted on a per annum basis, whereas the interest rate might be applied over a different time period, e.g. 3 months \([15]\). Here there arise a day count issue, namely how does one calculate the number of days, and how is this translated into a year basis. There are two widely used day count conventions; (i) Actual/360; (ii) Actual/365 \([11, 15]\), where Actual is the actual number of days between the two dates where interest rate is applied to, e.g.

May 1, 2017 - February 1, 2017 = 89 days

the number of days is then divided by 365 or 360 to convert into years. In Table 1 the difference between three day count conventions is displayed, the difference might seem insignificant, nonetheless this difference can have large affect when discounting future cash flows. Another day count convention is 30E/360, where the formula for counting days is

\[
\text{number of years} = \frac{\text{min}\{d_j, 30\} - \text{min}\{d_i, 30\} + 30 \cdot (m_j - m_i) + 360 \cdot (y_j - y_i)}{360}
\]

where the \( d_x \) is the day, \( m_x \) is the month and \( y_x \) denotes the year. The subscript \( i \) and \( j \) indicates
that two unique dates must be used where \( i < j \), since the number of days between "today" and "today" is 0, e.g.

\[
\text{February 1, 2017 - February 1, 2017} = 0.
\]

In the day count convention 30E/360 all months are considered to have 30 days. For example take the dates above, then \( d_j - d_i = 1 - 1 = 0 \).

**Table 1: Illustration of the difference between different day count convention for the period February 1 2017 to May 1 2017**

<table>
<thead>
<tr>
<th>Convention</th>
<th>Number of days</th>
<th>Years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Actual/360</td>
<td>89</td>
<td>0.247222222</td>
</tr>
<tr>
<td>Actual/365</td>
<td>89</td>
<td>0.243835616</td>
</tr>
<tr>
<td>30E/360</td>
<td>90</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Payments can solitary be executed on business days, hence one need a convention for determining if an apparent cash flow date is a non-business day. Modified following business day is widely used among practitioners. The implication is that the cash flow date are moved to the next business day, with exception if the next business day take place in a different month, then the previous business day is adopted. Modified previous business day is used in some markets and similarly to modified following business day cash flows are moved to the previous instead [15].

### 2.3.3 Bootstrapping yield curves

When constructing a risk free interest rate curve (zero rate curve) from observed prices of coupon bearing instruments on the market one need to strip the effect of accrued coupons. Accrued coupons are coupons earned but not collected, for example if you hold a coupon paying bond and you stand between two coupon dates, the accrued coupon is the accumulated interest earned from holding the bond since the last coupon until today. One widely used tool is the recursive bootstrap method which uses market quoted prices from coupon bearing instruments as input [11]. The output is a zero spot rate curve. A swap (see Section 2.4 and 2.5.2) which is valued at par, is to be valued by the following formula

\[
r_n \sum_{i=1}^{n} \delta_i \cdot DF(t; t_i) + DF(t; t_n) = 1,
\]  

(2.3.2)

where \( r_n \) is the par rate, \( \delta_i \) is the time in years from \( t_{i-1} \) to \( t_i \). Valued at par means that the price of the swap is equal to its nominal amount. In theory \( r_n \) is solved for, as

\[
r_n = \frac{1 - DF(t; t_n)}{\sum_{i=1}^{n} \delta_i \cdot DF(t; t_i)}.
\]
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If on the other hand we assume \( r_n \) and \( DF(t; t_i) \) is known for \( i = 1, \ldots, n - 1 \), it is possible to find \( DF(t; t_n) \) by rearranging Equation (2.3.2) which gives [14]

\[
DF(t; t_n) = \frac{1 - r_n \sum_{i=1}^{n-1} \delta_i \cdot DF(t; t_i)}{1 + r_n \cdot \delta_n}.
\] (2.3.3)

Further Equation (2.3.3) is known as the forward substitution formula when solving a system of equations which can be represented by a lower triangular matrix [16]. The process of using Equation (2.3.3) to generate discount factors is of great importance when pricing cross currency swaps (see Section 2.5).

### 2.4 Interest rate swaps

A swap is a derivative where two parties agree to exchange cash flows in the future. The first swap was settled in the early 1980s, and a forward rate agreement can be viewed as a simple example of a swap [11]. The agreement consists of dates when future cash flows are to be paid and how these cash flows should be calculated. A typical example of a swap is an interest rate swap (IRS), where the two parties exchange interest rates with each other. Usually one part pays a floating rate (usually LIBOR) and receives a fixed rate, the other part pays a fixed rate and receives a floating rate. Each payment is based on a principal amount [11, 15].

Consider a fixed-floating IRS where \( t_0 \) is the reference date (e.g. today or settlement date), cash flows (coupons) are paid on the dates \( t_1, \ldots, t_n \) where \( t_0 < t_1 \), and \( t_n \) is the maturity date. Let \( \delta_i \) be the length (day count fraction) of the period \([t_{i-1}, t_i]\). The floating rate is determined by the forward rate \( F(t; t_{i-1}, t_i) \) as defined in Equation (2.2.2) for period \([t_{i-1}, t_i]\). The forward rate is fixed at time \( t_{i-1} \), and paid at \( t_i \). The floating rate paid at time \( t_i \) is \( CF^\text{float}_i = N \cdot \delta_i \cdot F(t; t_{i-1}, t_i) \) where \( N \) is the notional amount (assume \( N = 1 \)). The fixed swap rate \( r_{\text{fix}} \) is determined to ensure a price of par at initiation. In Figure 3 a simple illustration of the floating leg of an IRS is found.

![Diagram of forward rates](time_line.png)

**Figure 3:** Time line describing the coupon characteristics of the floating leg of an interest rate swap.

The value today of a floating leg interest rate cash flow is its discounted forward rate [3], where the value of the whole floating leg at time \( t > t_0 \) is the sum of all future cash flows, namely

\[
PV^\text{float}_{\text{tot}}(t) = \sum_{i=1}^{n} CF^\text{float}_i \cdot DF(t; t_i).
\] (2.4.1)
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The value of the fixed leg is

\[ PV_{\text{fix}}^\text{tot}(t) = \sum_{i=1}^{n} CF_{i}^\text{fix} \cdot DF(t; t_i), \tag{2.4.2} \]

where \( CF_{i}^\text{fix} \) is a fixed rate cash flow defined as \( CF_{i}^\text{fix} = N \cdot \delta_i \cdot r_{\text{fix}} \). At inception the fixed rate is set so that it admits a price of par \([3, 13]\). The present value of the net cash flow at time \( t \) is given by \( \delta_i \cdot (F(t; t_{i-1}, t_i) - r_{f_{\text{fix}}}) \cdot DF(t; t_i) \). Hence the total value of the IRS at time \( t \) is the sum of all discounted cash flows, thus given by

\[ V_{\text{IRS}}(t) = \sum_{i=1}^{n} (CF_{i}^\text{float} - CF_{i}^\text{fix}) \cdot DF(t; t_i) \]

\[ = \sum_{i=1}^{n} \delta_i \cdot (F(t; t_{i-1}, t_i) - r_{f_{\text{fix}}}) \cdot DF(t; t_i). \tag{2.4.3} \]

2.5 Currency swaps

Currency swaps are an extension of IRS where the difference is that the interest rate legs are in different currencies. When exchanging two currencies on the market for immediate delivery the spot exchange rate, \( S_0 \), is defined as \([3, 13]\)

\[ S_0 = \frac{\text{units of the domestic currency}}{\text{unit of the foreign currency}} = \frac{N_d}{N_f}. \]

There are two widely used currency derivatives, foreign exchange swap (FX swap) and cross currency basis swap (CCY swap). The FX swap is a short term instrument (usually 3 months), and the CCY swap is a long term instrument (> 3 months) \([2]\).

2.5.1 FX swaps

The FX swap is a short term derivative where the parties swap notional in the two currencies at the issue date at the spot exchange rate, \( S_0 \), and at maturity the parties swap back the notional according to the forward exchange rate, \( f_0 \), which was agreed upon at the issue date. There are no interest rate payments between the parties in a FX swap \([2]\). The notional amounts in the two currencies must satisfy, \( N_d = S_0 \cdot N_f \), where \( N_d \) is the domestic notional, and \( N_f \) is the foreign notional. This relation of the notional amounts is used for the future exchange of notional \([13]\). The forward exchange rate \( f_0 \) is based on the assumption of no arbitrage where the following relation must hold \([4]\)

\[ -S_0 \cdot (1 + r_d)^T + f_0 \cdot (1 + r_f)^T = 0, \tag{2.5.1} \]

where \( r_d \) is the domestic default free spot rate, \( r_f \) is the foreign default free spot rate, and \( T \) is the time horizon for the forward exchange price. The forward exchange rate is then defined as \([2, 4, 5]\)

\[ f_0 = S_0 \cdot \frac{(1 + r_d)^T}{(1 + r_f)^T}. \tag{2.5.2} \]
Currency Basis Swap Valuation

This relation is often referred to as the classic interest rate parity, it is assumed that there is no counterparty default risk in forward or swap contracts [4].

In Figure 4 a schematic view of the cash flows between two parties engaged in a FX swap is presented. Let $t_0$ be today and $T$ be the maturity date, $N$ is the cash flow where the subscript $d$ and $f$ denotes domestic and foreign actions respectively. The future cash flow is determined by the forward exchange rate $f_0$.

![Figure 4: Illustration of the cash flows between two parties in a FX swap.](image)

Since the Global Financial Crisis 2007-2009 there are evidence that the interest rate parity has failed to hold [17]. In order to preserve the no arbitrage opportunity one leg is adjusted by adding a basis spread. In Equation (2.5.3) the basis spread is added on the foreign leg, hence assuming that the domestic leg is the liquidity benchmark [17]. The basis spread is normally added on the leg which is not the liquidity benchmark, where the basis spread can be viewed as the difference between the two underlying interest rate curves

$$f_0 = S_0 \cdot \frac{(1 + r_d)^T}{(1 + r_f + \gamma)^T}.$$  \hspace{1cm} (2.5.3)

2.5.2 Cross currency swaps

Cross currency swaps were developed from back-to-back loans in the late 1970s [15]. The cross currency swap is similar to the FX swap with the exception that interest payments are exchanged during the life of the contract. The interest rates are determined by either a fixed or floating rate (e.g. LIBOR, STIBOR depending on currency). Hence the interest rate curve is of great importance in the valuation process [2, 13, 15]. There are several possible types of interest flows in a cross-currency swap: floating versus floating, floating versus fixed, and fixed versus fixed, where the first one is of great interest and is called basis swap [13]. The others can be derived synthetically by combining a basis swap with an IRS.

The CCY swap can be seen as an exchange of two floating rate bonds one in the domestic currency with notional $N_d$ and one in a foreign currency with notional $N_f$. In Figure 5 you can find a time line depicting the cash flows of a cross currency basis swap.
Currency Basis Swap Valuation

From Section 2.5.1 we know that the two notional amounts are linked, \( N_d = S_0 \cdot N_t \) and the CCY swap is fair at inception if this is true. Fair in the meaning that each leg is priced at par. In theory this is true, nevertheless, market quoted CCY swaps are fair if a spread is added to one of the legs in the swap, a result of Equation (2.5.3). This spread is called cross currency basis spread [13]. This spread is added since the market charge a liquidity premium on one currency over the other for the transfer of assets/liabilities from one currency to another.

2.5.3 Simple valuation

Viewing the CCY swap as two floating rate bonds where the cash flow are made in two currencies, the forward rate is calculated for both currencies, hence the subscript \( x \) is introduced, which is substituted for \( d \), domestic or \( f \), foreign. The forward rate is calculated according to Equation (2.2.2)

\[
F_x(t; t_{i-1}, t_i) = \frac{DF_x(t; t_{i-1}) - DF_x(t; t_i)}{\delta_i \cdot DF_x(t; t_i)}, \quad t \leq t_{i-1} < t_i,
\]

where the discount factors \( DF_x(t; t_i) \) are calculated based on the prevailing risk free rate at \( t_i \) observed on the market at \( t \), and \( \delta_i \) is the year fraction between \( t_{i-1} \) and \( t_i \). The value at time \( t \), where \( t > t_0 \), for a payer of domestic currency is [11, 13]

\[
V_{CCY}(t) = -S_t \cdot N_t \left( \sum_{i=1}^{n} \delta_i \cdot F_f(t; t_{i-1}, t_i) \cdot DF_f(t; t_i) - DF_f(t; t_n) \right) + N_d \left( \sum_{i=1}^{n} \delta_i \cdot F_d(t; t_{i-1}, t_i) \cdot DF_d(t; t_i) - DF_d(t; t_n) \right).
\]  

Equation (2.5.4)

Since a CCY swap is the exchange of two floating rate bonds, each leg should have a value of \( N_x \), at the beginning of each interest rate period (see Section 2.4), where \( N_x \) are linked by \( S_t \). In a perfect world, this valuation holds [13]. In practice, the market quotes the basis spread on top of the leg which is not the currency benchmark, hence Equation (2.5.4) is just a theoretical valuation. This implies that there exists an arbitrage opportunity and it is necessary to incorporate the basis spread in the valuation [13]. In Section 2.5.4 the basis spread is included in the valuation.

Figure 5: Time line describing interest rate cash flows for the payer of foreign currency, i.e. receiver of domestic currency, where the floating coupon received is the forward rate \( F_d \). At \( t_0 \) there is an exchange of notional between the two parties in the swap.
2.5.4 Valuation based on a modified discount curve

This approach of valuing cross currency swaps was popular before the Global Financial Crisis 2007-2009 [12], nevertheless it is inconsistent with the standard IRS (single currency swap) [13]. Two discount factors are utilised in this approach, one for the construction of forward rates and the other for discounting final cash flows [13]. The forward rates, \( F_x(t; t_{i-1}, t_i) \), are calculated according to Equation (2.2.2) and the discount factors, \( DF_x(t; t_i) \) are calculated based on the prevailing risk free rate at \( t_i \) observed on the market at \( t \). The leg which is the currency benchmark is valued as in Equation (2.4.1) since no liquidity adjustment is required [13]. For currencies different from the liquidity benchmark, the basis spread \( \gamma_n \) is included. The basis spread is in fact a basis spread curve with several tenors, hence \( \gamma_n \) is not equal for all future cash flows in the swap. Let \( DF^n_x \) be the discount factor exclusively used for discounting cash flows where the defining condition at initiation is [13]

\[
1 = \sum_{i=1}^{n} \delta_i (F_x(t; t_{i-1}, t_i) + \gamma_n) DF^s_x(t; t_i) + DF^n_x(t; t_n), \quad n = 1, \ldots
\]  (2.5.5)

Recall from Section 2.3.3 that if a swap is valued at par one can derive unknown discount factors using a bootstrapping procedure. The defining condition in Equation (2.5.5) is similar to Equation (2.3.2) with the exception that the cash flow and the basis spread is included in the formula. Since \( DF^n_x(t; t_i) \) is unknown, a recursive bootstrapping formula derived from the defining conditions above is used

\[
DF^n_x(t; t_n) = \frac{1 - \sum_{i=1}^{n-1} \delta_i (F_x(t; t_{i-1}, t_i) + \gamma_n) DF^s_x(t; t_i)}{1 + \delta_n \cdot (F_x(t; t_{i-1}, t_i) + \gamma_n)}, \quad n = 1, \ldots
\]  (2.5.6)

where the discount factor \( DF^n_x \) is to assure that this leg is at par at inception, and at the beginning of each coupon period.

The value at time \( t \) using this methodology is similar to Expression (2.5.4) with the exception that the spread \( \gamma_n \) is added,

\[
V_{CCY}(t) = -S_t \cdot N_f \left( \sum_{i=1}^{n} \delta_i \cdot (F_f(t; t_{i-1}, t_i) + \gamma_n) \cdot DF^s_f(t; t_i) - DF^n_f(t; t_n) \right) + N_d \left( \sum_{i=1}^{n} \delta_i \cdot F_d(t; t_{i-1}, t_i) \cdot DF_d(t; t_i) - DF^n_d(t; t_n) \right)
\]  (2.5.7)

For the leg which is the currency benchmark (in this case the domestic leg), the discount factor \( DF \) is used, whereas for the leg to which the spread is added the cash flows are discounted by \( DF^n \).

2.6 Risk measures

As mentioned in Section 1.1, financial institutions must have adequate risk management functions for the risk they take. In order to estimate risk, we need a way to calculate and measure risk. There are plenty of risk measures, e.g. Value at Risk, volatility, spread, Expected Shortfall etc. In this section, basis spread risk, Value at Risk, and Expected Shortfall will be examined.
2.6.1 Basis spread risk

There are several types of spread risks, where the most famous is the bid-ask spread, which is a measure of market liquidity. A narrow spread indicates high liquidity and a wide spread indicates low market liquidity, i.e., a trader might experience difficulties to sell or buy on the market [18]. Basis spread risk refers to the impact of relative changes in underlying risk factors for financial instruments which; (i) use two different interest rate curves with equal tenors, e.g., 3M STIBOR vs 3M Government yield (reference rate basis risk); (ii) use different tenors based on the same interest rate curve, e.g., 3M STIBOR vs 6M STIBOR (tenor basis risk); (iii) equal tenors in different currencies, e.g., 3M STIBOR vs 3M EURIBOR, known as currency basis risk [19].

The currency basis spread is represented by a currency basis spread curve, which is the difference between the two underlying interest rate curves. Hence, currency basis spread risk is the risk of loss as a consequence of fluctuations in this spread [20], where the spread is quoted relative a liquidity benchmark, e.g., USD or EUR [13] (see Section 2.5). Fluctuations in the currency basis spread, further referred to as the basis spread, is a substantial risk for banks who fund their operations through the foreign capital market since it affect the result and capital requirement [9].

2.6.2 Value at Risk

A widely used risk measure is Value at Risk, which has been a market standard for over 20 years [6]. VaR is calculated for all market risks a bank is exposed to. VaR is an attempt to aggregate the total risk in the bank’s portfolio. This is done by focusing on the portfolios loss over time, where one can think of Value at Risk as in the following way [11].

The loss will be less or equal to VaR over the period T with α percent certainty.

Let $L$ be a random variable, which indicates the loss of a portfolio over the interval $\Delta t$, where

$$ L = -\Delta V = -(V(t + \Delta t) - V(t)) = V(t) - V(t + \Delta t), \quad (2.6.1) $$

and $V(t)$ is the value of a portfolio at a time $t$ [21]. Note that when we are working with losses a loss is a positive number. Let $F_L$ denote the cumulative distribution function of the random variable $L$, defined as [21]

$$ F_L(q) = P(L < q), $$

and let $f_L$ denote the probability density function of $L$. Given a confidence level $\alpha \in (0, 1)$, $VaR_{\alpha \cdot 100\%}(L)$ of a portfolio is given by the smallest number $q$, where the probability that the loss $L$ exceeds $q$ is smaller than $(1 - \alpha)$. When the distribution $F_L$ is discrete the following holds [22, 23]

$$ VaR_{\alpha \cdot 100\%}(L) = \inf\{q : P(L < q) \geq \alpha\} = \inf\{q : F_L(q) \geq \alpha\}. \quad (2.6.2) $$

With the notation above $VaR_{\alpha \cdot 100\%}(L) = F_L^{-1}(\alpha)$.

$^1\inf\{\cdot\}$ is the infimum, also known as greatest lower bound.
2.6.3 Expected Shortfall

Value at Risk does not give any information about the losses greater than $VaR_{\alpha \cdot 100\%} (L)$. Hence one need a risk measure which answers the question [11]

\[
\text{If things get bad, how much do we expect to lose?}
\]

Expected Shortfall (ES) answers this question and is refereed by some as Conditional VaR [22], where ES is defined as [24]

\[
ES_{\alpha \cdot 100\%} (L) = \frac{1}{1 - \alpha} \cdot E[L \cdot 1 \{ L > VaR_{\alpha \cdot 100\%} (L) \}], \quad (2.6.3)
\]

where $1 \{ \cdot \}$ is the indicator function. Since ES measures the expected loss if things go bad $ES_{\alpha \cdot 100\%} (L) \geq VaR_{\alpha \cdot 100\%} (L)$.

2.6.4 Historical simulation

In order to calculate VaR or ES, it is necessary to estimate the distribution of $L$. Historical simulation (HS) is a methodology to forecast risk under the assumption that history repeats itself [24]. HS estimates the loss distribution under the empirical distribution of historical data points [23]. Intuitively one can think of historical simulations like this:

By extracting $n$ historical risk factor changes and revaluing the current portfolio for each historical risk factor change one gets the portfolio development for the current portfolio as it would have been held static for the whole period.

Let $t$ be the valuation date. Given a time series vector $H = (h_0, \ldots, h_t)$ with historical risk factor observations, let $X = (x_1, \ldots, x_n)$ describe the change of the historical time series vector $H$, where $x_i = h_i - h_{i-1}$, $i \in \{1, \ldots, n\}$, and $n = t$. Denote the current portfolio depending on risk factor change $h_t$ as $V(t, h_t)$. From the historical risk factor changes generate a vector of historical losses $L = (l_1, \ldots, l_n)$ where [23]

\[
l_i(t) = -(V(t + i, h_t + x_i) - V(t, h_t)). \quad (2.6.4)
\]

A widely used technique to estimate e.g. VaR, is to use the vector of historical losses to sample quantiles of the data [6, 23]. In Procedure 1 the steps required to evaluate VaR and ES using historical simulation is presented.

**Procedure 1** Historical simulation procedure.

1. Generate a vector $L$ of historical losses, given a time series vector $H$, and risk factor changes $X$.
2. Sort the vector of historical losses, i.e. order the values by $l_{[1]} \leq \cdots \leq l_{[n]}$, where $l_{[i]}$ denotes the $i^{th}$ sorted loss.
3. Choose a confidence level $\alpha \in (0, 1)$.
4. Select the value on position $p = \alpha \cdot n$, if $\alpha \cdot n$ is not an integer than chose $p_{\text{roof}} = \lceil \alpha \cdot n \rceil$, $p_{\text{floor}} = \lfloor \alpha \cdot n \rfloor$ or interpolate between $p_{\text{roof}}$ and $p_{\text{floor}}$.
5. Let $VaR_{\alpha \cdot 100\%} (L) = l_p$ [23, 24].
For instance, let $n = 500$ and $\alpha = 0.99$ then VaR is the 5th largest value. Estimation ES is simply the average of the losses larger than VaR. In Figure 6 the probability density function of $L$ is displayed, where Expected Shortfall is the area exceeding VaR, i.e. the area $1 - \alpha$.

2.6.5 Price value of a basis point

When measuring the sensitivity of a derivative and a portfolio, one captures the change of value if an underlying risk factor fluctuates, also known as sensitivity analysis. PV01 is short for Price value of 1 basis point, and measures the sensitivity of a portfolio with respect to a change of 1 bp (0.01%) of an underlying risk factor, e.g. interest rate, yield curve, basis spread curve, etc. [25]. The formula for measuring PV01 is

$$PV01 = V(t; r_x + 0.0001) - V(t; r_x), \quad (2.6.5)$$

where $V(t; \cdot)$ is the present value of a portfolio or a derivative at time $t$, $r_x$ represents the underlying risk factor which the basis point is added. From Equation (2.6.5) it is clear that PV01 measures the change in market value of a portfolio with respect to 1 bp change.
3 Method

In this section, an account for how the study has been conducted is presented. The record below comprise the foundation to achieve the purpose and goal (see Section 1.3). The study has been conducted in three blocks, literature study found in Section 3.1, valuation replication and risk measure calculation explained in Section 3.2 and 3.3 respectively. In Section 3.4, the differences between the two systems are summarized.

3.1 Literature study

Through the course of this study, various scientific articles, books, legal documents, and internet resources have been read. These form the theoretical framework for the study. A comprehensive list of each resource is found in the reference list at the end of the paper. There are numerous scientific papers in the subject of pricing financial derivatives and a selection of those which cover the area of the study has been selected. When choosing each article, a critical mindset based on relevance, and trustworthiness has been applied.

Further, the two systems (front office and risk system) provide, to some extent, documentation regarding the valuation. The documentation was a guideline for the replication of each system.

3.2 Valuation replication

As mentioned in Section 1.2.1, the portfolio is valued by two systems independently, the front office system and the risk system. All trades are conducted in the front office system and then followed up in the risk system. The valuation process in the two systems follow a similar process namely; (a) determine instrument characteristics; (b) bootstrap the zero coupon curve (if necessary); (c) interpolate interest rates for each coupon date; (d) calculate forward rates; (e) discount cash flows. In the sections below, the valuation process for each system is explained in more detail.

When replicating the valuation performed by the two systems, one need to know the basic configuration such as coupon type, coupon frequency, underlying risk factors, etc. for the instrument. In Table 2 the basic information for the cross currency basis swap is displayed.

<table>
<thead>
<tr>
<th>Setting</th>
<th>Front office system</th>
<th>Risk system</th>
</tr>
</thead>
<tbody>
<tr>
<td>Date base</td>
<td>Actual/360</td>
<td>Actual/360</td>
</tr>
<tr>
<td>Underlying risk factors</td>
<td>Euro zero swap curve</td>
<td>Euro zero swap curve</td>
</tr>
<tr>
<td></td>
<td>SEK zero swap curve</td>
<td>SEK zero swap curve</td>
</tr>
<tr>
<td></td>
<td>SEK/EUR basis spread curve</td>
<td>SEK/EUR basis spread curve</td>
</tr>
<tr>
<td></td>
<td>SEK/EUR exchange rate</td>
<td>SEK/EUR exchange rate</td>
</tr>
<tr>
<td>Coupon frequency</td>
<td>3 months</td>
<td>3 months</td>
</tr>
<tr>
<td>Coupon type</td>
<td>Floating</td>
<td>Floating</td>
</tr>
<tr>
<td>Coupon fixing date</td>
<td>3 months prior coupon payment</td>
<td>-</td>
</tr>
<tr>
<td>Coupon fixing rate</td>
<td>EURIBOR</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>STIBOR</td>
<td>-</td>
</tr>
</tbody>
</table>
Currency Basis Swap Valuation

The underlying risk factors are observed on the market at any given valuation date. The two zero swap curves are used for discounting future cash flows and for calculating forward rates, the two interbank offered rates are used when fixing a coupon at its fixing date. In the front office system the Euro zero swap curve was derived from market quoted Euro swap curve using the recursive bootstrap formula found in Equation (2.3.3). This was necessary since the original Euro zero swap curve was not saved in the database as the other curves. In the risk system on the other hand the original curve is preserved. From Table 2 one can see that the two systems have almost identical set-up, where the main difference is when the coupon is fixed, i.e. the coupon must be determined in advance.

Quoted interest rates applies for a specific time period, e.g. from today to tomorrow (overnight). A time line illustrating the interest rate periods used in the front office system can be found in Figure 7.

![Figure 7: Time line depicting time period for which the quoted interest rate apply in the front office system, where t is the valuation date.](image)

In Figure 8 the interest rate periods used in the risk system, one can observe that the tenor T/N is not used in the risk system.

![Figure 8: Time line depicting quoted interest rates, and for which time period they apply, used in the risk system, t is the valuation date.](image)

When converting interest rates to discount factors, the discount factor apply for the same time period as the interest rate. Hence if one want to discount a cash flow which occurs at the tenor 1W, one must first discount it to T/N, then discount it in two steps further to get the present value of the cash flow. The same logic is applied in the risk system with the exception that the tenor T/N is excluded. In Table 3 a numerical example of the interest rate periods is presented. It is clear that \( r_{O/N} \) applies from the valuation date to the day after. As one can see the quoted rate \( r_{O/N} \) applies for the period \( t \rightarrow O/N \), where \( t \) is the valuation date and O/N is the day after;
Currency Basis Swap Valuation

$r_{TN}$ applies from O/N $\rightarrow$ T/N, where T/N is the day after O/N; for tenors greater than T/N the interest applies for T/N $\rightarrow$ chosen tenor, e.g. T/N $\rightarrow$ 3M.

**Table 3:** Numerical example over which time period a quoted interest rate apply for in the front office system, where the valuation date is February 1, 2017.

<table>
<thead>
<tr>
<th>Tenor</th>
<th>Date from</th>
<th>Date to</th>
<th>Interest rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>O/N</td>
<td>01 Feb 2017</td>
<td>02 Feb 2017</td>
<td>-0.58</td>
</tr>
<tr>
<td>T/N</td>
<td>02 Feb 2017</td>
<td>03 Feb 2017</td>
<td>-0.58</td>
</tr>
<tr>
<td>1W</td>
<td>03 Feb 2017</td>
<td>10 Feb 2017</td>
<td>-0.57</td>
</tr>
<tr>
<td>2W</td>
<td>03 Feb 2017</td>
<td>17 Feb 2017</td>
<td>-0.57</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3M</td>
<td>03 Feb 2017</td>
<td>03 May 2017</td>
<td>-0.54</td>
</tr>
</tbody>
</table>

In Table B.1 and B.2 in the appendix there is a more detailed overview of all quoted tenors for both systems.

**Interpolation**

To determine the discount factor and the forward rate for each coupon one must first find the prevailing interest rate for each coupon date. By interpolation, see Section 2.3, one can determine the interest rate on coupon dates which do not occur at a quoted tenor on the interest rate curve. In Figure 9 an example of when interpolation is required to determine the prevailing interest rate for the coupon.

![Figure 9: Illustration of when interpolation is necessary, the coupon occur between two quoted tenors and to determine the current interest rate interpolation is required.]

Given a set of quoted tenors $t_j$ with corresponding interest rates $r_j$, where $j \in \{1, \ldots, m\}$ and a set of coupon dates $t^*_i$ where $i \in \{1, \ldots, n\}$, interpolation is required to find $r^*_i$ where $t^*_i \neq t_j$. The following procedure was used to interpolate for each coupon date:
Currency Basis Swap Valuation

**Procedure 2 Interpolation procedure for finding interest rates.**

1: Let \( t \) be the valuation date, calculate the number of years from \( t \) to \( t^*_i \) \( \forall i \in \{1, \ldots, n\} \), according to the instruments date base. Denote the number of years with \( \ell^*_i \).

2: Calculate the number of years to \( t^*_i \) for each interest rate period according to the interest rate curves date base and denote the number of years with \( \ell^\min_j \).

3: For each coupon date where \( t^*_i \neq t^*_j \), find \( t^\max_j = \{t_j | max\{t_j\} < t^*_i\} \), and \( t^\min_j = \{t_j | min\{t_j\} > t^*_i\} \).

4: Let \( \ell^\min_j \) and \( \ell^\max_j \) be the number years corresponding to \( t^\min_j \) and \( t^\max_j \).

5: The interest rate \( r^*_i \) is evaluated using linear interpolate according to Equation (2.3.1), where the number of years \( \ell^\min_j \), \( \ell^\max_j \), \( r^\min_j \) and \( r^\max_j \) are together with \( \ell^*_i \). Here \( r^\min_j \) and \( r^\max_j \) are the corresponding interest rate for \( \ell^\min_j \) and \( \ell^\max_j \). This approach take into account the different date bases of the underlying risk factors and the swap.

6: If no \( t^\max_j \) could be found, flat interpolation is used, i.e. \( t^\min_j \) with corresponding \( r^j \) is used, and for \( t^\min_j \) vice versa.

This procedure is applied on both legs of the CCY swap. In Table 4, an example of the interpolation procedure is displayed. One can observe that the two first coupons fall on a quoted tenor, hence interpolation is unnecessary.

**Table 4: Example of the interpolation procedure. The valuation date is February 1, 2017, and February 3, 2017 is T/N.**

<table>
<thead>
<tr>
<th>Coupon date ( t^*_i )</th>
<th>( \ell^*_i )</th>
<th>( \ell^\min_j )</th>
<th>( \ell^\max_j )</th>
<th>( r^\min_j )</th>
<th>( r^\max_j )</th>
<th>( r^*_i )</th>
<th>( DF(t; t^*_i) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>03Apr2017</td>
<td>0.164</td>
<td>0.164</td>
<td>0.164</td>
<td>-0.578</td>
<td>-0.578</td>
<td>-0.578</td>
<td>1.00093504</td>
</tr>
<tr>
<td>03Jul2017</td>
<td>0.417</td>
<td>0.417</td>
<td>0.417</td>
<td>-0.436</td>
<td>-0.436</td>
<td>-0.436</td>
<td>1.001794882</td>
</tr>
<tr>
<td>02Oct2017</td>
<td>0.669</td>
<td>0.592</td>
<td>0.672</td>
<td>-0.345</td>
<td>-0.315</td>
<td>-0.316</td>
<td>1.002086547</td>
</tr>
<tr>
<td>02Jan2018</td>
<td>0.925</td>
<td>0.844</td>
<td>0.928</td>
<td>-0.335</td>
<td>-0.393</td>
<td>-0.391</td>
<td>1.003574764</td>
</tr>
<tr>
<td>02Apr2018</td>
<td>1.175</td>
<td>1.006</td>
<td>2.003</td>
<td>-0.456</td>
<td>-0.259</td>
<td>-0.422</td>
<td>1.004906554</td>
</tr>
</tbody>
</table>

**Calculating forward rates**

In order to calculate the forward rates (see Figure 5) for each coupon, the discount factor for each interpolated interest rate must be evaluated. The discount factors, \( DF(t; t^*_i) \), are calculated using Equation (2.2.1), with the modification that all underlying risk factors have the same yield date base, namely Actual/365 (see section 2.3). Since the interest rates apply for the period \( t^*_X \rightarrow t^*_i \) the number of days used when calculating Actual/365 is the number of days minus two. Hence the formula used for calculating the discount factors are

\[
DF(t^*_X; t^*_i) = \exp \left( -r^*_i \cdot \frac{t^*_i - t^*_X}{365} \right), \quad i \in \{1, \ldots, n\}.
\]

The forward rates for each period is then calculated using Equation (2.2.2),

\[
F(t; t^*_{i-1}, t^*_i) = \frac{DF(t^*_X; t^*_i) - DF(t^*_X; t^*_{i-1})}{\delta(t^*_{i-1}, t^*_i) \cdot DF(t^*_X; t^*_i)}.
\]

\(^2\)X/N is substituted for T/N in the front office system and O/N in the risk system.
Valuation

When the forward rates are in place one can determine the size of each coupon. As mentioned in Section 2.5 the instrument is set to have a price of par at inception. As a result there is a certain spread added to each leg. Note that this is not the basis spread. This spread can be viewed as a credit spread, to incorporate the counterparty risk. Further this spread is constant over the life of the instrument. Denote this spread as $\varepsilon_x$, where the subscript $x$ refers to the either leg.

Discounting the coupons on the leg which is the liquidity benchmark the discount factor $P$ is used since no spread is added on this leg. On the leg which is not the liquidity benchmark the basis spread must be added when the discount factor is calculated (see Section 2.5.2). The basis spread curve is observed on the market and for each coupon date one must interpolate as described in Procedure 2. Let $\gamma_i$ be the basis spread (interpolated if necessary) then the discount factor is calculated using the following formula

$$DF_s(t_{XN}; t_i) = \exp\left(-\left(r_i + \gamma_i\right) \frac{(t_i - t_{XN})}{365}\right), \quad i \in \{1, \ldots, n\}. \tag{3.2.1}$$

Since the discount factors $DF$ and $DF_s$ discounts the cash flows to $t_{XN}$ one must discount each cash flow from $t_{XN}$ to the valuation date $t$. This is done by multiplying the appropriate discount factor, in the front office system

$$DF_x(t; t_{TN}) = DF_x(t; t_{ON}) \cdot DF_x(t_{ON}; t_{TN})$$

where

$$DF_x(t; t_{ON}) = \exp\left(-\frac{r_{ON}}{365}\right),$$

$$DF_x(t_{ON}; t_{TN}) = \exp\left(-\frac{r_{TN}}{365}\right),$$

is multiplied with $DF_x(t_{TN}; \cdot)$ and $DF_s(t_{TN}; \cdot)$ and $DF_s(t_{TN}; \cdot)$. Similarly in the risk system $DF_x(t; t_{ON})$ is multiplied with $DF_x(t_{ON}; \cdot)$ and $DF_s(t_{ON}; \cdot)$.

Denote each coupon as $CF_{x,i} = \delta_i \cdot (F_x(t; t_{i-1}, t_i) + \varepsilon_x)$, where $\delta_i$ is the length of the period $[t_{i-1}, t_i]$. Then the value of the CCY swap at time $t$ where $t > t_0$ in the front office system is

$$V_{k}^{FOS}(t, N_{d,k}, N_{l,k}) = S_t \cdot N_{l,k} \cdot DF_l(t; t_{TN}) \left( \sum_{i=1}^{n} CF_{l,i} \cdot DF_l(t_{TN}; t_i) - DF_l(t_{TN}; t_n) \right)
- N_{d,k} \cdot DF_d(t; t_{TN}) \left( \sum_{i=1}^{n} CF_{d,i} \cdot DF_d(t_{TN}; t_i) - DF_d(t_{TN}; t_n) \right), \tag{3.2.2}$$

where $S_t$ is the spot exchange rate at time $t$, $N_x$ are the notional amounts which are connected at inception by the exchange rate $S_0$. The subscript $k$ denotes position in the portfolio. Similarly
the value of the CCY swap at time $t$ where $t > t_0$ in the risk system is

$$
V_{k}^{RS}(t, N_{d,k}, N_{f,k}) = S_t N_{f,k} \cdot D F_f(t; t_{O/N}) \left( \sum_{i=1}^{n} C F_{f,i} \cdot D F_f(t_{O/N}; t_i) - D F_f(t_{O/N}; t_n) \right) - N_{d,k} \cdot D F_d(t; t_{O/N}) \left( \sum_{i=1}^{n} C F_{d,i} \cdot D F_d(t_{O/N}; t_i) - D F_d(t_{O/N}; t_n) \right).
$$

(3.2.3)

To value the whole portfolio, aggregate the value of all positions in the portfolio. Let $l$ be the number of positions in the portfolio, then the value is

$$
V^{FOS}(t) = \sum_{k=1}^{l} V_{k}^{FOS}(t, N_{d,k}, N_{f,k}),
$$

(3.2.4)

and

$$
V^{RS}(t) = \sum_{k=1}^{l} V_{k}^{RS}(t, N_{d,k}, N_{f,k}).
$$

(3.2.5)

### 3.3 Risk measure calculation

Historical simulation (see Procedure 1) was used to determine $VaR_{\alpha \cdot 100\%}(L)$, where the number of observations is $n = 500$. The confidence level $\alpha$ is set to 99% according to regulations standards. When evaluating VaR using historical simulation with historical losses, the distribution function of the losses is discrete and Equation (2.6.2) applies.

With the Historical simulation procedure in place to evaluate VaR, the step to determine ES is the average of the losses greater than VaR, according to Equation (2.6.3).

When estimating the sensitivity of the portfolio with respect to 1 bp shift in the basis spread curve, 1 bp is added on the basis spread curve through the valuation. Then, PV01 was estimated as described in Section (2.6.5).

### 3.4 Replication differences

There are two main differences between the two systems in the valuation process namely, the number of tenors for each underlying risk factor and the coupon fixing. Recall from Table 2 that there are three interest curves used which both systems have in common. For the SEK zero swap curve, the front office system utilizes 27 tenors, i.e. 27 observed quotes from the market, while the risk system use 18 tenors. The front office system make use of 23 tenors for the Euro zero swap curve compared with 17 in the risk system. 9 and 7 tenors are used for the SEK/EUR basis spread curve in the front office and risk system respectively. For a detailed

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3In reality the front office system and risk system use the same Euro zero swap curve except for the number of tenors. Nonetheless the Euro zero swap curve is bootstrapped from the Euro swap curve due to the fact that the zero swap curve is not available in the data base.
Currency Basis Swap Valuation

overview over the underlying risk factors utilized in the two system see Appendix B.

When interpolating interest rates and further calculate discount factors there will be a discrepancy between the two systems since the number of tenors used by the two systems differ and the interest rates apply over different time periods (see Figure 7 and 8). An example illustrating this discrepancy is found in Figure 10.

Figure 10: Illustration of how the discounting works in the two systems. The dashed lines represent the discount factors used to value the cash flow. The different tenors used in the two systems when interpolating the same cash flow is also clear.

Recollect from Section 2.4 that a coupon is fixed one time period prior to the payment date. In the front office system this is three months, where the prevailing three month STIBOR and EURIBOR is used. Whereas the risk system does not fix the coupon at all. Hence both legs in the risk system will have one coupon less compared with the front office system.

Table 5: Interpolation example to illustrate the difference between the risk system and the front office system. Valuation date is is March 31, 2017.

<table>
<thead>
<tr>
<th>Tenor</th>
<th>Front office system Interest rate</th>
<th>Risk system Interest rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>9M</td>
<td>-0.29</td>
<td>-0.29</td>
</tr>
<tr>
<td></td>
<td>-0.33664</td>
<td>-0.337151</td>
</tr>
<tr>
<td>10M</td>
<td>-0.34</td>
<td>-</td>
</tr>
<tr>
<td>11M</td>
<td>-0.38</td>
<td>-</td>
</tr>
<tr>
<td>1Y</td>
<td>-0.43</td>
<td>-0.43</td>
</tr>
</tbody>
</table>
4 Result

The results are presented in two sections; in Section 4.1 the two systems valuations are compared with the true value over time; and in Section 4.2 the affect on risk measures are exhibited. In Appendix B, the risk factors used in the valuation are shown in detail. Since the front office system is where all trading and P&L analysis is conducted (see Section 1.2), this systems own portfolio value will be used as a benchmark when comparing the two replications. As a remainder, the front office system can not perform any advanced risk measure calculations, such as VaR and ES, a replication of this system was necessary to facilitate these calculations. In the result and discussion, the front office systems own portfolio value is referred to as the true value.

4.1 Portfolio development

To enable comparison between the two systems’ (RS and FOS) valuation and how this affect risk measures, it is of interest to see how the present value of the portfolio change over time. In Figure 11, the present value of the cross currency basis swap portfolio between September, 1st 2015 and March, 31st 2017 is presented together with the outstanding volume. The outstanding volume is the aggregated value of all nominal amounts in SEK.

![Figure 11: The present value of the cross currency basis swap portfolio over time together with the outstanding volume in SEK.](image)

Since the reporting currency is SEK, the outstanding volume in SEK is presented. One can observe that the value fluctuates between -1 100 M SEK and 4 000 M SEK over time. The fluctuations are a result of changes in underlying market conditions as well as new and matured trades. When depicting the foreign exchange rate together with the present value of the portfolio, one can observe how underlying risk factors affect the portfolio value. In Figure 12 this result is displayed, where it is clear that there is a high correlation between the exchange rate and and the portfolio value.
Currency Basis Swap Valuation

Figure 12: The present value of the cross currency basis swap portfolio over time together with the EUR/SEK exchange rate.

To determine if the replications are good enough one can compare the absolute deviation from the true value. In Figure 13 the absolute deviation for the two replications are presented together with the outstanding volume in SEK. One can observe that under periods of constant outstanding volume the deviation is small and the replications perform well. Whereas when the outstanding volume changes the deviation is higher.

Figure 13: The absolute deviation from the true present value of the portfolio.

The risk system generate a greater deviation than the front office system 75% of the observations. The larger deviations can be neglected when estimating risk exposure, due to the fact that historical simulation is used. Recall from Section 2.6.4, that in historical simulation, the portfolio is held static for each valuation date while the historical risk factor changes are used
as input to derive historical portfolio losses for the current portfolio. In Table 6, there is a sample of how the true value generated by the risk system deviates from the one generated by the front office system.

**Table 6:** Sample of one position in the portfolio displaying the true values from each system to illustrate the difference between the two systems. The valuation date is January, 31st 2017.

<table>
<thead>
<tr>
<th></th>
<th>True value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Front office system</td>
<td>25,836,373</td>
</tr>
<tr>
<td>Risk system</td>
<td>25,482,375</td>
</tr>
<tr>
<td><strong>Deviation</strong></td>
<td><strong>353,998</strong></td>
</tr>
</tbody>
</table>

### 4.2 Risk measure development

When calculating Value at Risk and Expected Shortfall historical simulation was used with a window of 500 days. Hence the period available for analysis is between October, 7th 2016 and March, 31st 2017. Since the replications perform good enough when the outstanding volume is constant, the extreme values can be neglected when calculating risk measures using historical simulation. Due to the fact that the portfolio is held constant during each simulation, i.e. during each simulation the portfolio is revalued using 500 historical risk factor observations.

Figure 14 and 15 depicts how VaR and ES change over time for each valuation date respectively. The outstanding volume is displayed together with the risk measures. One can see that when the outstanding volume increase VaR increase as well. A similar trend is observed for Expected Shortfall.

![Value at Risk of the portfolio over time](image)

**Figure 14:** Value at Risk over time.
The result of a parallel shift with 1 bp in the basis spread curve, i.e. the PV01 result is displayed in Figure 16 and Figure 17. It is displayed together with the portfolios outstanding volume in SEK. PV01 measures the change in portfolio value with respect to a parallel shift in the basis spread curve (see Equation (2.6.5)). If the curve is shifted up, i.e. the spread is increased, the value of the portfolio drops. This is a bad thing since the portfolio is worth less. If the spread narrows, the present value of the portfolio increase. A parallel shift in the basis spread curve would generate the value change depicted in Figure 16 and 17. Depending on what direction the change will have the opposite sign.

There is a similar trend for VaR and ES, that when that outstanding volume increases, the risk exposure increases. PV01 increases from around 19 M SEK to approximately 25 M SEK when the outstanding volume increases form 65 B SEK to 74 B SEK. This means that the percentage change in PV01 is greater than the percentage change in outstanding volume.
Figure 17: One base point added to the basis spread curve, i.e. PV01 of the RS.
5 Discussion

In this section a summary and a discussion about the discrepancy between the valuation in the two systems’ is given. Further, the affect on the value of the portfolio and risk measures is analysed.

5.1 Valuation discrepancies

From the results in Section 4.1 it is clear that the discrepancies between the two valuation methods affect the value of the portfolio. The differences between the two system are summarised in Section 3.4, where the two main inequalities are (i) the number of tenors used for each underlying risk factor; and (ii) how and when next coupon is fixed.

The foundation in the valuation process are the underlying risk factors found in Table 2. These are observed on the market hence they fluctuate daily (In Figure B.2 and B.3 in Appendix B, a snapshot of the underlying risk factors are presented). From Figure 11 one can observe how the present value of the portfolio changes over time. As mentioned the underlying risk factors together with new and matured trades drive the present value of the portfolio. When comparing Figure 11 and 12 it is clear that there is a strong correlation between the FX rate fluctuation and the value of the portfolio.

The first difference is the number of tenors used in the two systems’ differ, the impact on interpolated interest rates is significant, see Table 5. The difference seem to be small, but when the interest rate is converted to forward rates and discount factors, the effect becomes clear, especially when large cash flows are discounted, e.g. the nominal amount. The RS only use tenors up to 10 years whereas the FOS use tenors up to 30 years. When the risk system value cash flow greater than 10 years it extrapolates flat, i.e. using the same rate as the previous tenor (see Procedure 2). The number of positions in the portfolio with maturities exceeding 10 years are few, hence this affect is small. Another difference which has a great influence on the valuation is the discount factor O/N and T/N. Since the risk system only use O/N compared with the front office system, which use O/N and T/N (see Figure 10). The result of not using T/N in the RS, is for example that if a cash flow occur on a quoted tenor used by both systems (e.g. 3M) the present value will still differ.

Secondly, the risk system does not value the fixed coupons due to the fact that the fixing rates STIBOR and EURIBOR are not accounted for in the basic set-up for the instrument. The result of not fixing the next coupon is that one cash flow less on both legs compared with the front office system. A reason why there are less risk factor tenors in the risk system is because of computational time in the computer. Fewer tenors means less data that must be read and processed. Since the bank have other instruments, e.g. bonds and interest rate swaps (see Figure 2), which it must estimate risk for but also for, the process of estimating risk is time consuming. The risk exposure is calculated each day in the second line of defence (Section 1.2.1) and sent to the first line of defence, this should bee done early during the day to give the first line of defence to chance act on the information during the day.

When comparing the two valuations the risk system valuation generate a greater absolute deviation 3 out of 4 observations. Hence the replication of the FOS is closer the true value than the risk system replication. Another important point of interest is the bootstrapped Zero swap
EUR used in the front office system. The curve could not be perfectly replicated for tenors greater than 1 year which affect the valuation. This further explains why the replicated value of the front office system deviates from the true value.

These results are reasonable since the risk system originally deviates from the risk system, see Table 6.

5.2 Effect on risk measures

From Figure 14 it is clear that the risk system generate a greater VaR measure than the VaR generated from the front office system. This is also the case for Expected Shortfall which is displayed in Figure 15. Expected Shortfall generates a greater value than Value at Risk, which is no surprise since it answers the question if things goes bad, how bad can it get (see Section 2.6.3) while Value at Risk is how much we expect to lose or less with $\alpha \cdot 100\%$ certainty (see Section 2.6.2). What is interesting is that when the outstanding volume increases the risk exposure increases as well. This should be the case since with a larger portfolio the risk should be greater as well, at least when the same instruments are added. Since there is no risk measure generated by the front office system comparison with a true VaR and true ES is not possible. In other terms the replication of the risk system overestimates the risk exposure compared with the front office system.

The replicated value of the front office system gives a good estimate of how the true VaR and ES should be, it is reasonable that the risk system overestimate the measures since the replicated portfolio deviates more from the true value (see Figure 13). The fact that the risk system overestimate the risk exposure compared with front office is preferable, otherwise the bank might think it is possible to take on more risk, when it might not be the case.

The sensitivity measure PV01 for the two systems follow the same pattern and magnitude (see Figure 16 and 17). One can observe that as for the case with VaR and ES, when the outstanding volume increases the PV01 increases as well, and vice versa. As mentioned in Section 4.2 PV01 captures the change in the portfolios value with respect to a parallel shift in the basis spread curve with opposite sign as the parallel shifts. This mean that if the spread is added the value of the portfolio depreciate with PV01. PV01 increases when the outstanding volume increases due to the fact that there are more cash flows affected by the spread. The percentage change in PV01 is greater than the percentage change in outstanding volume. This mean that the larger the portfolio is the more sensitive it will be, in terms of PV01 (given that the duration\(^4\) of the portfolio is held constant).

PV01 is a good compliment to VaR and ES to capture the sensitivity of the portfolio given a constant stress, i.e. any change in outstanding volume or duration would be fully explained. Movements in VaR and ES, on the other hand, can also be affected by changes in the historical data sample, known as the ghost effect.

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\(^4\)Duration is a measure of the average life a bond/portfolio. Further it is also an approximation to the ratio of the proportional change in the bond price to the absolute change in its return [11].
6 Conclusion

In response to the Global Financial Crisis 2007-2009, regulators have imposed stricter risk management procedures (see Section 1.1). As a result, banks have established departments where risk is assessed and followed up. Banks have at least two systems, a front office system and a risk system, where financial instruments are valued. Further, the risk system is used to assess risk exposure. It is preferable that the risk system overestimates risk, but just to a certain degree.

In conclusion, the risk system and the front office system differ in their valuation of cross currency basis swaps, the number of tenors differ and when a coupon is fixed in the front office system it is not considered in the risk system. This affects the value of a portfolio containing cross currency basis swaps valued separately in the two systems. As displayed in Section 4 the risk system deviates from the true portfolio value. This impacts risk measures so that risk exposure is overestimated. As mentioned, it is preferable that the risk system overestimates the risk than the other way around. Both systems are equally sensitive to movements in the currency basis spread.

An extension to this thesis would be to validate the VaR and ES model, by implementing a back testing procedure where actual historical P&L of the portfolio is compared with loss forecasted by VaR and ES. Other VaR models, such as Delta-Gamma VaR, Delta VaR and Monte Carlo simulated VaR, could be investigated. Furthermore a variation of historical simulation where a period of historical stress from e.g. the Global Financial Crisis could be implemented to estimate the risk exposure under a period of great turmoil.
References


Currency Basis Swap Valuation


Appendices

A  Forward rate

Let $P(t; T)$ denote the price of a zero coupon bond with maturity $T$ valued at time $t$ where $t > t_0$ [12]. A zero coupon bond with maturity $T > 0$ is a contract that guarantees the holder a cash payment of one unit on the date $T$. Since a zero coupon bond can be viewed as a pure discount bond, let $P(t; T) = DF(t; T)$ [6]. The forward rate is derived from Equation (A.0.1)

$$DF(t; t_2) = DF(t; t_1) \cdot DF(t; t_1, t_2), \quad t \leq t_1 < t_2,$$

where $DF(t; t_1, t_2)$ denotes the forward discount factor from time $t_1$ to $t_2$, prevailing at time $t$. Think of Equation (A.0.1) as, that the value of a cash flow at time $t_2$ valued at time $t$ must be unique if we discount in one step or two steps. In Figure A.1 the forward discount factor $DF(t; t_1, t_2)$ is illustrated.

Figure A.1: Illustration of the forward discount factor, $DF(t; t_1, t_2)$, used to derive the forward rate, $F(t; t_1, t_2)$.

Denote the simply compounded forward rate as $F(t; t_1, t_2)$ associated with $DF(t; t_1, t_2)$ on the time interval $[t_1, t_2]$.

$$DF(t; t_1, t_2) = \frac{DF(t; t_2)}{DF(t; t_1)} = \frac{1}{1 + F(t; t_1, t_2) \cdot \delta(t_1, t_2)},$$

where $\delta(t_1, t_2)$ is the year fraction between $t_1$ and $t_2$. Rearranging Equation (A.0.2), we obtain the arbitrage free expression of the simply compounded forward rate, $F(t; t_1, t_2)$, prevailing at time $t$ for the time interval $[t_1, t_2]$ as [12]

$$F(t; t_1, t_2) = \frac{1}{\delta(t_1, t_2)} \left( \frac{1}{DF(t; t_1, t_2)} - 1 \right),$$

$$= \frac{DF(t; t_1) - DF(t; t_2)}{\delta(t_1, t_2) \cdot DF(t; t_2)},$$

The simply compounded spot rate, $L(t; T)$, is the constant rate an investment grows with on
Currency Basis Swap Valuation

the interval \([t, T]\), and is defined as \([3]\)

\[
L(t; T) = \frac{1 - DF(t; T)}{\delta(t, T) \cdot DF(t; T)},
\]

where \(\delta(t, T)\) is the year fraction between \(t\) and \(T\).

**B  Risk factors**

In Figure B.2 a snapshot of how the underlying risk factors used by the front office system on February 1st, 2017.

![Figure B.2](image)

**Figure B.2:** Overview of the underlying risk factors used in the valuation of the cross currency swap in the front office system. Valuation date is February 1st, 2017.

The underlying risk factors used by the risk system is presented in Figure B.3, one can observe that the number of tenors used are not as many as in the FOS.
Figure B.3: Underlying risk factor snapshot from used by the risk system, where the value date is February 1st, 2017.

The day count convention 30/360 is a variation of the 30E/360, where the main difference occurs when $d_i < 30$. Then the formula for 30/360 is

$$\text{number of years} = \frac{d_j - d_i + 30 \cdot (m_j - m_i) + 360 \cdot (y_j - y_i)}{360}.$$
Currency Basis Swap Valuation

Table B.1: Interest rate curves with their tenors and corresponding date base used in the front office system. O/N stands for over night, T/N is shorthand for tomorrow next, W, M, and Y denotes week, month, and year respectively.

<table>
<thead>
<tr>
<th>Tenor</th>
<th>Zero swap SEK Date base</th>
<th>Zero swap EUR Date base</th>
<th>Basis swap SEK/EUR Date base</th>
</tr>
</thead>
<tbody>
<tr>
<td>O/N</td>
<td>Actual/360</td>
<td>Actual/360</td>
<td></td>
</tr>
<tr>
<td>T/N</td>
<td>Actual/360</td>
<td>Actual/360</td>
<td></td>
</tr>
<tr>
<td>1W</td>
<td>Actual/360</td>
<td>Actual/360</td>
<td>Actual/360</td>
</tr>
<tr>
<td>2W</td>
<td>Actual/360</td>
<td>Actual/360</td>
<td>Actual/360</td>
</tr>
<tr>
<td>1M</td>
<td>Actual/360</td>
<td>Actual/360</td>
<td>Actual/360</td>
</tr>
<tr>
<td>2M</td>
<td>Actual/360</td>
<td>Actual/360</td>
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</tr>
<tr>
<td>3M</td>
<td>Actual/360</td>
<td>Actual/360</td>
<td>Actual/360</td>
</tr>
<tr>
<td>4M</td>
<td>Actual/360</td>
<td>Actual/360</td>
<td>Actual/360</td>
</tr>
<tr>
<td>5M</td>
<td>Actual/360</td>
<td>Actual/360</td>
<td>Actual/360</td>
</tr>
<tr>
<td>6M</td>
<td>Actual/360</td>
<td>Actual/360</td>
<td>Actual/360</td>
</tr>
<tr>
<td>7M</td>
<td>Actual/360</td>
<td>Actual/360</td>
<td>Actual/360</td>
</tr>
<tr>
<td>8M</td>
<td>Actual/360</td>
<td>Actual/360</td>
<td>Actual/360</td>
</tr>
<tr>
<td>9M</td>
<td>Actual/360</td>
<td>Actual/360</td>
<td>Actual/360</td>
</tr>
<tr>
<td>10M</td>
<td>Actual/360</td>
<td>Actual/360</td>
<td>Actual/360</td>
</tr>
<tr>
<td>11M</td>
<td>Actual/360</td>
<td>Actual/360</td>
<td>Actual/360</td>
</tr>
<tr>
<td>1Y</td>
<td>30E/360</td>
<td>Actual/365</td>
<td>Actual/360</td>
</tr>
<tr>
<td>2Y</td>
<td>30E/360</td>
<td>Actual/365</td>
<td>30/360</td>
</tr>
<tr>
<td>3Y</td>
<td>30E/360</td>
<td>Actual/365</td>
<td>30/360</td>
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<tr>
<td>4Y</td>
<td>30E/360</td>
<td>Actual/365</td>
<td>30/360</td>
</tr>
<tr>
<td>5Y</td>
<td>30E/360</td>
<td>Actual/365</td>
<td>30/360</td>
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<tr>
<td>6Y</td>
<td>30E/360</td>
<td>Actual/365</td>
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<td>7Y</td>
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<td>30/360</td>
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<td>30/360</td>
</tr>
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<td>12Y</td>
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<td>Actual/365</td>
<td>Actual/365</td>
</tr>
<tr>
<td>15Y</td>
<td>30E/360</td>
<td>Actual/365</td>
<td>30/360</td>
</tr>
<tr>
<td>20Y</td>
<td>30E/360</td>
<td>Actual/365</td>
<td>30/360</td>
</tr>
<tr>
<td>25Y</td>
<td>Actual/365</td>
<td>Actual/365</td>
<td>Actual/365</td>
</tr>
<tr>
<td>30Y</td>
<td>Actual/365</td>
<td>Actual/365</td>
<td>Actual/365</td>
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</table>
**Table B.2:** Interest rate curves with their tenors and corresponding date base used in the risk system. O/N stands for over night, T/N is shorthand for tomorrow next, W, M, and Y denotes week, month, and year respectively.

<table>
<thead>
<tr>
<th>Tenor</th>
<th>Zero swap SEK Date base</th>
<th>Zero swap EUR Date base</th>
<th>Basis swap SEK/EUR Date base</th>
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<td>Actual/360</td>
<td>Actual/360</td>
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</tr>
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<td>Actual/360</td>
<td></td>
</tr>
<tr>
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<td>Actual/360</td>
<td>Actual/360</td>
<td></td>
</tr>
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<td>Actual/360</td>
<td>Actual/360</td>
<td></td>
</tr>
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<td>Actual/360</td>
<td>Actual/360</td>
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<td>Actual/360</td>
<td>Actual/360</td>
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<td>Actual/360</td>
<td></td>
</tr>
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<tr>
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<td>Actual/365</td>
<td>30/360</td>
</tr>
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<td>30/360</td>
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