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Changes of scattering are observed as the grazing angle of incidence of an incoming beam increases and probes different depths in samples. A model has been developed to describe the observed intensity in grazing incidence small angle neutron scattering (GISANS) experiments. This includes the significant effects of instrument resolution, the sample transmission, which depends on both absorption and scattering, as well as the sample structure. The calculations are tested with self-organised structures of two colloidal samples with different size particles that were measured on two different instruments. The model allows calculations for various instruments with defined resolution, and can be used to design future improved experiments. The possibilities and limits of GISANS for different studies are discussed using the model calculations.

I. INTRODUCTION

The special role of interfaces is crucial in many applications of materials and in physical and chemical processes that include catalysis, lubrication and corrosion\textsuperscript{1,2}. Similarly in subjects such as biological and environmental science, interactions at boundaries are important\textsuperscript{3,4}. Surface and interface science has attracted considerable attention in recent decades and with increasing interest, many surface sensitive experimental methods have been developed such as scanning force microscopy\textsuperscript{5}, electron microscopy\textsuperscript{6}, fluorescence microscopy\textsuperscript{7} as well as X-ray photo-electron spectroscopy\textsuperscript{8}, surface plasmon spectroscopy\textsuperscript{9} and other surface sensitive X-ray\textsuperscript{10} and neutron scattering techniques\textsuperscript{11}. Understanding the advantages and limitations of each technique is important in order to obtain information about a specific structure.

Neutron and X-ray scattering experiments have several features in common and in contrast to microscopy that produces images by means of reconstruction with lenses or by scanning probes, they provide averaged statistical information from large areas or volumes of the sample\textsuperscript{12–14}. Neutron scattering benefits from contrast variation using isotopic labelling and has higher penetration through many materials compared to other techniques. This makes neutrons a powerful tool to determine structures at buried interfaces such as those between solids and liquids. Calculations are needed to interpret data quantitatively, the scale of structural features such as the spacing of particles is often readily identified from the position of peaks in momentum transfer space in scattering patterns. The degree of correlation between objects and details of structural arrangements usually relies on understanding the observed intensity distribution and relating this to model structures. The calculation of the scattering will also depend on the penetration of the radiation to different depths, and in this respect there are useful similarities of neutron studies with various near surface spectroscopy methods such as infra-red attenuated total reflection and fluorescence techniques as well as scattering experiments with visible both light and X-rays\textsuperscript{15–17}.

In recent decades, specular neutron reflection has developed rapidly as a valuable tool to study the structure of nanoscale films of hard and soft matter by providing information about the density profile in the direction of the interface normal. Additionally, the in-plane structure of the interface can be studied when illuminating the sample at low angles that are close to the critical angle, by observing the scattering away from the specular direction.

We have previously shown that colloidal particles can self-assemble and form large domains of oriented and regular structure near solid/liquid interfaces\textsuperscript{18}. These systems have potential applications as, for example, templates for patterning to create photonic devices\textsuperscript{19}. Studying self-organising systems can be used to understand particle/particle and particle/surface interactions, packing, ordering and dynamic behaviour of crystalline systems\textsuperscript{20,21}.

In this article we describe the use of grazing incidence neutron scattering (GISANS) in order to study structures of colloidal dispersions that self-assemble at interfaces, into crystalline structures with ordered and aligned region extending several micrometers from the solid/liquid interface. The penetration depth of the beam and scattering signal are modelled using known properties of the materials and with allowance for the effects of spread of wavelength and the angular divergence of the beam. This model will be compared to new experimental data for a colloidal dispersion of latex particles at a solid/liquid interface. Discussions on the accessibility and limitations of the GISANS technique to determine such structures will follow.

A. Theory

In GISANS experiments, the beam is tightly collimated so as to provide good angular resolution. In order to determine the structure of dispersions at solid/liquid interfaces, a sample geometry is chosen with the incident beam entering the edge face of a crystal substrate. It
then impinges on the solid/liquid interface at small angles close to the critical angle for total reflection where it is reflected and scattered. Figure 1 shows a GIMSANS experiment set up with vertical sample geometry. At incident angles, below the critical angle for total reflection, only an evanescent wave extends into the liquid phase and the penetration depth is determined by the interface ‘contrast’, that is the difference in refractive index for neutrons of the two bulk phases. Above the critical angle, a refracted beam enters the liquid and the penetration is limited generally both by the absorption of the beam by the sample and by the various scattering processes. There are a number of physical processes that can play important roles in these types of experiments that are less common in other scattering studies and the circumstances in which these can be important are mentioned briefly in the following paragraphs.

The decay of the amplitude of the wave due to absorption and scattering is characterised by an attenuation coefficient, (μ). The low angles and large sample lengths give rise to long paths through the materials and so the attenuation is often quite large. In the case of grazing incidence X-ray scattering (GISAXS), the absorption is normally dominant. For neutrons, the absorption is often smaller and the scattering (mostly incoherent scattering) is normally dominant. For neutrons, the absorption is often smaller and the scattering (mostly incoherent scattering) is normally dominant. For neutrons, the absorption is often smaller and the scattering (mostly incoherent scattering) is normally dominant.

The characteristic decay length of the wave in GISANS experiments is called the penetration depth and is defined as the distance at which the wave amplitude drops to (1/e) of the value at the interface. By careful choice of the materials forming the interface, the wavelength and the incident angle, it is possible to select the average depth of penetration of the beam in the sample and thus control the volume or mean depth of the sample that contributes to the measured scattering signal. This allows measurements of the structure at different depths. Knowledge of the wave amplitude is also required to calculate the scattering, which is obtained from the square of the sum of the amplitudes for the coherent scattering. The long path length of the beam in the sample has a further consequence when there is scattering, as the probability of multiple scattering can be significant.

In simple scattering theory, it is usually assumed that one can add the scattering from each nucleus in the sample in order to calculate the total scattering. This approach that treats the perturbation of the wave as weak is known as the Born approximation. This is also only valid if a neutron is scattered once passing through the sample. In the case of grazing incidence experiments, where neutrons are totally reflected from the surface, there is no longer weak interaction so the Born approximation is not adequate. The distorted wave Born approximation (DWBA) is an extension to the calculation that considers the perturbation of the incident wave and has been applied to calculate diffuse scattering, such as that from rough interfaces. Recently, packages for these calculations such as BornAgain and IsGISAXS have been developed in a way that scattering can be treated with the DWBA approach for various structures at different interfaces, particularly those of solids and air or vacuum. There are also algorithms treating samples as layers, such as the multi-slice DWBA approach that has been used to calculate X-ray scattering at grazing incidence. When measuring solid/air interfaces, it is often known where the structure has formed, so that other parameters are adjusted in order to measure the scattering pattern from that range of depth. In systems where the structure can form with an unknown separation from the interface, the experiment becomes more depth sensitive. Probing this spacing as well as the structure becomes a great challenge.

The penetration depth has been calculated in several different ways in the literature and various different expressions have been presented, which can be confusing both as regards the different notation that is used and the different approximations that are assumed. In the majority of the studies, the penetration depth can be expressed as:

\[ z_{1/e} = \frac{\lambda}{\sqrt{2\pi(l_i + l_f)}} \]  

(1)

with

\[ l_{i,f} = \left[ (\theta_i^2 - \theta_{i,f}^2) + \sqrt{(\theta_i^2 - \theta_{i,f}^2)^2 + \left(\frac{\lambda \mu}{2\pi}\right)^2} \right]^{1/2} \]  

(2)

where \( \lambda \) is the wavelength, \( \theta_i \) the incident beam angle, \( \theta_c \) the critical angle for the given interface and wavelength, and \( \mu \) is the attenuation coefficient. According to Snell’s law the critical angle is given by \( \cos \theta_c = n_2 / n_1 \), where \( n \) is the refractive index for neutrons of each material that is given by \( n = 1 - (\lambda^2 p / 2\pi) \). The equation can be simplified so that the scattering angle, \( \theta_f \), is equal to \( \theta_i \).

<table>
<thead>
<tr>
<th>Material</th>
<th>D_2O</th>
<th>H_2O</th>
<th>Silicon Polystyrene</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_{coh} ) (cm^{-1})</td>
<td>0.00</td>
<td>0.01</td>
<td>0.00</td>
</tr>
<tr>
<td>( \sigma_{inc} ) (cm^{-1})</td>
<td>0.51</td>
<td>0.00</td>
<td>0.11</td>
</tr>
<tr>
<td>( \sigma_{coh} ) (cm^{-1})</td>
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<td>5.37</td>
<td>0.00</td>
</tr>
<tr>
<td>( \sigma_{inc} ) (cm^{-1})</td>
<td>0.65</td>
<td>5.38</td>
<td>0.11</td>
</tr>
<tr>
<td>SLD /10^{-6} (A^{-2})</td>
<td>6.35</td>
<td>-0.56</td>
<td>2.07</td>
</tr>
</tbody>
</table>

TABLE I. Contributions of the various compounds of the sample to the absorption of neutrons.

<table>
<thead>
<tr>
<th>Material</th>
<th>D_2O</th>
<th>H_2O</th>
<th>Silicon Polystyrene</th>
</tr>
</thead>
<tbody>
<tr>
<td>SLD /10^{-6} (A^{-2})</td>
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<tr>
<td>SLD /10^{-6} (A^{-2})</td>
<td>-0.03</td>
<td>-0.03</td>
<td>-0.45</td>
</tr>
</tbody>
</table>

TABLE II. Contributions of the various compounds of the sample to the absorption of X-ray (data corresponds to Cu Kα E = ~8 keV).
FIG. 1. Schematic geometry of the GISANS experiment with vertical sample holder (D22 and NG3-SANS). The dispersion is sealed between the silicon and sapphire crystals. $\theta_i$ is the incident angle of the beam; $\theta_f$ is the angle of the scattered beam on the horizontal plane; $\alpha$ is the angle of scattered beam on the vertical plane. Scattered beam with $\theta_f = \theta_i$ is shown with red arrows and scattered beam with $\theta_f \neq \theta_i$ in green.

and then the penetration depth of the beam varies only with respect to the incident beam:

$$z_{1/e} = \sqrt{2/\lambda} \left( \frac{\lambda}{2\pi} \right)^{1/2} \left[ \sqrt{\left( \frac{\lambda^2}{2\pi} \right)^2 - \left( \frac{\lambda^2}{2\pi} - \frac{\lambda^2}{2\pi} \right)^2} \right]^{1/2}$$

For X-rays $\mu$ is usually taken as the absorption coefficient.

Some reports such as $^{38,39}$ describe the penetration depth only below the critical angle. Their expression gives a value for $z_{1/e}$ that is twice that calculated by equation 3 and would correspond to the decay of intensity rather than amplitude, which is not expressly stated in those papers.

The penetration depth has been introduced as a straightforward description of the depth sensitivity of the GISANS and GISAXS techniques in various papers. However as we will show, depending on the resolution of the instrument, the scattering signal from the sample beneath the interface can appear even when the mean angle of incidence is below the critical angle because of the spread of wavelength and the angular divergence. In systems where accurate investigation of structure at different depths in the sample is important, the penetration must be modelled more precisely.

The magnitude of the attenuation coefficient can be assessed from the known properties of materials for neutrons and X-rays:

$$\mu = \sigma_{tot}/V,$$

where $\sigma_{tot}$ is the total cross-section and is given as the sum over various wavelength dependent components of the cross-section for a volume, $V$:

$$\sigma_{tot} = \sigma_{abs} + \sigma_{coh} + \sigma_{inc}$$

and the three terms correspond to the absorption, the coherent scattering cross-section and the spin incoherent scattering cross-section respectively. For X-rays the last term is not relevant. For neutron experiments with a wide range of wavelengths such as with time-of-flight white beam instruments, would require the calculations to be made with the wavelength dependence of the absorption and scattering.

A further factor that needs general consideration is the effect of refraction on distorting the scattering that can be significant in the case of near small-angle surface scattering (as described by Hamilton et al$^{40}$). The refraction of the scattered neutrons, as the angle with the interface is different to that of the incident beam, is not simply an inversion of the effect that occurs with the incident beam. For the case of neutron instruments the wavelength spread is often significant and as the refraction
is wavelength dependent this will give rise to a spatial spread of the beam that is correlated with wavelength. These effects are particularly large near the critical angle.

II. SAMPLES AND MEASUREMENTS

A. Materials

Colloidal dispersions of charge-stabilised polystyrene latices, PS3 and PS11, dispersed in D2O have been used for the experiments in this study. The particles are spherical and monodisperse and have been characterised using several methods including zeta potential, scanning electron microscopy, atomic force microscopy, light scattering, small angle X-ray and neutron scattering, etc. The PS3 latex particles have 72 nm radius (polydispersity < 1%), about -30 mV surface potential and a density of 1.05 g cm$^{-3}$. The concentration of the sample was measured by drying to constant weight to be 9% wt.

The PS11 latex particles have 35 nm radius (polydispersity < 5%) and about -35 mV surface potential with a density of 1.05 g cm$^{-3}$. The concentration of the sample was 7% wt.

SANS and GISANS measurements show several orders of Bragg diffraction peaks suggesting that particles dispersed in water order to a depth of at least 1 µm into the bulk forming face centred cubic structure with (111) plane parallel to the solid surface. A study using a quartz crystal microbalance with dissipation (QCM-D) for various different particle and salt concentrations indicated that the particles are close to the interface but not bound at the solid surface [Hellsing et al. 2017, manuscript].

B. Measurements

In the present experiments, 50 × 50 × 10 mm sapphire (0001) and silicon (111) crystals were used as the solid surfaces. The crystals were cleaned with dilute detergents in H2O for a few minutes and then rinsed repeatedly with pure water. The cleaning was continued by sulfuric acid and water (as described in the supporting information) until the surfaces became hydrophilic with no measurable contact angle for water. A sample holder was used where the sample was contained by a 2 mm thick polytetrafluoroethylene (PTFE) gasket between the two crystals. One side was silicon which provides sufficient contrast with D2O for the GISANS experiment and the other side transparent sapphire crystal which allows checking the condition of the sample. The combined influence of solid/liquid and liquid/air interfaces with Poiseuille flow can influence the ordering of particles. Sample handling was performed as reported in our previous paper. It has been observed experimentally that gentle flow of the air/latex meniscus across the crystal surface can advance ordering of the particles. The sample was injected at ∼ 1 ml min$^{-1}$ (average Peclet number < $10^{-3}$) into the empty cell when the cell was rotated 45° with the filling port at the bottom.

The sample cell was placed with the solid-liquid interface vertical on the sample stage as shown in Figure 1. The z axis is defined perpendicular to the interface, with positive direction increasing away from the reflecting surface. Increase in depth into the sample is represented by larger negative z values. Following the conventional right hand rule in this geometry, the y axis is the direction of the beam and the x axis is in the vertical direction. Consequently for all incident angles, $Q_x$ will be mapped on to the vertical axis of the detector and $Q_z$ on the horizontal axis.

Measurements on PS3 latex were performed on the D22 instrument at the Institut Laue-Langevin (ILL), Grenoble, with a source to sample distance of 17.6 m and source aperture 20 mm (circular). With a wavelength 1.4 nm, sample to detector distance of 12 m and slit size 0.3 mm, the footprint of the beam was 38 mm×38 mm at the critical angle and sufficiently high resolution could be obtained for the experiment. The wavelength spread of the incident direct beam was measured on D22 using a small chopper and time-of-flight with the collimation used for the scattering experiment and is shown to have a triangular distribution with Δλ/λ (FWHM) = 0.097 (Figure S1 supplementary document). The measurements on PS11 that are shown were performed on the NG3 SANS instrument at the National Institute of Standards and Technology (NIST) Center for Neutron Research. The wavelength was set to 0.8 nm, source aperture 14.4 mm and source to sample distance of 15.7 m was chosen and sample to detector distance was set to 13.1 m. Wavelength spread was measured separately and shown to have a triangular distribution with FWHM = 0.124 (Figure S2 supplementary document).

III. RESULTS AND DISCUSSIONS

Figure 2 shows an example of the detector image for the PS11 latex dispersed in D2O measured on the NG3 instrument. The different scattering regions and the axes, which are similar to those for the measurements of PS3 on D22, are marked on the figure.

The conversion to $Q_x$ and $Q_z$ at each measured angle is made using the following expressions:

$$Q_x = \frac{2\pi}{\lambda} (\cos \theta_f \sin \alpha)$$

(6)

$$Q_y = \frac{2\pi}{\lambda} (\cos \theta_f \cos \alpha - \cos \theta_i)$$

(7)

$$Q_z = \frac{2\pi}{\lambda} (\sin \theta_i + \sin \theta_f)$$

(8)
FIG. 2. Detector image recorded for PS11 latex in D₂O at \( \theta_i/\theta_c = 1.4 \) indicating scattering regions (NG3-SANS, NIST), the critical angle for this sample, measured with 0.55 nm was 0.7°. Streaking of the scattering around the labeled Bragg peaks shows the effect of instrument resolution. This effect is more pronounced with NG3-SANS than D22 due to the slightly larger wavelength spread and better pixel resolution. The region around the transmitted beam shows the refraction and the scattered signal with clear diffraction peaks from a highly ordered sample structure but these are subject to heavy multiple scattering. There is also strong diffuse background scattering from the sample cell and sealing ring in this region. The intensities are normalized and shown on a log scale.

where \( \alpha \) is the angle between the projection of scattered beam on the vertical plane and the direct beam. It can be important to note that each detector image contains some three-dimensional information but as \( Q_y \) is small the data is essentially two-dimensional in \( (Q_x \) and \( Q_z \) ) space, however the signal with different \( Q_z \) values also contain information from different \( Q_x \). Figure 3 shows the scattering pattern at several selected angles close to the critical angle recorded for the PS3 latex at silicon/ D₂O interface at D22 instrument. Similar scattering patterns (different \( Q \) values) were recorded for PS11 with NG3 SANS.

Several orders of Bragg peaks which are equally spaced in \( Q_x \) are an indication of a highly ordered sample. The Bragg peak positions correspond to a d-spacing of 1.1 planes (with the uncertainty of 15%) due to the detector pixel resolution for a face centred cubic structure that are aligned parallel with the interface, as observed in a previous study\(^{18}\).

For a single wavelength and no angular dispersion, below the critical angle, the evanescent wave can penetrate a few nm into this sample, which is not sufficient to obtain a signal from the particles. However, scattering from the sample appears below the critical angle. This occurs because neutron scattering instruments do not work with a single wavelength but rather, for a 'monochromatic' instrument, a distribution around the set wavelength in order to provide sufficient intensity and there is also a range of incident angles. The shorter wavelengths will penetrate into the sample below the critical angle and can be scattered while the mean wavelength is still below the critical angle and would be totally reflected. As a result, the penetration depth and consequently the intensity close to the critical angle is smeared depending on the shape of angle and the wavelength distributions. The longer wavelength neutrons with lower energies are also, generally, absorbed more in the sample.

In order to determine the intensity that corresponds to scattering from a certain depth of the sample, our model assumes the scattered intensity to be a product of:

- the contrast in scattering length density at each depth of the sample \( (\Delta \rho) \)
- the intensity profile of the wave, either evanescent or transmitted, at that specific depth

Knowing the concentration, shape and the structure of particles, the fractional occupancy of the material in each distance from the interface can be calculated. Slicing the sample into planes parallel to the reflecting surface, scattering length density (SLD) of each plane is then calculated as \( \rho_z \) and SLD contrast as a function of depth, \( z \), is then:

\[
\Delta \rho(z) = \rho_{D_2O} - \rho(z)
\]  \( (9) \)

Figure 4 shows the SLD profile calculated for 9% wt PS3 (R = 72 nm) latex in D₂O, i.e. lattice parameter \( a = 400 \) nm which is separated with a gap, \( z_g = 50 \) nm, from the solid interface.

The intensity of the evanescent wave at the depth of \( 'z' \) as a function of wavelength and angle is calculated as:

\[
A(\lambda, \theta) = A_0(\lambda, \theta) \exp(-z_{1/\epsilon}(|z|)) \quad , \quad z < 0 \]  \( (10) \)

where \( z_{1/\epsilon} \) is the penetration depth of the evanescent wave, \( \theta \) is the angle in the distribution around \( \theta_c \) and \( A_0(\lambda, \theta) \) is the initial intensity of the wave. Note that the reason we have absolute value of \( z \) is that in this paper, \( z \) increases moving away from the reflecting interface towards the crystal and the depths of different parts of the sample are shown with negative values (Figure 4).

As described by equation 3, the penetration depth is a function of the incident angle and the critical angle, the attenuation coefficient and the wavelength of the beam. The different components that contribute to the attenuation coefficient are listed in Table I. Values for X-ray scattering experiment are shown as a comparison in Table II. For a neutron experiment, the attenuation coefficient of this system is 1.6 cm\(^{-1}\), whereas that of a X-ray experiment is more than two orders of magnitude larger, which cause the limited penetration depth of X-rays and increased surface sensitivity of GISAXS.
FIG. 3. Scattering intensities recorded close to the critical angle from 9% wt PS3 latex in D₂O against a silicon surface with D₂2. Density measurements showed that the particles were dispersed in 15:85 mixture of H₂O:D₂O. Using the scattering length density of this combination critical angle measured with 1.4 nm was 0.8°. The rectangles around the first order Bragg peaks represent the regions of interest chosen to estimate relative intensities plotted in Figure 5. The intensities are normalized to the measurement time and shown on a log_e scale. Vertical strip along the central part of Qₓ on the right side of the figure shows the integrated peaks as an example for θ_i/θ_c =1.07.

FIG. 4. SLD profile for a dispersion of particles with radius R, lattice parameter a, and a gap between the structure and interface labeled as z_g, in D₂O next to a silicon surface. Note that z_g is defined as the distance between the center of the first layer particles and the interface.

The model described in this paper accounts for the wavelength spread and the angular divergence of the beam. Both instruments used in this study have the expected triangular function for the distribution of wavelengths from a mechanical velocity selector⁴⁶,⁴⁷. For simplicity in the calculations, a triangular distribution is considered for angular dispersion as well; but these can be changed according to shape of resolution functions. By creating a triangular distribution around the wavelength and each incident angle (θ_i), a matrix for probability function P(λ, θ) is defined accordingly. The probability function consists of the probabilities for possible angles in columns and possible wavelength in rows.

Angular divergences of 0.065° for D₂2 and 0.063° for NG3-SANS were calculated, using the following equation:

\[ \Delta \theta = \arctan \left( \frac{w}{l} \right) \tag{11} \]

where w is the collimation width and l is sample to detector distance of the instrument.

Our code implementing the model creates a three dimensional matrix for the penetration depth in all incident angles and wavelengths within the triangular distribution around the central value of these quantities. Then the relative scattered intensity for each incident angle is calculated as:

\[ I_S(\theta_i) = \left[ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (A(\lambda, \theta) \Delta \rho(z)) P(\lambda, \theta) \, d\theta \, d\lambda \, dz \right]^2 \tag{12} \]

our calculated intensity comes from an addition of coherent scattering amplitudes. The calculation could readily be altered to allow for a finite longitudinal coherence length that would tend to flatten the decay of intensity with incident angle.

In a system where the structure forms with a gap from the interface that is unknown, understanding instruments artefacts becomes crucial to be able to determine from where in the sample the signal is being recorded. This is usually not a problem in GISAXS experiments, since
X-rays do not penetrate deep into the sample and scattering is not recorded from such a structure. The change of the intensity as a function of incident angle, estimated by the model, is shown in Figure 5. Intensities are calculated for particles forming cubic close packed structure with a (111) face parallel to the solid surface with lattice parameter, \( a = 400 \) nm. Different curves represent the expected scattering intensity when the closest layer of particles are found at distances \( z_g = 0, 100 \) and 140 nm away from the solid surface. In order to compare the model with experimental data, first order Bragg peaks on a line at constant \( Q_x \), were chosen as regions of interest. These regions of interest are shown as rectangles around the peaks in Figure 3. This choice for region of interest allows analysis of the signal dominated by the first layer of particles near the interface, and avoids some of the complication of the three dimensional nature of the structure.

Regions of interest were chosen to be in the same position relative to specular reflection peak and have the same number of detector pixels for each of the incident angles. The regions were chosen to include the whole peak, this allows the program to estimate the shape of the peak and make a model fit to the peak intensity. The Bragg peaks, for both negative and positive \( Q_x \) values, at each angle were fitted with both Gaussian and Lorentzian functions keeping a flat constant background. The mean values for the integral of the Gaussian fit between top and bottom regions of interest are plotted against the model in Figure 5. The estimated uncertainties are calculated from the difference between the Gaussian and Lorentzian integrals.

If no instrument effects are included, the intensity as a function of incident angle would have a sharp increase at the critical angle, while depending on the instrument resolution this change is smeared close to the critical angle. This smearing has not been modelled previously. Our model calculates this effect and demonstrates that the changes the observed intensity are very significant. It provides a good match to the experimental data. The model can be used for other instruments by simply changing the parameters such as dimensions of the collimation apertures and the wavelength spread that determine the instrument resolution. The model has also been tested for data measured on a different latex sample on the NG3-SANS instrument at NCNR that uses more relaxed wavelength resolution than D22. The peak intensities from this sample recorded with NG3-SANS also show results that follow the calculations of the model (Figure S3 supplementary document). Since the wavelength distribution has a larger FWHM for the NG3-SANS instrument, the smearing of the curves is more pronounced for these measurements.

Some other features that arise from resolution and instrument effects can be identified in the scattering. The refraction of the incident beam produces a streak between the observed direct beam on the detector and the ‘horizon’ that represents the interface between the silicon and the liquid in the sample. Similar streaking/distortion caused by refraction has been discussed previously\(^{40,48}\). This is more marked for measurements with a larger wavelength spread. This wavelength dependent angular dispersion can also modify the observed shape of the diffraction spots and cause them to be streaked. The alignment of these streaked artefacts will depend on both the spread of the incident beam and the angle of the scattered beam at the interface. These features are indicated in Figure 2.

As shown in Figure 5, curves with \( z_g = 0, 100 \) and 140 nm differ from each other significantly below the critical angle. In that region however, the intensity that would be recorded by the instrument become orders of magnitude weaker than that which can be measured readily. This means that even increasing measurement times to hundreds of hours would not provide sufficient data to distinguish these models as the background will restrict the precise determination of the signal. It is however useful to consider both what is determined from the present simple experiments and what might be measured for other samples and contrasts that are specifically designed to identify different depths of the sample.

One could design an experiment by adding strongly absorbing materials such as boron which can increase the attenuation coefficient of the sample two orders of magnitude. The motivation for such an experiment is that the separation of the intensity curves, corresponding to different distances from the interface, will extend to higher angles and may be measurable. According to our model, for this choice of materials, the relative intensity would decrease by two to three orders of magnitude and even with longer measurement time direct measuring of the separation of the curves for different \( z_g \) values will be challenging (see Figure S4 supplementary information). Proposed bright new sources may provide sufficient flux for such an experiment in the future.

**IV. CONCLUSIONS**

A model has been developed that explicitly takes into account not just the optical effects of the evanescent wave with absorption, but also including scattering that may be the dominant influence in many neutron experiments. The penetration depth is calculated accounting for instrument resolution and sample effects. The calculations can be used for any instrument when the resolution function and the structure of the sample are defined. These calculations highlight the difference between experiments with X-rays and with neutrons.

As a test system for our model, the self-assembly of latex particles close to solid/liquid interfaces was studied using GISANS experiments. The particles give rise to scattering from ordered arrays and the calculations show that this arises from depths up to several micrometres from the interface. The scattering shows a powder-like pattern without a preferred orientation when illuminated.
FIG. 5. Relative GISANS intensity calculated by the model for PS3 latex with lattice parameter, a, at different separation, \( z_g \) from the interface are shown with lines and the experimental data from two measurements of the same sample shown as scattered point with errorbars (top figure). The corresponding penetration depth at each angle is represented in the bottom plot with (dashed line) and without (solid line) wavelength spread.

with a beam in normal incidence \( (\theta_i = 90 \text{ deg.}) \) that passes through the full depth of 2 mm of the dispersion in the sample holder. These GISANS results indicate that the orientationally ordered structure of the particles is found to depths of several micrometres but they cannot directly determine the thickness of this relatively deep layer. The range of measurable intensity and the instrument resolution do not permit the determination of the structure in regions of just a few nanometres at the interface.

GISANS is a powerful technique to study the structures at solid/liquid interfaces. When illuminating the sample at grazing incidence, structures up to several micrometres can be studied (Figure S5 supplementary document). Only a small scattering volume is probed directly by the evanescent wave which is typically less than 10 nm from the interface. This gives weak scattering compared to that observed from transmitted wave which can provide signal from distances of 10 \( \mu \text{m} \) or more. Local measurement techniques such as QCM-D and ultrasonic thickness measurements can be more effective than neutron scattering for depth of a few nm. Highly ordered structures that provide strong diffraction peaks, particularly those that occur at high angles can be suitable for neutron experiments. Quantitative fitting of the measured intensity always depends on the effects that have been described. All factors that contribute to the attenuation coefficient, which for the case of neutrons may frequently be dominated by scattering rather than absorption, need to be included and this is an important difference to GISAXS.

There are few widely available programs that make quantitative interpretation of GISANS data and these have not been optimised for the type of experiment that has been described in this paper. Improved software is needed to analyse GISANS data for structures that are buried such as those near solid/liquid interfaces. The present work has provided a simple means to estimate the changes of GISANS intensity as the incident beam angle increases. The calculations have been tested against
measurements on model systems with two different sizes of particles that were measured on two different instruments. However the application of the model is not limited to these samples or instruments. Apart from the application in the interpretation of measured data, this approach could be incorporated readily into computer simulation software and used to predict results prior to an experiment, thus improving the design of the experiment by optimising the sample and instrument settings so as to observe specific features.

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