Implementation of a solenoidal magnetic field map in FLUKA

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Abstract

This report describes the creation of a solenoidal magnetic field in MATLAB and the process of integrating said field into the physics simulation program FLUKA as a magnetic field map. The implementation of an externally created field map in FLUKA was mainly accomplished through programming of the magfld user defined routine which is responsible for reading the map and for conveying the information to the simulation. In order to validate the magnetic field interpretation in FLUKA, charged pions were tracked through a solenoid magnet. The FLUKA simulation was compared with theoretical descriptions of the characteristics shown by a particle travelling inside a solenoid magnet in order to validate that the magnetic field behaved as intended. The method can be further developed to investigate the suitability of alternative pion collectors for producing well collimated, high intensity neutrino beams.
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1 Introduction

1.1 Background

At the European Spallation Source (ESS) the world’s most intense proton linear accelerator is being constructed and is projected to start operation by 2023 [1]. In parallel to the acceleration of protons, the linac could be used for delivering a neutrino super beam with tremendous intensity. Utilizing this opportunity is the ESS neutrino Super Beam (ESSνSB) project which will be dedicated to the measuring of CP violation in the leptonic sector.

The conventional method to produce a neutrino super beam is by employing a magnetic horn to cover the neutrino target where charged pions are produced by impinging proton bunches from the linear accelerator. A magnetic horn is of a toroidal structure with the magnetic field enclosed, which provides charge separation and focusing of the pions before they decay into neutrinos to form the neutrino Super Beam. To generate the magnetic field, a very high current needs to be passed through the walls of the structure, which can only be maintained for a very short time period due to the resulting heating. Making the horn superconducting would enable DC operation but is greatly complicated due to the energy deposition from the pions when passing through the structure's walls. The duration of the nominal pulse from the ESS linac is much longer than what the horn can handle without overheating and thus the pulse duration needs to be shortened. The compression of the proton pulse can be accomplished in an accumulator ring but to reach a satisfying efficiency in pulse compression the linac also has to be equipped to accelerate $^4\text{He}$ pulses to enable charge exchange injection to the ring. These required extensions to the ESS facility could be omitted if an alternative pion collector, that can handle the nominal proton pulses, was to be used instead of the horn. A superconducting solenoid could provide focusing, but does not provide charge separation, for the long pulses and is considered to be a key component for a viable option to the magnetic horn. An investigation of how different magnetic structures, such as the solenoid, can handle the neutrino Super Beam needs to be performed in, for example, the physics simulation program FLUKA where particle transport through magnetic fields can be simulated.

1.2 Objective

The objective of this project is to implement a solenoidal magnetic field in the simulations program FLUKA by appending an external field map. A magnetic field can be implemented in FLUKA by programming the user defined routines in order to read an external magnetic field map. By creating a solenoidal magnetic field in MATLAB using a finite difference method and established electromagnetic theory, the magnetic field map could be designed according to the simulation requirements and implemented in FLUKA as intended. Once the magnetic field had been implemented the outcome of the simulations was compared to relevant theory to assure its consistency.

1.3 Outline

In Section 2 we begin by formulating the theoretical description of the solenoid’s magnetic field through its axial and radial components. The equations that we attain are then to be further used to create a simplified model which is applicable in FLUKA. We also cover the characteristics shown by particles when moving in the solenoid’s field along with a description of how the matrix system is constructed for the later use in comparing simulation to theory. Further on, in Section 3 the focus lies in how the simulation is built by describing the construction of the field map in MATLAB as well as the necessary programming of the user defined routines. Shown in Section 4 is the results from comparing the simulated solenoid to the matrix model for situations where a point-to-parallel focusing is predicted by theoretical relations.
2 Solenoid

2.1 The Solenoidal Magnetic Field

A solenoid consists of a tightly wound coil which forms a long open cylinder that gives rise to a characteristic magnetic field when a current is led through its windings. The magnetic field considered in this report exists inside the cylinder and extends in the axial direction as fringe fields. Since the cylinder is open the particles does not have to pass through the structure’s wall in order to be affected by the magnetic field which increases the chances of the solenoid collector to be operated in a continuous DC state as a superconductor.

The shape of the solenoid provides compelling reason for using a cylindrical coordinate system when looking at its magnetic field. Its axial symmetry allows for a complete description of the magnetic field using the radial and axial components, $r$ and $z$ respectively, while the angular component $\phi$ cancels out. This means that it is necessary and sufficient to determine the axial magnetic field component $B_z$ and the radial component $B_r$. In Figure 1 a representation of the two magnetic field components are shown.

![Figure 1: Schematic showing the components $B_z$ and $B_r$ of a solenoidal magnetic field and the dimensional variables $L$, $z$ and $R$ used in eq. (1)](image)

The axial field can be determined to the first order by initially defining a solenoid of infinite length. In the first order approximation this solenoid would have a homogeneous magnetic field inside with a strength that is independent of the radial position. In the infinite length abstraction the axial field is given by $B_{z,inf} = \mu_0 n I$ where $\mu_0$ is the permeability of vacuum, $n$ the number of windings per unit length and $I$ the current.
The on-axis field for a solenoid of finite length can be expressed in terms of $B_{z,\text{inf}}$ by applying the Biot-Savart law or through use of a simplified expression. The expression reads [2]

$$B_z = \frac{B_{z,\text{inf}}}{2} \left[ \frac{(L/2) - z}{\sqrt{(z - (L/2))^2 + R^2}} + \frac{(L/2) + z}{\sqrt{(z + (L/2))^2 + R^2}} \right]$$

which is a first order description of the axial field. In eq. (1) we define the length $L$ of the solenoid as the distance between the entrance and exit of the solenoid with a radius $R$ as shown in Figure 1. The desired field strength $B_{z,\text{inf}}$ at the centre of the solenoid can then be chosen and one decides the width of the field, centred in the middle of the solenoid, by the variable $z$ which determines how far the axial field extends from the solenoid as is also shown in Figure 1. A visualization of the resulting axial field $B_z$ from eq. (2) is presented in Figure 2.

![Figure 2: The axial magnetic field component $B_z$ calculated using eq. (1) for a $L = 1$ m long solenoid of radius $R = 10$ cm and a field width $z = 600$ cm at a maximum field strength $B_{z,\text{inf}} = 9$ T, with corresponding hard edge approximate field $B_0$.](image)

The radial field can be found by following the reasoning given in [3] where the Laplace equation for a scalar potential is solved by applying a serial form of the solution, which provides a first order linear term for the radial component. This component is given as

$$B_r = \frac{-r}{2} B'_z(z)$$

(2)
where \( r \) is the radial distance from the solenoidal axis. It is evident in eq. (2) that the radial field depends on the change in the axial field, \( B'_z(z) \), and increases in strength further from the central axis. The strength of the radial magnetic field will therefore be at a maximum at the entrance and exit of the solenoid where the largest changes in \( B_z \) occur and will increase in magnitude further from the solenoid axis. Shown in Figure 3 is the radial field calculated at three different radial positions \( r \) for the \( B_z \) field generated by the solenoid defined in Figure 2.

![Figure 3: Radial magnetic field component \( B_r \) at radial positions \( r = 0, 5, 10 \text{ cm} \), calculated using eq. (2) for the solenoid defined in Figure 2.](image)

### 2.2 Characteristics of Particles Moving in a Solenoidal Magnetic Field

We now go onto looking at the effect of a solenoid on the trajectory of a charged pion. For a simpler analytical description of the solenoid we look at its hard edge equivalent. This means that the axial field is assumed to be constant inside the solenoid and zero outside. This, in turn, implies that the radial field is zero everywhere except at the entrance and exit. The equivalent hard edge axial field is calculated by integrating the axial field \( B_z \) of the solenoid over the total field width and then dividing the result by the length of the solenoid, giving the approximated field strength \( B_0 \) as shown in Figure 2.

A charged particle inside the solenoid field follows a helical path. As a charged particle approaches the entrance of the solenoid its transverse momentum will couple to the axial field and produce a rotation about the helical axis. When the charged particle is inside the solenoid, where
the axial field can be considered constant, the radius of the rotation will remain constant until the exit of the solenoid is reached.

2.2.1 Helical Radius

Consider a positively charged pion approaching the entrance of a solenoid. In the impulse approximation a particle will be given an increase in its azimuthal momentum, from the steep change in the radial field, when it enters the magnet [4]. Using this in addition to the paraxial approximation where we assume very small deviation in the particle’s direction and position relative to the solenoid’s symmetry axis. The transverse momentum of a particle, resulting from the kick, can be found by determining the angular and radial components of the momentum for a particle with total momentum $P$ [4]. The angular component is given by

$$P_{\phi_1} = -\frac{eB_0 r_1}{2}$$

where $r_1$ is the radial position of the particle upon entering the magnet and $B_0$ is the axial magnetic field. The radial momentum can be calculated as

$$P_{r_1} = P\theta_1$$

with a production angle $\theta_1$ which is the direction of the particle relative to the solenoid axis. With these components found, the total transverse momentum of the particle is given by

$$P_\perp = \sqrt{P_{\phi_1}^2 + P_{r_1}^2}$$

The radius $R_h$ of the helical path made by the particle when travelling inside the magnet is then given by

$$R_h = \frac{P_\perp}{eB_0}$$

2.2.2 Helical Rotation Angle and Direction

It is possible to calculate the rotation angle $\phi_h$ of a particle about the helical axis as it travels through the solenoidal magnet [4]. By determining the Larmor frequency

$$\omega = \frac{qB_0}{\gamma m}$$

where $\gamma$ is the Lorentz factor and $m$ is the rest mass of the particle, we can multiply the frequency by the time it takes for the particle to traverse the length of the magnet

$$t = \frac{L}{P_\perp/\gamma m}$$

so that the rotation angle can be found through

$$\phi_h = \omega t$$

thus providing the second characteristic of the helical trajectory. By changing the sign of the charge, $q = \pm e$ where $e$ is the elementary charge, the particle rotation changes direction in eq. (7), from moving in a counter-clockwise direction for a positive pion to a clockwise direction for a negatively charged pion when seen along the $z$ axis, as is predicted by the Lorentz force law.
2.3 Solenoid as a Focusing Magnet

A solenoid can be used as a focusing magnet. We will discuss two distinct cases. The first is when the source is placed outside of the solenoidal magnet and the second is when the source is placed inside. In both cases the goal is to produce a beam with near zero divergence relative to the solenoid axis when originating from a point source, denoted point-to-parallel focusing.

2.3.1 Source Outside

To produce a parallel beam from a point source located outside of the solenoid we begin by considering a charged particle leaving the solenoid at some radial position \( r_2 \) which will cross the magnetic axis at a distance \( d_2 \). When \( d_2 \) is positive we may treat the magnet as a thick focusing lens [4] and as the distance, \( d_2 \), increases, the angle, \( \theta_2 \), of which the particle diverges from the magnetic axis, decreases according to

\[
\tan \left( \frac{r_2}{d_2} \right) = \theta_2
\]

so that we may accomplish a point-to-parallel focusing as \( d_2 \to \infty \). We calculate where the particle crosses the magnetic axis through

\[
d_2 = \frac{2P}{cB_0 \tan(\beta)}
\]

where \( P \) is the particle’s momentum and \( \beta \) is given by

\[
\beta = \frac{L}{2\lambda} - \tan^{-1} \left( \frac{2\lambda}{d_1} \right)
\]

with \( d_1 \) being the distance between the source and the entrance of the solenoid. The Larmor wavelength \( \lambda \) is defined as [4]

\[
\lambda = \frac{P}{cB_0}
\]

For the case of point-to-parallel focusing, i.e. with \( d_2 \to \infty \), the focal length of the solenoid can be expressed as

\[
f = \frac{2\lambda}{\tan \left( \frac{r_2}{d_2} \right)}
\]

A particle with momentum \( P \) starting a distance \( f \) from the entrance of the magnet, will leave the magnet travelling in parallel to the symmetry axis of the solenoid, irrespective of its initial angle.
2.3.2 Source Inside

When placing the source inside of the solenoid, a parallel beam can be produced only for particles with certain discrete momenta given in [4] as

\[ P_n = \frac{P_0}{2n + 1}, \quad (n = 0, 1, 2, \ldots) \]  \hspace{1cm} (15)

where

\[ P_0 = \frac{eB_0L'}{\pi} \]  \hspace{1cm} (16)

with the length \( L' \) defined as the distance between the source and the exit of the solenoid. The point-to-parallel focusing for the particles is achieved irrespective of initial angle, as for the case with the source placed outside.

2.4 Matrix Description

To describe the propagation of a particle or particle beam through a solenoid it is possible to use the transfer-matrix method. In our case the matrix method will be used to set up a system for the propagation of particles from a point source, located outside or inside the solenoid, as they pass through the solenoid and continue into free space.

We are interested in finding the position and direction properties of the particles some distance after they leave the solenoid therefore a four dimensional column vector representing the source is created

\[ U_0 = \begin{pmatrix} x_0 \\ x'_0 \\ y_0 \\ y'_0 \end{pmatrix} \]  \hspace{1cm} (17)

corresponding to the particle’s position, \( x \) and \( y \), as well as the direction relative to the \( z \) axis, \( x'_0 \) and \( y'_0 \). To determine the coordinates of the particle after passing through a system, we seek to calculate

\[ U_1 = M U_0 \]  \hspace{1cm} (18)

were \( M \) is the matrix representation of the system and the resulting column vector \( U_1 \) contains necessary information for comparing the focusing effect when employing the transfer-matrix method to the results from simulations or numerical calculations.

The transfer matrix of a solenoid is based on the same hard-edge model as previous calculations. This means that it can be treated as a combination of the transfer matrices representing the entrance and exit regions of the magnet and the central uniform field [3]. In the hard edge model the entrance region will give an azimuthal momentum which in matrix form is expressed as

\[ M_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \frac{eB_0}{2P} & 0 \\ 0 & 0 & 1 & 0 \\ -\frac{eB_0}{2P} & 0 & 0 & 1 \end{pmatrix} \]  \hspace{1cm} (19)

under the condition that \( P \approx P_2 \) from applying the paraxial approximation. The particle will encounter a kick in the opposite direction at the exit which cancels out the azimuthal momentum given in \( M_1 \). The matrix form of the exit region will therefore be the same as in eq. (19) with the
kick given in the opposite direction as

\[
M_3 = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & -eB_0/2P & 0 \\
0 & 0 & 1 & 0 \\
eB_0/2P & 0 & 0 & 1
\end{pmatrix}
\]

(20)

To complete the matrix description we need to consider the central part of the solenoid containing the uniform magnetic field. The central matrix describes the helical trajectory inside the uniform field and is given as

\[
M_2 = \begin{pmatrix}
1 & \frac{L}{\theta} \sin(\theta) & 0 & \frac{L}{\theta} (1 - \cos(\theta)) \\
0 & \cos(\theta) & 0 & \sin(\theta) \\
0 & -\frac{L}{\theta} (1 - \cos(\theta)) & 1 & \frac{L}{\theta} \sin(\theta) \\
0 & -\sin(\theta) & 0 & \cos(\theta)
\end{pmatrix}
\]

(21)

with \( \theta = eB_0 L/P \) being the rotation angle completed by the particle inside the solenoid. We can combine the three matrices to form a complete description of the solenoid

\[
M_{\text{sol}} = M_3 M_2 M_1 = \begin{pmatrix}
C^2 & CS/\alpha & CS & S^2/\alpha \\
-CS\alpha & C^2 & -S^2/\alpha & CS \\
-CS & -S^2/\alpha & C^2 & CS\alpha \\
S^2\alpha & -CS & -CS\alpha & C^2
\end{pmatrix}
\]

(22)

by setting \( C = \cos(\theta/2) \), \( S = \sin(\theta/2) \), \( \theta = 2L/\alpha \) and \( \alpha = eB_0/2P \). When we are to use the matrix method for a source placed inside the solenoid the particle will be affected by the centre field over the distance between the source and the exit edge and will thereafter receive the kick from the exit fringe field. The effect of the half-solenoid can in this case be described by

\[
M_{\text{sol, inside}} = M_3 M_2 (L')
\]

(23)

where the distance between the source and the exit of the solenoid is given by \( L' \).

With the transfer matrix of the solenoid defined we can continue to build the system by adding the propagation matrices for when the particle is travelling through free space, a so called drift. The drift matrix is given by

\[
M_{\text{drift}}(s) = \begin{pmatrix}
1 & s & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & s \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

(24)

where the variable \( s \) indicates the length of the drift. The two systems can now be given as

\[
M_{\text{sys, outside}} = M_{\text{drift2}} M_{\text{sol}} M_{\text{drift1}}
\]

\[
M_{\text{sys, inside}} = M_{\text{drift2}} M_{\text{sol, inside}}
\]

(25)

(26)

depending on the source placement, and used in eq. (18) for calculating \( U_1 \).

3 Implementation of Magnetic Field Map

3.1 MATLAB

A magnetic field map is a discrete representation of a continuous magnetic field in a given region. In this case, the magnetic field of the solenoid is described by the two components \( B_z \) and \( B_r \). These components were calculated numerically in MATLAB, see appendix A for details.
The MATLAB program detailed in Appendix A begins with the declaration of the variables \( B_{z,\text{inf}}, L, R \) and \( z \), as previously defined, to be used in eq. (1) for calculating the \( B_z \) field. This will at the same time define a magnetic field region for the solenoid given by the two variables \( z \) and \( R \) corresponding to the total width and the radius in a cylindrical coordinate system. We then continue by setting the number of measuring points in the axial and radial direction, \( N_z \) and \( N_r \) respectively, so that a discretization of the magnetic field region can be made which is illustrated in Figure 4.

![Figure 4](image)

\begin{align*}
\text{Figure 4: Discretization of the magnetic field region showing measuring points in radial } N_r \text{ and axial direction } N_z. \text{ The axial points begin at position } -\frac{z}{2} \text{ and continue along the positive } z \text{ axis to } \frac{z}{2}.
\end{align*}

The position coordinates of the measuring points are saved in two vectors named \( ZB(k) \) and \( RB(i) \) which are used as reference points when extracting the magnetic field values inside the simulation program. To assign the magnetic field values to the axial measuring points the function MagFieldBZ, seen in Appendix B, is used to return the vector \( BZ(k) \) which is a discrete representation of eq. (1) with a vector length equal to \( N_z \). A derivative of this vector is then calculated and used to determine the discrete valued radial component of the magnetic field according to eq. (2). This results in the matrix \( BR(i,k) \) containing \( N_r \) rows and \( N_z \) columns due to the dependence on both radial and axial position in the radial magnetic field component \( B_r \).

The last step of the program is to save the data in a plain text format which is readable by the simulation program. Both the position vectors \( ZB(k) \) and \( RB(i) \) are put in separate files as single column vectors creating an ascending indexation.

### 3.2 FLUKA

FLUKA [5] [6] is a physics simulation program for particle transport and interactions with matter. It has a wide range of applications and can be further diversified through programming of the user interface routines if special requirements are necessary for the simulation. FLUKA can track particles in regions with a magnetic field. A homogeneous magnetic field can be defined directly in the program. In addition, a magnetic field map can be imported into the program. In this case, the user routine magfld.f is reprogrammed in order to instruct FLUKA on how to interpret the magnetic field map.

#### 3.2.1 magfld.f

The programming written in the magfld.f user routine is presented in Appendix C and is conducted to provide the simulation with the information contained within the field map. FLUKA receives
information from the magfld.f routine in terms of the total field strength \( B \) and its direction cosines in Cartesian coordinates. Therefore we must convert the information in the field map from a cylindrical to a Cartesian description within the routine before calculating the total field strength and its direction. The way to achieve this can be generalized to a few steps. Every time the particle moves a small incremental step within the magnetic field region the simulation will make a call to the user routine. During the first call to the routine it opens and reads the files containing the field map. Thereafter, each time the routine is called, it compares the current particle position given by FLUKA to the discrete positions stored in the vectors \( ZB(i) \) and \( RB(k) \) and extracts the magnetic field values from the closest point represented.

At initiation, a section of the routine that is to run only once following the first call from FLUKA is handled by the LFIRST command. This section therefore includes the opening and reading of the plain text files that were generated in the MATLAB program to form the field map. When the routine is called beyond the first time the process of opening and reading the files is effectively disregarded.

With the data retrieved the next step is to structure the part which is to run during each call to the routine. The position of the particle inside the simulation is given by its Cartesian coordinates, \( X \) and \( Y \), and must be related to the cylindrical system used in the field map. Note that the field is independent of the angle \( \tan^{-1}(Y/X) \). We introduce the radial position variable \( R = \sqrt{X^2 + Y^2} \) inside the routine. We now compare the radial distance given by \( R \) to the discrete positions stored inside the vector \( RB(i) \). This is done by stepping through the discrete values until the closest representation of \( R \) inside \( RB(i) \) is found. The corresponding procedure is performed for the axial component to determine the representation of the simulated axial position \( Z \) in the vector \( ZB(k) \). Once the discrete representation of the particle’s position has been identified in \( RB(i) \) and \( ZB(k) \) the indices \( i \) and \( k \) are used to access the magnetic field values stored in the \( BZ(k) \) and \( BR(i,k) \) vectors, as described below.
To provide the simulation with the information of the axial magnetic field contained in the discrete $BZ(k)$ vector we note that the magnitude is only dependent on the axial position. The direction will be along the positive z-axis and we obtain the value from $BZ(k)$ by inserting the $k$ index which was determined in the previous procedure.

In the case of the radial component we extract the magnetic field value $B_r$ from $BR(i,k)$ by inserting the found $i$ and $k$ indices. This provides a radial field magnitude with a direction pointing radially towards or away from the symmetry axis of the solenoid for the entrance and exit fringe fields respectively, see Figure 3. In the routine we resolve the radial component into its $x$- and $y$-components by using the particle’s current position and the conversion properties between a cylindrical and Cartesian system

$$BRX = BR(i,k) \frac{X}{R} \quad (27)$$
$$BRY = BR(i,k) \frac{Y}{R} \quad (28)$$

where the variables $X$, $Y$ and $R$ are the position coordinates of the particle given by FLUKA. With the radial component resolved we obtain the magnetic field in terms of $x$- and $y$-components defined as $BRX$ and $BRY$ in the magfld.f routine. The process is illustrated in Figure 5.

![Figure 5: Vector resolve of the radial field vector $BR(i,k)$ into the Cartesian components $BRX$ and $BRY$ in the magfld.f routine.](image)

Finally, the total field intensity is calculated from the three resulting field components through $B = \sqrt{BZ(k)^2 + BRX^2 + BRY^2}$ which is then used to divide each of the components to effectively normalize and provide the direction cosines for the magnetic field in three dimensions which is compulsory to the routine. An example of the total magnetic field vector and its components for an arbitrary point is presented in Figure 6.
Figure 6: The magnetic field for an arbitrary point inside the simulation. The three components $BRX$, $BRY$ and $BZ(k)$ are needed to give the direction and magnitude for the total magnetic field vector $B$.

### 3.3 The Graphical Interface Flair

The advanced interface program for FLUKA called flair [7] was used to construct the project and to run the simulations. This interface provides tools for handling and visualizing the many input options and provides an overview of the complete set-up. In order to run a simulation with the magnetic field map there are settings that need to be present and set in a specific way. A magnetic field region has to be defined that has the same width and radius as the magnetic field generated from the MATLAB code to form the field map. In this case a circular cylinder was used and positioned to coincide with the coordinates of the $RB(i)$ and $ZB(k)$ position vectors. The cylindrical region is then assigned to consist of a vacuum containing a magnetic field. To activate the call to the user routine magfld.f, the MGNFIELD section needs to be present and its magnetic field parameters set to zero. The MGNFIELD section also provides the possibility to dictate the step length of the particle transport through the magnetic field region. It can therefore be used to decide the number of times the program simulates the particle interaction with the magnetic field and the number of times the user routine is called. It therefore controls the speed and precision of the simulation.
4 Comparison of Simulation and Analytical Description

We are interested in the focusing effect of the solenoid on charged particles and the agreement between analytical descriptions to the simulation. The matrix description provides an opportunity to compare the propagation of particles throughout a system corresponding to the simulation whilst we use the analytical expressions for helical radius and rotation angle to compare parameters inside the magnet.

Throughout this section the initial direction of the particles when leaving the source are given by their direction cosines. The direction cosines are defined as the cosines of the angles between a vector and the three Cartesian coordinate axes. The divergence in direction relative to the z axis has been kept low as to fit the paraxial approximation so that the direction of the particles corresponding to $dx/dz$, $dy/dz$ or $dr/dz$ is interchangeable with the corresponding.

4.1 Simulation and Matrix Comparison with Source Outside

The parameters for the comparison consists of those that define the solenoid and those that governs the particles. We set the solenoid to a fixed geometry, $L = 1\, \text{m}$, $R = 0.1\, \text{m}$, $z = 6\, \text{m}$, with three different field strengths, $B_{z,\text{inf}} = 7, 9, 11\, \text{T}$ as defined in eq. (1). The number of sample points are set to $N_z = 6001$ and $N_r = 301$.

We then consider eqs. (11)-(13) to determine the momentum of the particles in each of the three cases, meaning that we may treat the magnet as a thick focusing lens when $d_2$ is positive and approaches infinity or is very large. This is accomplished by finding the maximum value of $d_2$ as a function of momentum, $P$, and the distance between source and entrance, $d_1$. From the maximum point of $d_2$ we found that the momentum should be set to $P = 1.360, 1.305, 1.950\, \text{GeV/c}$ for the respective $B_{z,\text{inf}} = 7, 9, 11\, \text{T}$ solenoids. For the given values of the momentum, the focal length in eq. (14) is equal to the value of $d_1$.

4.1.1 Varying the Source Placement

We proceed with the comparison by firstly looking at the focusing effect on the outgoing direction of the particles while varying the source’s placement. In doing so, we should see an improved focusing when the placement of the source is located close to the focal length. The corresponding matrix model is set up by varying the first drift length, $s_1$, so that it equals the distance between the current source position and the entrance of the solenoid. The measurements for the direction of the particles after they passed through the solenoid is made at a distance $s_2 = 2.5\, \text{m}$ behind the exit of the solenoid in both simulation and matrix model. The particles are given their respective momentum for the three cases and the initial directions are set to $x_0 = 0.01$ and $y_0 = 0.01$ giving a radial direction $r'_0 = \sqrt{(x'_0)^2 + (y'_0)^2}$. From eq. (14) the focal lengths, $f$, are found to be 1.22, 0.66, 1.05 m for the $B_{z,\text{inf}} = 7, 9, 11\, \text{T}$ case respectively. In Figure 7 the direction of the particle is presented in its radial representation with $r'_1 = \sqrt{(x'_1)^2 + (y'_1)^2}$ for when the source is placed at -250 cm to when the source is placed at -50 cm, corresponding to the entrance of the solenoid.
Figure 7: Direction of the particles after passing through the solenoid, $r'_1$ is shown as a function of source position where -50 cm corresponds to the entrance of the solenoid. Numerical noise is due to finite step length in the simulation. a) $P = 1.360$ GeV/c, $B_{z,inf} = 7$ T. b) $P = 1.305$ GeV/c, $B_{z,inf} = 9$ T. c) $P = 1.950$ GeV/c, $B_{z,inf} = 11$ T.

The matrix model predicts a perfect focusing when the source is placed at the focal length that was found through analytical calculations. We see that the simulation behaves similarly but has a few centimetres longer focal length and does not drop entirely to zero.

4.1.2 Varying the Initial Direction

We now seek to investigate how the focusing of the solenoid affects the particles when the initial direction $r'_0$ is increased while keeping the source fixed at three different positions. Since the focusing effect should be irrelevant of the initial direction of the particle, we expect that $r'_1/r'_0$ remains constant in the matrix case. The three source placements are chosen from Figure 7-b) at -180, -140 and -100 cm with $B_{z,inf} = 9$ T and the momentum $P = 1.360$ GeV/c. At -180 cm the simulation proves $r'_1$ to be less than what the matrix model predicts, at -100 cm we see the opposite. Near -140 cm the matrix model and simulation predict very close results. Presented in Figure 8 is the results from the three different source placements.
Figure 8: The focusing effect of the solenoid with the direction of the particle upon leaving the solenoid divided by the initial direction $r'_1/r'_0$ is shown for an increasing initial direction $r'_0$. a) Source positioned at -100 cm. b) Source positioned at -140 cm. c) Source positioned at -180 cm.

We see that the increasing initial direction does not affect the efficiency of the solenoid. The difference between simulation and matrix model from the three source placements remain constant for an increasing $r'_0$. 
4.1.3 Characteristics Inside the Solenoid

The particle will follow a helical trajectory inside the magnet, giving rise to a circular motion with a well-defined radius in the x-y plane. The radius and rotation angle are extracted from the simulation through evaluation of the position coordinates of the particle inside the solenoid. In Table 1, we look at the difference between the helical radius and rotation angle extracted from the FLUKA simulation and that of the expected radius $R_h$ in eq. (6) and the total rotation angle $\phi_h$ in eq. (9) for three different conditions. The chosen conditions are when the source is placed at -180, -140 and -100 cm, $P = 1.360 \text{ GeV}/c$ and $B_{z,\text{inf}} = 9 \text{ T}$, with the initial directions $x'_0 = y'_0 = 0.038$. Note that a change in the source position results in a change in the radial coordinate $r$ at the entrance. This in turn affects the helical radius. Compared to the radius measured from simulation data, the analytical calculations predict a larger radius in the two latter cases and a smaller radius in the first. The rotational angle from analytical calculations are larger in all three cases.

Table 1: Difference in helical radius and rotation angle between simulation and analytical calculations for three source placements.

<table>
<thead>
<tr>
<th>Source Position [cm]</th>
<th>$R_h$ [cm]</th>
<th>$R_{\text{sim}}$ [cm]</th>
<th>$\phi_h$ [rad]</th>
<th>$\phi_{\text{sim}}$ [rad]</th>
</tr>
</thead>
<tbody>
<tr>
<td>-100</td>
<td>3.0</td>
<td>2.6</td>
<td>1.99</td>
<td>1.69</td>
</tr>
<tr>
<td>-140</td>
<td>3.6</td>
<td>3.5</td>
<td>1.99</td>
<td>1.66</td>
</tr>
<tr>
<td>-180</td>
<td>4.4</td>
<td>4.5</td>
<td>1.99</td>
<td>1.63</td>
</tr>
</tbody>
</table>

4.2 Simulation and Matrix Comparison with Source Inside

With the source placed inside the solenoid, at $z = 0$, the point-to-parallel focusing should be accomplished for certain discrete values of the momentum given by eq. (15) irrespective the initial direction of the particles. We look at the comparison between simulation and matrix description for a solenoid with $L = 2 \text{ m}$, $R = 0.20 \text{ m}$ and $z = 6 \text{ m}$ at three initial field strengths $B_{z,\text{inf}} = 8, 12, 16 \text{ T}$ with the number of measuring points set to $N_z = 6001$ and $N_r = 301$. The particles, after passing through the solenoid, are evaluated 2 m from the exit of the solenoid, corresponding to an exit drift length $s_2 = 2 \text{ m}$ in the matrix description.

4.2.1 Varying the Particle’s Momentum

The initial direction is set to $x'_0 = y'_0 = 0.05 \text{ rad}$ and we vary the momentum, $P$, of the particles between 0.1 and 1.1 GeV/c. We formulate the matrix system in eq. (26) using $L' = 1 \text{ m}$ in the $M_{\text{sol, inside}}$ matrix from eq. (23). In Figure 9 the radial direction of the particles is shown for an increasing momentum with the vertical lines corresponding to the $P_n$ values from eq. (15).
Figure 9: Direction of the particles after leaving the solenoid, $r'_1$, with the source placed in the centre of the solenoid shown as a function of increasing momentum. The vertical lines indicate the $P_n$ values calculated in eq. (15). a) $B_{z,inf} = 8 \text{T}$. b) $B_{z,inf} = 12 \text{T}$. c) $B_{z,inf} = 16 \text{T}$.

The matrix model's prediction for point-to-parallel focusing and the analytical calculations for the $P_n$ values coincides accurately. The simulation shows that effective focusing occurs when the momentum of the particle is a bit smaller than what is predicted in the matrix model, meaning that the magnet has a weaker focusing effect.

4.2.2 Varying the Initial Direction

We continue by looking at the difference in outgoing direction of the particles when the momentum $P$ is set to three different values, $P = 0.300, 0.365, 0.400 \text{ GeV/c}$, for the $B_{z,inf} = 12 \text{T}$ solenoid and increase the initial direction of the particles. These values for the momentum corresponds to when there is a large difference, and a very small difference, between simulation and matrix model in Figure 9-b). We should see a constant focusing efficiency when plotting $r'_1/r'_0$, as in the case with the source placed outside of the solenoid. Figure 10 shows the radial focusing, $r'_1/r'_0$, for when initial directions, $x'_0$ and $y'_0$, are set between 0.001 and 0.100 with $x'_0 = y'_0$. 

20
Figure 10: The focusing effect of the solenoid with the direction of the particle upon leaving the solenoid divided by the initial direction $r'/r'_0$ is presented for an increasing initial direction $r'_0$. a) $P = 0.3$ GeV/c. b) $P = 0.365$ GeV/c. c) $P = 0.4$ GeV/c.

The difference between simulation and matrix model from the three momenta cases is close to constant for an increasing $r'_0$. 

\[ r'/r'_0 \]
5 Discussion and Conclusion

It has been shown that a magnetic field map created in MATLAB can be successfully integrated in FLUKA by programming of the user-defined routines. The process is shown step-by-step and provides sufficient information to form a basis for more advanced research. Since the simulation program FLUKA lacks the built-in tools to generate certain magnetic fields it is essential that an external field map can be introduced.

5.1 Analysis of the Results

We have seen from the comparison that the simulation match the matrix model to a satisfactory level. We showed in Figure 7 that the simulation provides a weaker focusing than the hard-edge model. The simulation showed that the best focusing would occur when the source was placed a few centimetres further away from the entrance of the solenoid, than that predicted from the hard-edge model, but the final divergence never reaches zero. When we lower the hard-edge field value $B_0$ is lowered in the analytical calculations, the waists of the two curves coincide. The results in Figure 8 show that the focusing is independent of the initial angle $r_0'$ as expected and that the difference, between matrix and simulation, again can be attributed to the weaker focusing power of the realistic field in the simulation.

In Figure 9 we observed that the point-to-parallel focusing occur at discrete momenta, as predicted. In the matrix model, these discrete momenta are higher than what the simulation showed. By lowering the hard-edge value, $B_0$, the matrix model would provide point-to-parallel focusing at lower momenta and agree accurately with the simulation. From the variation of initial direction in Figure 10, we see the same behaviour as for the case when the source is placed outside.

The matrix model can be considered unrealistic when compared to the simulation. It can however be shown that by lowering the $B_0$ value in the analytical calculations, the simulation and matrix model would agree more accurately. The simulation is more credible since we include the fringe fields, a distributed azimuthal kick and a non-homogeneous axial field inside the solenoid. We do not expect the matrix model and simulation to conform perfectly since FLUKA is more realistic and does not rely on approximations. However, the agreement is enough for us to conclude that the magnetic field map has been successfully implemented in FLUKA.

5.2 Outlook

As a result of this project, we have provided a framework for detailed studies of the focusing capabilities of a solenoid collector. In addition, it offers the opportunity to further develop the simulations to involve combinations of magnets and more complex fields, such as a dipole field which would provide a stage of charge separation.

In the upcoming masters thesis, a complete investigation of a magnetic field map provided by researchers from CERN will be performed by using the same principles covered here. This includes incorporating an external field map and, in addition, an in-depth analysis of beam characteristics and focusing. By building a basis for an advanced simulation environment, this project concludes the first step towards a general investigation of a new pion collector.
References


6 Appendix

6.1 Appendix A

%% Solenoidal Magnetic Field Map %

% The dimension of the solenoid is given by the parameters % r and L, where r denotes the radius and L is the total length % of the solenoid itself. In order to calculate the magnetic field % the parameter B_zero needs to be set and corresponds to the desired % magnitude of the magnetic field vector at the center of the solenoid. % The region in which the magnetic field is calculated is given % by the parameter r which gives the maximum radial distance from % the solenoidal centered z-axis. The width of the magnetic field % along the z-axis is given by the parameter z and is centered % at z = 0.

clear all

%% Dimensions & field intensity %%

r = 10; % Radius of solenoid and magnetic field map [cm] z = 600; % Width of magnetic field centered at z = 0 [cm] L = 100; % Length of solenoid magnet to be used in calculating the magnetic field [cm] B_zero = 9; % Solenoid initial magnetic field magnitude [T]

%% Discretization %%
Nr = 301; % Number of measuring points on radial axis [#]
Nz = 6001; % Number of measuring points on z-axis [#]

dr = r/(Nr-1); % Step length in radial dimension [cm]
RB = 0:dr:r; % Measuring points position in radial direction

dz = z/(Nz-1); % Step length in z direction [cm]
ZB = -z/2:dz:z/2; % Measuring points position in z direction

%% Magnetic field in z direction, BZ %%

BZ = MagFieldBz(L, r, z, Nz, B_zero); % Returns the magnetic field strength for each measuring point in z direction

%% Derivative of BZ %%
% Calculates the changes between points in BZ using a centred FDM approximation in order to further calculate the radial component of the magnetic field

k = 2;
for k = 2:1:Nz-1
    BZ_prime(1, k) = (BZ(1, k+1)-BZ(1, k-1))/(2*dz);
k = k+1;
end
BZ_prime(1, Nz) = 0;

%% Magnetic field in radial direction, BR %%
% Calculates the radial component of the magnetic field using the previously determined BZ_prime vector

BR = zeros(Nr, Nz); i = 1;
for i = 1:Nr;
    BR(i, 1:Nz) = (-RB(i))/2*BZ_prime(1, 1:Nz);
i = i+1;
end

%% Write data to file %%
% Writes the calculated magnetic field vectors BR, BZ, and the measuring point coordinates to files and places them in the folder shared between the virtual machine running FLUKA and the current MATLAB workstation

filename_br = 'C:\Users\Patrik\Desktop\FlukaShared\INDATA\BRDATA.txt';
filename_bz = 'C:\Users\Patrik\Desktop\FlukaShared\INDATA\BZDATA.txt';
filename_rb = 'C:\Users\Patrik\Desktop\FlukaShared\INDATA\RBDATA.txt';
filename_zb = 'C:\Users\Patrik\Desktop\FlukaShared\INDATA\ZBDATA.txt';

fid_br = fopen(filename_br, 'w');
fid_bz = fopen(filename_bz, 'w');
fid_rb = fopen(filename_rb, 'w');
fid_zb = fopen(filename_zb, 'w');

formatSpec = '%+.6f %
'; fprintf(fid_br, formatSpec, BR);
fprintf(fid_bz, formatSpec, BZ);
fprintf(fid_rb, formatSpec, RB);
fprintf(fid_zb, formatSpec, ZB);
fclose(fid_br);
fclose(fid_bz);
fclose(fid_rb);
fclose(fid_zb);

6.2 Appendix B

function [BZ] = MagFieldBz(L, r, z, Nz, B_zero)
% Returns a vector of length Nz with the magnetic field values of Bz on the solenoid-centered z-axis. The field is given in a region which extends ±z/2 from z = 0, for a solenoid of length L, radius r and magnetic field intensity B_zero.
% L - Length of solenoid in cm
% r - Radius of solenoid in cm
% z - Total width of field in cm
% Nz - Number of measuring points
% B_zero - Field intensity in Tesla

dz = z/(Nz-1);
BZ = zeros(1, Nz);
i = 1;
for z = -z/2:dz:z/2
BZ(1, i) = 1/2*B_zero*(((z+L/2)/(sqrt((z+L/2)^2+r^2))) - ((z-L/2)/(sqrt((z-L/2)^2+r^2))));
i = i+1;
end
end

6.3 Appendix C

*==magfld================================================================================================================================================*
* SUBROUTINE MAGFLD ( X, Y, Z, BTX, BTY, BTZ, B, NREG, IDISC ) INCLUDE 'DBLPRC'
INCLUDE 'DIMPAR'
INCLUDE 'IOUNIT'
*-------------------------------------------------------------------------------------------------------------------------------------*
* Copyright (C) 1988-2010 by Alberto Fasso` & Alfredo Ferrari
* All Rights Reserved.
* Created in 1988 by Alberto Fasso`
* Last change on 06-Nov-10 by Alfredo Ferrari
* Input variables:
* x, y, z - current position
* nreg - current region
* Output variables:
* btx, bty, btz - cosines of the magn. field vector
*
* B = magnetic field intensity (Tesla) *
* idisc = set to 1 if the particle has to be discarded *
* *
*----------------------------------------------------------------------*
*----------------------------------------------------------------------*
* Parameters Section *
PARAMETER (NR = 301)
PARAMETER (NZ = 6001)
DIMENSION BR(NR, NZ)
DIMENSION RB(NR)
DIMENSION BZ(NZ)
DIMENSION ZB(NZ)
INTEGER i, k
REAL dr, dz, BRX, BRY
*----------------------------------------------------------------------*
* LFIRST Section *
* Reads the magnetic field map and saves values in respective vectors *
* RB(i) - Position vector in radial direction *
* BR(i, k) - Magnetic field magnitude in radial direction *
* ZB(k) - Position vector in axial direction *
* BZ(k) - Magnetic field magnitude in axial direction *
LOGICAL LFIRST
SAVE LFIRST
DATA LFIRST / .TRUE. /
IF (LFIRST) THEN
OPEN(UNIT = 20, FILE = '/home/flupix/Desktop/FlukaShared/OUTDATA &FLUKAOUTPUT.txt')
OPEN(UNIT = 40, FILE = '/home/flupix/Desktop/FlukaShared/INDATA &RBDATA.txt')
OPEN(UNIT = 41, FILE = '/home/flupix/Desktop/FlukaShared/INDATA &ZBDATA.txt')
OPEN(UNIT = 50, FILE = '/home/flupix/Desktop/FlukaShared/INDATA &BRDATA.txt')
OPEN(UNIT = 51, FILE = '/home/flupix/Desktop/FlukaShared/INDATA &BZDATA.txt')
DO i = 1, NR
READ(40, *) RB(i)
END DO
DO k = 1, NZ
DO i = 1, NR
READ(50, *) BR(i, k)
END DO
END DO
DO k = 1, NZ
READ(41, *) ZB(k)
READ(51, *) BZ(k)
END DO
CLOSE(40 41 50 51)
LFIRST = .FALSE.
END IF
*----------------------------------------------------------------------*
* Algorithm Section - Finds closest representation of simulated particle
* in the position vectors RB(i) and ZB(k) thereafter returns the index
* of the found position

i = 1
k = 1
dr = RB(NR)/(NR-1)
dz = (ZB(NZ)-ZB(1))/(NZ-1)

DO WHILE (Z.GT.ZB(k)+dz/2)
k = k+1
END DO

R = SQRT(X**2 + Y**2)
DO WHILE (R.GT.RB(i)+dr/2)
i = i+1
END DO

*----------------------------------------------------------------------*
* Direction Section - Resolves radial magnetic field into x-y components
* and calculates the direction cosines for each field component in
* Cartesian coordinates

IF (BR(i, k).EQ.ZER) THEN
BRX = ZER
BRY = BRX
ELSE
BRX = BR(i, k)*X/R
BRY = BR(i, k)*Y/R
ENDIF

B = SQRT(BZ(k)**2 + BRX**2 + BRY**2)
BTX = BRX/B
BTY = BRY/B
BTZ = BZ(k)/B

*----------------------------------------------------------------------*
* Output Section - Writes and saves the position of the particle moving
* through the magnetic field region

WRITE(20, 100) X, Y, Z
100 FORMAT(1X, F9.6, 3X, F9.6, 3X, F12.6)
RETURN
ENDIF

*----------------------------------------------------------------------*
*====================== End of subroutine Magfld ======================*
END