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Bo Källsner, Ove Ditlevsen, Kirsi Salmela

Variation of bending strength along structural timber members

Three papers presented in 1997

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VARIATION OF BENDING STRENGTH ALONG STRUCTURAL TIMBER MEMBERS

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Variation of bending strength along structural timber members - Three papers presented in 1997

Preface

In this publication three papers on the variation of bending strength along structural timber members are presented. All papers are dealing with an ongoing research project where the second phase has just been finished.

The first paper focuses on describing the evaluation of the test results from the second phase of the project. Measured bending strengths are compared with values obtained from a proposed hierarchical model.

In the second paper a summary of the project is given and the obtained test results are compared with results from other investigations.

Finally, the third paper gives a summary of the project and focuses on the possibility of introducing the proposed model for the bending strength variation in a reliability based design code for timber structures.

Variationen i böjhållfasthet längs konstruktionsvirke - Tre konferensbidrag presenterade under 1997

Förord

I denna rapport presenteras tre konferensbidrag om variationen i böjhållfasthet längs konstruktionsvirke. Samtliga bidrag behandlar ett pågående forskningsprojekt där den andra fasen nyligen avslutats.

I det första konferensbidraget ligger tyngdpunkten på att beskriva utvärderingen av försöksresultaten från den andra fasen av projektet. Uppmätta böjhållfastheter jämförs med beräknade värden enligt en tidigare föreslagen hierarkisk modell.

I det andra bidraget ges en sammanfattning av projektet och de erhållna provningsresultaten jämförs med resultat från andra undersökningar.

Slutligen presenteras i det tredje konferensbidraget en sammanfattning av projektet varvid en fokusering sker på möjligheten att införa den föreslagna modellen för böjhållfasthetens variation i en sannolikhetsbaserad norm för dimensionering av träkonstruktioner.

IUFRO S 5.02 Timber Engineering

Experimental Verification of a Weak Zone Model for Timber in Bending

B. Källsner[1], O. Ditlevsen[2], K. Salmela[1]

[1]Swedish Institute for Wood Technology Research
[2]Department of Structural Engineering and Materials
Technical University of Denmark

Copenhagen
Denmark

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Summary
In order to verify a stochastic model for the variation of bending strength within and between structural timber members, tests with long members subjected to constant bending moment have been performed. The span with constant moment contained between five and nine weak zones, i.e. zones with a cluster of knots. In a previous investigation test specimens, each containing one weak zone, have been tested in bending separately. Based on these tests a hierarchical model with two levels was formulated.

The test results show that the bending strength of the long timber members on the average is 5 to 15% lower than is predicted by the proposed hierarchical model. Energy considerations show that the reduction in strength of long beams may not be solely a statistical effect caused by an increased number of possible failure modes in the long beams as compared to the short test specimens. The large elastic energy released in a long highly bent beam at the onset of failure may mean that a later higher external load level cannot be realised as in a controlled slowly progressing failure.

1 Background

In a previous investigation (Källsner & Ditlevsen, 1994), the variation of bending strength within and between structural timber members has been studied. The experimental approach was to determine the bending moment capacity of each weak zone, i. e. zone with a cluster of knots, separately. For this purpose the timber members were cut into pieces, each containing one weak zone, see Figure 1. Test specimens were manufactured by finger jointing these pieces together with timber of high strength at each end. The specimens were tested in standard four point loading, with the weak zones in the middle of the span, see Figure 2.

To investigate if the distance between the weak zones did affect the bending strength, the timber members were grouped with reference to the number of weak cross-sections within each member.

In order to model the variation of the bending strength, a hierarchical model with two levels was used, see chapter 4. This is the simplest type of probabilistic model for describing equicorrelation between the bending strength values of the weak zones within the same timber member. The experimental data were reasonably well represented by the formulated model.

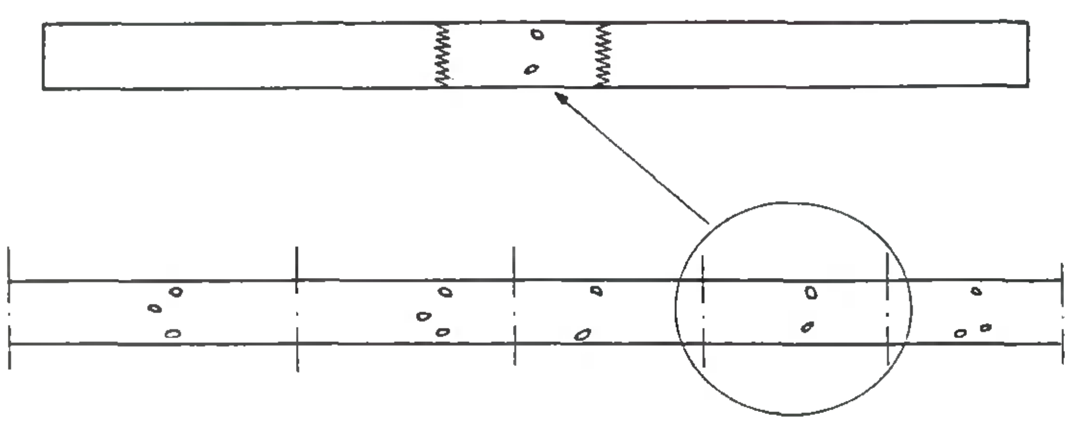

Figure 1 *Example of how a timber member was cut into pieces and finger jointed with pieces of timber of high strength at the ends.*

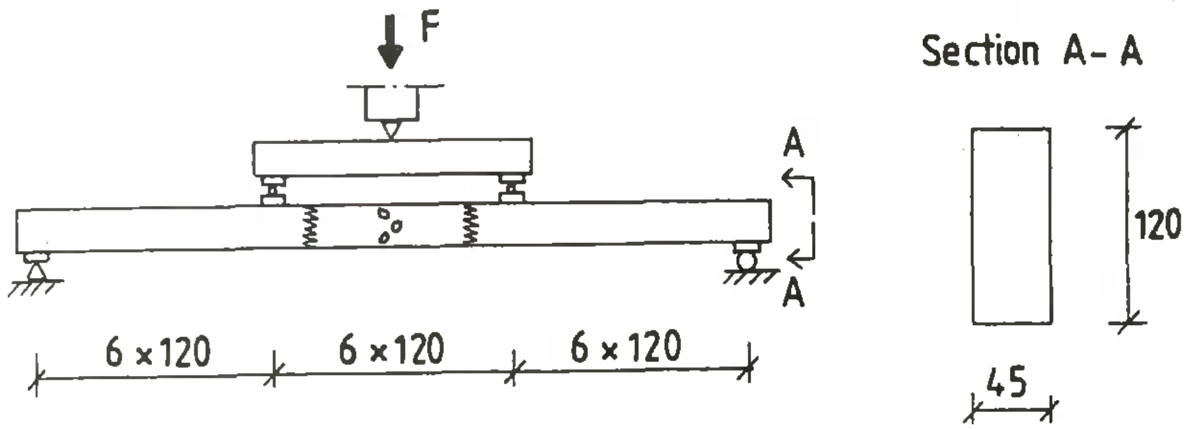

Figure 2 *Principal test arrangement when one weak zone was tested at a time. All sizes in mm.*

To further examine the validity of the proposed model, a second phase of the project is in progress. This paper gives a situation report. A more detailed report will be published as a master thesis by Salmela (1997).

2 Objective

The purpose of this second phase of the project is to investigate if the hierarchical model of the bending strength variation can be used to determine the strength of long timber members subjected to constant bending moment.

3 Experimental investigation

3.1 Selection of timber

All wood material for this second phase of the investigation was taken from the same population of timber members which was selected during the first phase of the project. This means that the wood had been harvested in a forest west of lake Vänern in Sweden. All wood originated from 93 trees of spruce (Picea Abies). The trees were cut in an area of about 10 000 m^2. One selection criterion was that at least two logs, 4800 mm long, could be cut from each tree and that at least two timber members could be sawn from each log. Another criterion was that logs with rot were excluded. All logs were sawn through the pith. After drying in a chamber kiln, the timber members were planed to their final dimension 45 mm x 120 mm. The total number of timber members was more than 500.

In this second phase of the project, as well as in the first phase, it was decided that only timber members sawn from the second log from the ground level and which were from the centre of the log closest to the pith should be selected.

The bending tests with long timber members consisted of two series of specimens:

Test series 1 comprised of timber members that were "twins" (i.e. situated beside each other) of the 26 timber members which were tested during phase 1 when the bending strength of each weak zone was determined separately.

Test series 2 comprised of 14 pairs of timber members, where each pair was sawn from each side of the pith.

3.2 Material parameters

The following material parameters were measured:

- Modulus of elasticity in edgewise bending: The timber members were subjected to a constant bending moment and the modulus of elasticity was determined by using a movable device for measurement of deflection within a gauge length of 900 mm at every 10th mm along the timber members.
- Modulus of elasticity in flatwise bending: Two methods were used: Firstly the same device was used as mentioned above but with a gauge length of 600 mm, secondly a Cook-Bolinder stress grading machine was used, recording data every 10th mm along the timber members.
- Knots: The positions and sizes of the knots in the weak zones were measured.
- Pith: The position in every weak zone was measured.
- Density, moisture content and annual ring width: A test specimen was taken out in a defect free zone close to the location of failure.

3.3 Test equipment and test procedure

During phase 1 of the project, when each weak zone was tested separately (Figure 2), the test procedure given in the standard ISO 8375 was followed. Denoting the depth of the timber member by h, the distance subjected to constant bending moment was 6 h and the total span was equal to 18 h.

Since the purpose with the bending tests in phase 2 was to test timber members with many weak zones, the test arrangements had to be modified according to Figure 3. The total span was increased from 18 h to 36 h, and the span with constant moment was increased from 6 h to 30 h. To prevent lateral buckling, the test specimens were equipped by lateral stiffeners at the third points.

The load was applied under constant rate of displacement. The deflections of the test specimens were recorded at the supports, at the point loads and at the centre of the span. Due to the long test specimens special attention had to be taken to large displacements. Thus the measured bending moments had to be corrected in this respect.

Each timber member belonging to test series 1 was tested with the same edge in tension as was used for the corresponding "twin" timber member in phase 1. For the timber members belonging to test series 2, each pair was tested with the same edge in tension. The tension side for each pair was chosen randomly.

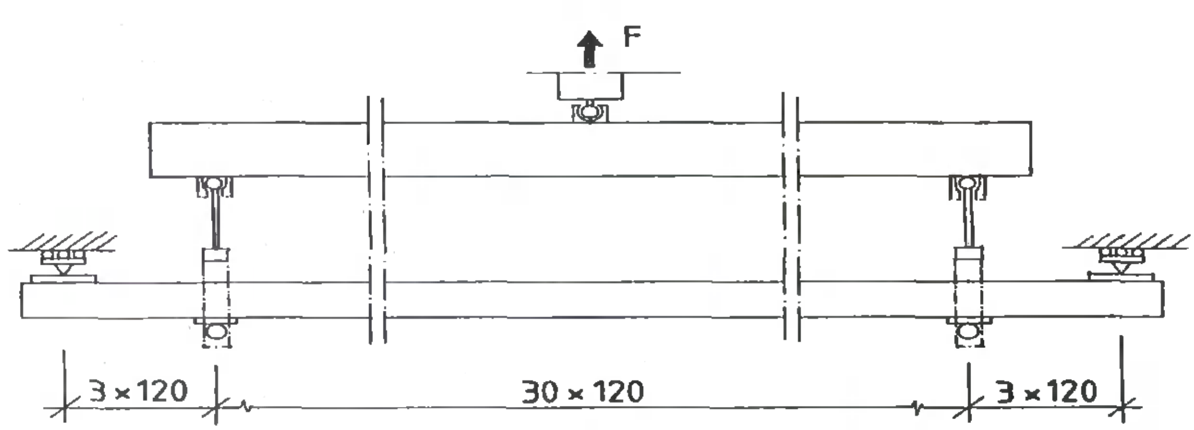

Figure 3 Principal test arrangement when timber members containing many weak zones were tested. All sizes in mm.

3.4 Reinforcement of test specimens

To prevent failures occurring in the area between the supports and the point loads, all test specimens were reinforced with plywood at the ends according to Figure 4. The plywood consisted of five veneers with a total thickness of 7 mm. The plywood was glued to both sides of the timber and extended 50 mm into the centre span at both point loads. The plywood was placed so that the distance from the edge of the plywood to the first weak zone in the centre span should be the same at both ends. The unreinforced part of the centre span was 3500 mm long.

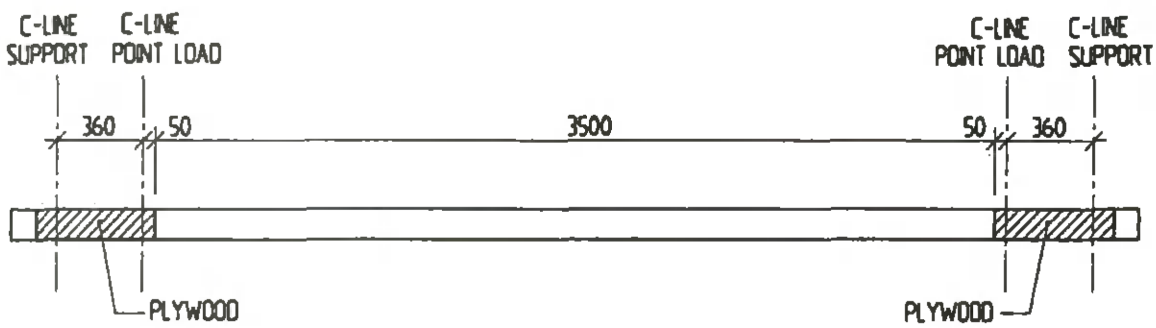

Figure 4 Test specimen reinforced with plywood. All sizes in mm.

4 Probabilistic modelling

This chapter gives a brief description of the model used in (Källsner and Ditlevsen 1994).

4.1 Hierarchical probabilistic model of weak cross-section strengths

In connection with the description of the models a weak zone is idealised to be concentrated as a point. Thus the terminology "weak zone" is replaced by "weak cross-section".

Reported results show indications of equicorrelation (i.e., constant correlation independent of separation) between the bending strengths of the weak cross-sections within the same beam, Riberholt and Madsen (1979), Williamson (1994), Isaksson (1996). The simplest type of probabilistic model with this property is the so-called hierarchical model with two levels. This model is as follows.

For a given beam with k identified weak cross-sections, it is assumed that the bending strengths of the k cross-sections are

$$X+Y_1, \dots, X+Y_k \tag{1}$$

where $Y_1, \dots, Y_k$ are mutually independent random variables of zero mean and standard deviation σ_Y. The variable X is a random variable over the population of beams. If the variables $Y_1, \dots, Y_k$ are assumed to be independent of X, the variance of the bending strength of a weak cross-section is

$$\mathrm{Var}[X+Y_i] = \mathrm{Var}[X]+\mathrm{Var}[Y_i] = \sigma_X^2+\sigma_Y^2 \tag{2}$$

The covariance between the strengths of two weak cross-sections in the same beam is

$$\mathrm{Cov}[X+Y_i, X+Y_j] = \mathrm{Var}[X] = \sigma_X^2 \tag{3}$$

such that the correlation coefficient becomes

$$\rho[X+Y_i, X+Y_j] = \frac{\sigma_X^2}{\sigma_X^2 + \sigma_Y^2} \tag{4}$$

Thus the correlation coefficient is independent of i and j for $i \neq j \leq k$, that is, the correlation is independent of the distance between the two weak cross-sections. Moreover, if it is assumed that the distributions of X and Y are independent of $k \geq 1$, it follows that the model is an equicorrelation model.

4.2 Probabilistic model for beam subjected to constant moment

The physical model for the beam strength is based on the assumption that failure can occur only at a finite number of weak cross-sections along the beam. The bending strength S of a beam subjected to a constant bending moment along the entire length and containing $k > 0$ weak cross-sections can be described by the random variable

$$S = X + \min\{Y_1, \ldots Y_k\} \tag{5}$$

where X and Y are defined as in the previous section. Assuming that $X, Y_1, \ldots, Y_k$ all have normal distributions (not a crucial assumption) it follows that the distribution function of the bending strength S is

$$F_S(z|k) = P(X + \min\{Y_1, \ldots, Y_k\} \leq z) = \frac{1}{\sigma_X} \int_{-\infty}^{\infty} \left[1 - \Phi\left(-\frac{z-x}{\sigma_Y} \right)^k \right] \varphi\left(\frac{x-\mu_X}{\sigma_X} \right) dx \tag{6}$$

where $\varphi(\cdot)$ is the standardised normal density function, $\Phi(\cdot)$ is the standardised normal distribution function and μ_X is the mean of X. Since $F_S(z|0) = 0$ this formula is also valid for $k = 0$. Assuming next that K is a random variable with $k \in \{0,1,2,\ldots\}$ as outcome with probability p_k, the distribution function of S becomes

$$F_S(z) = \sum_{k=0}^{\infty} F_S(z|k)\, p_k = 1 - \frac{1}{\sigma_X} \int_{-\infty}^{\infty} E\left[\Phi\left(\frac{x-z}{\sigma_Y} \right)^K \right] \varphi\left(\frac{x-\mu_X}{\sigma_X} \right) dx \tag{7}$$

in which the expectation

$$E\left[\Phi\left(\frac{x-z}{\sigma_Y} \right)^K \right] = \sum_{k=0}^{\infty} \Phi\left(\frac{x-z}{\sigma_Y} \right)^k p_k \tag{8}$$

can be expressed by the so-called probability generating function $\psi(x) = E[x^K]$ of the integer random variable K.

5 Results

A summary of the results from the bending tests are shown in Appendix 2. Two main types of failure modes were observed, namely, tensile (T) and compression (C) failure. Only in a few cases were permanent deformations caused by excessive compressive stresses observed before the test specimens collapsed. The final failures developed very fast. They often started with a fissure propagating from one weak zone (e.g. a knot on the tensile side of the specimen) to an adjacent weak zone where the timber member was broken. This first failure was often followed by secondary failures in the weak zones closest to the point loads. Consequently the test specimens were often broken into three or even four pieces.

To give some idea of the quality of the timber the results from two methods of grading of the timber members are also given in the appendix. An appearance grading according to the "Guiding principles for grading of Swedish sawn timber" gave the qualities shown in the second column. A visual strength grading according to the Swedish T-rules resulted in the strength classes shown in the third column. Each of the strength classes is designated by a number indicating the value of the characteristic bending strength in N/mm^2. Since the specimens were taken from an ungraded sample of timber members, two of the specimens in test series 1 did not fulfil the requirements of the lowest strength class K12.

6 Analysis of test results

For a beam subjected to constant moment along the entire length and containing $k > 0$ weak zones, the distribution function of the bending strength $F_S(z|k)$ is given by equation (6). By using the bending strength values from phase 1 of the project (Källsner and Ditlevsen 1994) the curves in Figure 5 are obtained (In this case it has been assumed that all B and BF failures reported were true bending failures). It is obvious that the bending strength decreases with an increased number of weak sections.

Since test series 1 and 2 consist of timber members with different number of weak zones in the test span, it is better to compare the test results with a weighted distribution function $F_S(z)$ according to equation (7). In Figure 6 and Figure 7 the bending strength values obtained from test series 1 and 2 are plotted together with the weighted distribution functions (the curves to the left). The weighted distribution functions for test series 1 and 2 are almost identical and they are also very close to the distribution function $F_S(z|k)$ corresponding to 6 weak cross-sections. The curves to the right in the figures show the distribution function $F_S(z|k)$ according to equation (6) with $k = 1$, i.e. with one weak zone in the span subjected to constant moment.

The plots show that the empirical distribution functions are shifted more to the left of the predicted distribution function than can be explained solely by referring to the statistical uncertainty related to the small sample size. This uncertainty is crudely evaluated below. Also it is seen that the deviation to the left is largest for test series 1, where the test specimens are "twins" of the ones tested during phase 1. However, the difference between the positions of the two empirical distribution functions cannot be excluded as being a result of statistical uncertainty combined with the fact that the predicted distribution functions are also affected by statistical uncertainty. In fact, the predictions are based on a special statistical analysis model that corrects for the observation that 42% of the test results of the specimens from phase 1 corresponded to finger joint failures. Moreover, since the sample of series 2 consists of 14 correlated pairs with an estimated correlation coefficient of 0.68 (see Figure 8 in which the 14 pairs are plotted), the effective sample size with respect to statistical uncertainty is no more than $28/(1+0.68) = 16.7$. The standard deviation of the individual failure stress random variable is about 8.8 N/mm^2. With an effective sample size of 16.7 the standard deviation of the estimator of the mean (which is close to the 50% fractile) is $8.8/\sqrt{16.7} = 2.2$ N/mm^2. This number indicates the size of the statistical uncertainty. Thus the statistical uncertainty may very well explain that the independent results of series 2 are shifted to the right relative to the results of series 1, and also the size of the shift.

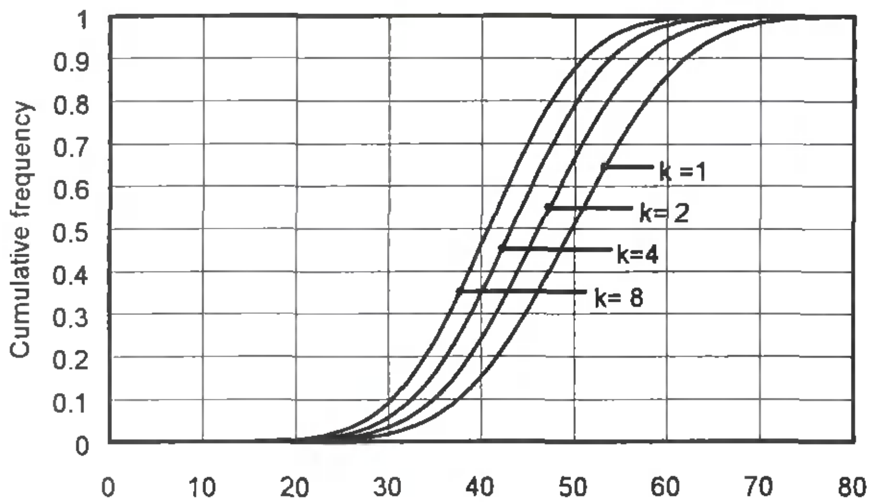

Figure 5 *Distribution function for bending strength in N/mm^2 of timber members under constant bending moment and k weak zones.*

In part the general shift to the left relative to the predicted distribution functions can be explained as follows. During phase 1, where only one weak zone was tested at a time, two main types of failures were observed. These were bending failure in the weak zone and failure in the finger joint. The bending failures always took place or started at the weak zone in the test specimen. One type of failure that could be observed was when a fissure propagated from a knot on the tension side of the weak zone. Sometimes this fissure led to an immediate collapse at the weak zone and sometimes the fissure stopped propagating and the load could be further increased. Since there were no adjacent weak zones in the test specimens, there was no influence from fissures that started from other weak zones propagating to the weak zone tested. It is obvious that this kind of fissures would have led to lower strength values and that such failure types may be possible in long beams.

The reason for the shift to the left may also partly be that the long timber members tested during phase 2 of the project contained more elastic energy than the standard beams tested during phase 1. As argued in Appendix 1, it can be assumed that the large elastic energy stored in a long highly bent beam before the onset of failure causes the failure to progress to total failure directly at the first decrease of the load. At least partly, the shift to the left is then explainable by the external force variation measured during the deformation controlled bending test. During the slowly progressing failure the external force is observed to vary upwards and downwards with its largest global value not necessarily coinciding with the first occurring local maximal value. The appearance of a shift to the left is then a consequence of the applied definition of the carrying capacity as being the globally largest value of the measured load during the test.

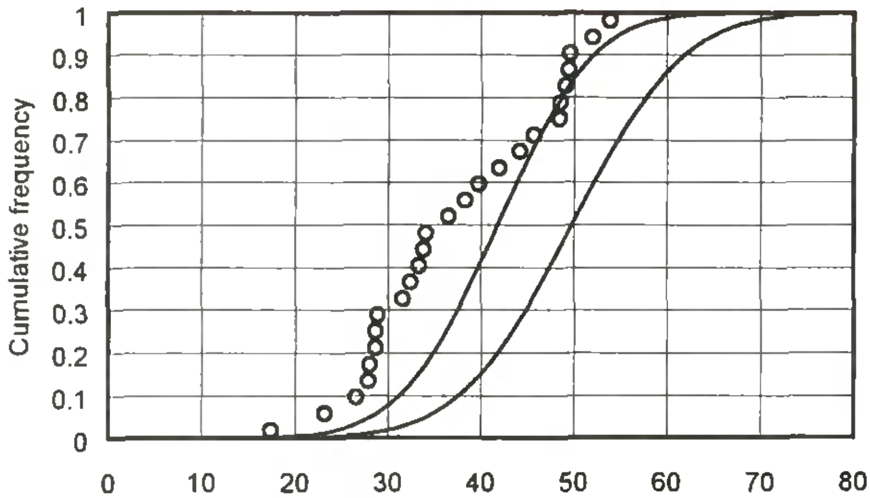

Figure 6 *Empirical distribution function for the bending strength data (in N/mm²) of test series 1 plotted together with the estimated distribution function (7) obtained from the hierarchical model (left curve). The curve to the right shows the estimated distribution function (6) for one weak zone.*

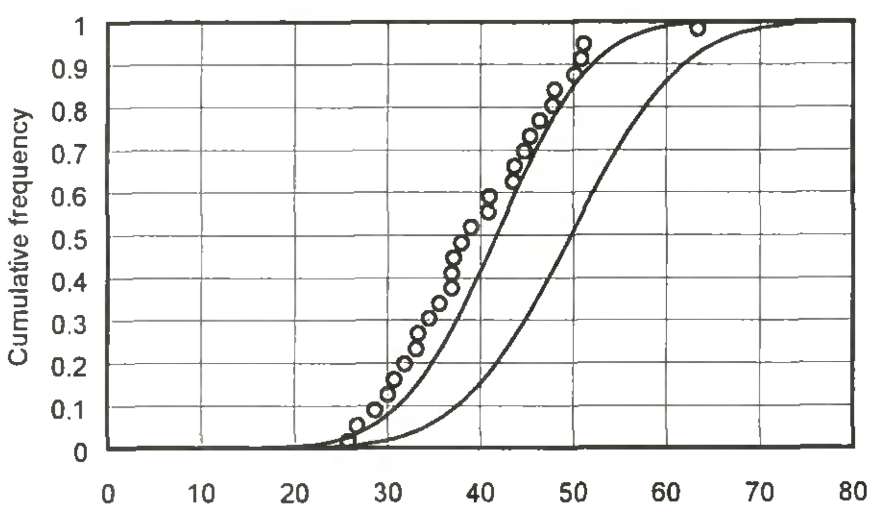

Figure 7 *Empirical distribution function for the bending strength data (in N/mm²) of test series 2 plotted together with the estimated distribution function (7) obtained from the hierarchical model (left curve). The curve to the right shows the estimated distribution function (6) for one weak zone.*

It is seen that there is a tendency to a better fit between measured and theoretical values in the upper strength range than in the lower and medium ranges. This indicates that during the process of controlled slowly progressing failure, as observed in the phase 1 experiments, there are larger differences between the first observed local maximum of the external load and the global maximum of all the following local maxima for the low strength timber than for the high strength timber. For the high strength timber this confirms the expectation that the first local maximum of the load is also the global maximum. Such timber has only small knots on the tension side. The failure may start as a compression failure but the final failure is usually a tension failure. This type of failure develops as a total collapse of the timber and the external load cannot be increased further. Thus it makes no difference whether it is a short or a long beam. Moreover, the empirical distribution function in Figure 6 shows a decrease of the slope (that is, the probability density) in the medium range. This indicates that the carrying capacity distribution may be bimodal. Such bimodality can be caused by some separation of the carrying capacity distribution of the compression failure mode from the lower placed carrying capacity distribution corresponding to the tension failure mode. The same bimodality tendency is not seen in the data of series 2 plotted in Figure 7.

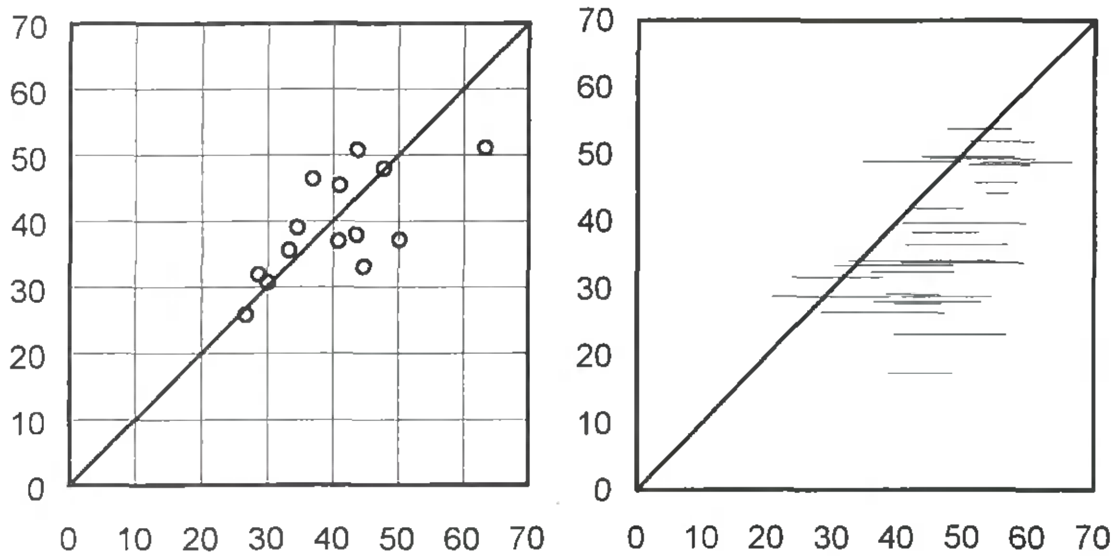

Figure 8 *Left: Bending strength values (in N/mm²) in test series 2 for pairs of timber members belonging to the same log.*
Right: Bending strength values (in N/mm²) in test series 1 for "twin" members to short test piece specimens (ordinate axis). The end points of the line pieces mark the smallest and the largest measured bending strengths of the pieces cut from the other "twin" member.

The 14 pairs of "twin" members of test series 2 have been selected such that they are all deemed to contain 6 weak zones (except one pair with 7 weak zones). As expected the strength results plotted in Figure 8 (left) are seen to be correlated. The correlation coefficient is estimated at $\rho_2 = 0.68$. This correlation coefficient complies with the equicorrelation coefficient given by (4), which from the results of the phase 1 tests is estimated at $\rho_1 = 0.55$. In fact, the hierarchical model implies that for a given number k of weak cross sections the correlation coefficient between the strengths of the "twins" becomes

$$\begin{aligned}\rho_2 &= \frac{\mathrm{Var}[X] + \mathrm{Cov}[\min\{Y_{11},\dots,Y_{1k}\},\min\{Y_{21},\dots,Y_{2k}\}]}{\mathrm{Var}[X] + \mathrm{Var}[\min\{Y_{11},\dots,Y_{1k}\}]} \\ &\le \frac{\mathrm{Var}[X] + \zeta_k\, Var[Y] P(\min\{Y_{11},\dots,Y_{1k}\} = \min\{Y_{21},\dots,Y_{2k}\})}{\mathrm{Var}[X] + \zeta_k\, Var[Y]}\end{aligned} \tag{9}$$

where ζ_k is a variance reduction factor that can be calculated for any given distribution of Y. For the normal distribution it has the approximate value 0.65 for $k = 6$. The probability $P(\min\{Y_{11},\dots,Y_{1k}\} = \min\{Y_{21},\dots,Y_{2k}\})$ can be estimated as the ratio of the number of pairs that failed at the same cross section to the total number of pairs. In the series 2 experiments this ratio was observed to be 3/14 = 0.21 (and 5/26 = 0.19 in test series 1). Using these numbers and the formula (4) the right side of the inequality becomes 0.73. If the covariance in the numerator on the left side of the inequality as an example is put to $0.5\zeta_6\mathrm{Var}[Y]$, the left side becomes 0.69.

The test results of series 1 are plotted to the right in Figure 8. The horizontal line pieces indicate by their end points the smallest and the largest measured strength of the small test specimens of the "twin". According to the discussion about elastic energy release at starting failure and the possibility that weak zone interacting failure modes can occur in the long beams, it should be expected that all the left end points of the line pieces should be to the right of the diagonal of the diagram. That this is not the case is consistent with the fact that coinciding failure cross-sections are only observed in 5 out of the 26 pairs of series 1.

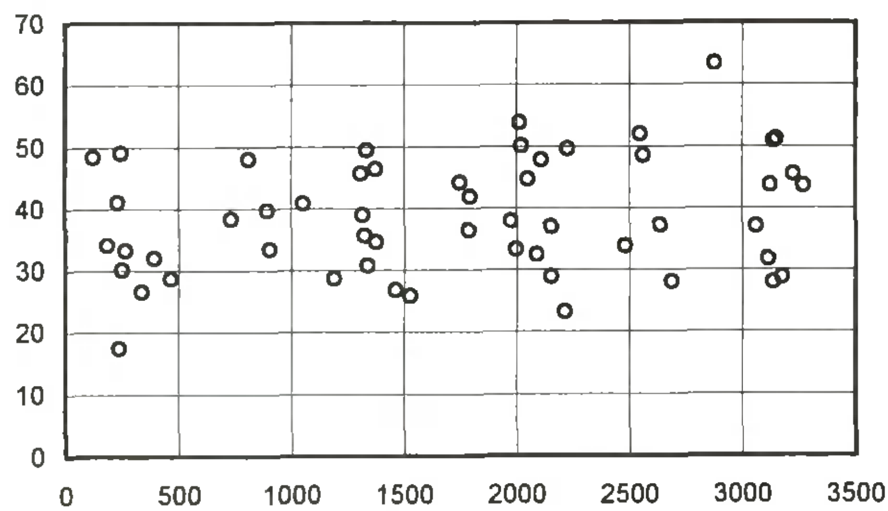

Figure 9 *Measured bending strength (in N/mm^2) as a function of the distance (in mm) from the plywood reinforcement in the root end of the timber member to the position where failure occurred.*

To see whether the positions of the failures are uniformly distributed along the span subjected to constant bending moment, all strength values from test series 1 and 2 are plotted in Figure 9 as a function of the distance from the root end of the span to the actual location of failure. The mean failure position is found to be very close to the centre of the span, only 36 mm towards the root end. Consequently there seems to be no indication of a root-top effect in the data.

7 Conclusions

Tests with long timber members subjected to constant bending moment indicate that the bending strength is 5 to 15% lower than was predicted by the proposed hierarchical model. There are indications that the reduction in strength of long beams is not solely a statistical effect but also an effect of the high elastic energy released at the onset of failure.

The test results are still subject to analysis, and the conclusions of this paper are therefore preliminary.

8 Acknowledgements

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Niels Jacob Tarp-Johansen at the Department of Structural Engineering and Materials, DTU is acknowledged for computer assistance under support from the Danish Technical Research Council.

9 References

Isaksson, T. 1996: Variability of bending strength within timber elements. Licentiate thesis at Department of Structural Engineering, Lund Institute of Technology, Sweden

Källsner, B.; Ditlevsen, O. 1994: Lengthwise bending strength variation of structural timber. Paper presented at the IUFRO/ S5.02 Meeting, Sydney, Australia

Riberholt, H.; Madsen, P. H. 1979: Strength distribution of timber structures, measured variation of the cross sectional strength of structural timber. Structural Research Laboratory, Technical University of Denmark, Report No. R 114

Salmela, K. 1997: Bending strength of timber in long-span testing (in Swedish). Master thesis at Building Materials, Department of Buildings and Building Services, Royal Institute of Technology, Stockholm, Sweden (under preparation)

Appendix 1: Effect of elastic energy redistribution at the onset of bending failure

Let the span in which bending failure can take place have a length of L and let the bending flexibility of the beam before failure be C. The beam is subject to a deflection controlled test such that the bending moment M is the same all through the span and such that the rotation ω of the one end of the span relative to the other end of the span is controlled. As long as the beam behaves linear-elastically the rotation is $\omega = CLM$. If a bending failure starts to develop at position x, the failure mechanism may be modelled by a hinge placed at x. If the developing failure stops before the beam has lost its carrying capacity, the result is that an angular rotation θ has taken place in the hinge. This implies that $\omega = CLM + \theta$. The hinge is a mathematical point idealisation of a failure process taking place over a small domain around the hinge point. This domain can in a less idealised model be taken as an interval of length λh, where h is the beam height and λ is a number of the order of size of 1. Within this interval it is assumed that the bending flexibility is increased corresponding to a decrease of the effective beam height by a reduction factor r. Then the flexibility increases to Cr^{-3}. The elastic energy stored in the beam on either side of the hinge is then $E(\omega,0) = CM^2L/2$ before the failure starts to develop, and after stop of the failure development it is

$$E(\omega,\theta) = \frac{1}{2}CM^2L[1-\frac{\lambda h}{L}(1-r^{-3})] = \frac{1}{2CL}[1+(r^{-3}-1)\delta](\omega-\theta)^2 \tag{10}$$

where $\delta = \lambda h/L$. If the excess energy $E(\omega,0) - E(\omega,\theta)$ is larger than the total energy dissipation D in the hinge during the failure in progress, then the failure will continue, otherwise an equilibrium position will exist before total failure occurs. In that case the carrying capacity may be larger than that given by the measured value of M_f of the bending moment immediately before the occurrence of its first decrease indicating that a failure is in progress. The dissipation D depends on the material properties in the weak zone. It can be written as

$$D = \alpha(1 - r^3)\frac{\omega_f}{CL}\theta \qquad (11)$$

where α is a factor between 0 and 1, and where $(1 - r^3)\omega_f/CL$ is the excess bending moment over the moment $r^3 M_f = r^3\,\omega_f/CL$ carried by the reduced cross-section. For $\alpha = 1$ a full ideal plastic resistance is assumed (dropping to zero, though, when the failure progress stops). The value $\alpha = 0.5$ more realistically, perhaps, corresponds to a linear decrease of the resistance from the full value at the onset of the failure process to zero when the failure progress has stopped. Of course, the value of α (which like λ may be modelled as a random variable) can only be determined from experiments.

The angular rotation θ in the hinge is determined as the extra rotation needed to come from the rotation over the beam piece of length λh without and with increased flexibility. Thus

$$\theta + \frac{\lambda h}{L}\omega_f = r^{-3}\frac{\lambda h}{L}(\omega_f - \theta) \qquad (12)$$

giving

$$\frac{\theta}{\omega_f} = (1 - r^3)\frac{\delta}{r^3 + \delta} \qquad (13)$$

Substituting this into the condition $E(\omega_f, 0) - E(\omega_f, \theta) > D$ that equilibrium cannot be obtained after the first onset of the failure process then gives the condition

$$(1 - r^3)[\delta^2 + (2\alpha - 1)(r^{-3} - 1)\delta + 2\alpha(1 - r^3) - 1] < 0 \qquad (14)$$

For $\alpha = 0.5$ this gives the simple condition $\delta < r^{3/2}$. Figure 11 shows the domains of (δ, r) in which equation (13) is satisfied for a given value of α. For supposed values of α and $\lambda h/L$ it can be seen from the corresponding bounding curve whether or not there is a value of the reduction factor r for which equilibrium can be obtained.

The value of $\delta = \lambda h/L$ can be assessed by observing the extension of the damage of the failure zone after failure. Typically this damaged length is about the distance between two adjacent clusters of knots, that is, about 50 cm in the tests considered. For the long beams a reasonable assessment of the value of δ is therefore that it is about 0.1 to 0.2, while it for the short test specimens it is about 0.8 to 1. Even though nothing is known about the possible dependence of α on the cross-section reduction factor r, it can be seen from the diagram in Figure 10 that if $\alpha \geq 0.5$, then the considered idealised beam model will reach an equilibrium state at some value of $r > 0$. However, for $\alpha < 0.5$ there may not be an equilibrium state. Under the assumption that α does not depend on r, it is seen that there is a value of below δ which there is no value of $r > 0$ that gives equilibrium. Thus the reported test results indicate that $\alpha < 0.5$ for the bending failures in the types of wood beams considered.

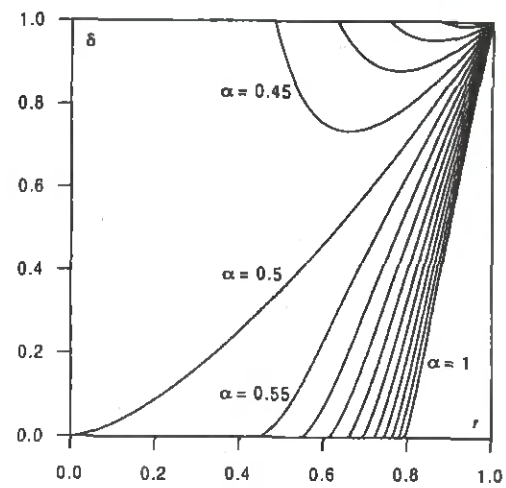

Figure 10 *Curves that according to equation (14) for each given α upward bound the domains of occurrence of total failure after the first onset of local failure in the deformation controlled bending test described herein.*

Appendix 2: Experimental data

Test series 1:

Timber member No.	Quality	Strength class	Number of weak zones	Failure mode	Bending strength N/mm^2
621	V	K15	5	T	41.9
821	VI	K15	7	T	39.7
1222	IV	K15	7	T	48.4
1321	V	K12	5	T	31.6
1422	III	K30	5	T	45.7
1511	VI	K18	5	T	49.6
1822	V	K15	9	T	33.8
2022	IV	K24	7	T	28.8
2722	IV	K18	5	T	49.0
3321	IV	K24	5	T	44.1
3621	V	-	6	T	28.0
4022	V	K18	5	T	28.7
4121	VI	-	6	T	26.6
4721	IV	K24	6	T	23.2
4921	IV	K18	5	T	48.5
5022	VI	K12	5	C	49.4
5221	VI	K12	6	T	36.5
5522	V	K15	8	T	34.1
5622	VI	K15	5	T	28.7
6021	V	K15	8	T	33.3
6122	V	K12	6	T	53.8
7022	V	K15	7	T	32.4
7321	IV	K24	8	T	38.3
8121	IV	K18	6	T	27.9
8621	III	K24	6	T	52.0
8921	VI	K12	6	T	17.4

Test series 2:

Timber member No.	Quality	Strength class	Number of weak zones	Failure mode	Bending strength N/mm^2
121	V	K15	6	T	50.1
122	V	K15	6	T	37.1
1921	IV	K24	6	T	43.6
1922	V	K15	6	T	37.9
2621	IV	K18	6	T	33.3
2622	V	K12	6	T	35.6
2921	V	K18	6	T	36.9
2922	IV	K24	6	T	46.4
3721	IV	K15	6	T	30.1
3722	IV	K24	6	T	30.8
4621	VI	K12	6	C	44.7
4622	IV	K24	6	T	33.1
5321	VI	K12	6	T	43.7
5322	V	K15	6	T	50.8
6921	IV	K24	6	C	63.4
6922	IV	K18	6	T	51.1
7121	IV	K15	6	T	28.7
7122	VI	K12	6	T	31.9
7221	VI	K12	7	T	40.8
7222	IV	K24	7	T	36.9
7421	VI	K12	6	T	26.8
7422	V	K12	6	T	25.8
7621	IV	K15	6	T	34.5
7622	V	K15	6	T	39.0
9221	IV	K15	6	T	41.0
9222	IV	K18	6	C	45.4
9321	III	K24	6	T	47.8
9322	IV	K15	6	C	48.0

Bo Källsner, Ove Ditlevsen, Kirsi Salmela

Experimental Verification of a Weak Zone Model for Timber in Bending

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Timber Engineering Meeting,
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Sammanfattning

Trä kännetecknas av en jämfört med andra byggnadsmaterial förhållandevis stor variation i hållfasthetsegenskaperna. Hållfastheten är starkt knuten till förekomsten av kvistar i virket. Kvistarna uppträder ofta i grupper mer eller mindre regelbundet utmed virkets längd. Dessa kvistgrupper kan karakteriseras som "svaga zoner" i den meningen att brotten initieras i dessa områden med fiberstörningar.

I en tidigare etapp har provkroppar som endast innehöll vardera en svag zon i provningsspannet böjts till brott. Den experimentella metodiken bestod i att kapa virket i segment så att varje segment endast innehöll en svag zon. Provkropparna tillverkades genom att fingerskarva segmenten med virke av hög kvalitet i ändarna. Resultaten från böjprovningarna analyserades med en hierarkisk modell med två nivåer.

Syftet med denna etapp har varit att ytterligare undersöka giltigheten hos den framtagna hierarkiska modellen för böjhållfasthetens variation inom och mellan olika virkesdelar. Därför har långa provkroppar som innehåller ett flertal svaga zoner inom provningsspannet böjts till brott.

Provningsresultaten visar att böjhållfastheten hos de långa virkesdelarna i medeltal är 5 till 15% lägre än vad den hierarkiska modellen förutsäger. Energibetraktelser visar att minskningen i bärförmåga inte enbart behöver vara en effekt orsakad av ett större antal möjliga brottställen i de långa virkesdelarna jämfört med de korta. Den stora mängden elastisk energi som frigörs i en lång böjbelastad virkesdel i samband med ett begynnande brott kan innebära att en högre lastnivå ej kan uppnås såsom är fallet i ett mer kontrollerat långsamt framväxande brott.

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A WEAK ZONE MODEL FOR TIMBER IN BENDING

by

B Källsner
K Salmela
Swedish Institute for Wood Technology Research
Sweden
O Ditlevsen
Technical University of Denmark
Denmark

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A weak zone model for timber in bending

B. Källsner[1], O. Ditlevsen[2], K. Salmela[1]
[1]Swedish Institute for Wood Technology Research
[2]Department of Structural Engineering and Materials
Technical University of Denmark

Summary

In order to verify a stochastic model for the variation of the bending strength within and between structural timber members, tests with long members subjected to constant bending moment have been performed. The span with constant moment contained between five and nine weak zones, i.e. zones with a cluster of knots. In a previous investigation test specimens, each containing one weak zone, have been tested in bending separately. Based on these tests a hierarchical model with two levels was formulated.

The test results show that the bending strength of the long timber members on the average is 5 to 15% lower than is predicted by the proposed hierarchical model. Energy considerations show that the reduction in strength of long beams may not be solely a statistical effect caused by an increased number of possible failure modes in the long beams as compared to the short test specimens. The large elastic energy released in a long highly bent beam at the onset of failure may mean that a later higher external load level cannot be realised as in a controlled slowly progressing failure.

It should be reflected in the codes that the elastic energy plays a role with respect to the size effect. Therefore, if the hierarchical model is codified, information should be given both on the largest load-carrying capacity of a weak zone, and on the capacity at the first occurring weakening of the zone.

1 Background

Compared to other structural materials, timber is characterised by a rather large variation of the strength properties. The grading of structural timber in Europe is usually based on the so called "grade determining defect" i.e. the defect which is believed to give the lowest strength. In traditional design of structural timber in bending, it is assumed that the load-carrying capacity is equal to the strength of this weakest part all over the length. A more accurate design based on a probabilistic method makes it possible to take advantage of the timber having higher strength in other parts. In order to do so, it is necessary to know the strength variation both within and between the timber members.

It is reasonable to assume that the variation of the bending strength within structural timber members to a large extent is connected to the occurrence of knots along the members. Structural timber in Sweden is produced from spruce (Picea Abies) and pine (Pinus Silvestris). In both species the branches grow in a rather regular pattern characterised by the branches forming whorls along the stem. Between these groups of branches, singular smaller branches exist, but these knots do not affect strength significantly.

It is complicated to determine the bending strength of all weak zones within a timber member experimentally since bending failure is often accompanied by fissures propagating along the member. Consequently it is difficult to obtain separate bending failures of sections close to each other without any mutual influence on the load-carrying capacity by the adjacent weak zones. There seem to be very few investigations where direct measurements have been carried out of the bending strength of weak zones along timber members.

Källsner and Ditlevsen (1994) presented a test method where the finger joint technique was used. In order to model the variation of the bending strength, a hierarchical model with two levels was used, see chapter 4. The experimental data were reasonably well represented by the formulated model. In order to further examine the validity of the model, the strength of long timber members subjected to a constant bending moment is investigated in a second phase of that project (Källsner, Ditlevsen and Salmela, 1997). The evaluation of these test results has not yet been finalised. A more detailed report will be published as a master thesis by Salmela (1997).

2 Objective

The purpose of this paper is to give a brief overview of the project and to present the most important results so far obtained and to compare the results with results obtained from other investigations.

3 Experimental investigation

3.1 Selection of timber

In order to obtain a test material with not too high a variation of the mechanical properties which could arise from different growth conditions, the timber was selected from a limited area of about 10 000 m^2. The intention is, in later phases of the project, to sample timber from other regions in order to see if the proposed model needs some modification with respect to different growth conditions. Only spruce was cut, and the trees were randomly taken from the area. One selection criterion, however, was that at least two logs, 4800 mm long, could be cut from each tree, and that at least two timber members could be sawn from each log. Another criterion was that logs with rot were excluded. All logs were sawn through the pith. After drying in a chamber kiln, the timber members were planed to their final dimension 45 mm x 120 mm. The total number of members was more than 500. Prior to testing all timber members were conditioned in a climate of 20 °C and 65 % relative humidity.

During the first two phases of the project only the two central twin members sawn from the second log (number 2 from the ground level) and closest to the pith were selected. In order to study the influence of the knot cluster frequency on the bending strength, all planks were grouped with respect to the number of weak zones. The number of weak zones was determined with all timber members belonging to the same log placed next to each other. This meant that a weak zone in one of the timber members always had a corresponding weak zone in the twin member. Consequently it often happened that, in a specific weak zone, there was only small knots or grain deviations while the corresponding weak zone of the twin member contained big knots. It must be pointed out that it was sometimes difficult to judge whether there was a weak zone or not, especially in the case of slow-grown timber with short distances between the knot whirls.

3.2 Method and scope

The main idea during the first phase of the project was to determine the bending strength of the timber at each weak zone (cluster of knots) separately. For that reason the timber members were cut into pieces, each of them containing one knot cluster. Each of these pieces were then finger jointed together with pieces of timber without defects, thus forming a specimen containing only one weak zone in the centre, see Figure 1. By this procedure the test specimens could be tested in the normal way. As has already been mentioned the timber members were grouped with reference to the number of weak zones within each member. The intention was to achieve about the same number of strength values in each group. Thus more timber members were taken from the groups with a few number of weak zones. The members were selected randomly from each group. The total number of timber members selected for the tests was 26.

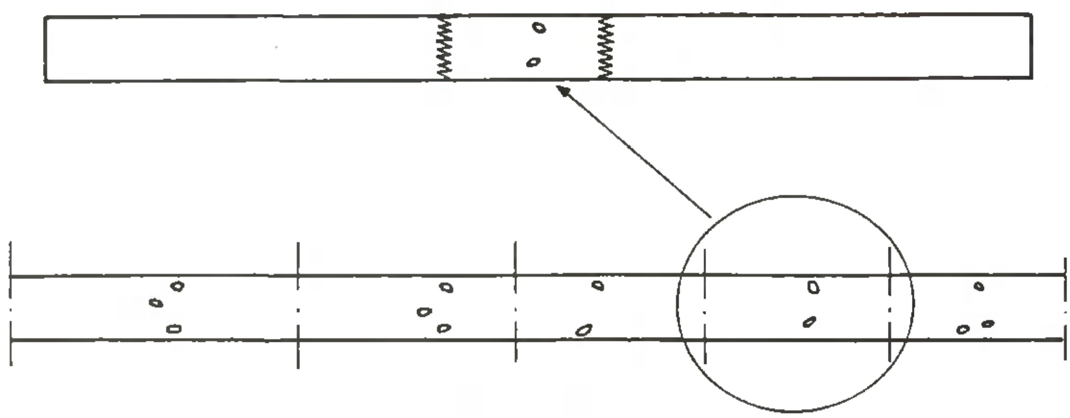

Figure 1 *Example of how a timber member was cut into pieces and finger jointed with pieces of timber of high strength at the ends.*

The purpose of the second phase of the project was to investigate if the hierarchical model of the bending strength variation, developed during the first phase, could be used to determine the strength of long timber members subjected to a constant bending moment. For that reason tests with long timber members containing many weak zones were performed. The bending tests with long timber members consisted of two series of specimens:

Test series 1 comprised the twin members of the 26 ones which were tested during phase 1 when the bending strength of each weak zone was determined separately.

Test series 2 comprised 14 pairs of twin members, each pair being sawn from each side of the pith.

3.3 Material parameters

A number of material parameters were measured. The most important ones were: The moduli of elasticity in edgewise and flatwise bending, knots, position of pith, annual ring width, density and moisture content.

3.4 Test equipment and test procedure

During phase 1 of the project, when each weak zone was tested separately (Figure 2), the test procedure given in the standard ISO 8375 was followed. Denoting the depth of the timber member by h, the distance subjected to constant bending moment was 6 h and the total span was equal to 18 h. All the test specimens cut from the same timber member were tested with the same edge in tension. The tension side was chosen at random.

During phase 2 the test arrangements had to be modified according to Figure 3. The total span was increased from 18 h to 36 h, and the span with constant moment was increased from 6 h to 30 h. To prevent lateral buckling, the test specimens were braced at the third points. Due to the long span of the test specimens, the influence of large displacements had to be taken into account by correcting the measured bending moments. Each timber member belonging to test series 1 was tested with the same edge in tension as the corresponding twin member of phase 1. For the timber members belonging to test series 2, each pair was tested with the same edge in tension. The tension side for each pair was chosen randomly.

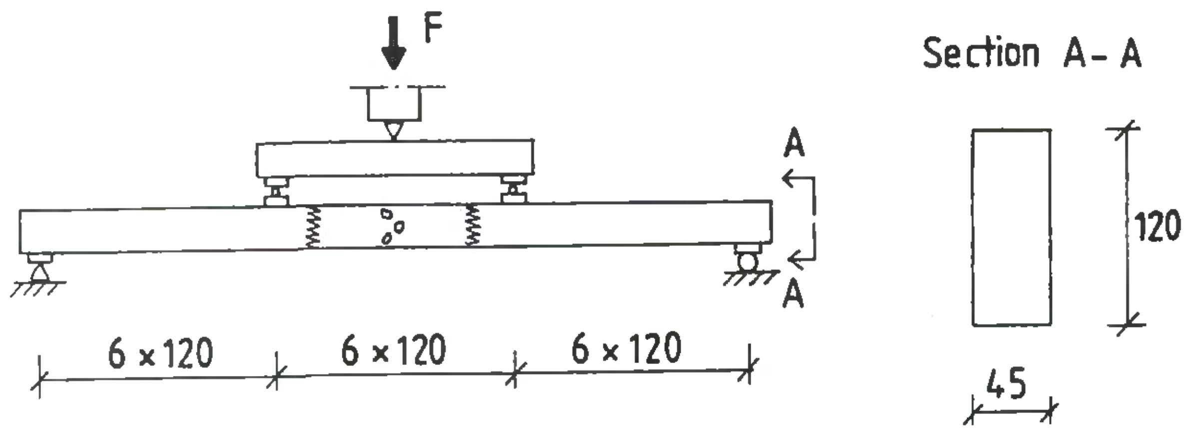

Figure 2 *Principal test arrangement when one weak zone was tested at a time. All sizes in mm.*

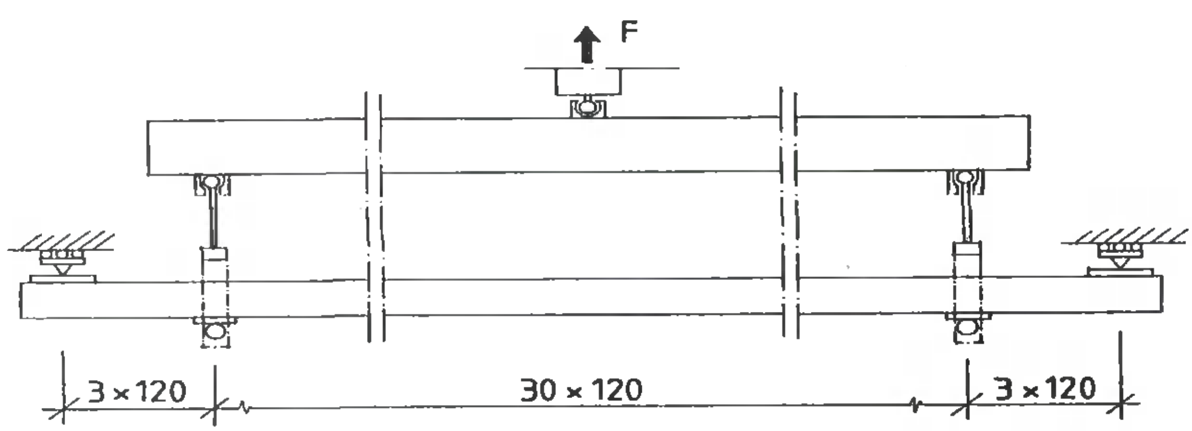

Figure 3 *Principal test arrangement when timber members containing many weak zones were tested. All sizes in mm.*

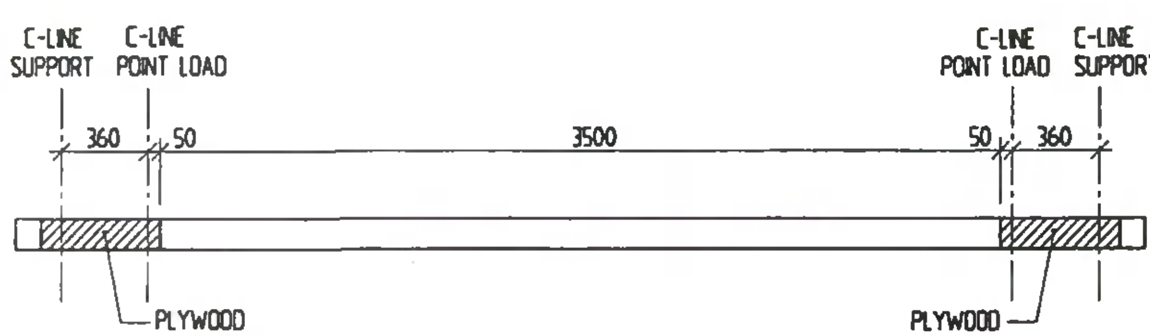

Figure 4 *Test specimen reinforced with plywood. All sizes in mm.*

To prevent failure in the zone between the supports and the point loads, all test specimens of phase 2 were reinforced with plywood at the ends according to Figure 4. The plywood was placed such that the distance between the edge of the plywood and the first weak zone in the centre span should be the same at both ends.

In both phase 1 and phase 2, the load was applied at a constant rate of displacement.

4 Probabilistic modelling

This chapter gives a brief description of the model used by Källsner and Ditlevsen (1994). This is the simplest type of probabilistic model for describing equicorrelation between the bending strength values of the weak zones within the same timber member.

4.1 Hierarchical probabilistic model of weak cross-section strengths

In connection with the description of the models, a weak zone is idealised to be concentrated as a point. Thus the terminology "weak zone" is replaced by "weak cross-section".

Reported results show indications of equicorrelation (i.e., a constant correlation independent of separation) between the bending strengths of the weak cross-sections within the same beam, Riberholt and Madsen (1979), Williamson (1994), Isaksson (1996). The simplest type of probabilistic model with this property is the so-called hierarchical model with two levels. This model is as follows.

For a given beam with k identified weak cross-sections, it is assumed that the bending strengths of the k cross-sections are

$$X + Y_1, \ldots, X + Y_k \tag{1}$$

where $Y_1, \ldots, Y_k$ are mutually independent random variables of zero mean and standard deviation σ_Y. The variable X is a random variable over the population of beams. If the variables $Y_1, \ldots, Y_k$ are assumed to be independent of X, the variance of the bending strength of a weak cross-section is

$$\mathrm{Var}[X + Y_i] \;=\; \mathrm{Var}[X] + \mathrm{Var}[Y_i] \;=\; \sigma_X^2 + \sigma_Y^2 \tag{2}$$

The covariance between the strengths of two weak cross-sections in the same beam is

$$\mathrm{Cov}[X + Y_i, X + Y_j] \;=\; \mathrm{Var}[X] \;=\; \sigma_X^2 \tag{3}$$

such that the correlation coefficient becomes

$$\rho[X + Y_i, X + Y_j] \;=\; \frac{\sigma_X^2}{\sigma_X^2 + \sigma_Y^2} \tag{4}$$

Thus the correlation coefficient is independent of i and j for $i \neq j \leq k$, that is, the correlation is independent of the distance between the two weak cross-sections. Moreover, if it is assumed that the distributions of X and Y are independent of $k \geq 1$, it follows that the model is an equicorrelation model.

4.2 Probabilistic model for beam subjected to constant moment

The physical model for the beam strength is based on the assumption that failure can occur only at a finite number of weak cross-sections along the beam. The bending strength S of a beam subjected to a constant bending moment along the entire length and containing $k > 0$ weak cross-sections can be described by the random variable

$$S = X + \min\{Y_1, \ldots Y_k\} \tag{5}$$

where X and Y are defined as in the previous section. Assuming that $X, Y_1, \ldots, Y_k$ all have normal distributions (not a crucial assumption) it follows that the distribution function of the bending strength S is

$$F_S(z|k) = P(X + \min\{Y_1, \ldots, Y_k\} \leq z) = \frac{1}{\sigma_X} \int_{-\infty}^{\infty} \left[1 - \Phi\left(-\frac{z-x}{\sigma_Y} \right)^k \right] \varphi\left(\frac{x-\mu_X}{\sigma_X} \right) dx \tag{6}$$

where $\varphi(\cdot)$ is the standardised normal density function, $\Phi(\cdot)$ is the standardised normal distribution function and μ_X is the mean of X. Since $F_S(z|0) = 0$ this formula is also valid for $k = 0$. Assuming that K is a random variable with $k \in \{0, 1, 2, \ldots\}$ as outcome with probability p_k, the distribution function of S becomes

$$F_S(z) = \sum_{k=0}^{\infty} F_S(z|k)\, p_k = 1 - \frac{1}{\sigma_X} \int_{-\infty}^{\infty} E\left[\Phi\left(\frac{x-z}{\sigma_Y} \right)^K \right] \varphi\left(\frac{x-\mu_X}{\sigma_X} \right) dx \tag{7}$$

in which the expectation

$$E\left[\Phi\left(\frac{x-z}{\sigma_Y} \right)^K \right] = \sum_{k=0}^{\infty} \Phi\left(\frac{x-z}{\sigma_Y} \right)^k p_k \tag{8}$$

can be expressed by the so-called probability generating function $\psi(x) = E[x^K]$ of the integer random variable K.

5 Results and analysis

Three main types of failure modes were identified during the tests in phase 1, namely:

- B Bending failure in the centre part between the finger joints
- F Failure in one of the finger joints
- S Failure in one of the end parts outside the finger joints

A combined type of failure mode BF could also be observed which normally started as a fissure in the vicinity of a knot and propagated to one of the finger joints where it caused a failure in the finger joint.

The test results obtained in phase 1 indicate that there exists an equicorrelation between the bending strength values of the weak zones within the same piece of structural timber. The simplest type of probabilistic model with this property is the hierarchical model with two levels. Due to failure in the finger joints in a great number of the tests the hierarchical model had to be extended in the evaluation of the results (Källsner and Ditlevsen, 1994). The experimental data were reasonably well described by this extended model. By assuming that the failure modes B and BF were true bending failures, the following estimates of the parameters were obtained:

$$\mu_X = 49.77 \text{ N/mm}^2$$
$$\sigma_X = 7.03 \text{ N/mm}^2$$
$$\sigma_Y = 6.33 \text{ N/mm}^2$$
$$\rho = 0.55$$

Two main types of failure modes, tensile and compression failure, were observed during the testing of the long-span timber members in phase 2. Only in a few cases were permanent deformations caused by excessive compressive stresses observed before the test specimens collapsed. The final failures developed very fast. They often started with a fissure propagating from one weak zone (e.g. a knot on the tensile side of the specimen) to an adjacent weak zone where the timber member was broken. This first failure was often followed by secondary failures in the weak zones closest to the point loads. Consequently the test specimens were often broken into three or even four pieces.

For a beam subjected to constant moment along the entire length and containing $k > 0$ weak zones, the distribution function of the bending strength $F_S(z \mid k)$ is given by equation (6). By using the bending strength values from phase 1 of the project, the curves in Figure 5 are obtained. It is obvious that the bending strength decreases with an increased number of weak sections.

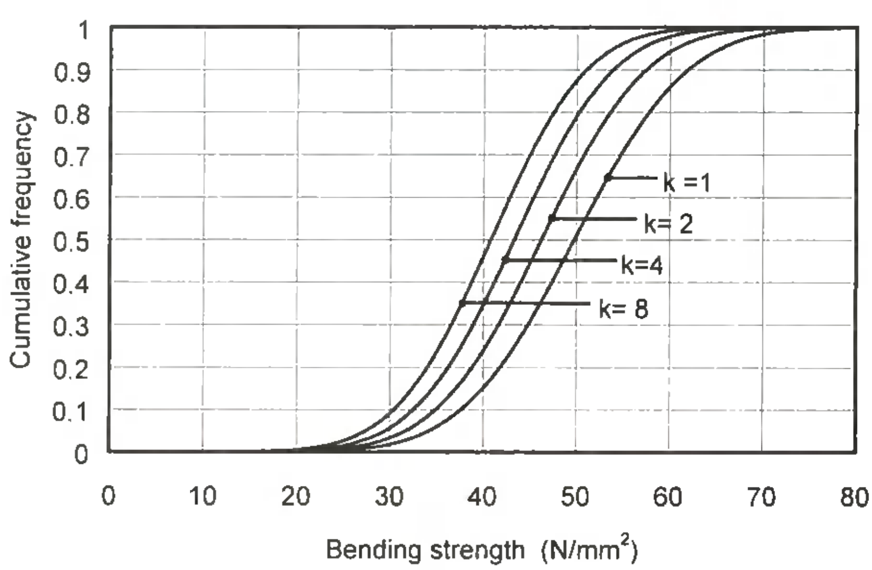

Figure 5 *Distribution function for bending of timber members under constant bending moment and k weak zones.*

Since test series 1 and 2 consist of timber members with different number of weak zones within the test span, it is better to compare the test results with a weighted distribution function $F_S(z)$ according to equation (7). In Figure 6 and Figure 7 the bending strength values obtained from test series 1 and 2 are plotted together with the weighted distribution functions (the curves to the left). The weighted distribution functions for test series 1 and 2 are almost identical, and they are also very close to the distribution function

$F_S(z \mid k)$ corresponding to 6 weak cross-sections. The curves to the right in the figures show the distribution function $F_S(z \mid k)$ according to equation (6) with $k = 1$, i.e. with one weak zone in the span subjected to constant moment.

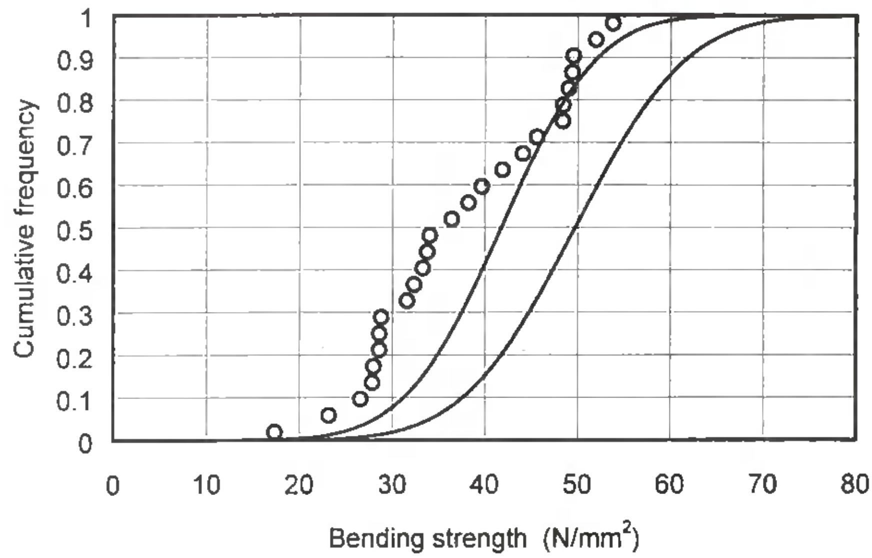

Figure 6 *Empirical distribution function for the bending strength data of test series 1 plotted together with the estimated distribution function (7) obtained from the hierarchical model (left curve). The curve to the right shows the estimated distribution function (6) for one weak zone.*

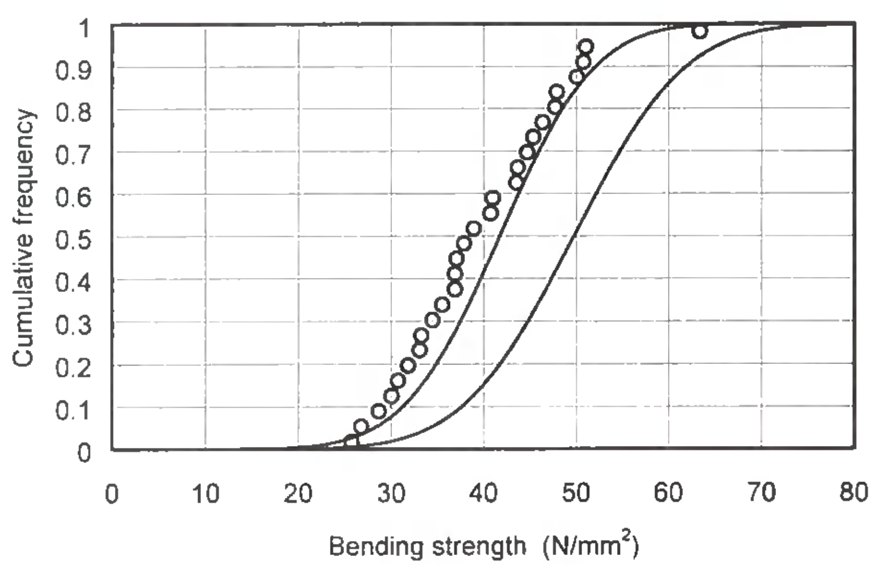

Figure 7 *Empirical distribution function for the bending strength data of test series 2 plotted together with the estimated distribution function (7) obtained from the hierarchical model (left curve). The curve to the right shows the estimated distribution function (6) for one weak zone.*

The plots show that the empirical distribution functions are shifted further to the left of the predicted distribution function than can be explained solely by referring to the statistical uncertainty related to the small sample size. It is also seen that the deviation to the left is largest for test series 1, where the test specimens are twins of the ones tested during phase 1. However, the difference between the positions of the two empirical distribution functions cannot be excluded to be a result of statistical uncertainty combined with the fact that the predicted distribution functions also are affected by statistical uncertainty.

In part the general shift to the left relative to the predicted distribution functions can be explained as follows. During phase 1, where only one weak zone was tested at a time, two main types of failures were observed. These were bending failure in the weak zone and failure in the finger joint. The bending failures always took place or started at the weak zone in the test specimen. One type of failure that could be observed was when a fissure propagated from a knot on the tension side of the weak zone. Sometimes this fissure led to an immediate collapse at the weak zone and sometimes the fissure stopped propagating and the load could be further increased. Since there were no adjacent weak zones in the test specimens, there was no influence from fissures that started from other weak zones propagating to the weak zone tested. It is obvious that this kind of fissures would have led to lower strength values and that such failure types may be possible in long beams.

The reason for the shift to the left may also partly be that the long timber members tested during phase 2 of the project contained more elastic energy than the standard beams tested during phase 1. As argued in (Källsner, Ditlevsen and Salmela, 1997), it can be assumed that the large elastic energy stored in a long highly bent beam before the onset of failure causes the failure to progress to total failure directly at the first decrease of the load. At least partly, the shift to the left is then explainable by the external force variation measured during the deformation controlled bending test. During the slowly progressing failure the external force is observed to vary upwards and downwards with its largest global value not necessarily coinciding with the first occurring local maximal value. The appearance of a shift to the left is then a consequence of the applied definition of the load-carrying capacity as being the globally largest value of the measured load during the test.

It is seen that there is a tendency to a better fit between measured and theoretical values in the upper strength range than in the lower and medium ranges. This indicates that during the process of controlled slowly progressing failure, as observed in the experiments of phase 1, there are larger differences between the first observed local maximum of the external load and the global maximum of all the following local maxima for the low strength timber than for the high strength timber. For the high strength timber this confirms the expectation that the first local maximum of the load is also the global maximum. Such timber has only small knots on the tension side. The failure may start as a compression failure but the final failure is usually a tension failure. This type of failure develops as a total collapse of the timber and the external load cannot be increased further. Thus it makes no difference whether it is a short or a long beam. Moreover, the empirical distribution function in Figure 6 shows a decrease of the slope (that is, the probability density) in the medium range. This indicates that the carrying capacity distribution may be bimodal. Such bimodality can be caused by some separation of the carrying capacity distribution of the compression failure mode from the lower placed carrying capacity distribution corresponding to the tension failure mode. The same bimodality tendency is not seen in the data of series 2 plotted in Figure 7.

6 Comparison with other investigations

6.1 Size effect

From tests of structural timber in bending it is well-known that the strength of members decreases with increasing span and that the strength depends on the load configuration.

This behaviour of the structural timber is often described as a "size effect". See for example Madsen (1992) and Barrett and Fewell (1990). A major contribution to the basic theory for analysing of this phenomenon was developed by Weibull (1939a,b). The two basic assumptions of this "weakest link theory" are that the material is perfectly brittle and that the strength variation is locally homogeneous with appearance like white noise, that is, with statistical independence of the strengths of any set of non-coinciding cross-sections. Thus the Weibull theory does not take account of the existence of at most a finite random number of weak cross-sections and the possible observation of their positions along the beam. Nor does it take the strength correlation (equi-correlation, according to the two-level hierarchical model) along the beam into account.

The size factor found in different investigations varies considerably. Rouger and Fewell (1994) demonstrate that the "size effect" could be significantly influenced by the sawing pattern and by the grading method. For a beam in bending tested at constant depth but with different lengths L, the size effect is often expressed by an exponent S_L which is calculated from the relation

$$\frac{f_m}{f_{m,ref}} = \left(\frac{L_{ref}}{L}\right)^{S_L} \tag{9}$$

where f_m is the bending strength and where the subscript "ref" refers to a reference length of the beam. Madsen (1992) reports that the exponent S_L at the 50th percentile is in average equal to 0.18.

To be able to compare this value of S_L with the results presented in this paper, it seems reasonable to modify equation (9) so that the length L is replaced by the number of weak zones k, i.e.:

$$\frac{f_m}{f_{m,ref}} = \left(\frac{k_{ref}}{k}\right)^{S_L} \tag{10}$$

By considering the bending strength of one weak zone as the reference level, it is obvious that $k_{ref} = 1$ and $f_{m,ref} = 49.77$ N/mm². If the 50th percentile of the empirical distribution function for the bending strength data of test series 1 and 2 are put into equation (10) assuming 6 weak zones (which is very close to the mean value), the exponent S_L obtains the values given in Table 1. These values are close to the value 0.18 reported by Madsen (1992). It is of special interest to compare these values with values obtained by the hierarchical model. Assuming 6 weak zones gives $S_L = 0.097$. This value will come out

Test series	f_m N/mm²	S_L
1	35.27	0.192
2	38.44	0.144

Table 1 Exponent S_L for test series 1 and 2.

larger when taking into account that the release of elastic energy may trigger the total failure during the period of damage development before reaching the maximal static carrying capacity of the weakest of the single weak zones of the beam. According to this the strength reduction by length may possibly still be solely a statistical effect. Further analysis of the data including the measured force-deflection curves will reveal whether this conjecture can be confirmed.

6.2 The Lund investigation

Isaksson (1996) has presented results of an investigation similar to the one reported in this paper. Specially designed equipment for clamping of the structural timber when being tested in bending was used. The timber was sampled at a sawmill. The nominal dimension of the timber was 45 mm x 145 mm. For a timber length of 5.1 m it was possible to test 4-7 weak zones depending on how the distances between the zones were distributed.

The mean bending strength of the weak zones was estimated to 57.4 N/mm^2 i.e. considerably higher than in the investigation reported in this paper (49.8 N/mm^2). The standard deviation of the bending strength was estimated to 13.4 N/mm^2 compared to 9.5 N/mm^2 reported here. The higher standard deviation found by Isaksson is probably an effect of a higher σ_X i.e. variation between the timber members. The reason for this may be a consequence of the sampling. The timber was probably taken from a larger area than the timber reported here.

In a simulation based on the test data from the separate timber members, Isaksson obtains $S_L = 0.106$ at the 50th percentile. This value is very close to the value 0.097 found by the hierarchical model and may indicate that the strength variation within the members is of the same magnitude as reported in this paper.

7 Conclusions

Tests with long timber members subjected to constant bending moment indicate that the bending strength is 5 to 15% lower than was predicted by the proposed hierarchical model.

It should be noted that there is a fundamental difference between the Weibull modelling and the discrete weak zone modelling. According to the Weibull model the number of weak zones is infinite, which for small timber members and members with peak moments means that the timber in the design is not fully utilised.

There are indications that in order to use the statistical information from single weak zone bending strength testing to predict the bending strength distribution for long beams, the release of stored elastic energy should be taken into consideration. It should therefore be reflected in the codes that the elastic energy plays a role with respect to the size effect. Therefore, if the hierarchical model is codified, information should be given both on the largest load-carrying capacity of a weak zone, and on the capacity at the first occurring weakening of the zone.

The test results are still subject to analysis, and the conclusions of this paper are therefore preliminary.

Acknowledgements

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References

Barrett J.D.; Fewell A.R. (1990): Size factors for the bending and tension strength of structural timber. Proc of the CIB-W18 Meeting, Lisbon, Portugal, Paper 23-10-3.

Isaksson, T. (1996): Variability of bending strength within timber elements. Licentiate thesis at Department of Structural Engineering, Lund Institute of Technology, Sweden.

Källsner, B.; Ditlevsen, O. (1994): Lengthwise bending strength variation of structural timber. Paper presented at the IUFRO/ S5.02 Meeting, Sydney, Australia.

Källsner, B.; Ditlevsen, O.; Salmela, K. (1997): Experimental verification of a weak zone model for timber in bending. Paper presented at the IUFRO/ S5.02 Meeting, Copenhagen, Denmark.

Madsen B. (1992): Structural behaviour of timber. Timber Engineering Ltd., North Vancouver BC, Canada.

Riberholt, H.; Madsen, P. H. (1979): Strength distribution of timber structures, measured variation of the cross sectional strength of structural timber. Structural Research Laboratory, Technical University of Denmark, Report No. R 114.

Rouger F.; Fewell T. (1994): Size effects in timber. Novelty never ends. Proc. of the CIB-W18 Meeting, Sydney, Australia, Paper 27-06-2.

Salmela, K. (1997): Bending strength of timber in long-span testing (in Swedish). Master thesis at Building Materials, Department of Buildings and Building Services, Royal Institute of Technology, Stockholm, Sweden (under preparation).

Weibull W. (1939a): A statistical theory of the strength of materials. Royal Swedish Institute for Engineering Research, Proceedings, No. 141, p. 45.

Weibull W. (1939b): The phenomenon of rupture in solids. Royal Swedish Institute for Engineering Research, Proceedings, No. 153, p. 55.

Williamson J.A. (1994): Statistical dependence of timber strength. Paper presented at the IUFRO/ S5.02 Meeting, Sydney, Australia.

Bo Källsner, Ove Ditlevsen, Kirsi Salmela

A Weak Zone Model for Timber in Bending

Paper presented at the CIB/W18 Meeting,
August 1997,
Vancouver, Canada

Sammanfattning

För att verifiera en stokastisk modell som beskriver variationen i böjhållfasthet inom och mellan olika virkesdelar har provningar av långa virkesdelar belastade med konstant moment utförts. Provningsspannet har innehållit mellan fem och nio svaga zoner. Med begreppet "svag zon" menas ett område där ett brott kan initieras. De svaga zonerna utgörs normalt av kvistgrupper som uppträder mer eller mindre regelbundet i virkets längdriktning. I en tidigare etapp har korta virkesdelar som endast innehåller en svag zon inom området med konstant moment böjprovats till brott. Baserat på dessa provningsresultat formulerades en hierarkisk modell med två nivåer för variationen i böjhållfasthet.

Resultaten från provningarna av de långa virkesdelarna visar att böjhållfastheten hos de långa virkesdelarna i medeltal är 5 till 15% lägre än vad den hierarkiska modellen förutsäger. Energibetraktelser visar att minskningen i bärförmåga inte enbart behöver vara en effekt orsakad av ett större antal möjliga brottställen i de långa virkesdelarna jämfört med de korta. Den stora mängden elastisk energi som frigörs i en lång böjbelastad virkesdel i samband med ett begynnande brott kan innebära att en högre lastnivå ej kan uppnås såsom är fallet i ett mer kontrollerat långsamt framväxande brott.

En jämförelse av resultaten med nordamerikanska provningsresultat visar att minskningen i böjhållfasthet vid långa spännvidder är av samma storleksordning. En jämförelse av resultaten med provningsresultat framtagna i Lund visar att den rent statistiskt betingade minskningen i böjhållfasthet vid långa spännvidder är av samma storleksordning.

Stochastic model for bending strength of structural timber

B. Källsner[1], O. Ditlevsen[2]
[1]Swedish Institute for Wood Technology Research
[2]Department of Structural Engineering and Materials
Technical University of Denmark

Summary

The variation of the bending strength of structural timber members is to a large extent related to the occurrence of knots along the members. The knots often appear in clusters with a fairly constant distance between them. Based on tests of timber specimens each containing one single weak zone (a group of knots) a hierarchical stochastic model with two levels has been set up to describe the strength variation. Verifying tests with long timber members containing several weak zones indicate that the reduction in bending strength of long beams is mainly due to the statistical effect caused by the presence of several weak zones. It can be argued that the statistical effect is influenced by the large elastic energy released in a long highly bent beam at the onset of failure.

The proposed hierarchical model for the bending strength is judged to be sufficiently simple to be codified for practical use.

1 Introduction

Compared to other structural materials timber is characterised by a rather large variation of the strength properties. The grading of structural timber in Europe is usually based on the so called "grade determining defect" i.e. the defect which is believed to give the lowest strength. In traditional design of structural timber in bending it is assumed that the load-carrying capacity is equal to the strength of this weakest part all over the length. A more accurate design based on a probabilistic method makes it possible to take advantage of the timber having higher strength in other parts. To do so it is necessary to know the strength variation both within and between the timber members.

The within-member variation of bending strength has been studied in only few investigations. Riberholt and Madsen (1979) formulated a model where it was assumed that the timber was composed of short weak zones connected by sections of clear wood. Failure was assumed to occur only in the centre of the weak zones. The model included two mutually independent random variables, namely the strength of the weak zones and the distance between them. Czmoch et al. (1991) extended the Riberholt-Madsen model to include both correlation between the strengths of the weak zones and correlation between the distances between the weak zones.

It is complicated to determine experimentally the bending strength of each and every weak zone within a timber member since bending failure is often accompanied by fissures propagating along the member. Consequently it is difficult to obtain separate bending failures of sections close to each other without any mutual influence on the load-carrying capacity by the adjacent weak zones. There seem to be very few investigations where direct measurements have been carried out of the bending strength of weak zones along timber members. Källsner and Ditlevsen (1994) have presented a test method where the finger joint technique was used. The principle will be given in this report. Isaksson (1996) has presented results of a similar investigation where specially designed equipment for clamping of the structural timber when being tested in bending was used.

2 Objective

The purpose of this paper is to present a stochastic model that can be easily codified to describe the variation of the bending strength within and between structural timber members.

3 Definition of weak zone

It is reasonable to assume that the variation of the bending strength within structural timber members is to a large extent related to the occurrence of knots along the members. Structural timber in Sweden is produced from spruce (Picea Abies) and pine (Pinus Silvestris). In both species the branches grow in a fairly regular pattern characterised by the branches forming whorls along the stem. Between these groups of branches, single smaller branches exist, but these knots do not affect strength significantly.

When a log is sawn into planks it often happens that one of the planks only contains small knots or grain deviations in some of the places where groups of knots on the surface of the log were observed. This makes it difficult to judge whether there is a weak zone or not when inspecting a single plank. Therefore it is best to determine the weak zones when all planks belonging to the same log are placed next to each other. Even if there are only small knots or grain deviations in some of the weak zones, the strength of the material in these zones is likely to be lower than in the surrounding material. Defining weak zones in this way there will be a natural link to the growth of the trees. In the selection of the test specimens this definition of a weak zone has been used.

4 Probabilistic model

This chapter gives a brief description of the model used by Källsner and Ditlevsen (1994).

4.1 Hierarchical probabilistic model of weak cross-section strengths

In the description of the model a weak zone is idealised as concentrated to a point. Thus the terminology "weak zone" is replaced by "weak cross-section".

Reported results show indications of equicorrelation (i.e., a constant correlation independent of separation) between the bending strengths of the weak cross-sections within the same beam, Riberholt and Madsen (1979), Källsner and Ditlevsen (1994), Williamson (1994) and Isaksson (1996). The simplest type of probabilistic model with this property is the so-called hierarchical model with two levels. This model is as follows.

For a given beam with k identified weak cross-sections it is assumed that the bending strengths of the k cross-sections are

$$X + Y_1, \ldots, X + Y_k \tag{1}$$

where $Y_1, \ldots, Y_k$ are mutually independent random variables of zero mean and standard deviation σ_Y. The variable X is a random variable over the population of beams. If the variables $Y_1,...,Y_k$ are assumed to be independent of X, the variance of the bending strength of a weak cross-section is

$$\mathrm{Var}[X+Y_i] = \mathrm{Var}[X]+\mathrm{Var}[Y_i] = \sigma_X^2+\sigma_Y^2 \tag{2}$$

The covariance between the strengths of two weak cross-sections in the same beam is

$$\mathrm{Cov}[X+Y_i, X+Y_j] = \mathrm{Var}[X] = \sigma_X^2 \tag{3}$$

such that the correlation coefficient becomes

$$\rho[X+Y_i, X+Y_j] = \frac{\sigma_X^2}{\sigma_X^2+\sigma_Y^2} \tag{4}$$

Thus the correlation coefficient is independent of i and j for $i \neq j \leq k$, that is, the correlation is independent of the distance between the two weak cross-sections. Moreover, if it is assumed that the distributions of X and Y are independent of $k \geq 1$, it follows that the model is an equicorrelation model.

4.2 Probabilistic model for beam subjected to constant moment

The physical model for the beam strength is based on the assumption that failure can occur only at a finite number of weak cross-sections along the beam. The bending strength S of a beam subjected to a constant bending moment along the entire length and containing $k > 0$ weak cross-sections can be described by the random variable

$$S = X+\min\{Y_1,...Y_k\} \tag{5}$$

where X and Y are defined as in the previous section. Assuming that $X, Y_1,...,Y_k$ all have normal distributions (not a crucial assumption) it follows that the distribution function of the bending strength S is

$$F_S(z|k) = P(X+\min\{Y_1,...,Y_k\} \leq z) = \frac{1}{\sigma_X}\int_{-\infty}^{\infty}\left[1-\Phi\left(-\frac{z-x}{\sigma_Y}\right)^k\right]\varphi\left(\frac{x-\mu_X}{\sigma_X}\right)dx \tag{6}$$

where $\varphi(\cdot)$ is the standardised normal density function, $\Phi(\cdot)$ is the standardised normal distribution function and μ_X is the mean of X. Since $F_S(z \mid 0) = 0$, this formula is also valid for $k = 0$. Assuming that K is a random variable with $k \in \{0,1,2,...\}$ as outcome with probability p_k, the distribution function of S becomes

$$F_S(z) = \sum_{k=0}^{\infty} F_S(z|k)\,p_k = 1-\frac{1}{\sigma_X}\int_{-\infty}^{\infty} E\left[\Phi\left(\frac{x-z}{\sigma_Y}\right)^K\right]\varphi\left(\frac{x-\mu_X}{\sigma_X}\right)dx \tag{7}$$

in which the expectation

$$E\left[\Phi\left(\frac{x - z}{\sigma_Y}\right)^K\right] = \sum_{k=0}^{\infty}\Phi\left(\frac{x - z}{\sigma_Y}\right)^k p_k \tag{8}$$

can be expressed by the so-called probability generating function $\psi(x) = E[x^K]$ of the integer random variable K.

4.3 Some aspects on the hierarchical model

The main advantage with the model is that it is very simple. The proposed hierarchical model includes two levels: one for the strength variation between the timber members and one for strength variation within the timber members. If needed, the model can easily be expanded to include more levels, e.g. the strength variation between different regions.

In the model each member is given a mean strength characterised by the random variable X and a within member strength variation characterised by the random variable Y. There are several reasons for these assumptions. Since the logs are normally sawn parallel to the pith the density will be relatively constant along the members. If the members are not too long the knots will be of similar size along the members. Certainly there are some trends of changes in the mechanical properties along the timber members, but these trends can normally be neglected in comparison with the random variations.

No distinction between compression and tension failures has been included in the model. The within-member variation seems not to be significantly influenced by different failure modes.

At grading of structural timber the distribution function of the strength will be truncated. The result will be a lower σ_Y and other values of μ_X.

5 Experimental verification

5.1 Tests of single weak zones

In the investigation by Källsner and Ditlevsen (1994) the main idea was to determine the bending strength of the timber at each weak zone (cluster of knots) separately. For this reason the timber members were cut into pieces, each of them containing one knot cluster. Each of these pieces were then finger jointed together with pieces of timber without defects, thus forming a specimen containing only one weak zone in the centre, see Figure 1. By this procedure the specimens could be tested in the normal way according to the standard ISO 8375, see Figure 2. In order to obtain a test material with not too high a variation of the mechanical properties which could arise from different growth conditions, the timber was selected from a limited area. Only spruce was cut. All logs were sawn through the pith. The timber members were planed to their final dimension 45 mm x 120 mm. All the test specimens cut from the same timber member were tested with the same edge in tension. The total number of timber members selected for the tests was 26. The tension side was chosen at random. Due to failure in the finger joints in a great number of the tests the hierarchical model had to be extended in the evaluation of the results. The experimental data were reasonably well described by this extended model. No evident mutual dependency between the bending strength and the distance between the weak zones was revealed. The following estimates of the parameters were obtained:

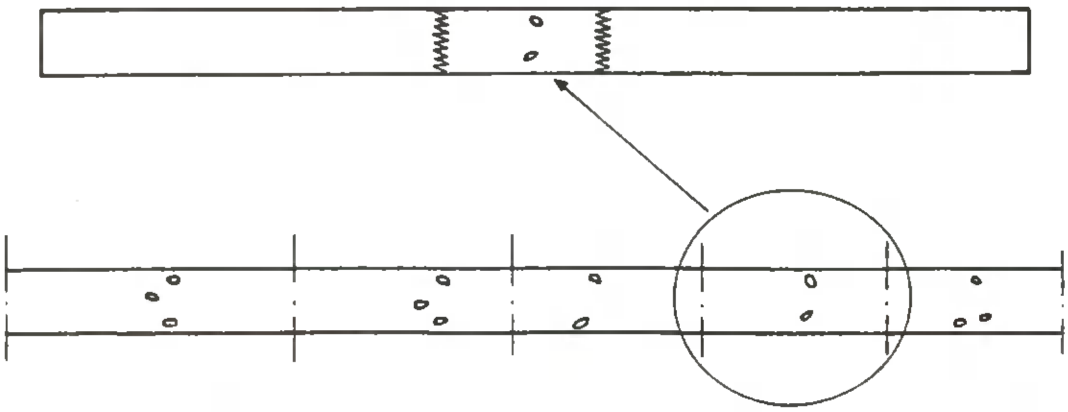

Figure 1 *Example of how a timber member was cut into pieces and finger jointed with pieces of timber of high strength at the ends.*

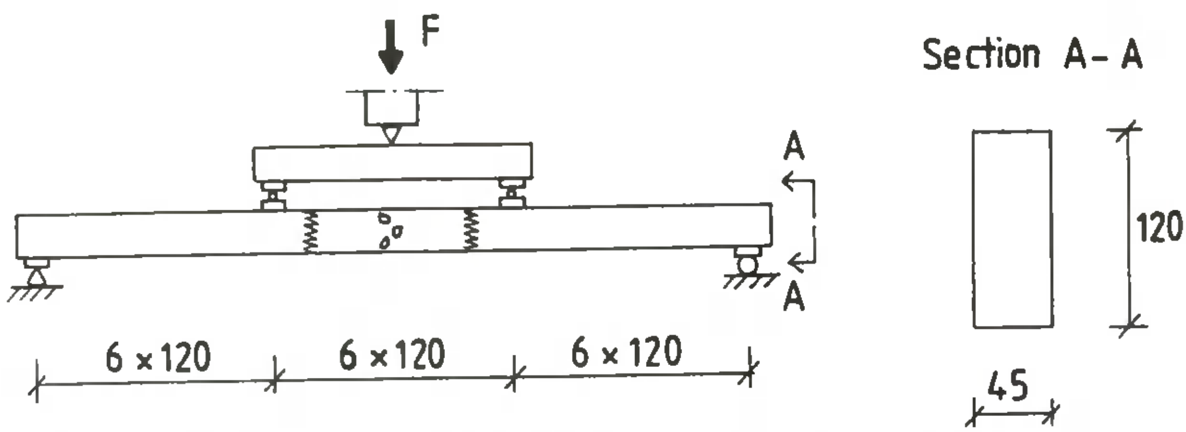

Figure 2 *Principal test arrangement when one weak zone was tested at a time. All sizes in mm.*

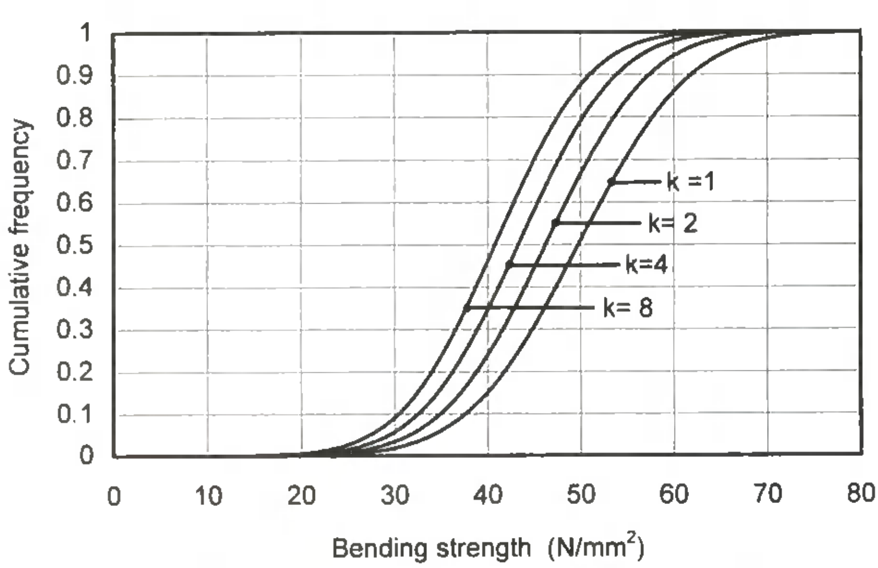

Figure 3 *Distribution function for bending of timber members under constant bending moment and k weak zones.*

$\mu_X = 49.77$ N/mm^2
$\sigma_X = 7.03$ N/mm^2
$\sigma_Y = 6.33$ N/mm^2
$\rho = 0.55$

For a beam subjected to constant moment along the entire length and containing $k > 0$ weak zones, the distribution function of the bending strength $F_S(z \mid k)$ is given by equation (6). By using the estimates of the bending strength values above, the curves in Figure 3 are obtained. It is obvious that the bending strength decreases with an increased number of weak sections.

5.2 Tests of long beams

To further verify the hierarchical model tests with long members subjected to constant moment have been performed (Källsner, Ditlevsen and Salmela, 1997). The span with constant moment contained between five and nine weak zones, i.e. zones each with a single cluster of knots. In this phase 2 of the project the members were taken from the same population as was used in phase 1. The bending tests with long timber members consisted of two series of specimens:

Test series 1 comprised the twin members of the 26 members tested during phase 1 when the bending strength of each weak zone was determined separately. A twin member is a member sawn from the same log but on the other side of the pith.

Test series 2 comprised 14 pairs of twin members, each pair being sawn from each side of the pith.

The principal test arrangement for timber members containing several weak zones is shown in Figure 4. The span with constant moment was 30 times the depth. Each timber member belonging to test series 1 was tested with the same edge in tension as the corresponding twin member of phase 1. For the timber members belonging to test series 2 each pair was tested with the same edge in tension. The tension side for each pair was chosen at random.

To prevent failure in the zone between the supports and the point loads all test specimens in phase 2 were reinforced with plywood at the ends as shown in Figure 5.

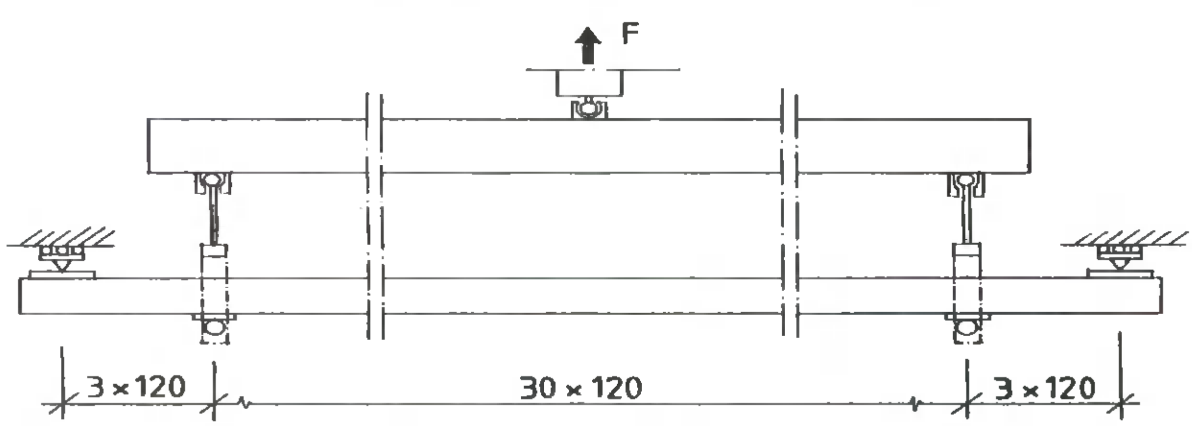

Figure 4 Principal test arrangement for timber members containing several weak zones. All sizes in mm.

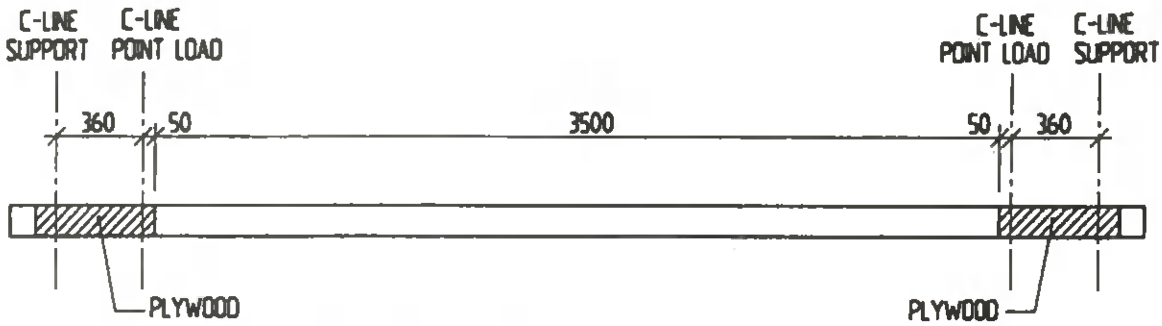

Figure 5 *Test specimen reinforced with plywood. All sizes in mm.*

Since test series 1 and 2 consist of timber members with different numbers of weak zones within the test span the test results are compared with the weighted distribution function $F_S(z)$ according to equation (7). In Figure 6 and Figure 7 the bending strength values obtained from test series 1 and 2 are plotted together with the weighted distribution functions (the curves to the left). The weighted distribution functions for test series 1 and 2 are almost identical, and they are also very close to the distribution function $F_S(z \mid k)$ corresponding to 6 weak cross-sections. The curves to the right in the figures show the distribution function $F_S(z \mid k)$ according to equation (6) with $k = 1$, i.e. with one weak zone in the span subjected to constant moment.

The plots show that the empirical distribution functions are shifted further to the left of the predicted distribution function than can be explained solely by referring to the statistical uncertainty related to the small sample size. It is also seen that the deviation to the left is largest for test series I, where the test specimens are twins of the members tested during phase 1.

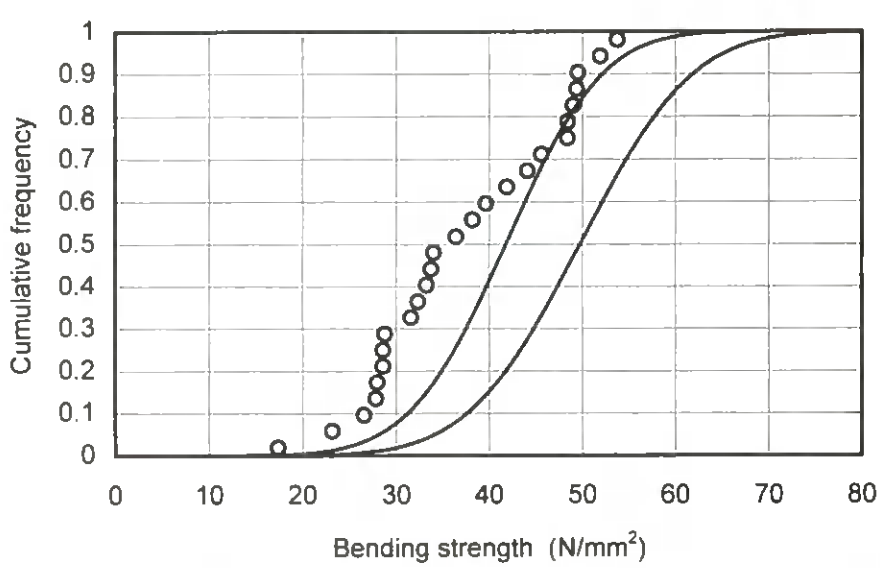

Figure 6 *Empirical distribution function for the bending strength data of test series 1 plotted together with the estimated distribution function (7) obtained from the hierarchical model (left curve). The curve to the right shows the estimated distribution function (6) for one weak zone.*

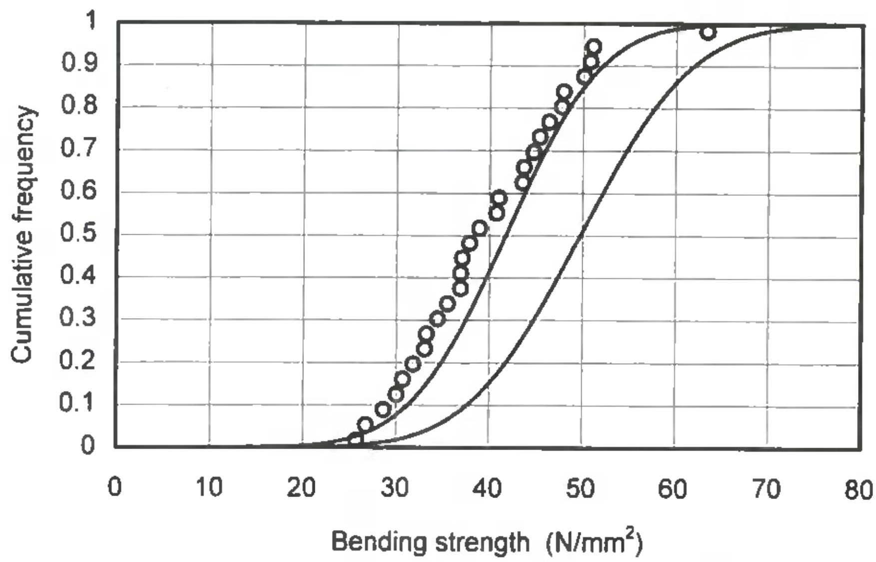

Figure 7 *Empirical distribution function for the bending strength data of test series 2 plotted together with the estimated distribution function (7) obtained from the hierarchical model (left curve). The curve to the right shows the estimated distribution function (6) for one weak zone.*

A preliminary evaluation of the tests results was presented in (Källsner, Ditlevsen and Salmela, 1997). It was concluded that the test results show that the bending strength of the long timber members on the average is 5 to 15% lower than is predicted by the proposed hierarchical model. Energy considerations show that the reduction in strength of long beams may not be solely a statistical effect caused by an increased number of possible failure modes in the long beams as compared to the short test specimens. The large elastic energy released in a long highly bent beam at the onset of failure can mean that a later higher external load level cannot be realised as in a controlled slowly progressing failure.

6 Conclusions

For the random variation of the bending strength within and between structural timber members a stochastic hierarchical model with two levels has been proposed in a previous paper.

Tests with long timber members subjected to constant bending moment indicate that the bending strength is 5 to 15% lower than was predicted by the hierarchical model.

There are indications that in order to use the statistical information from single weak zone bending strength testing to predict the bending strength distribution for long beams, the release of stored elastic energy should be taken into consideration.

The test results are still subject to analysis, and the conclusions of this paper are therefore preliminary.

Acknowledgements

Kirsi Salmela has done the testing of the long beams and also performed a great deal of the evaluation of the test results. This work will be published as a master thesis (Salmela, 1997) at the Royal Institute of Technology in Stockholm.

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References

Czmoch I., Thelandersson S. and Larsen H.J. (1991): Effect of within member variability on bending strength of structural timber. Proc. of the CIB-W18 Meeting, Oxford, United Kingdom, Paper 24-6-3.

Isaksson, T. (1996): Variability of bending strength within timber elements. Licentiate thesis at Department of Structural Engineering, Lund Institute of Technology, Sweden.

Källsner, B.; Ditlevsen, O. (1994): Lengthwise bending strength variation of structural timber. Paper presented at the IUFRO/ S5.02 Meeting, Sydney, Australia.

Källsner, B.; Ditlevsen, O.; Salmela, K. (1997a): Experimental verification of a weak zone model for timber in bending. Paper presented at the IUFRO/ S5.02 Meeting, Copenhagen, Denmark.

Riberholt, H.; Madsen, P. H. (1979): Strength distribution of timber structures, measured variation of the cross sectional strength of structural timber. Structural Research Laboratory, Technical University of Denmark, Report No. R 114.

Salmela, K. (1997): Bending strength of timber in long-span testing (in Swedish). Master thesis at Building Materials, Department of Buildings and Building Services, Royal Institute of Technology, Stockholm, Sweden (under preparation).

Williamson J.A. (1994): Statistical dependence of timber strength. Paper presented at the IUFRO/ S5.02 Meeting, Sydney, Australia.

Bo Källsner, Ove Ditlevsen

Stochastic Model for Bending Strength of Structural Timber

Paper presented at the seminar "Reliability Based Design of Timber Structures",
October 2/3, 1997
Paris, France

Sammanfattning

Variationen i böjhållfasthet hos konstruktionsvirke är i stor utsträckning relaterad till förekomsten av kvistar längs med virket. Kvistarna uppträder ofta i grupper med ett förhållandevis konstant avstånd mellan dem. Med utgångspunkt från böjprovningar av virkesdelar som innehåller enbart en svag zon (område med fiberstörningar orsakade av en kvistgrupp) inom provningsspannet har en hierarkisk stokastisk modell med två nivåer ställts upp som beskriver variationen i böjhållfasthet. Verifierande böjförsök med långa virkesdelar som innehåller flera svaga zoner inom provningsspannet indikerar att minskningen i böjhållfasthet hos långa virkesdelar i huvudsak orsakas av den statistiska effekten till följd av att flera svaga zoner uppträder i spannet. Det kan hävdas att minskningen i böjhållfasthet vid långa virkeslängder även orsakas av den stora elastiska energi som frigörs då brott initieras.

Den föreslagna hierarkiska modellen för böjhållfastheten bedöms vara tillräckligt enkel för att kunna ingå i en norm för dimensionering av träkonstruktioner.

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Trätek

INSTITUTET FÖR TRÄTEKNISK FORSKNING

Box 5609, 114 86 STOCKHOLM
Besöksadress: Drottning Kristinas väg 67
Telefon: 08-762 18 00
Telefax: 08-762 18 01

Åsenvägen 9, 553 31 JÖNKÖPING
Telefon: 036-30 65 50
Telefax: 036-30 65 60

Skeria 2, 931 77 SKELLEFTEÅ
Besöksadress: Laboratorgränd 2
Telefon: 0910-652 00
Telefax: 0910-652 65