Modelling the temperature dependences for Silicon Carbide BJTs

ALEJANDRO D. FERNÁNDEZ S.
Modelling the temperature dependences for Silicon Carbide BJTs

Alejandro D. Fernández S.

Supervisors: Prof. B. Gunnar Malm, Muhammad Waqar Hussain
Examiner: Prof. Ana Rusu
Abstract

Silicon Carbide (SiC), owing to its large bandgap, has proved itself to be a very viable semiconductor material for the development of extreme temperature electronics. Moreover, its electrical properties like critical field ($E_{crit}$) and saturation velocity ($v_{sat}$) are superior as compared to the commercially abundant Silicon, thus making it a better alternative for RF and high power applications.

The in-house SiC BJT process at KTH has matured a lot over the years and recently developed devices and circuits have shown to work at temperatures exceeding 500°C. However, the functional reliability of more complex circuits requires the use of simulators and device models to describe the behavior of constituent devices. SPICE Gummel Poon (SGP) is one such model that describes the behavior of the BJT devices. It is simpler as compared to the other models because of its relatively small number of parameters.

A simple semi-empirical DC compact model has been successfully developed for low voltage applications SiC BJTs. The model is based on a temperature-dependent SiC-SGP model. Studies over the temperature dependences for the SGP parameters have been performed. The SGP parameters have been extracted and some have been optimized over a wide temperature range and they have been compared with the measured data. The accuracy of the developed compact model based on these parameters has been proven by comparing it with the measured data as well. A fairly accurate performance at the required working conditions and correlation with the measured results of the SiC compact model has been achieved.

**Keywords:** silicon carbide (SiC), wide bandgap, bipolar junction transistor (BJT), SPICE modelling, SPICE Gummel Poon (SGP), compact modelling, transistor characterization, high temperature, integrated circuits.
# Table of Contents

Abstract

Acronyms and symbols

Acknowledgements

1 Introduction .............................................................................................................................. 1
1.1 Background .......................................................................................................................... 1
1.2 Problem ............................................................................................................................... 1
1.3 Purpose ................................................................................................................................ 1
1.4 Goal ..................................................................................................................................... 2
1.5 Methodology ........................................................................................................................ 2
1.6 Delimitations ....................................................................................................................... 2
1.7 Thesis Organization ............................................................................................................. 3

2 Theoretical Background ......................................................................................................... 4
2.1 Physical models of SiC ......................................................................................................... 4
   2.1.1 Incomplete dopant ionization ......................................................................................... 4
   2.1.2 Bandgap and bandgap narrowing .................................................................................. 6
   2.1.3 Carrier mobility ............................................................................................................. 7
   2.1.4 Limitations of the physical models ................................................................................ 8
2.2 The SPICE Gummel Poon model ...................................................................................... 9
   2.2.1 Model description .......................................................................................................... 9
   2.2.2 Limitations of the SGP model ....................................................................................... 15

3 Methodology .......................................................................................................................... 18
3.1 Measurement plan and setup ............................................................................................. 19
3.2 Parameter extraction and optimization .............................................................................. 20
   3.2.1 Extraction methodology ............................................................................................... 20
   3.2.2 Optimization methodology .......................................................................................... 31
3.3 Parameter scaling process .................................................................................................. 32

4 Parameters temperature scaling .......................................................................................... 34

5 Experimental results ............................................................................................................. 41

6 Conclusions and future outlook ........................................................................................... 52

References .................................................................................................................................. 53
Acronyms and symbols

β  Current gain for NPN transistors (If it isn’t specified differently)
β_M  Maximum value of the current gain
β_NPN  Forward current gain of NPN transistor
µ_n  Electron mobility
µ_p  Hole mobility
A  Ampere
AC  Alternating current
Al  Aluminium
Ar  Argon
BF  Ideal forward current gain
BJT  Bipolar junction transistor
BR  Ideal reverse current gain
BV  Breakdown voltage
C  Carbon
DC  Direct current
E_A  Acceptor ionization energy
E_a-b  Effective activation energy for transistor current gain
SGP  SPICE Gummel Poon
SiC  Silicon Carbide
E_{cr}  Critical electric field
E_D  Donor ionization energy
E_G  Energy bandgap
FGP  Forward Gummel Plot
IC  Integrated Circuit
i_B  Base current
i_C  Collector current
i_E  Emitter current
k  Boltzmann constant
n  Electron concentration
N  Nitrogen
N_B  Transistor base doping concentration
N_C  Density of states in conduction band
N_D  Net doping concentration for n-type doping
N_E  Transistor emitter doping concentration
n_i  Intrinsic carrier concentration
NMOS  n-channel MOSFET
NPN  Bipolar junction transistor where emitter, base and collector are
      n-, p- and n-doped respectively
N_V  Density of states in valence band
p  Hole concentration
PNP  Bipolar junction transistor where emitter, base and collector are
      p-, n- and p-doped respectively
q  Elementary charge
RB  Transistor base resistance
RBM  Transistor minimum base resistance
RC  Transistor collector resistance
RE  Transistor emitter resistance
RGP  Reverse Gummel Plot
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Si</td>
<td>Silicon</td>
</tr>
<tr>
<td>SiC</td>
<td>Silicon carbide</td>
</tr>
<tr>
<td>T</td>
<td>Temperature</td>
</tr>
<tr>
<td>V</td>
<td>Voltage</td>
</tr>
<tr>
<td>$V_{BC}$</td>
<td>Base-collector voltage</td>
</tr>
<tr>
<td>$V_{BE}$</td>
<td>Base-emitter voltage</td>
</tr>
<tr>
<td>$V_{CE}$</td>
<td>Collector-emitter voltage</td>
</tr>
<tr>
<td>$V_{EC}$</td>
<td>Emitter-collector voltage</td>
</tr>
<tr>
<td>$W_E$</td>
<td>Emitter width</td>
</tr>
</tbody>
</table>
Acknowledgements

I would like to thank my supervisor Prof. B. Gunnar Malm, for his guidance in terms of the SiC BJTs physical phenomena and technology, developing some ideas wouldn’t have been possible without it. On the same page I’m really thankful to my supervisor Muhammad Waqar Hussain, for his great dedication and patience. Answering many questions, discussing ideas, revising my work and introducing me to the laboratory measurements, is an effort that I must be grateful for.

I would like to show my deepest appreciation to my examiner Prof. Ana Rusu, for giving me the great opportunity to work at her research group and examining my work. I feel very honoured for being granted the chance to proof myself at working in a very interesting, taxing and rewarding project.

Many thanks to Raheleh Hedayati for introducing me to the high temperature measurement probe station. Thanks to my office roommate Anders Eklund for the nice discussions of our projects and random subjects in our spare time.

I would also like to thank the Circuit and Systems research group, for their feedback and support on my work.

Without naming anyone in particular, I’d like to thank my friends and classmates at my home university USB, and KTH for all their support and interest during my time working in this project.

And last, but not least, I’m more than grateful to my parents, my sister, my family and my girlfriend in Venezuela, who have supported me immensely in countless ways during the hardest of times in my home country. This work is dedicated to them.
1 Introduction

1.1 Background
In the last decades, electronics have found more niches in highly demanding systems, such as combustion monitoring, process control, nuclear energy production and the automotive, aerospace and naval industry [1]. These applications require reliable high temperature operation, which can overcome the use of dedicated cooling systems that increase the complexity (and therefore chance of failure), size, weight and cost. Moreover, high temperature measurement and data conversion systems would benefit largely from being as close as possible to the critical zone, since this enhances significantly signal integrity, which is critical for an accurate transmission to low temperature data storage/processing systems.

SiC based electronics offer a suitable solution in terms of reliable high temperature operation, based on the fact that the well-developed Si technology has its physical limitations. Wide bandgap semiconductors, such as SiC, are capable of overcoming Si technology limitations for extreme environments, by having a much lower intrinsic carrier concentration, lower p-n junction leakage and thermionic leakage [1]. For this reason, research on high temperature electronics has been motivated and different studies on SiC-IC technology have been reported recently.

1.2 Problem
The design and operational reliability of complex electronic systems requires the use of circuit and system level simulators that are able to replicate the behavior of constituent devices. But how can one predict and simulate the behavior of these devices?

Compact models of closed form equations describe the behavior of linear and nonlinear devices. For the particular case of semiconductor devices, the models used for simulations are also called “compact models”. SPICE Gummel Poon (SGP) is one of the simplest models that have been successfully used to describe the behavior of SiC Bipolar Junction Transistors (BJTs) [22]. Particularly, SGP modelling of high power SiC-BJTs have been proven to be accurate with respect to the physical device operation [14]. This being the case, would it be feasible to successfully model SiC-BJTs for low voltage, high temperature ICs over a wide temperature range with a SGP-based compact model?

1.3 Purpose
In general terms, following up the last question stated in section 1.2, this thesis presents the characterization and modelling of SiC-BJTs. More specifically it has the purpose to illustrate the development flow of a SGP-based DC compact model of SiC-BJTs for low voltage, high temperature ICs over a wide temperature range. Discussions over the accuracy of the model and its limitations with a focus on the temperature dependences will be addressed as well. In addition, recommended follow-up work and future outlook on the subject will be mentioned.
1.4 Goal
The main goal of the project is to successfully model the temperature dependences of the DC parameters of SGP model for SiC-BJTs. As a result, an accurate SGP-based DC compact model over a wide temperature range will be delivered. The relevant analytical temperature dependent expressions for the SGP parameters will be obtained. Plots of the model equations against temperature will be shown for comparison purposes against the measured data of the physical performance of the device.

1.5 Methodology
There are different ways to approach temperature modelling of devices. For the particular case of SiC-BJTs a good starting point is to take an existing compact model, and then modify and include, where necessary, the temperature scaling equations of model parameters. This process should be done in such a way that the model equations resemble the measured data over the covered temperature range [23].

This approach is not fault free, because certain physically measured device features may lead to unreasonable parameters values, due to the limitations of the compact models. Therefore, it can be found that a model parameter extracted at different temperatures doesn’t show any physically meaningful temperature dependence. On the other hand, several different sets of parameters could lead to fairly accurate fitting [23].

In addition, it can be found that the model could not possibly fit certain characteristics, independently of the variation of the parameters. For this reason, new temperature-dependent equations must be developed. Device physics theoretical equations may not be able to accurately describe the measured temperature features, making empirical or semi-empirical equations required. Therefore, it was needed to assume functional forms of these equations, based on the understanding of device physics, or observation of the experimental data. Then the parameters were extracted over the whole temperature range and finally the functional forms were updated iteratively [23].

Consequently, developing the temperature dependences for a particular compact model is not a straightforward task. Extracting model parameters over the temperature range and developing new temperature mapping equations for these model parameters is not enough and more complex algorithms must be used [23].

1.6 Delimitations
Certain device phenomena at high current injections, such as self-heating effects and quasi-saturation effects [15], base recombination on the parasitic pnp transistor [15] and base punch-through at low current injections, all have a strong temperature dependence, due to their relation with minority carrier injection and diffusion. Therefore it isn’t possible to know beforehand the functional form that the high current and low current parameters will take. In addition, the SGP model has its own limitation due to its simplicity; hence it doesn’t model these strong temperature-dependent effects. For this reason
these mentioned phenomena will not be taken into account for the compact modeling of devices. But it's recommended that follow-up works and studies on these matters are performed to obtain a more accurate compact model for any working condition.

1.7 Thesis Organization
This thesis is divided in five main sections: theoretical background, methodology, compact model development, experimental results and conclusions.

In the theoretical background section, fundamentals of the physical models of SiC, the SGP model description and parameter extraction will be explained. In this way, a clearer understanding of the parameter extraction and optimization methods and compact model development for a wide temperature range can be achieved. Limitations of the SiC physical models at high temperatures for device modelling are discussed. Special attention is dedicated to their inability to reproduce the experimentally observed curves of the temperature dependence of the model's parameters. Limitations of the SGP model in terms of its incapability of modeling certain features of the measured characteristic curves over the whole temperature range are addressed as well.

In the methodology section, first a detailed description of the measurement plan and setup will be presented. Then the parameter extraction and optimization methodologies will be explained, giving special attention to the principles behind every extraction technique and the reasons behind the optimization of certain parameters. Finally the data processing plan and the curve fit methodologies over the whole temperature range will be addressed, emphasizing on the relation of the extracted parameters and the functional form of the temperature-dependent equations.

The compact modelling section will provide a detailed explanation of the iterative process of the temperature scaling for the device. The focus is on the final set of equations and how they are able to provide a meaningful physical description of the temperature dependence of the parameters. The reasoning behind the choice of the functional forms for the model parameters is shown.

In the experimental results section, the functional forms used to model the physically meaningful temperature dependent parameters will be compared with the experimental data over the whole temperature range. Additionally, the simulated DC compact model will be compared with the physical device behavior. Discussions over the limitations of the developed DC compact model and the functional forms of the closed form set of equations are included in this section. The accuracy of the developed model with respect to the physical device behavior is discussed in this section as well.

Finally, in the conclusion and future outlook section, the results of this work and the highlights of the future challenges and follow-up work will be summarized.
2 Theoretical Background

For modelling the temperature dependences of SiC-BJTs it’s fundamental to understand the relevant physical principles of SiC and the physical models that describe its electronic properties, and therefore the device characteristics. This is paramount to determine the temperature-dependent behavior of the model’s parameters.

Moreover, understanding semiconductor device physics, modelling and parameter extraction of modern BJT devices is also paramount. It will enable to approach the temperature-dependences in a physically meaningful way and not just from a mathematical point of view.

In this section, an overview of the fundamental background of the thesis will be provided. Related work and discussions on the perspective from which the theory will be applied for the developed model will be given.

2.1 Physical models of SiC

Although many physical models have to be included to be able to accurately simulate the performance of SiC BJTs, in the following only the relevant models for the SGP parameters and their temperature dependences will be discussed.

The most relevant models for the SGP DC parameters are those related to incomplete dopant ionization, bandgap and bandgap narrowing and carrier mobility. For this reason, these mentioned physical models will be the ones treated in this section. It’s important to highlight that additional models must be included to describe all of the BJT characteristics with precision, and therefore it’s encouraged to do follow-up work in this matter. Moreover, the mentioned physical models have their own limitations to describe the BJT characteristics and these limitations will be discussed.

2.1.1 Incomplete dopant ionization

SiC has a hexagonal crystal lattice. When dopants atoms are added in SiC these substitute the position of the Si or C atoms. Donors and acceptors can show more than one energy level depending on the site they are incorporated, due to the stacking sequence of the SiC polytype, where not all Si or C atoms positions are the same in terms of the lattice point conditions [2]. Being able to accurately model this behavior is paramount for a proper prediction of the device temperature dependences, since the equivalent ionized doping concentrations in the emitter and the base affects considerably the transistor “emitter efficiency” and therefore the current gain. It also affects significantly the bandgap and carrier mobility temperature dependence and therefore the temperature dependence of the model’s parameters such as the saturation and leakage currents and the junction voltages.

During the KTH in-house fabrication process, Al is used as p-type dopant in 4H-SiC. Al substitutes Si on a hexagonal or a cubic position, having ionization energies of 197.9 and 201.3 meV respectively [2][3]. For n-type dopants, N
atoms are used. N typically substitutes C on a hexagonal position having ionization energy of 61.4 meV [2]. But N can take place on hexagonal or cubic C-sites on 4H-SiC meaning that two different ionization levels exist. Values for the ionization energies have been reported for both levels, ranging from 45-66 meV for hex-sites and 92-124 meV for cubic sites [5][6][7][8]. Nevertheless, in this work, only the first mentioned energy level will be the one used for modelling the device, accounting for both levels. This could add up to the model limitations.

Because of its high ionization energy with respect to the thermal energy (kT, where k is the Boltzmann constant), Al and N dopants are not completely ionized at room temperature for 4H-SiC [3]. From now on, $N_A^-$ and $N_D^+$ will represent the density of ionized acceptors and donors, respectively [2].

For the following equations, numerical values for 4H-SiC are considered. The equilibrium hole and electron concentrations in extrinsic material when $n_i \ll N_{A,D}$ are given by $p = N_A^- \ (p$-type material) or $n = N_D^+ \ (n$-type material). From the charge neutrality condition, the equilibrium density of electrons and holes in $n$-type or $p$-type material can be obtained [2] and the ionized dopant concentrations $N_A^-$ or $N_D^+$ are given by:

$$p = N_A^- = \frac{\eta}{2} \sqrt{1 + \frac{4N_A}{\eta}} - 1 \quad (2.1)$$

$$n = N_D^+ = \frac{\gamma}{2} \sqrt{1 + \frac{4N_D}{\gamma}} - 1 \quad (2.2)$$

where $\eta$ and $\gamma$ is given by:

$$\eta = \frac{N_V}{g_A} \exp\left(\frac{E_V - E_A}{kT}\right) \quad (2.3)$$

$$\gamma = \frac{N_C}{g_D} \exp\left(\frac{E_D - E_C}{kT}\right) \quad (2.4)$$

here $E_A$ and $E_D$ are the energy level of the acceptors and donors impurities, $g_A$ and $g_D$ are the degeneracy factor for acceptors and donors (typically taken as 4 and 2 respectively) [2]. $N_V$ and $N_C$ are the effective density of states in the valence and conduction band, given by:

$$N_V = 2 \left(\frac{2\pi m_{vh}^* kT}{\hbar^2}\right)^{1/2} \quad (2.5)$$

$$N_C = 2 \left(\frac{2\pi m_{ve}^* kT}{\hbar^2}\right)^{1/2} \quad (2.6)$$
where \( m_{dh}^* \) and \( m_{de}^* \) are the density-of-states effective mass for holes and electrons, \( k \) is the Boltzmann constant and \( h \) is Planck’s constant.

Figure 1 shows the ionization fraction for Al acceptors and N or P donors for 4H-SiC, computed by using this model at ionization energy of 200 meV and 61 meV respectively [2]. It’s clear from Figure 1 that for high doping concentrations of donors or acceptors, a considerable fraction of them aren’t ionized at room temperature.

![Figure 1. Ionization fraction for Al acceptors (left) and N or P donors (right) in 4H-SiC, computed using ionization energy of 200 meV and 61 meV respectively [2].](image)

### 2.1.2 Bandgap and bandgap narrowing

To be able to predict accurately the current gain [9], the saturation and leakage currents and junction voltages of the BC and BE regions temperature scaling it is paramount to model the temperature dependence of the bandgap and the bandgap narrowing.

The temperature dependent equation for the bandgap of 4H-SiC, used in this work is given as follows:

\[
E_g(T) = E_g(300K) + \left( \frac{300^2}{300 + \beta} - \frac{T^2}{T + \beta} \right) \alpha
\]  

(2.7)

Parameters \( \alpha \) and \( \beta \) have been reported with different values in [10] and [11] resulting in contrasting dependences at higher temperature. For this particular work, the values of \( \alpha \) and \( \beta \) from [11] have been used.

Besides the temperature dependence of the nominal bandgap, bandgap narrowing (formation of a smaller “effective bandgap”), due to variations in the local doping concentration, formation of an impurity band in the bandgap or interactions between electrons, holes and ionized impurities has to be considered for strongly doped layers [3]. This effect has been modelled in SiC by Lindefelt [12] where the ionized doping concentration model and the band-edge displacements of both conduction and valence band are included. For n-type semiconductors this is described as follows:
\[
\Delta E_C = A_{nc} \left( \frac{N_p^+}{10^{18}} \right)^{\frac{1}{3}} + B_{nc} \left( \frac{N_p^+}{10^{18}} \right)^{\frac{1}{2}} \\
\Delta E_v = A_{nv} \left( \frac{N_p^+}{10^{18}} \right)^{\frac{1}{4}} + B_{nv} \left( \frac{N_p^+}{10^{18}} \right)^{\frac{1}{2}}
\]

(2.8) (2.9)

While for p-type semiconductors equations (2.9) and (2.10) describe the phenomena as follows:

\[
\Delta E_C = A_{pc} \left( \frac{N_A^-}{10^{18}} \right)^{\frac{1}{3}} + B_{pc} \left( \frac{N_A^-}{10^{18}} \right)^{\frac{1}{2}} \\
\Delta E_v = A_{pv} \left( \frac{N_A^-}{10^{18}} \right)^{\frac{1}{4}} + B_{pv} \left( \frac{N_A^-}{10^{18}} \right)^{\frac{1}{2}}
\]

(2.10) (2.11)

For both n-type and p-type materials, the band edge displacements and the ionized doping concentration are included, and therefore, these equations are indirectly related to the temperature [3]. The values of all coefficients are collected in Table 1.

**Table 1.** Band edge displacements for n-type and p-type 4H-SiC according to Lindefelt model: parameter values for equations (2.8)-(2.11) [12].

<table>
<thead>
<tr>
<th></th>
<th>(A_{nc})</th>
<th>(B_{nc})</th>
<th>(A_{nv})</th>
<th>(B_{nv})</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.5 (10^{-2}) eV</td>
<td>-2.93 (10^{-3}) eV</td>
<td>-1.9 (10^{-2}) eV</td>
<td>-8.74 (10^{-3}) eV</td>
<td></td>
</tr>
<tr>
<td>(A_{pc})</td>
<td>(B_{pc})</td>
<td>(A_{pv})</td>
<td>(B_{pv})</td>
<td></td>
</tr>
<tr>
<td>-1.5 (7) (10^{-2}) eV</td>
<td>-3.87 (10^{-4}) eV</td>
<td>-1.3 (10^{-2}) eV</td>
<td>-1.15 (10^{-3}) eV</td>
<td></td>
</tr>
</tbody>
</table>

2.1.3 Carrier mobility

For high doping concentrations in SiC, temperature dependences have been proposed by Balachandran et al. in [11] for both electrons and holes mobility. In this work, the electrons and holes mobility were modelled as follows:

\[
\mu_n = \mu_n^{\text{min}} + \frac{(T/T_o)^{\alpha_n}}{1 + (T/T_o)^{x_{\text{nl}}}} - \mu_n^{\text{min}} = 40 + \frac{950 (T/T_o)^{-2.4}}{1 + (T/T_o)^{-0.76}} - 40
\]

(2.12)

\[
\mu_p = \mu_p^{\text{min}} + \frac{(T/T_o)^{\alpha_p}}{1 + (T/T_o)^{x_{\text{pl}}}} - \mu_p^{\text{min}} = 53 + \frac{105 (T/T_o)^{-2.1}}{1 + (N/2.2 \times 10^{18})^{0.73}} - 53
\]

(2.13)

In equation (2.12) \(N\) refers to the ionized doping concentration, while for equation (2.13) \(N\) refers to the neutral doping concentration [3].

For an adequate prediction of the temperature dependence of the transistor current gain [3] and the saturation and the leakage currents, an accurate
temperature modelling of the highly doped epitaxial layers is paramount. More specifically, an accurate prediction of the emitter layer is fundamental because it determines the emitter efficiency and therefore the temperature scaling of the mentioned parameters.

### 2.1.4 Limitations of the physical models

The main concerns about the previously described physical models are related to their incapability to accurately describe by classical semiconductor devices theoretical equations the temperature dependence over a wide temperature range of the relevant model’s parameters. Equation 2.14 shows the theoretical final form of the saturation current for a uniformly doped base.

\[
I_S = \frac{A k T \mu_n n_i^2}{N_{AB} W_b} \tag{2.14}
\]

Where A is the effective area of the emitter layer, k is the Boltzmann constant, \( n_i \) is the intrinsic concentration, \( W_b \) is the base thickness and \( N_{AB}^- \) is the dopant concentration of acceptors in the base.

Attempts at modelling the saturation and leakage currents (the leakage currents are indirectly related to the saturation current equation) by using equation 2.14, didn’t result in good agreement with the extracted parameter data; especially on the temperature range above 300°C. From equation 2.12 one can intuit that the incomplete ionization model and the carrier mobility model fail to accurately describe the actual behavior of the device at high temperatures.

Previous results obtained by L. Lanni in [3] suggest that using a single energy level for N in 4H-SiC may be the cause of significant deviations over wide temperature ranges. Moreover, the modelling of the bandgap and transistor current gain becomes questionable.

For the previously mentioned reasons, the semi-empirical equations that describe the temperature scaling of the saturation current and the leakage currents in the SGP model, were used. By adjusting the respective temperature scaling coefficient of the equations accordingly, a better agreement with the extracted data, over the whole temperature range, could be easily obtained. Even though these equations use the bandgap and bandgap narrowing model, as will be seen afterwards, and therefore indirectly the incomplete ionization model, the temperature scaling coefficient provide a tuning capability that the theoretical equation cannot provide. Therefore, the semi-empirical equations provide better fitting to the extracted parameter data. The SGP model equations and their consequences will be treated in the following section.

In [3], it was shown that concerning the transistor current gain, accurate simulated values with respect to the measured data has been achieved up to 300 °C. The simulated \( \beta \) decreases monotonically with temperature when the previously mentioned mobility model is used, while the measured \( \beta \) actually increases above 300°C [3]. This experimental behavior can be associated to
two effects affecting $\beta$. When the temperature raises the minority carrier lifetime increases, while the emitter efficiency decreases until the base dopants are fully ionized [13]. Therefore, the reduction of the emitter efficiency is the dominant mechanism from 27 to 300 °C. Whereas all or almost all base dopants have been ionized above 300°C and that the increasing carrier lifetime has become the dominant mechanism.

Studies over more accurate physical models should be performed for temperatures above 300°C to be able to predict theoretically the measured current gain over the whole temperature range. But for the particular case of the developed SGP-based compact model, a reasonable fit to the measured current gain over the whole temperature range was obtained by using the semi-empirical equations from the SGP model and new empirical equations developed for the temperature dependent behavior of the forward, reverse and leakage emission coefficients. Hence, even when the physical models provide a lacklustre description of the current gain behavior above 300°C the developed temperature scaling of the SGP parameters provide a sufficiently good agreement with the measured behavior, due to certain circumstances of the parameter extraction procedures and the model’s equations that will be properly explained in section 4.

2.2 The SPICE Gummel Poon model

The SPICE Gummel Poon (SGP) [22] is one of many models that can be used as a starting point for the device modeling. Due to its simplicity, because of its relatively small number of parameters, this existing compact model is an adequate candidate for a first approach to model SiC-BJT for low power ICs. Previous works has shown successfully that this model can be used to describe the behavior of SiC Bipolar Junction Transistors (BJT). Particularly, SGP modelling of high power applications SiC-BJTs have been proven to be accurate with respect to the physical device operation [14]. For this reason the SGP model is a perfect starting point to model the temperature dependences of the DC characteristics of low-power, high temperature BJTs for ICs.

2.2.1 Model description

The representation of the SGP model when large signal conditions are considered is showcased in Figure 2. This model is a physical representation of the transistor. It includes a current-controlled output current source, two two-diode structures with their respective capacitors, and the ohmic parasitics of the device [22]. Figure 3 shows the cross section of the modelled device.
Figure 2. Gummel-Poon large signal schematic of the bipolar transistor [4]

Figure 3. Cross section of the modelled device [3]

B', E' and C' represent the base, emitter and collector intrinsic terminals of the transistor respectively. iB'C' and iB'E' represent the base-collector and base-emitter intrinsic currents through their respective diode structures. iC'E' represents the collector-emitter intrinsic currents as an ideal current source. CB'C' and CB'E' represent the space charge and diffusion capacitances of the base-collector and base-emitter diode structures. RBB', RC and RE represent the base, collector and emitter ohmic parasitics respectively. iB, iC and iE represent the base, collector and emitter current through their respective ohmic parasitics.
The following equations that describe the Gummel Poon model take into account that no voltage drops over the ohmic parasitics occur, i.e. $V_{BE'}=V_{BE}$ and $V_{BC'}=V_{BC}$, where $V_{BE}$ is the base-emitter junction voltage, $V_{BC}$ is the base-collector junction voltage.

The base current $i_B$ can be expressed as the addition of the base-emitter and the base-collector current as follows:

$$i_B = i_{BE} + i_{BC} \quad (2.15)$$

where the BE current is expressed as the addition of its ideal and a non-ideal component:

$$i_{BE} = \frac{i_F}{BF} + i_{BErec} \quad (2.16)$$

Similarly, the BC current is expressed as the addition of its ideal and non-ideal component:

$$i_{BC} = \frac{i_R}{BR} + i_{BCrec} \quad (2.17)$$

Here $i_F$ is the ideal forward diffusion current, BF is the ideal forward maximum current gain and $i_{BErec}$ is the BE recombination effect current. Similarly $i_R$ is the ideal reverse diffusion current, BR is the ideal reverse maximum current gain and $i_{BCrec}$ is the BC recombination effect current.

The ideal components of the BE and BC currents are a result of the recombination of minority charge carriers (electrons) in the quasi-neutral base region as well as the injection of majority charge carriers (holes) from the base into the emitter region. The non-ideal components owe their existence to the recombination/generation in the space charge regions as well as the surface recombination [22].

The ideal forward and reverse diffusion currents are defined as:

$$i_F = IS \left( \exp \left[ \frac{V_{BE}}{NF V_{th}} \right] - 1 \right) \quad (2.18)$$

$$i_R = IS \left( \exp \left[ \frac{V_{BC}}{NR V_{th}} \right] - 1 \right) \quad (2.19)$$

where IS is the saturation current, $V_{th}$ is the thermal voltage, and NF and NR are the forward and the reverse current emission coefficients, respectively.

The BC and BE recombination currents are defined as:

$$i_{BErec} = IS \left( \exp \left[ \frac{V_{BE}}{NE V_{th}} \right] - 1 \right) \quad (2.20)$$
\[ i_{BC\text{rec}} = ISC \left( \exp \left( \frac{V_{BC}}{NC V_{th}} \right) - 1 \right) \]  \hspace{1cm} (2.21)

where ISE and ISC are the emitter and collector leakage currents, and NE and NC are the emitter and collector current emission coefficients, respectively.

Therefore, the base current can be expressed as:

\[ i_B = \frac{IS}{BF} \left( \exp \left( \frac{V_{BE}}{NF V_{th}} \right) - 1 \right) + ISE \left( \exp \left( \frac{V_{BE}}{NE V_{th}} \right) - 1 \right) \]
\[ + \frac{IS}{BR} \left( \exp \left( \frac{V_{BC}}{NR V_{th}} \right) - 1 \right) + ISC \left( \exp \left( \frac{V_{BC}}{NC V_{th}} \right) - 1 \right) \]  \hspace{1cm} (2.22)

The collector current \( i_C \) can be expressed as the subtraction of the collector-emitter current and the base-emitter current as follows:

\[ i_C = i_{CE} - i_{BE} = \frac{(i_F - i_R)}{N_{qb}} - \frac{i_R}{BR} - i_{BC\text{rec}} \]  \hspace{1cm} (2.23)

where \( N_{qb} \) is the base charge equation and it models non-idealities of the device, such as the base-width modulation and the hi-level injection effects. It is defined as the base charge at a given bias normalized to its unbiased value [24]. This term is expressed as follows:

\[ N_{qb} = \frac{q_{1s}}{2} \left( 1 + \sqrt{1 + 4q_{2s}} \right) \]  \hspace{1cm} (2.24)

Parameter \( q_{1s} \) models the base-width modulation and \( q_{2s} \) the hi-level injection effect as follows:

\[ q_{1s} = \frac{1}{1 - \frac{V_{BE}}{VAR} - \frac{V_{BC}}{VAF}} \]  \hspace{1cm} (2.25)

\[ q_{2s} = \frac{IS}{IKF} \left( \exp \left( \frac{V_{BE}}{NF V_{th}} \right) - 1 \right) + \frac{IS}{IKR} \left( \exp \left( \frac{V_{BC}}{NR V_{th}} \right) - 1 \right) \]  \hspace{1cm} (2.26)

where VAF and VAR are the forward and reverse early voltages respectively and IKF and IKR are the forward and reverse knee currents respectively.

Finally, the emitter current can be expressed as the addition of the base and the collector current:

\[ i_E = i_B + i_C \]  \hspace{1cm} (2.27)

The equations representing the relationship between the intrinsic junction voltages \( V'B'E' \) and \( V'B'C' \) and the respective terminal voltages \( VBE \) and \( VBC \) are given as follows:
\[ V_{B\!E\!E'_1} = V_{BE} - (i_B R_{BB}(i_B) + i_E R_E) \]  
\[ V_{B\!C\!C'_1} = V_{BC} - (i_B R_{BB}(i_B) - i_C R_C) \]  

(2.28)

(2.29)

The bias dependence of base resistance is due to the emitter current crowding effect and the base-conductivity modulation. Both these effects tend to decrease the base resistance [15]. However, in SGP model, bias dependence of base resistance is based on a simplified version of Hauser's model [16] which only considers current crowding effect. The base resistance in SGP model is represented by the following equation:

\[ RBB(i_B) = RBM + 3 \left( RB - RBM \left[ \tan \frac{z}{z} - \frac{z}{z} \right] \right) \]  

(2.30)

with:

\[ z = \sqrt{1 + \left( \frac{12}{\pi^2} \right)^2 \frac{i_B}{IRB} - 1} \]  

(2.31)

where \( RB \) is the zero bias base resistance, \( RBM \) is the minimum base resistance at high base current and \( IRB \) is the base current at which the base resistance \( RBB(i_B) \) drops to the average of \( RB \) and \( RBM \). Consequently, correct modelling of the base resistance involves determination of \( RBM \), \( RB \) and \( IRB \) [15].

The depletion capacitances of the BE and BC junctions are defined as their respective contribution of the space charge capacitance and the diffusion capacitance. Since the diffusion capacitance can’t be measured from DC methods, only the space charge contribution will be measured in this work. The space charge capacitance equation for the BE and BC junctions are given as follows:

\[ C_{SBE} = \frac{C_{JE}}{\left( 1 - \frac{V_{BE}}{V_{JE}} \right)^{M_{JE}}} \]  

(2.32)

\[ C_{SBC} = \frac{C_{JC}}{\left( 1 - \frac{V_{BC}}{V_{JC}} \right)^{M_{JC}}} \]  

(2.33)

where \( C_{JE} \) and \( C_{JC} \) are the BE and BC zero bias capacitances, \( V_{JE} \) and \( V_{JC} \) are the BE and BC junction voltages (Also known as junction built-in potentials), and \( M_{JE} \) and \( M_{JC} \) are the BE and BC junction exponential factors.

The relevant DC parameters, ohmic parasitics, and space charge capacitances parameters of the Gummel Poon model are summarized in table 2.
Table 2. Relevant SGP parameters for this work

<table>
<thead>
<tr>
<th>Parameter name</th>
<th>Parameter explanation</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>DC:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>IS</td>
<td>Saturation current</td>
<td>A</td>
</tr>
<tr>
<td>BF</td>
<td>Ideal forward maximum current gain</td>
<td>-</td>
</tr>
<tr>
<td>BR</td>
<td>Ideal reverse maximum current gain</td>
<td>-</td>
</tr>
<tr>
<td>VAF</td>
<td>Forward early voltage</td>
<td>V</td>
</tr>
<tr>
<td>VAR</td>
<td>Reverse early voltage</td>
<td>V</td>
</tr>
<tr>
<td>NF</td>
<td>Forward current emission coefficient</td>
<td>-</td>
</tr>
<tr>
<td>NR</td>
<td>Reverse current emission coefficient</td>
<td>-</td>
</tr>
<tr>
<td>NE</td>
<td>Base-emitter leakage emission coefficient</td>
<td>-</td>
</tr>
<tr>
<td>NC</td>
<td>Base-collector leakage emission coefficient</td>
<td>-</td>
</tr>
<tr>
<td>ISE</td>
<td>Base-emitter leakage saturation current</td>
<td>A</td>
</tr>
<tr>
<td>ISC</td>
<td>Base-collector leakage saturation current</td>
<td>A</td>
</tr>
<tr>
<td>IKF</td>
<td>Forward knee current</td>
<td>A</td>
</tr>
<tr>
<td>IKR</td>
<td>Reverse knee current</td>
<td>A</td>
</tr>
<tr>
<td><strong>Ohmic parasitic:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>RB</td>
<td>Zero bias base resistance</td>
<td>Ω</td>
</tr>
<tr>
<td>IRB</td>
<td>Current at medium base resistance</td>
<td>A</td>
</tr>
<tr>
<td>RBM</td>
<td>Minimum base resistance at high current</td>
<td>Ω</td>
</tr>
<tr>
<td>RE</td>
<td>Emitter resistance</td>
<td>Ω</td>
</tr>
<tr>
<td>RC</td>
<td>Collector resistance</td>
<td>Ω</td>
</tr>
<tr>
<td><strong>C\textsubscript{SBE}:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CJE</td>
<td>Base-emitter zero bias depletion capacitance</td>
<td>F</td>
</tr>
<tr>
<td>VJE</td>
<td>Base-emitter built-in potential</td>
<td>V</td>
</tr>
<tr>
<td>MJE</td>
<td>Base-emitter junction exponential factor</td>
<td>-</td>
</tr>
<tr>
<td><strong>C\textsubscript{SBC}:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CJC</td>
<td>Base-collector zero bias depletion capacitance</td>
<td>F</td>
</tr>
<tr>
<td>VJC</td>
<td>Base-collector built-in potential</td>
<td>V</td>
</tr>
<tr>
<td>MJC</td>
<td>Base-collector junction exponential factor</td>
<td>-</td>
</tr>
</tbody>
</table>

When the device is wanted to be simulated at a temperature that is different from the extraction temperature (typically, room temperature), the SGP model has its own temperature scaling equations, based on semi-empirical and theoretical models for both the cases of the parameter scaling and physical phenomena such as the bandgap and the intrinsic concentration. These equations are presented now, and some of the parameter temperature scaling equations forms will serve as a starting point for modelling the temperature dependences for the SiC-BJTs parameters.

The SGP model temperature dependent auxiliary variables are the following:

\[ V_{th} = \frac{kT}{q} \]  \hspace{1cm} (2.34)

\[ E_G = 1.16 - \frac{7.02 \times 10^{-4}T^2}{T + 1108} \]  \hspace{1cm} (2.35)
\[ n_i = 1.45 \times 10^{10} \left( \frac{T}{T_0} \right)^{1.5} \exp \left[ \frac{q}{2k} \left( -\frac{E_G}{T} + \frac{1.1151}{T} \right) \right] \]  \hspace{1cm} (2.36)

Here \( V_{th} \) is the thermal voltage, \( E_G \) is the bandgap, \( n_i \) is the intrinsic carrier concentration, \( q \) is the electron charge and \( k \) is the Boltzmann constant. For the purpose of this work, these temperature dependent auxiliary variables will be updated with the previously mentioned physical model for SiC, to accurately predict the temperature scaling of the BJTs.

The SGP model temperature dependent modelling parameters are the following:

\[ IS(T) = IS(T_0) \left( \frac{T}{T_0} \right)^{X_{TI}} \exp \left[ \frac{E_G}{V_{th}} \left( \frac{T}{T_0} \right)^{1.1151} \left( \frac{T}{T_0} - 1 \right) \right] \]  \hspace{1cm} (2.38)

\[ BF(T) = BF(T_0) \left( \frac{T}{T_0} \right)^{X_{TB}} \]  \hspace{1cm} (2.39)

\[ BR(T) = BR(T_0) \left( \frac{T}{T_0} \right)^{X_{TB}} \]  \hspace{1cm} (2.40)

\[ ISE(T) = ISE(T_0) \left( \frac{T}{T_0} \right)^{-X_{TB}} \frac{IS(T)^{\frac{1}{NE}}}{IS(T_0)} \]  \hspace{1cm} (2.41)

\[ ISC(T) = ISC(T_0) \left( \frac{T}{T_0} \right)^{-X_{TB}} \frac{IS(T)^{\frac{1}{NC}}}{IS(T_0)} \]  \hspace{1cm} (2.42)

\[ VJE(T) = VJE(T_0) \left( \frac{T}{T_0} \right) + 2V_{th} \log \left[ \frac{1.45 \times 10^{10}}{n_i} \right] \]  \hspace{1cm} (2.43)

\[ VJC(T) = VJC(T_0) \left( \frac{T}{T_0} \right) + 2V_{th} \log \left[ \frac{1.45 \times 10^{10}}{n_i} \right] \]  \hspace{1cm} (2.43)

Due to the semi-empirical nature of the temperature dependent modelling parameters, they will be slightly modified for the purpose of this work, to be able to properly predict the temperature scaling of them while modelling SiC-BJTs.

### 2.2.2 Limitations of the SGP model

Limitations of the SGP model can contribute to the difference between simulated and measured output characteristics and Gummel plots (both in the forward and reverse active regions) [15].

For the case of the output characteristics, one limitation is the assumption that the device has constant output resistance at all current levels [22]. Another limitation is the lack of parameters to describe self-heating [22]. The
lack of parameters to describe quasi-saturation effects is another limitation of the model as well [22].

For the case of the Gummel plots, the lack of parameters to describe quasi-saturation effects is also an issue [22]. In addition, lacking the parameters to describe base punch-through effects adds up to the model limitations [22].

The self-heating effect can manifest itself in one of the two ways at large reverse biased output junction voltages and large output currents: as a rapid increase in the output current due to avalanche multiplication or as a decrease in the output current (negative conductance) due to the beta degradation with temperature [15].

It has been demonstrated on [15] that the real culprit for the negative conductance for the in-house SiC-BJTs is the carrier multiplication in the base-collector junction. The positive charges produced as a result of the carrier multiplication decreased the base current [17] and consequently, the collector current. The carrier multiplication is not modelled in the SGP [22], so there will always be a mismatch between the measured and the simulated output characteristics when the device is operating at high power, irrespective of the accuracy of the extracted early parameters.

The quasi-saturation effect (Also known as Kirk effect) is caused when the charge density related with the current going through base-collector junction is larger than the ionized doping concentration in the base-collector depletion region. Therefore, the effective width of the base layer becomes the width of the base-collector layer, thus increasing the carrier transit time considerably, reducing the current gain [18].

This effect can be observed on the forward output characteristics as a second slope on the linear region, as can be seen in Figure 4. It can also be observed in the forward Gummel plot as an increasing base current at high injection levels and therefore a reduction of the current gain, as can be seen in Figure 5.
Figure 4. Forward output characteristics at 500°C. The quasi-saturation effect can be observed at high current injections at collector-emitter voltages approximately between 1.75V and 2.25V, before the saturation region of the device.

Figure 5. Forward Gummel plot at 500°C. The quasi-saturation effect can be observed at current injections for base-emitter voltages above 3V at this temperature.
3 Methodology

Existing compact models for SiC-BJTs for ICs does not include temperature scaling equations for most parameters, meaning that they can be used for a specific temperature exclusively. For this reason, simulations of more complex circuits are difficult to be carried out, requiring to extract and optimize the whole set of parameters at each of the desired temperatures. This approach isn’t practical since extracting and optimizing all the SGP parameters over a wide temperature range is very time consuming.

Moreover, since there wasn’t an existing temperature scaling model for these devices, the extraction and optimization of the SGP parameters has been performed separately at equally spaced temperatures in the 25 to 500°C temperature range. Previous work approaches to model SiC-BJTs for ICs can be summarized in the flow chart shown in Figure 6. This general work flow is the one that was followed initially to understand the temperature dependences of the device, and therefore it is the starting point to model its temperature scaling.

![Flow chart of previous works modelling approach](image)

**Figure 6.** Flow chart of previous works modelling approach, from previous works, used as a starting point in this work.

From the previous procedure shown in Figure 6, the obtained data of the SGP parameters at each temperature point was processed on Matlab by fitting functional forms of the temperature dependent equations to the parameters over the whole temperature range. Then the SPICE simulator (IC-CAP) was updated with the parameter values obtained from the functional forms and then certain parameters had to be optimized for proper fitting of the simulation to the measured characteristic plots. Then the functional forms were updated with the new parameter values after optimization and this process was repeated iteratively until acceptable fit was obtained.
Finally, a new methodology for modelling the device has been proposed by using a model of temperature scaling equations of the SGP parameters, where physically meaningful dependences could be observed. The proposed methodology is summarized in a flow chart in Figure 7.

![Flow chart of the proposed modelling approach using the developed model.](image)

**Figure 7.** Flow chart of the proposed modelling approach using the developed model.

This new approach is considerably less time consuming since only an initial extraction and optimization of the whole set of SGP DC parameters needs to be done at room temperature, and the iterative optimization over the whole temperature range only needs to be carried out for exclusively four parameters. In addition this process guarantees that the scaled SGP parameters have physically meaningful values while also achieving an acceptable fit of the measured characteristic plots data.

### 3.1 Measurement plan and setup

The chosen temperature range for modelling the device is 25 to 500 °C, since this would be the typical temperature of operation. Ten data points for each parameter were obtained at equally spaced temperatures. It could be observed that at temperatures above 450°C the parameters tend to behave differently than at temperatures below this one (more on this in the experimental results section). For this reason, more data points could be considered for modelling the device between 450 and 500 °C.

The setup for performing the measurements consisted of an interface between IC-CAP and the electrical characterization module of the probe station, where the device was physically being measured. The measurement routines were set on the IC-CAP software (voltages and currents sweeps on the device terminals), and the characterization tool executed the programmed routines on the device. The programmed routines on IC-CAP are based on the extraction methods and characteristic plots of the SGP model, which will be explained in detail in section 3.2.
Once all the required measurements were performed on one device, measurements at 25, 150, 350 and 500°C on different devices at completely opposite sides of the wafer were performed using the same setup, to demonstrate the accuracy of the model using the SPICE simulator.

3.2 Parameter extraction and optimization

For the extraction of the model parameters at each temperature, a routine graphical parameter extraction methodology from [4] has been used. The following extraction procedures have been implemented on a Matlab algorithm, where linear regression and nonlinear curve fitting methods for the respective extractions were performed.

For the optimization of most model parameters at each temperature with respect to the measured characteristic plots, routine optimization procedures were followed from [4]. For certain parameters, where routine optimization procedures didn’t reach proper fit of the device simulation to the measured data, new developed optimization procedures were used. This new procedures are base on the theoretical understanding of the model and its limitations and experimental observation of the device data. The optimization algorithms were implemented in IC-CAP, which provides with a graphical optimization and parameter tuning interface that facilitates the understanding of the relationship between the model parameters and the different characteristic plots features.

3.2.1 Extraction methodology

The order in which the following extraction procedures were carried out is the same order as they are shown in this section.

- Ohmic Parasitics:
  - Collector Resistance RC:
    Collector resistance is a function of bias because of collector conductivity modulation [19]. In short, the value of collector resistance is smaller in the saturation region as compared to its value in the active region. However, in SGP model, RC is considered as a constant. This limitation may result in a mismatch between the measured and the simulated output characteristics [25]. Most of the RC extraction methods use the collector flyback method or variants of it. In the flyback method the device is operating in the saturation region. As a result, the measured value of RC can be smaller than the value intended to be measured [25]. Therefore, a large base current (deep saturation) was used for unambiguous extraction of RC [25].

    The principle of the collector flyback method is to make the emitter current zero by opening the emitter contact and at the same time measuring the collector-emitter voltage ($V_{CE}$) [25]. The measured $V_{CE}$ can be written as:

    $$V_{CE} = V_{CEI} + i_B RC$$  \hspace{1cm} (3.1)

    For large base currents, the collector-emitter junction voltage is a constant [20]. Therefore, RC is extracted as a slope of $V_{CE}$ versus $I_B$ plot. The measured
values of $V_{CE}$ are fitted to a first order polynomial and the slope of it will give RC [25].

- **Emitter Resistance $RE$:**
  RE, similar to RC is not a constant. It is a function of emitter current and increases at high current levels due to the emitter current crowding effect. However, similar to RC it is also modelled as a constant in the SGP model [25].

The emitter flyback method, similar to the one described for the extraction of RC, is the one used for the extraction of RE, but instead of an open emitter, one has an open collector. Thus, equation for $V_{CE}$ can be modified to:

$$V_{CE} = V_{CEE} + i_B RE$$  \hspace{1cm} \text{(3.2)}$$

Therefore, RE can be extracted as a slope of $V_{CE}$ versus $I_B$ curve [25] as shown in the Figure 8. High base currents were used to minimise the ambiguity in the measured value [20].

Figure 8 summarizes the measurement setup and extraction method of RE. A similar setup is used for extracting RC, where the emitter current $i_E$ is set to zero and $V_{CE}$ is measured as $i_B$ is swept.
Figure 8. Measurement setup and determination of RE from the emitter flyback method [4].

- **Base Resistance RB:**
  Due to difficulties and inaccuracy of the DC extractions of the base resistance, the base resistance RB is set to the minimum base resistance RBM, in order to reduce possible errors introduced by the default values of RB. This is a reasonable assumption as the sheet resistance of the base layer is usually very large in this technology [3].

The flyback method for the emitter resistance is the one used for the extraction of RBM. In this method, $V_{BE}$ in addition to $V_{CE}$ is also measured during the open-collector experiment [4]. Then RBM is measured as a y-intercept of the following equation [4]:

$$\frac{V_{BE} - V_{CE}}{i_B} = \frac{\text{Constant}}{i_B} + RBM$$  \hspace{1cm} (3.3)

The typical plot of $\frac{V_{BE}-V_{CE}}{i_B}$ versus $I_B$ and the measurement setup for determination of RBM out of transformed emitter flyback measured data, is shown in Figure 9.

Figure 9. Measurement setup and determination of RBM out of transformed emitter flyback measured data [4].
• **VAF, VAR:**

VAF and VAR extraction technique makes use of the assumption that the injection levels are low. This assumption implies that the current levels are too low to result in a significant voltage drops across the parasitics resistances, i.e. \( V_{BE'} = V_{BE} \) and \( V_{BC'} = V_{BC} \). It also implies that the base conductivity modulation effect which results in a decrease of collector (or emitter) current in the forward (or reverse) active region is negligible, i.e. \( q2 = 0 \) [25].

For low injection levels, \( N_{qb} = q1 \), since \( q2 = 0 \). Moreover, \( V_{BE'} = V_{BE} \) and \( V_{BC'} = V_{BC} \). Hence, \( i_E \) can be written as:

\[
 i_E = -\frac{IS}{q1} \left( \exp \left( \frac{V_{BC}}{NRV_{th}} \right) - 1 \right)
\]  

(3.5)
\[ = -I_S \left( \exp \left[ \frac{V_{BC}}{N R V_{th}} \right] - 1 \right) \left( 1 - \frac{V_{BE}}{V_{AR}} - \frac{V_{BC}}{V_{AF}} \right) \]
\[ \approx -I_S \left( \exp \left[ \frac{V_{BC}}{N R V_{th}} \right] \right) \left( 1 - \frac{V_{BE}}{V_{AR}} - \frac{V_{BC}}{V_{AF}} \right) \]  

Figure 10 summarizes the measurement setup and the measurement result of the forward and reverse output characteristics.

![Measurement setup and result](image)

**Figure 10.** Measurement setup and result of the forward and reverse output characteristics [4].

Rewriting (3.4) at two different values of \( V_{BE} \) and for same value of \( V_{BC} \):

\[ i_{E1} = -I_S \left( \exp \left[ \frac{V_{BC}}{N R V_{th}} \right] \right) \left( 1 - \frac{V_{BE1}}{V_{AR}} - \frac{V_{BC}}{V_{AF}} \right) \]  
\[ i_{E2} = -I_S \left( \exp \left[ \frac{V_{BC}}{N R V_{th}} \right] \right) \left( 1 - \frac{V_{BE2}}{V_{AR}} - \frac{V_{BC}}{V_{AF}} \right) \]  

Dividing these two equations will result in the following equation:

\[ \frac{i_{E2}}{i_{E1}} = \frac{\left( 1 - \frac{V_{BE2}}{V_{AR}} - \frac{V_{BC}}{V_{AF}} \right)}{\left( 1 - \frac{V_{BE1}}{V_{AR}} - \frac{V_{BC}}{V_{AF}} \right)} \]  

From the forward output characteristics, equations of the same form can be written for two different values of \( V_{BC} \) for a single value of \( V_{BE} \).

\[ \frac{i_{C2}}{i_{C1}} = \frac{\left( 1 - \frac{V_{BE}}{V_{AR}} - \frac{V_{BC2}}{V_{AF}} \right)}{\left( 1 - \frac{V_{BE}}{V_{AR}} - \frac{V_{BC1}}{V_{AF}} \right)} \]  

These two equations are linear in two variables, \( \frac{1}{V_{AF}} \) and \( \frac{1}{V_{AR}} \) and are solved simultaneously for these parameters [26].
• **IS, NF, ISE, NE, BF:**
All these parameters can be extracted from the FGP at low injection levels once the early parameters are extracted.

Figure 11 summarizes the extraction setup to obtain the FGP parameters IS, NF, ISE, NE and BF. A similar setup is used to obtain the RGP parameters. For that case the terminals voltages sweep is done in such a way that $V_{BE}=0V$, instead of making $V_{BC}=0V$.

**Figure 11.** Measurement setup and extraction principles of IS, NF, ISE, NE and BF [4].
**IS, NF:**
The value of parameters IS and NF can be extracted from the manipulation of the collector current equation at low injection levels as follows:

\[
i_C = \frac{IS}{q} \left( \exp \left[ \frac{V_{BE}}{NF V_{th}} \right] \right)
= IS \left( \exp \left[ \frac{V_{BE}}{NF V_{th}} \right] \right) \left( 1 - \frac{V_{BE}}{VAR} \right)
= \frac{i_C}{\left( 1 - \frac{V_{BE}}{VAR} \right)} = IS \left( \exp \left[ \frac{V_{BE}}{NF V_{th}} \right] \right)
\] (3.11)

Taking natural logarithm on both sides:

\[
\ln \left[ \frac{i_C}{\left( 1 - \frac{V_{BE}}{VAR} \right)} \right] = \ln[IS] + \frac{V_{BE}}{NF V_{th}} \] (3.12)

This equation implies that in order to extract IS and NF, the measured value of IC has to be divided by the term \( \left( 1 - \frac{V_{BE}}{VAR} \right) \), where VAR is the value extracted during the last step. It is then followed by taking the natural logarithm. This corrected collector current plotted against \( V_{BE} \) is then fitted to a straight line. The y-intercept of that line would be equal to \( \ln[IS] \) and the slope would be equal to \( \frac{1}{NF V_{th}} \).

**ISE, NE:**
Rewriting the base current equation of a bipolar transistor operating in the forward active mode:

\[
i_B = \frac{IS}{BF} \left( \exp \left[ \frac{V_{BE}}{NF V_{th}} \right] - 1 \right) + ISE \left( \exp \left[ \frac{V_{BE}}{NE V_{th}} \right] - 1 \right)
\] (3.13)

The first term in the equation above is the ideal component of the base current and has a slope which is equal to the one of \( i_C \) (before the onset of base-conductivity modulation and the Kirk effect) because they have similar emission coefficient NF. The additional second term in the base current can be due to the following mechanisms: the recombination of carriers in the base-emitter junction, the recombination of carriers at the surface and the formation of emitter-base channels [21]. Usually channel and surface recombination currents can both be made small by careful processing, leaving junction recombination current as the dominant non-ideal component.

This theoretical insight is very helpful to determine ISE and NE for the devices in which the base current knee (initial bend in the \( i_B \) curve of FGP) is not visible. In such cases, either the knee current is too low to be measured or is too large and is overlaid by the parasitic resistances. For in-house SiC devices, the knee in the base current is not visible. It has been previously demonstrated
that the reason for this is that the knee current is too large and is overlaid by the parasitic resistances since NF for \( i_B \) is always larger than 2.

For this case, \( i_B \) at low injection levels can be written as:

\[
\ln[i_B] = \ln[ISE] + \frac{V_{BE}}{NE \, V_{th}}
\]  

(3.13)

Therefore, a straight line can be fitted in this region of \( i_B \) to extract ISE and NE in a similar way as it was done with IS and NF.

\( \textbf{BF:} \)

The ratio of the collector current to base current in FGP can be mathematically written as:

\[
\beta = \frac{i_C}{i_B} = \frac{IS}{N_q b} \left( \exp \left[ \frac{V'_{BE'}}{NF \, V_{th}} \right] \right)
\]

\[
= \frac{IS}{BF} \left( \exp \left[ \frac{V'_{BE'}}{NF \, V_{th}} \right] - 1 \right) + ISE \left( \exp \left[ \frac{V'_{BE'}}{NE \, V_{th}} \right] - 1 \right)
\]  

(3.14)

Dividing the denominator by numerator and rearranging:

\[
\beta = \frac{1}{N_q b \left[ \frac{1}{BF} + \frac{ISE}{IS} \exp \left[ \frac{V'_{BE'}}{NF \, V_{th}} \right] \left( \frac{1}{NE} - \frac{1}{NF} \right) \right]}
\]  

(3.15)

An assumption is made at this point, that the second bracketed term is negligible as compared to the still unknown \( 1/BF \) term. The validity of this assumption has been already proven in [15] and the term containing ISE/IS will be negligible for \( V_{BE} \) a few 100 mV greater than \( V'_{BE'} \) of 2.9 V.

Once the validity of this assumption is confirmed, the equation for BF can be simplified to:

\[
BF = \beta N_q b
\]  

(3.15)

Where the equation describing the normalized base charge, \( N_q b \), is given by equation (2.24).

Moreover, in the forward active region and for \( V_{BC}=0 \):

\[
q1 = \frac{1}{1 - \frac{V'_{BE'}}{VAR}} \quad \text{and} \quad q2 = \frac{i_f}{IKF}
\]  

(3.16)

Therefore, the correct extraction of BF requires VAR, IKF and parasitic resistances. VAR was extracted before [15]; however, the latter two quantities are still unknown.
Fortunately, for in-house SiC devices, it can be safely assumed that $IKF >> IF$ or $q2 = 0$ in the forward active region. The validity of assumption has also been already verified in [15] where it’s shown that the main cause for the current gain reduction is the quasi-saturation effect and not the knee currents, despite the overlay of both effects at high injection levels.

Hence, it can be safely assumed that conductivity modulation is negligible or $q2 = 0$. This will further simplify the equation describing $BF$ as follows:

$$BF = \beta q1$$

$$= \frac{\beta}{1 - \frac{V_{BE'}}{VAR}}$$

(3.17)

The value of $BF$ at the $V_{BE}$ at which $\beta$ becomes equal to $\beta_M$ (Maximum value of the current gain) is the extracted value of the parameter [15].

- **NR, ISC, NC, BR:**
  All these parameters can be extracted from the RGP at low injection levels once early parameters are extracted [15].

  - **NR:**
    The extraction of NR follows the same principle as the extraction of NF. The equation of emitter current in RGP for low level injection ($VBC = VB'C'$ and $q2 = 0$) is given as:

    $$i_E = IS \left( \exp \left[ \frac{V_{BC}}{NR \ V_{th}} \right] \right) \left( 1 - \frac{V_{BC}}{VAF} \right)$$

    (3.18)

    The extracted values of VAF for in-house SiC devices were much larger than applied input voltage, $VBC$. Therefore, the equation can be simplified to:

    $$i_E = IS \left( \exp \left[ \frac{V_{BC}}{NR \ V_{th}} \right] \right)$$

    (3.19)

    $$\ln(i_E) = \ln(IS) + \frac{V_{BC}}{NR \ V_{th}}$$

    (3.20)

    At low injection levels, a line is fitted to $\ln(i_E)$. The y-intercept of that line would be $\ln(IS)$ and the slope of the fitted line would be $\frac{1}{NR \ V_{th}}$.

  - **ISC, NC:**
    The value of parameters ISC and NC can be extracted from the RGP by using the same technique that was discussed for the extraction of ISE and NE. If the normalized inverse of the slope of $i_B$ is equal to the normalized inverse of the slope of $i_E$ (extracted NR) at low injection levels, ISC can be set to zero and NC can be set to any arbitrary value. On the other hand, if the normalized inverse of the slope of $i_B$ is close to or equal to 2, the plot of $ln(i_B)$ is fitted against a
straight line. The y-intercept of that line would give \( \ln(\text{ISC}) \), whereas, the slope would give \( \frac{1}{\text{NC} \cdot V_{th}} \) [15]. For simplicity of the methodology over the whole temperature range, where the normalized inverse of the slope of \( i_B \) could acquire values between 1 and 2, it was decided to fit the plot of \( \ln(i_B) \) against a straight line and extract ISC from the y-intercept and NC from the slope of it.

- **BR:**
In reverse mode, B is defined as the ratio of the emitter current to the base current. The value of \( \text{BR} \) can be extracted using this definition for \( V_{B'E'}=0 \) (using the currents from RGP) as follows:

\[
\beta = \frac{i_E}{i_B} = \frac{IS}{N_{qb}} \left( \exp \left[ \frac{V_{B'C_I}}{NR \cdot V_{th}} \right] \right) - \frac{IS}{\text{BR}} \left( \exp \left[ \frac{V_{B'C_I}}{NR \cdot V_{th}} \right] - 1 \right) + ISC \left( \exp \left[ \frac{V_{B'C_I}}{NC \cdot V_{th}} \right] - 1 \right)
\]  

(3.21)

Ignoring the non-ideal base current component (Section-2.4.3), the value of \( \text{BR} \) can be written as:

\[
\text{BR} = \beta N_{qb}
\]

(3.22)

During the extraction of early parameters for in-house SiC devices, it has been observed that the parameter VAF was very large and hence \( q_1 \) becomes 1. Moreover, \( q_2 \) can also be calculated now as all the parameters that define \( q_2 \) have been extracted already during the previous steps [15]. Once \( q_1 \) and \( q_2 \) are known, \( N_{qb} \) can also be calculated using (2.24). Finally, the value of \( \text{BR} \) parameter can be extracted using the technique discussed for the extraction of BF.

- **IKF, IKR:**
For extraction purposes IKF is defined as the collector current where the \( \beta \) of the device drops to \( 1/\sqrt{2} \) of its maximum value [4]. Both collector and base currents are affected by the resistive drops in a similar way, so their ratio is independent of the deviation due to parasitic effects [15].

It was described previously that no base-conductivity modulation was observed in the forward active mode for the in-house SiC devices. Consequently, IKF can be kept to its default SPICE value. However, this value can be optimized to fit the current gain at the slight decrease of its value right before the quasi-saturation effect takes place.

IKR is defined in the similar way as IKF but for the reverse mode of operation. It has been showcased in [15] that the decrease of the inverse current gain is due to the base-conductivity modulation. Therefore, the parameter IKR is extracted from the \( \beta \) plot as the emitter current where the value of reverse \( \beta \)
drops to $1/\sqrt{2}$ of its maximum value. However, as for IKF this value can be optimized to fit the current gain at high injection levels.

- **CJE, CJC, MJE, MJJC, VJE, VJC (CV measurements):**
The base-emitter and base-collector junction space charge capacitance measurements setup and extraction principle are exactly the same (Except for the terminals where the stimulus are applied) and for simplicity only the base-emitter junction capacitance measurement and parameters extraction will be explained. Figure 11 summarizes the measurement setup, result and extraction technique of the space charge capacitance parameters.

![Figure 11](image1)

**Figure 11.** Measurement setup, result and extraction techniques of the space charge capacitance parameters [4].

As can be observed in Figure 11, for the measurement of the base-emitter space charge capacitance, the base-emitter junction is biased while the collector terminal is left open. Similarly, for the measurement of the base-collector space charge capacitance, the base-collector junction is biased while the emitter is left open. A voltage sweep of $V_{BE}$ and $V_{BC}$ respectively from negative to positive values is done while the space charge capacitance of the junction is measured [4].

Equation (2.32) describes the behavior of the space charge capacitor when $V_{BE} < F_C V_{JE}$ where $F_C$ is the forward capacitance switching coefficient with a default value of 0.5. For simplicity purposes, only the measured capacitance values from the negative bias region are used to guarantee that the previously mentioned condition for equation (2.32) is fulfilled [4].

The logarithmic conversion of equation (2.32) yields:

$$\ln[C_{SB}] = \ln[C_{JE}] - M_{JE} \ln \left[1 - \frac{V_{BE}}{V_{JE}}\right]$$

(3.23)
Therefore, a first order polynomial can be fitted to the negative bias region of the measurement and the parameters are extracted from a typical linear regression method [2]. CJE can be extracted from the y-intercept of the transform ($C_{SRC}$ at $V_{BE} = 0$). MJIE can be initially set to 0.5 which is the typical value for an abrupt junction or can be extracted as the slope of fitted curve (Both approaches give relatively good fit and similar values), and VJE is optimized afterwards from the curve fit of the $1 - \frac{V_{BE}}{V_{JE}}$ data transform.

### 3.2.2 Optimization methodology

The optimization of the whole set of SGP DC parameters was performed on IC-CAP by using the Levenberg-Marquardt algorithm (LMA), which in this case was utilized for nonlinear least-squares curve fitting of the simulated data to the characteristic curves measured data (FGP, RGP, forward and reverse output characteristics and CV measurements).

As will be explained further on in this section, certain parameters were optimized individually, and some were optimized simultaneously for a characteristic plot. Moreover, some parameters were also optimized simultaneously for more than one characteristic plot simultaneously.

- **Forward Gummel Plot (FGP) optimization:**
  The optimization steps of the SGP parameters for the FGP are the following:
  1. Optimize IS and NF simultaneously at low injection levels of the $i_C$ curve, where they were initially extracted.
  2. Optimize ISE and NE simultaneously at low injection levels of the $i_B$ curve, or alternatively, at the lower knee of the current gain plot (Transform of the $i_C$ and $i_B$ data from the FGP).
  3. Optimize BF in the flat region of the current gain plot.
  4. Optimize ohmic parasitics $RE$, $RC$ and $RBM$ to fit both the $i_C$ and the $i_B$ curves at high injection levels where the quasi-saturation effect takes place. In this way, a more accurate value of $RBM$ than the one obtained from the inaccurate DC extraction method can be obtained.
  5. Optimize $IKF$ to fit the current gain plot at the slight decrease of its value, right before the quasi-saturation effect takes place.

- **Reverse Gummel Plot (RGP) optimization:**
  The optimization steps of the SGP parameters for the RGP are the following:
  1. Optimize NR at low injection levels of the $i_E$ curve, where it was initially extracted, to guarantee that for the only saturation current that the SGP model supports (IS), both the FGP and RGP low injection levels fit the measured data.
  2. Optimize ISC and NC simultaneously at low injection levels of the $i_B$ curve, where they were initially extracted.
  3. Optimize BR at the flat region of the current gain plot (Transform of the $i_E$ and $i_B$ data from the RGP).
  4. Optimize $IKR$ and $RC$ simultaneously to fit both the $i_E$ and the $i_B$ curves at high injection levels. In this way, a more accurate value of $RC$ than the one obtained from the inaccurate DC extraction method can be obtained. Moreover, a more accurate value of IKR is obtained as well.
• **Forward and reverse output characteristics optimization:**
  The optimization steps of the SGP parameters for the forward and reverse output characteristics are the following:
  1- Optimize RE at the saturation region of the forward output characteristics and at the linear region of the reverse output characteristics simultaneously,
  2- Optimize RC at the linear region of the forward output characteristics and at the saturation region of the reverse output characteristics simultaneously,

• **Base-emitter and base-collector CV measurement optimization:**
  For the case of the base-emitter and base-collector CV measurements, only VJE and VJC were optimized to fit the simulation of the CV curve to their respective measurement of the space charge capacitance, particularly at the positive bias region, where the largest mismatch could be observed after extraction of the junction parameters.

3.3 **Parameter scaling process**

Once all the SGP DC parameters were extracted and optimized at each temperature point, a particular temperature scaling behavior for every SGP parameter is obtained. As it was already mentioned, the obtained SGP parameters at each temperature point were processed with a Matlab algorithm by fitting functional forms of the temperature dependent equations to the parameters over the whole temperature range. Then the SPICE simulator (IC-CAP) was updated with the parameter values obtained from the functional forms at every temperature and then certain parameters had to be optimized for proper fitting of the simulation to the measured characteristic plots. Then the functional forms were updated with the new parameter values after optimization and this process was repeated iteratively until acceptable fit was obtained.

This iterative process to obtain a physically meaningful temperature scaling of the parameters with the smallest error between the measured SGP parameters and the functional forms, and with the smallest mismatch of the model simulations to the measured characteristic plots, is summarized in Figure 12.

![Parameter scaling processing iteration loop](image)

**Figure 12.** Parameter scaling processing iteration loop.
Let’s explain in-depth the iterative process. Once all the SGP parameters are extracted and individually optimized with respect to the characteristic plots of the SGP model for the first time, the temperature scaling of the SGP parameters is approached with theoretical equations (if applicable) that describe the temperature behavior of the parameters. It could be observed by simulating the theoretical temperature scaling, based on the physical models described on section 2.1, that there is a considerable mismatch of them with respect to the measured data for most SGP parameters. In section 4 it will be seen that the theoretical equations were only kept for the junction voltages and fitting coefficients were needed for proper match to the measured data.

Afterwards, the temperature scaling of the SGP parameters is approached by using the SGP temperature scaling equations, showcased in section 2.2. For many parameters such as the saturation and leakage currents, the SGP temperature scaling functional forms appeared to be good candidates for modelling the measured SGP parameter scaling, but the fitting temperature scaling coefficients had to be redefined and added in some cases for proper match to the measured data.

For some physically meaningful temperature dependences, such as the ones of the emission coefficients and ohmic parasitics, where either no theoretical equations or SGP temperature scaling were available or provided a good match, new empirical equations had to be developed based on the observed behavior of the experimental measured data of the SGP parameters.

After all the physically meaningful temperature dependences had a temperature scaling equation associated to them, their respective temperature scaling fitting parameters were optimized to fit the measured SGP parameters over the whole temperature range.

Then the SPICE simulator (IC-CAP) is updated with the SGP parameter values obtained from the optimized functional forms at each temperature. Then the required parameters are optimized. For instance, the emission coefficients have strong influence on the simulated curves for their exponential behavior in the SGP model and therefore they require to be optimized every iteration. Then the process is repeated until the errors between the functional forms and the measured SGP parameters is minimized, and as a consequence of the iteration process, the best match between the model and the SGP characteristic plots will be obtained.
4 Parameters temperature scaling

The developed temperature scaling equations of the SGP-based compact model are a mixture of modified or tuned theoretical equations, SGP temperature scaling equations and empirical equations based on semiconductor devices physics and observation of the parameter behavior as a function of temperature. These equations are the result of many iterations of the process described in section 3.3.

The listing of the equations will be done by separating them in three main groups: existing SGP equations (with redefined temperature scaling parameters), theoretical equations (with new defined fitting parameters) and new empirical equations.

The existing equations taken from the SGP temperature scaling model are the following:

\[ I_S = I_S(T_0) \left( \frac{T}{T_0} \right)^{X_T I} \exp \left[ \frac{E_G}{V_{th}} \left( \frac{T}{T_0} \right) \right] \]

\[ X_{TI} = -3.5876 \]

\[ I_{SE} = I_{SE}(T_0) \left( \frac{T}{T_0} \right)^{X_{TE}} \exp \left[ \frac{E_G}{V_{th}} \left( \frac{T}{T_0} \right) \right] \]

\[ X_{TE} = 3.461 \]

\[ I_{SC} = I_{SC}(T_0) \left( \frac{T}{T_0} \right)^{X_{TC}} \exp \left[ \frac{E_G}{V_{th}} \left( \frac{T}{T_0} \right) \right] \]

\[ X_{TC} = 3.2806 \]

It can be observed that the functional forms of these equations are the same as for equations (2.38), (2.41) and (2.42), with the difference that for the latter two, the mutual dependence of the temperature scaling coefficients \( X_{TI} \) and \( X_{TE} \) are redefined for new coefficients \( X_{TE} \) and \( X_{TC} \) respectively, to guarantee proper fitting to the measured data. It’s also important to note that for these equations \( NE(T_0) \) and \( NC(T_0) \) are the room temperature (nominal) values of the emitter and collector emission coefficients, as done in the SGP temperature scaling equations for the emitter and collector leakage currents. The reason for not using the temperature scaling equations developed in this work for \( NE \) and \( NC \) - as will be seen afterwards- is that as the emission coefficients increase with temperature, the exponential terms would decrease significantly and the values of the temperature fitting scaling coefficients \( X_{TE} \) and \( X_{TC} \) would compensate with unreasonably high values (around 25 and 45, respectively), which not only isn’t physically meaningful, but also could affect the usage of the model.
outside the modelled temperature range, by obtaining excessively large values of the leakage currents.

In addition, in these equations, the physical variables $E_G$ and $V_{th}$ are the bandgap and the thermal voltage respectively which are based on the physical models described in section 2.1. The values of the temperature fitting parameters $X_{TI}$, $X_{TE}$ and $X_{TC}$ are listed below the equations. A graphical comparison between these model equations and the measured data of their respective parameters is shown in Figure 13.

![Figure 13. Comparison between equations (4.1)-(4.3) and the measured data of their respective parameters.](image)

The theoretical equations, with the newly defined fitting parameters are the following:

\[
V_{JE} = V_{th}^{XTVJE_1} \log \left[ \frac{N_{AB}N_{DE}^+}{n_i^2} \right]^{XTVJE_2}
\]

\[XTVJE_1 = 0.0382; \quad XTVJE_2 = 0.2697\]

\[
V_{JC} = V_{th}^{XTVJC_1} \log \left[ \frac{N_{AB}N_{DC}^+}{n_i^2} \right]^{XTVJC_2}
\]

\[XTVJC_1 = -1.7567; \quad XTVJC_2 = -1.1368\]

Equations (4.6) and (4.7) are the theoretical equations of the built-in potential, but the temperature fitting parameters $XTVJE_{1,2}$ and $XTVJC_{1,2}$ were added to obtain a proper fit to the measured data. The values of the temperature fitting parameters $XTVJE_{1,2}$ and $XTVJC_{1,2}$ are listed below the respective equations. Moreover, the physical variables $N_{AB}^-$, $N_{DE}^+$, $N_{DC}^+$ in
these equations, are the ionized base, emitter and collector doping concentration, respectively; whereas $n_i$ is the intrinsic carrier concentration. These equations were formulated previously in section 2.1. It’s important to address that the nominal doping concentrations used to calculate the ionized doping concentrations are $N_{AB} = 5 \times 10^{17}$, $N_{DE} = 10^{19}$ and $N_{DC} = 10^{19}$, with the last one being the doping concentration of the buried collector. A graphical comparison between equations (4.6) and (4.7) and the measured data of the respective parameters is shown in Figure 15.

![Figure 15](image-url)

**Figure 15.** Comparison between equations (4.6)-(4.7) and the measured data of their respective parameters.

The newly developed empirical equations for the SGP-based compact model are the following:

$$BF = BF(T_0)[XTBF_2(T - T_0)^2 + XTB_1(T - T_0) + 1]$$

$$XTBF_1 = 4.9349 \times 10^{-6}; \quad XTB_2 = -0.0033$$

$$BR = BR(T_0)[XTBR_2(T - T_0)^2 + XTB_1(T - T_0) + 1]$$

$$XTBR_1 = 4.6188 \times 10^{-6}; \quad XTB_2 = 0.0019$$

$$NF = NF(T_0)\left(\frac{T}{T_0}\right)^{XTNF}$$

$$XTNF = 0.8868$$

$$NE = NE(T_0)\left(\frac{T}{T_0}\right)^{XTNE}$$

$$XTNE = 1.0329$$

$$NR = NR(T_0)\left(\frac{T}{T_0}\right)^{XTNR}$$

$$XTNR = 0.8489$$
\[ NC = NC(T_0) \left( \frac{T}{T_0} \right)^{XTNC} \quad (4.11) \]

\[ XTNC = 0.9473 \]

\[ RBM = RBM(T_0)[XTRB_2(T - T_0)^2 + XTRB_1(T - T_0) + 1] \quad (4.12) \]

\[ XTRB_1 = 6.169 \cdot 10^{-6}; \quad XTRB_2 = -0.0041 \]

\[ RE = RE(T_0)[XTRE_2(T - T_0)^2 + XTRE_1(T - T_0) + 1] \quad (4.13) \]

\[ XTRE_1 = 5.7063 \cdot 10^{-6}; \quad XTRE_2 = -0.0027 \]

\[ RC = RC(T_0)[XTRC_2(T - T_0)^2 + XTRC_1(T - T_0) + 1] \quad (4.14) \]

\[ XTRC_1 = 2.5841E \cdot 10^{-6}; \quad XTRC_2 = 8.0878 \cdot 10^{-4} \]

\[ CJ = CJ(T_0) \left( \frac{T}{T_0} \right)^{XTCJE} \quad (4.15) \]

\[ XTCJE = 0.1411 \]

\[ CJ = CJ(T_0)[XTJC_2(T - T_0)^2 + XTJC_1(T - T_0) + 1] \quad (4.16) \]

\[ XTJC_1 = 1.1616 \cdot 10^{-6}; \quad XTJC_2 = -8.5085 \cdot 10^{-5} \]

Equations (4.4) and (4.5) are based on the observed behavior of the transistor current gain, as shown in [3], where a second order polynomial could lead to acceptable fitting of the measured BF and BR. Also, similar temperature dependent equations for BF and BR already exist for the HSPICE model described in [26]. New temperature fitting parameters XTBF\(_1\), XTBF\(_2\), XTBR\(_1\) and XTBR\(_2\) were defined to obtain a proper fit to the measured data. The values of the temperature fitting parameters XTBF and XTBR are listed below the equations. A graphical comparison between equations (4.4) and (4.5) and the measured data of their respective parameters is shown in Figure 14.

Attempts of modelling the ohmic parasitics and the space charge capacitances by using theoretical equations, failed to work, probably due to the inaccuracies of both the mobility model for SiC and the SGP model (more specifically, the used extraction methodologies). Therefore, an empirical approach to these dependences had to be elaborated.

Equations (4.8)-(4.16) were all developed based on the observed behavior of the parameters after a considerable amount of iterations of the process described in section 3.3. The values of the temperature fitting parameters highlighted in red are listed below the equations. Similar temperature dependent equations for the ohmic parasitics already exist for the HSPICE model described in [26]. A graphical comparison between equations (4.8)-(4.11), (4.12)-(4.14), (4.15)-(4.16) and the measured data of their respective parameters is shown in Figure 16, Figure 17 and Figure 18 respectively.
Figure 14. Comparison between equations (4.4)-(4.5) and the measured data of their respective parameters.

Figure 16. Comparison between equations (4.8)-(4.11) and the measured data of their respective parameters.
Finally, as shown in Figure 19, parameters VAR, VAF, IKF and IKR didn’t show any meaningful temperature dependence, which means that the obtained data didn’t follow a particular behavior that could be modelled by relatively simple mathematical expressions. Moreover, parameters VAR, IKF, and IKR didn’t change considerably over the measured temperature range. In addition, parameter VAF resulted in very high values over the measured temperature range, which means that the base-width modulation is negligible in the forward active region, and therefore it can be set to a very high value (Or infinity) for the purpose of the SGP model. For this reason, these values should be kept as their room temperature value over the whole temperature range.
range when working with the developed model, and optimizations of these parameters can be done if required.

**Figure 19.** Temperature dependence of parameters VAF, VAR, IKF and IKR
5 Experimental results

The similarity between the parameters obtained from the temperature scaling equations and the measured parameters over the whole temperature range is demonstrated by calculating the relative error between them. The aforementioned relative errors at every temperature are shown in Figure 20.

![Figure 20](image)

**Figure 20.** Relative errors between the temperature scaling equations and the measured parameters over the whole temperature range.

It’s noticeable that a good fit of the temperature scaling equations has been obtained for most parameters since most errors are below 20%. Considerable errors have been obtained for RBM and RC at high temperatures, meaning that the values of these two parameters at high temperature may be physically meaningless since they were used to fit the FGP at the quasi-saturation region that by definition is not modelled by the SGP model.

As it was mentioned in section 3.1, to demonstrate the accuracy of the model using the SPICE simulator, the developed model was tested over the measurements at 25, 150, 350 and 500°C on different devices at completely opposite sides of the wafer.

A comparison of the simulations and the measurements of the FGP at the mentioned temperatures, after exclusive optimization of the emission coefficients, can be observed in Figure 21, Figure 22, Figure 23 and Figure 24.
Figure 21. Measured (m) and simulated (s) FGP at 25°C.

Figure 22. Measured (m) and simulated (s) FGP at 150°C.
As shown in Figures 21-24, fairly accurate simulation results of the FGP are achieved with the parameters obtained with the developed model over the whole temperature range. As it was to expect from the limitations of the SGP model described in section 2.2.2, the quasi-saturation region and the base punch-through are not properly modelled, despite the efforts of emulating the observed behavior by optimizing the ohmic parasitics at high injection levels and the emission coefficients at low injection levels. It can be observed that the quasi-saturation effect in simulations becomes more prominent at higher temperatures.
One way to model the quasi-saturation effect using the SGP model is by adding an extra BJT in parallel with a saturation collector resistor [4]. By tuning the area of the added BJT and the value of the resistor, one could emulate the quasi-saturation effect. This is not included in the compact model because it wasn’t possible to develop a particular methodology to tune the parameters of the extra BJT for each temperature; and is only recommended for high power applications. On the other hand, modelling base punch-through isn’t possible by using the SGP model and therefore this effect is completely excluded out of the developed compact model.

A comparison of the simulations and the measurements of the RGP at the mentioned temperatures, after exclusive optimization of the emission coefficients, can be observed in Figure 25, Figure 26, Figure 27 and Figure 28.

**Figure 25.** Measured (m) and simulated (s) RGP at 25°C.

**Figure 26.** Measured (m) and simulated (s) RGP at 150°C.
Figure 27. Measured (m) and simulated (s) RGP at 350°C.

Figure 28. Measured (m) and simulated (s) RGP at 500°C.

Fairly accurate simulations of the RGP are achieved with the parameters obtained with the developed model over the whole temperature range, especially at high injection levels, as shown in Figures 25-28. Similarly, as for the FGP, the low injection effects in the reverse active region were not possible to be simulated due to limitations of the SGP model. But it’s believed that this effect could be caused by an excessive leakage current through the isolation substrate, instead of a device phenomenon (such as base punch-through), that could be solved if the isolation layer of the device is kept at the same potential as the collector.

A comparison of the simulations and the measurements of the forward output characteristics at the mentioned temperatures, after exclusive optimization of the emission coefficients, can be observed in Figure 29, Figure 30, Figure 31 and Figure 32.
**Figure 29.** Measured (m) and simulated (s) forward output characteristics at 25°C.

**Figure 30.** Measured (m) and simulated (s) forward output characteristics at 150°C.
Fairly accurate simulations of the forward output characteristics are achieved with the parameters obtained with the developed model over the whole temperature range, especially at the linear region of the device, as shown in Figures 29-32. Similarly, just like the FGP, the quasi-saturation effect is observed at the edge of the saturation region of the device more prominently as the temperature increases it could not be modelled properly due to limitations of the SGP model. Adding an extra BJT in parallel with a saturation collector resistor, as explained previously, can solve this modelling
issue. The BJT area and the value of the saturation collector resistor should be optimized simultaneously in the quasi-saturation region of the FGP and the forward output characteristics for optimal fitting of the simulation.

A comparison of the simulations and the measurements of the reverse output characteristics at the mentioned temperatures, after exclusive optimization of the emission coefficients, are shown in Figure 33, Figure 34, Figure 35 and Figure 36.

**Figure 33.** Measured (m) and simulated (s) reverse output characteristics at 25°C

**Figure 34.** Measured (m) and simulated (s) reverse output characteristics at 150°C
As shown in Figures 29-32, fairly accurate simulations of the reverse output characteristics are achieved with the parameters obtained with the developed model over the whole temperature range as well. The slight mismatch at the saturation and linear region of the reverse output characteristics is believed to be caused by the inaccuracy of the base resistance RB (Set to RBM due to the inaccuracy of the flyback measurement for extracting RB), and therefore it’s suggested that in future work the ohmic parasitics are extracted from S-parameters, as recommended in [4].

Figure 35. Measured (m) and simulated (s) reverse output characteristics at 350°C

Figure 36. Measured (m) and simulated (s) reverse output characteristics at 500°C
Finally, a comparison of the simulations and the measurements of the space charge capacitor at the mentioned temperatures, after exclusive optimization of base-collector and the base-emitter built-in potentials, can be observed in Figure 37, Figure 38, Figure 39 and Figure 40.

**Figure 37.** Measured (dotted) and simulated (straight), base-emitter (top) and base-collector (bottom) space charge capacitance vs voltage at 25°C.

**Figure 38.** Measured (dotted) and simulated (straight), base-emitter (top) and base-collector (bottom) space charge capacitance vs voltage at 150°C.
Figure 39. Measured (dotted) and simulated (straight), base-emitter (top) and base-collector (bottom) space charge capacitance vs voltage at 350°C.

Figure 40. Measured (dotted) and simulated (straight), base-emitter (top) and base-collector (bottom) space charge capacitance vs voltage at 500°C.

Fairly accurate simulations of the space charge capacitors of the base-collector and base-emitter junctions are achieved with their respective parameters obtained with the developed model over the whole temperature range.
6 Conclusions and future outlook

A simple set of temperature scaling equations for the SGP model has been successfully developed for low voltage SiC-BJTs. Comparisons between the modelled temperature scaling equations and the measured parameters have been done. Moreover, comparisons between the simulated and measured characteristics plots have been performed and the model accuracy has been demonstrated. The implementation of the model will enable the design of complex electronic systems based on SiC BJTs over a wide temperature range of operation. Moreover, the development flow of a SGP-based DC compact model of SiC-BJTs for low voltage, high temperature ICs over a wide temperature range has also been demonstrated.

Advantages and disadvantages of the parameter extraction methods have been showcased. Particularly, the disadvantages of using DC extraction methods for the ohmic parasitics are believed to cause some of the modelling mismatches to the measured data.

Follow-up work on modelling the temperature dependences of the AC parameters of the SGP model is suggested. Moreover, the usage of more complex models such as VBIC, HiCUM or MEXTRAM to model modern SiC BJTs is recommended, since these provide more flexibility for modelling diverse physical phenomena. Some of the transistor characteristics weren’t modelled. More specifically there aren’t any parameters to describe the temperature scaling of the quasi-saturation and base punch-through effects, and therefore more in depth studies of the device physics would solve the modelling issues. Follow-up work on implementing the developed model on SPICE or VerilogA is encouraged.
References


