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Conference Publication

LIU LINKÖPINGS UNIVERSITET

ACHIEVABLE UPLINK RATES FOR MASSIVE MIMO WITH COARSE QUANTIZATION

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ABSTRACT

The high hardware complexity of a massive MIMO base station, which requires hundreds of radio chains, makes it challenging to build commercially. One way to reduce the hardware complexity and power consumption of the receiver is to lower the resolution of the analog-to-digital converters (ADCs). We derive an achievable rate for a massive MIMO system with arbitrary quantization and use this rate to show that ADCs with as low as 3 bits can be used without significant performance loss at spectral efficiencies around 3.5 bpcu per user, also under interference from stronger transmitters and with some imperfections in the automatic gain control.

***Index Terms*—** ADC, channel estimation, low resolution, massive MIMO, quantization.

1. INTRODUCTION

Massive MIMO is a promising technology for the improvement of today's wireless infrastructure [1]. The huge number of transceiver chains required in massive MIMO base stations, however, makes their hardware complexity and cost a challenge that has to be overcome before the technology can become commercially viable [2]. It has been proposed to build each transceiver chain from low-end hardware to reduce the complexity [3].

In this paper, we perform an information theoretical analysis of a massive MIMO system with arbitrary ADCs and derive an achievable rate, which takes quantization into account, for a linear combiner that uses low-complexity channel estimation. The achievable rate is used to draw the conclusion that ADCs with 3 bits are sufficient to achieve a rate close to that of an unquantized system, see Section 6 for more detailed conclusions. This analysis is an extension of work in [4], where we only study one-bit ADCs.

Previous work has studied the capacity of the one-bit quantized frequency-flat MIMO channel [5,6], developed detection and channel estimation methods for the frequency-flat multiuser MIMO channel [7–9] and for the frequency-selective channel [10,11]. Low-resolution ADCs were studied in [12] and the use of a mix of ADCs with different resolutions in [13]. While the methods for frequency-flat channels are hard to extend to frequency-selective channels and the methods for frequency-selective channels either have high computational complexity, require long pilot sequences or imply impractical design changes to the massive MIMO base station, the linear detector and channel estimator that we study is the same low-complexity methods that has been proven possible to implement in practical testbeds [14, 15].

A parametric model for hardware imperfections was proposed in [16], where the use of low-resolution ADCs in massive MIMO also was suggested. The parametric model is used in [17] to show that 4–5 bits of resolution maximizes the spectral efficiency for a given power consumption. Several system simulations have been performed to analyze low-resolution ADCs, e.g. [18, 19], where the conclusions coincide with the conclusions in this paper: that three-bit ADCs are sufficient to obtain a performance close to an unquantized system.

2. SYSTEM MODEL

The uplink transmission from K single-antenna users to a massive MIMO base station with M antennas is studied. The transmission is based on pulse-amplitude modulation and, for the reception, a matched filter is used for demodulation. It is assumed that the matched filter is implemented as an analog filter and that its output is sampled at symbol rate by an ADC with finite resolution. Because the nonlinear quantization of the ADC comes after the matched filter, the transmission can be studied in symbol-sampled discrete time.

Each user k transmits the signal $\sqrt{P_k}x_k[n]$, which is normalized,

$$\mathsf{E}\big[|x_k[n]|^2\big] = 1, \tag{1}$$

so that P_k denotes the transmit power. The channel from user k to antenna m at the base station is described by its impulse response $\sqrt{\beta_k}h_{mk}[\ell]$, which can be factorized into a large-scale fading coefficient β_k and a small-scale fading impulse response $h_{mk}[\ell]$. The large-scale fading varies slowly in comparison to the symbol rate and it is assumed that it can be accurately estimated with little overhead by both user and base station. How the large-scale fading is estimated with low-resolution ADCs is left for future research. It is therefore assumed to be known throughout the system. The small-scale fading, in contrast, is *a priori* unknown to everybody. It is independent across ℓ and follows the power delay profile

$$\sigma_k^2[\ell] \triangleq \mathsf{E}\big[|h_{mk}[\ell]|^2\big], \tag{2}$$

however, is assumed to be known. It is also assumed that $\sigma_k^2[\ell] = 0$ for all taps $\ell \notin [0, \dots, L-1]$, where L is the number of nonzero channel taps. Since variations in received power should be described by the large-scale fading only, the power delay profile is normalized such that

$$\sum_{\ell=0}^{L-1} \sigma_k^2[\ell] = 1, \quad \forall k. \tag{3}$$

Base station antenna m receives the signal

$$y_m[n] = \sum_{k=1}^{K} \sqrt{\beta_k P_k} \sum_{\ell=0}^{L-1} h_{mk}[\ell]x_k[n-\ell] + z_m[n]. \tag{4}$$

The thermal noise of the receiver $z_m[n]$ is modeled as a white stochastic process, for which $z_m[n] \sim \mathscr{CN}(0, N_0)$. The received power is denoted

$$P_{\mathrm{rx}} \triangleq \mathsf{E}\big[|y_m[n]|^2\big] = \sum_{k=1}^{K} \beta_k P_k + N_0. \tag{5}$$

Transmission is assumed to be done with a cyclic prefix in blocks of N symbols. The received signal can than be given in the frequency domain as

$$y_m[\nu] \triangleq \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} y_m[n] e^{-j2\pi n\nu/N} = \sum_{k=1}^{K} h_{mk}[\nu] x_k[\nu] + z_m[\nu], \quad (6)$$

The Fourier transforms $x_k[\nu]$ and $z_m[\nu]$ of the transmit signal $x_k[n]$ and noise $z_m[n]$ are defined in the same way as $y_m[\nu]$. The frequency response of the channel is defined as

$$h_{mk}[\nu] \triangleq \sum_{\ell=0}^{L-1} h_{mk}[\ell] e^{-j2\pi\ell\nu/N}. \quad (7)$$

3. QUANTIZATION

The inphase and quadrature signals are assumed to be quantized separately by two identical ADCs with quantization levels given by $\mathcal{Q}_{\mathfrak{Re}} \subseteq \mathbb{R}$. The set of quantization points is denoted $\mathcal{Q} \triangleq \{a+jb : a, b \in \mathcal{Q}_{\mathfrak{Re}}\}$ and the quantization by

$$[y]_{\mathcal{Q}} \triangleq \arg\min_{q \in \mathcal{Q}} |y - q|. \quad (8)$$

To adjust the input signal to the dynamic range of the ADC, an automatic gain control scales the input power by A. The ADC outputs:

$$q_m[n] \triangleq \left[\sqrt{A} y_m[n]\right]_{\mathcal{Q}}. \quad (9)$$

Using the orthogonality principle, any signal with finite power can be partitioned into one part $\rho y_m[n]$ that is correlated to the transmit signal and one part $e_m[n]$ that is uncorrelated:

$$q_m[n] = \rho y_m[n] + e_m[n]. \quad (10)$$

The constant ρ and the variance of the uncorrelated part are given by:

$$\rho = \frac{\mathsf{E}[q_m[n] y_m^*[n]]}{\mathsf{E}[|y_m[n]|^2]}, \quad (11)$$

$$\mathsf{E}[|e_m[n]|^2] = \mathsf{E}[|q_m[n]|^2] - \frac{|\mathsf{E}[q_m[n] y_m^*[n]]|^2}{\mathsf{E}[|y_m[n]|^2]}. \quad (12)$$

The normalized mean-square error (MSE) of the quantization is denoted by:

$$Q \triangleq \frac{1}{|\rho|^2} \mathsf{E}[|e_m[n]|^2] \quad (13)$$

$$= P_{\mathrm{rx}} \left(\frac{\mathsf{E}[|q_m[n]|^2] \mathsf{E}[|y_m[n]|^2]}{|\mathsf{E}[q_m[n] y_m^*[n]]|^2} - 1 \right). \quad (14)$$

An ADC with b-bit resolution has $|\mathcal{Q}_{\mathfrak{Re}}| = 2^b$ quantization levels. In [20], the quantization levels that minimize the MSE for a Gaussian input signal with unit variance are derived numerically for 1–5 bit ADCs, both with arbitrarily and uniformly spaced quantization levels. The normalized MSE of the quantization has been computed numerically and is given in Table 1 for the optimized quantizers. To obtain the MSE in Table 1 with the quantization levels from [20], the input should be a unit-variance Gaussian signal and the automatic gain control $A = A^\star \triangleq 1/P_{\mathrm{rx}}$. Figure 1 shows how the quantization MSE in a four-bit ADC changes with imperfect gain control. Even if the gain control varies between −8 and 5 dB from the optimal value, the MSE is still better than that of a three-bit ADC.

Table 1: Normalized quantization mean square-error Q/P_{rx}

resolution b	1	2	3	4	5
optimal levels	0.5708	0.1331	0.03576	0.009573	0.002492
uniform levels	0.5708	0.1349	0.03889	0.01166	0.003506

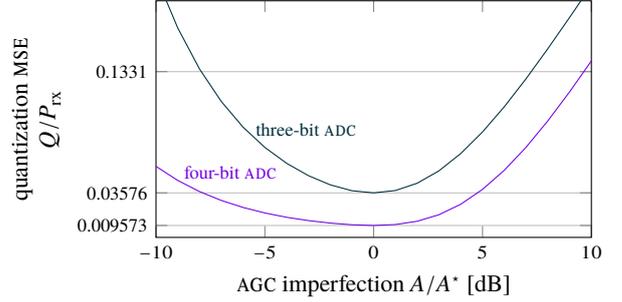**Fig. 1**: Quantization MSE for optimal four-bit ADC with imperfect AGC.

4. CHANNEL ESTIMATION

Channel estimation is done by receiving $N = N_{\mathrm{p}}$-symbol long orthogonal pilots from the users, i.e., pilots $x_k[n]$ such that:

$$\sum_{n=0}^{N_{\mathrm{p}}-1} x_k[n] x_{k'}^*[n+\ell] = \begin{cases} N_{\mathrm{p}}, & \text{if } k = k', \ell = 0 \\ 0, & \text{if } k \neq k', \ell = 1, \dots, L-1 \end{cases}, \quad (15)$$

where the indices are taken modulo N_{p}. To fulfill (15), $N_{\mathrm{p}} \geq KL$. We will call the factor of extra pilots $\mu \triangleq N_{\mathrm{p}}/(KL)$ the *pilot excess factor*. As remarked upon in [4], not all sequences fulfilling (15) result in the same performance. Here we use the pilots proposed in [4]. Using (10) and (15), an observation of the channel is obtained by correlation:

$$r_{mk}[\ell] = \frac{1}{\rho\sqrt{N_{\mathrm{p}}}} \sum_{n=0}^{N_{\mathrm{p}}-1} q_m[n] x_k^*[n+\ell] \quad (16)$$

$$= \sqrt{\beta_k P_k N_{\mathrm{p}}} h_{mk}[\ell] + e'_{mk}[\ell] + z'_{mk}[\ell], \quad (17)$$

where

$$e'_{mk}[\ell] \triangleq \frac{1}{\rho\sqrt{N_{\mathrm{p}}}} \sum_{n=0}^{N_{\mathrm{p}}-1} e_m[n] x_k^*[n+\ell], \quad (18)$$

$$z'_{mk}[\ell] \triangleq \frac{1}{\sqrt{N_{\mathrm{p}}}} \sum_{n=0}^{N_{\mathrm{p}}-1} z_m[n] x_k^*[n+\ell] \sim \mathcal{CN}(0, N_0). \quad (19)$$

The linear minimum MSE estimate of the frequency response of the channel is thus

$$\hat{h}_{mk}[\nu] = \sum_{\ell=0}^{L-1} \frac{\sqrt{\beta_k P_k N_{\mathrm{p}}} \sigma_k^2[\ell]}{\beta_k P_k N_{\mathrm{p}} \sigma_k^2[\ell] + Q + N_0} r_{mk}[\ell] e^{-j2\pi\ell\nu/N} \quad (20)$$

and the error $\epsilon_{mk}[\nu] \triangleq \hat{h}_{mk}[\nu] - h_{mk}[\nu]$ has the variance $1 - c_k$, where the variance of the channel estimate is given by

$$c_k \triangleq \mathsf{E}[|\hat{h}_{mk}[\nu]|^2] = \sum_{\ell=0}^{L-1} \frac{\sigma_k^4[\ell] \beta_k P_k N_{\mathrm{p}}}{\sigma_k^2[\ell] \beta_k P_k N_{\mathrm{p}} + Q + N_0}. \quad (21)$$

Figure 2 shows the variance of the channel estimate. A resolution of 2 bit is enough to obtain a channel estimate that is only 0.5 dB worse than in an unquantized system. With a resolution of 3 bit or higher, the variance of the channel estimate is practically the same as that
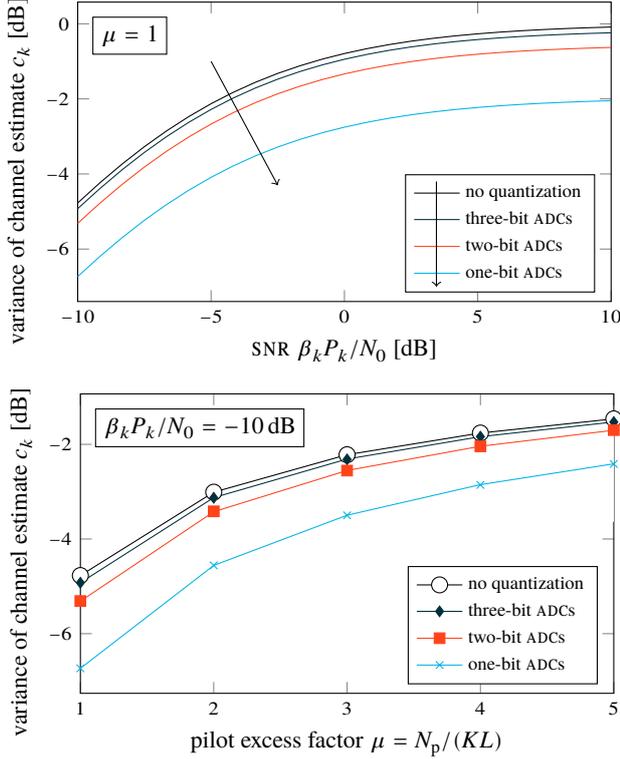
Fig. 2: The quality of the channel estimate with 5 users and a uniform power delay profile $\sigma_k^2[\ell] = 1/L$, for all k, ℓ, and with equal received power $\beta_k P_k = \beta_1 P_1$ from all users k. The optimal quantization levels derived in [20] are used. The markers show how simulated results with $L = 20$ channel taps corroborate the theory. When $c_k = 0$ dB, perfect channel state information is obtained.

of the unquantized system. Increasing the pilot length, increases the quality of the channel estimation in all systems. The improvement is, however, the largest when going from $\mu = 1$ to $\mu = 2$; thereafter the improvement gets smaller.

5. DATA TRANSMISSION

The uplink data is transmitted in a block of length $N = N_{\mathrm{u}}$, which is separated from the pilot block in time. The received signal is processed in a linear combiner and an estimate of the transmitted signal is obtained by

$$\hat{x}_k[\nu] = \frac{1}{\rho} \sum_{m=1}^{M} w_{mk}[\nu] q_m[\nu], \qquad (22)$$

where the Fourier transform $q_m[\nu]$ of $q_m[n]$ is defined in the same way as $y_m[\nu]$ in (6) and the combiner weights $w_{mk}[\nu]$ are chosen as a function of the channel estimate. For example, the maximum-ratio and zero-forcing combiners can be used, see [4].

If we code over many channel realizations, an achievable ergodic rate, i.e. lower bound on the channel capacity, is given by [4], independent of ν:

$$R_k = \log_2 \left(1 + \frac{\left| \mathsf{E}\left[\hat{x}_k^*[\nu] x_k[\nu] \right] \right|^2}{\mathsf{E}\left[|\hat{x}_k[\nu]|^2 \right] - \left| \mathsf{E}\left[\hat{x}_k^*[\nu] x_k[\nu] \right] \right|^2} \right). \qquad (23)$$

To compute the expected values in (23), the estimate of the transmit signal in (22) can be expanded by using the relation in (10) and writing

the channel as $h_{mk}[\nu] = \hat{h}_{mk}[\nu] - \epsilon_{mk}[\nu]$:

$$\hat{x}_k[\nu] = x_k[\nu] \sqrt{\beta_k P_k} \sum_{m=1}^{M} \mathsf{E}\left[w_{mk}[\nu] \hat{h}_{mk}[\nu] \right]$$

$$+ x_k[\nu] \sqrt{\beta_k P_k} \sum_{m=1}^{M} \left(w_{mk}[\nu] \hat{h}_{mk}[\nu] - \mathsf{E}\left[w_{mk}[\nu] \hat{h}_{mk}[\nu] \right] \right)$$

$$+ \sum_{k' \neq k} x_{k'}[\nu] \sqrt{\beta_{k'} P_{k'}} \sum_{m=1}^{M} w_{mk}[\nu] \hat{h}_{mk'}[\nu]$$

$$- \sum_{k'=1}^{K} x_{k'}[\nu] \sqrt{\beta_{k'} P_{k'}} \sum_{m=1}^{M} w_{mk}[\nu] \epsilon_{mk}[\nu]$$

$$+ \sum_{m=1}^{M} w_{mk}[\nu] z_m[\nu] + \frac{1}{\rho} \sum_{m=1}^{M} w_{mk}[\nu] e_m[\nu], \qquad (24)$$

where the Fourier transform $e_m[\nu]$ of $e_m[n]$ is defined as in (6). Note that only the first term is correlated to the desired signal. By assuming that the channel is i.i.d. Rayleigh fading, it can be shown that the other terms in (24)—channel gain uncertainty, interference, channel estimation error, thermal noise, quantization error—are mutually uncorrelated and the variance of each term can be evaluated. In [4], it is shown, for one-bit ADCs, that, in limit, the last term does not combine coherently and its variance equals

$$\mathsf{E}\left[\left| \frac{1}{\rho} \sum_{m=1}^{M} w_{mk}[\nu] e_m[\nu] \right|^2 \right] \to Q, \quad L \to \infty, \qquad (25)$$

if the combiner is normalized such that $\sum_{m=1}^{M} \mathsf{E}\left[|w_{mk}[\nu]|^2 \right] = 1$, which will be assumed here. This can be generalized to any quantization in a similar way. The rate in (23) can then be written as

$$R_k \to \log_2 \left(1 + \frac{\beta_k P_k c_k G}{\sum_{k'=1}^{K} \beta_{k'} P_{k'} (1 - c_{k'}(1 - I)) + Q + N_0} \right), \qquad (26)$$

as $L \to \infty$, where the *array gain* and *interference* terms are defined as

$$G \triangleq \left| \sum_{m=1}^{M} \mathsf{E}\left[w_{mk}[\nu] \hat{h}_{mk}[\nu] \right] \right|^2, \qquad (27)$$

$$I \triangleq \mathrm{Var}\left(\sum_{m=1}^{M} w_{mk}[\nu] \hat{h}_{mk'}[\nu] \right), \qquad (28)$$

where

$$G = \begin{cases} M \\ M - K \end{cases}, \quad I = \begin{cases} 1, & \text{for maximum-ratio combining} \\ 0, & \text{for zero-forcing combining} \end{cases}. \qquad (29)$$

Only the quantization error in (25) depends on the number of channel taps L. In the unquantized case, R_k thus does not depend on L. In other cases, the dependency on L vanishes as $L \to \infty$ and the energy of the received signal no longer depends on the energy of the detected symbol [4, Lemmata 1, 3]; this argument is valid for any quantizer.

The dependency on L quickly becomes negligible, and the limit in (26) can approximate the rate with negligible error, also for practical delay spreads L. The approximation can even be good for some frequency-flat channels ($L = 1$) when the received power $\sum_{k=1}^{K} \beta_k P_k$ is small relative to the noise power N_0 or when the number of users is large and there is no dominant user, i.e., no user k for which $\beta_k P_k \gg \sum_{k' \neq k} \beta_{k'} P_{k'}$. For general frequency-flat channels, however, it is *not* true that the variance of the quantization error vanishes with increasing number of antennas, as it does in the limit $L \to \infty$ in (26); this seems to be overlooked in some of the literature [21–24].

The rate R_k is plotted in Figure 3 for maximum-ratio and zero-forcing combining. The transmit powers are allocated proportionally

(a) Maximum-ratio combining

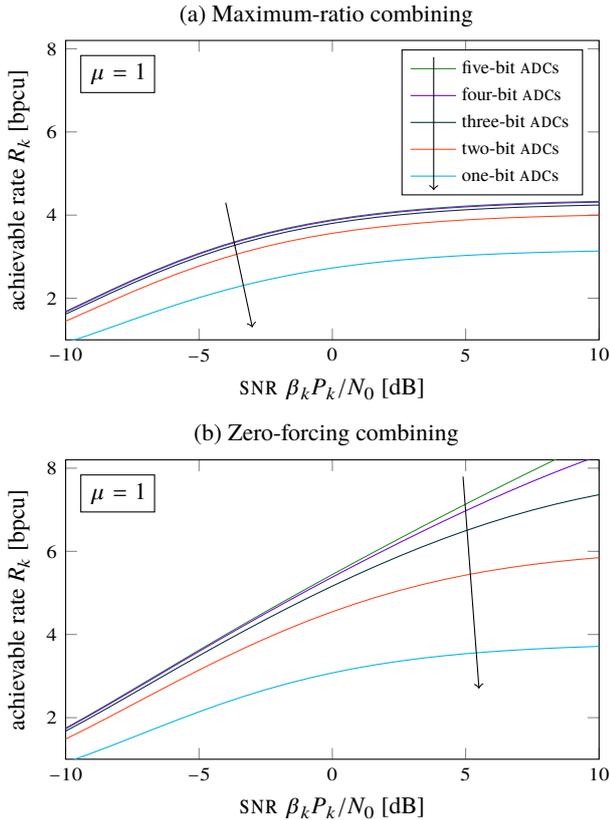

(b) Zero-forcing combining

Fig. 3: Rate of a system with 100 antennas and 10 users, where the power is proportional to $1/\beta_k$ and training is done with $N_p = KL$ pilots. The channel is i.i.d. Rayleigh fading with uniform power delay profile $h_{mk}[\ell] \sim \mathscr{CN}(0, 1/L)$. The optimal quantization levels derived in [20] are used.

Fig. 4: Per-user rate R_k for users $k = 2, \ldots, K$ when user $k = 1$ has a different receive SNR. The system has 100 antennas and $K = 10$ users, the channel is i.i.d. Rayleigh with uniform power delay profile and is estimated with $N_p = KL$ pilots. The optimal quantization levels derived in [20] are used.

around 10 dB higher than that of the weak users, where the data rate is degraded by approximately 15 % in the unquantized system. The degradation is larger in the quantized systems but the additional degradation due to quantization is almost negligible when the resolution is 3 bits or higher. With one-bit ADCs and one strong user with 10 dB larger received power, the degradation of the data rate increases to almost 50 %. Proper power control among users, however, can eliminate the near–far problem altogether.

6. CONCLUSION

We have derived an achievable rate for a single-cell massive MIMO system that takes quantization into account. The derived rate shows that ADCs with as low resolution as 3 bits can be used with negligible performance loss compared to an unquantized system, also with interference from stronger users. For example, with three-bit ADCs, the data rate is decreased by 4 % at spectral efficiencies of 3.5 bpcu in a system with 100 antennas that serves 10 users. It also shows that four-bit ADCs can be used to accommodate for imperfect automatic gain control—imperfections up to 5 dB still result in better performance than the three-bit ADCs. One-bit ADCs can be built from a single comparator and do not need a complex gain control (which ADCs with more than one-bit resolution need), which simplifies the hardware design of the base station receiver and reduce its power consumption. The derived rate, however, shows that one-bit ADCs lead to a significant rate reduction. For example, one-bit ADCs lead to a 40 % rate reduction in a system with 100 antennas that serves 10 users at spectral efficiencies of 3.5 bpcu. In the light of the good performance of three-bit ADCs, whose power consumption should already be small in comparison to other hardware components, the primary reason for the use of one-bit ADCs would be the simplified hardware design, not the lower power consumption.

7. ACKNOWLEDGMENTS

to $1/\beta_k$ and channel estimation is done with $N_p = KL$ pilots, i.e., the pilot excess factor $\mu = 1$. It can be seen that low-resolution ADCs cause very little performance degradation at spectral efficiencies below 4 bpcu. One-bit ADCs deliver approximately 40 % lower rates than the equivalent unquantized system and the performance degradation becomes practically negligible with ADCs with as few as 3 bit resolution. Assuming that the power dissipation in an ADC is proportional to 2^b, the use of one-bit ADCs thus reduces the ADC power consumption by approximately 6 dB at the price of 40 % performance degradation compared to the use of three-bit ADCs, which deliver almost all the performance of an unquantized system.

In [18], it is pointed out that low-resolution ADCs create a *near–far* problem, where users with relatively weak received power drown in the interference from stronger users. This is illustrated with a zero-forcing combiner in Figure 4, where it can be seen how the performance of the weak users degrades if there is a stronger user in the system. Note that the performance degrades also in the unquantized system, where the imperfect channel estimates prevent perfect suppression of the interference from the strong user. In the quantized systems, there is a second cause of the performance degradation: With quantization, the pilots are no longer perfectly orthogonal and the quality of the channel estimates is negatively affected by interference from the strong user. This effect can be seen in (21), where Q scales with the received power P_{rx} and thus with the power of the interferer.

Figure 4, however, shows that the near–far problem does not become prominent until the received power from the strong user is
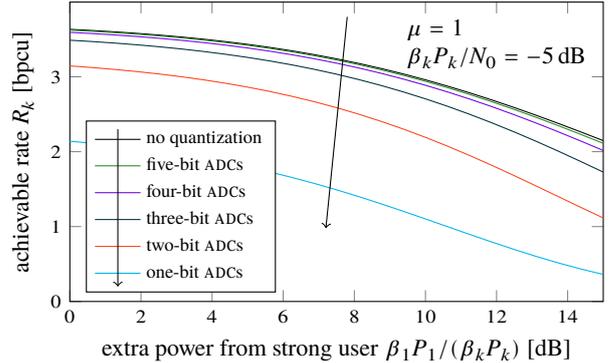
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