Efficient Modelling Techniques for Vibration Analyses of Railway Bridges

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Doctoral thesis
Stockholm, Sweden 2017
Akademisk avhandling som med tillstånd av Kungliga Tekniska högskolan framlägges till offentlig granskning för avläggande av teknologie doktorsexamen i brobyggnad fredagen den 24 februari 2017 klockan 13:30 i sal Kol, Kungliga Tekniska högskolan, Brinellvägen 8, Stockholm.

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Tryck: Universitetsservice US-AB
Abstract

The world-wide development of new high-speed rail lines has led to more stringent design requirements for railway bridges, mainly because high-speed trains can cause resonance in the bridge superstructure. Dynamic simulations, often utilising time-consuming finite element analysis (FEA), have become essential for avoiding such problems. Therefore, guidelines and tools to assist structural engineers in the design process are needed.

Considerable effort was spent at the beginning of the project, to develop simplified models based on two-dimensional (2D) Bernoulli–Euler beam theory. First, a closed-form solution for proportionally damped multi-span beam, subjected to moving loads was derived (Paper I). The model was later used to develop design charts (Paper II) and study bridges on existing railway lines (Paper III). The model was then extended to non-proportionally damped beams (Paper IV) in order to include the effects of soil–structure interactions. Finally, the importance of the interaction between the surrounding soil and the bridge was verified by calibrating a finite element (FE) model by means of forced vibration tests of an end-frame bridge (Paper V).

Recommendations on how to use the models in practical applications are discussed throughout the work. These recommendations include the effects of shear deformation, shear lag, train–bridge and soil–structure interactions, for which illustrative examples are provided. The recommendations are based on the assumption that the modes are well separated, so that the response at resonance is governed by a single mode.

The results of the work show that short span bridges, often referred to as ‘simple’ bridges, are the most problematic with respect to dynamic effects. These systems are typically, non-proportionally damped systems that require detailed analyses to capture the ‘true’ behaviour. Studying this class of dynamic system showed that they tend to contain non-classical modes that are important for the structure response. For example, the bending mode is found to attain maximum damping when its undamped natural frequency is similar to that of a non-classical mode.

Keywords: Railway bridge; High-speed train; Closed-form solution; Non-proportional damping; Complex mode
Sammanfattning

Utbyggnaden av nya höghastighetsbanor i världen har resulterat i striktare krav vid dimensionering av järnvägsbroar. Orsaken är främst att tåg vid höga hastigheter kan orsaka resonans i brons överbyggnad. För att kontrollera dessa effekter krävs omfattande dynamiska simuleringar, vilket ofta utförs med tidskrävande finita element analyser. På grund av detta är det nödvändigt att formulera rekommendationer och utveckla verktyg som hjälper konstruktören i arbetet.


Resultatet från arbetet visar att korta broar, vilka ofta omnämns som ”enkla”, är problematiska med avseende på dynamiska effekter. Dessa är normalt icke-proportionellt dämpade system och kräver detaljerade analyser för att fånga strukturers ”riktiga” beteende. Genom att studera denna klass av dynamiska system har det konstaterats att systemen ofta innehåller icke-klassiska moder som är viktiga för strukturen. Det har som ett exempel påvisats att en böjmod blir maximalt dämpad när den odämpade frekvensen är nära den av en icke-klassisk mod.

Nyckelord: Järnvägsbro; Höghastighetståg; Sluten lösning; Icke-proportionell dämpning; Komplex mod
Preface

The work presented in this doctoral thesis was financed by Formas and the Swedish Transport Administration (Trafikverket). All the research work was conducted at the Department of Civil and Architectural Engineering of the KTH Royal Institute of Technology over a period of seven years. Along with my academic career, I have also been employed by ELU Konsult as a structural engineer, meaning that I could devote only half of my time to research.

The work was supervised by Adjunct Professor Costin Pacoste, to whom I am grateful for encouragement and guidance throughout my research. We worked side-by-side over the years and have become good friends. There have been many valuable discussions in the office, car or pub. I would also like to thank my co-supervisor, Professor Raid Karoumi, for providing support to my research work and creating a good work environment at the department. Many thanks also to Professor Jean-Marc Battini, for our many valuable discussions and for taking the time to review this thesis before submission.

I would also like to take the opportunity to thank my colleagues at KTH and ELU Konsult, for making the office a fun place to be in and for inspiring me with their ideas. I would especially like to thank Andreas Andersson, Mahir Ülker-Kaustell, Therese Arvidsson and Abbas Zangeneh, who always supported me in my work.

At the beginning of 2014 I had the pleasure of visiting KU Leuven. I would therefore like to thank Professor Guido De Roeck for inviting me, and Professor Stijn François for his assistance during my brief stay.

Lastly, but without a doubt most importantly, I thank my wife, Charlotte, and daughter, Julia, for always keeping me busy and filling my soul with love, joy and meaning.

Stockholm, January 2017
Christoffer Svedholm
Publications

Appended Journal Papers:


All papers were planned, implemented and written by Svedholm (formerly Johansson). The co-author provided guidance throughout the work and reviewed the draft before submission. Svedholm did not perform the field measurements utilized in paper V.

Other Relevant Publications:


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Part A

Introduction and general aspects
Chapter 1

Introduction

1.1 Background

Predicting the dynamical response of bridges is essential during the design of new, high-speed rail lines. Modern numerical techniques such as finite element analysis (FEA) can be used for this purpose, but they often result in large models that are computationally demanding and provide limited insights into fundamental mechanisms that govern the dynamic response of the bridge. The basic difference between bridges on a conventional railway line and those on a high-speed line is that higher speeds may cause resonance conditions that amplify vibrations. From a theoretical point of view, this means that the response is governed by a few modes, or by a single mode for bridges with well-separated modes. Therefore, studying how a specific aspect alters the modal properties (such as shear deformation, shear lag, etc.), and knowing how a set of modes vibrates under moving loads, might be more fruitful than studying the complete problem.

Figure 1.1: High-speed project in Sweden. European Corridor (left) and East Link Project (right).
CHAPTER 1. INTRODUCTION

At the time of writing, the rail sector in Sweden is facing the major challenge of constructing the new, high-speed European Corridor, which should be in place by 2035. This corridor will connect Sweden’s three main cities (Stockholm, Gothenburg and Malmö) with the European high-speed rail network (Fig. 1.1).

The design of the East Link Project (150 km high-speed rail from Järna to Linköping), which is the first part of the European Corridor, is currently ongoing. The construction will start in 2017. This link includes approximately 200 bridges, all of which must be designed for high-speed trains. Simplified methods and guidelines for estimating the vibration amplitudes are important for such a task. The following must generally be considered:

- How should the train–bridge interaction be modelled?
- Is it sufficient to use a beam model?
- When does soil–structure interaction influence the results?
- How much damping should be assumed into the model?

Answering all those questions is beyond the scope of this research. Therefore, rather than attempting to provide precise answers, this study presents a general overview of the problem and derives appropriate design guidance.

1.2 Aims and scope

The overall aim of this thesis is to give practical tools and recommendations regarding the design of bridges for high-speed trains, more precisely, how to accurately predict bridge responses to high-speed rail traffic. Specific objectives are to: 1) identify factors that affect the dynamic response; 2) develop simplified methods for the analysis and assessment of bridges on high-speed lines; 3) apply statistical methods for the dynamic analysis of existing railway bridges; and 4) perform structural identification based on finite element (FE) models and dynamical measurements.

All calculations are performed based on the recommendations in EN1991-2 (2003) and ERRI D-214 (1999b). The design limit criteria for the serviceability limit state (e.g. maximum displacement and acceleration) are not questioned, but taken for granted. The analytical models and design charts are aimed to be used at an early stage in the design, when there is limited knowledge of both the structure and the construction site. This preliminary analysis will support the engineer in choosing a suitable bridge type before a more detailed analysis is performed using numerical models. Techniques for building a computationally efficient FE model are also discussed in the thesis.
1.3 Research contribution

The work presented in this thesis has resulted in the following research contributions:

- Two simplified models for the analysis and assessment of bridges on high-speed rail lines.
- Design curves to calculate the maximum displacement, acceleration and angular rotations of bridges under high-speed loads.
- A methodology for the preliminary dynamical assessment of bridges on existing high-speed lines.
- Efficient techniques for model updating of an existing bridge structure, including the soil–structure interaction effects, using measured frequency response functions.
- Guidelines and recommendations on how to deal with three-dimensional (3D) effects, train–bridge interaction and soil–structure interaction.

Illustrative examples are presented throughout the thesis and in the appended papers that demonstrate the abovementioned contributions.

1.4 Outline of the thesis

The thesis is divided into an introductory part and five appended papers. The introductory part provides readers with a general overview of the research field and highlights how certain aspects can be accounted for in a simple model. The text is organised as follows: Chapter 2 provides the general theoretical background for linear time-invariant systems. Several aspects of railway bridge dynamics are further discussed in Chapter 3. Some new results are presented, showing that the train–bridge interaction, 3D-effects and soil–structure interaction can be accounted for by adjusting the structural properties and damping ratios. Chapter 4 summarises the research work. Finally discussion and conclusions are presented in Chapter 5.

The introductory part is followed by five appended papers. Below is a brief summary of each of these papers:

Paper I develop a closed-form solution to determine the response of a proportionally damped multi-span beam. The governing differential equation is solved for each normal mode in the frequency domain after a Laplace transformation. This method provides the exact solution of the response for a multi-span Euler–Bernoulli beam under a moving load.
CHAPTER 1. INTRODUCTION

Paper II develop design curves for the preliminary dynamic assessments of railway bridges at higher speeds. The design curves are based on more than 58 million train passages, which were performed on 73,440 beams with the closed-form model developed in Paper I. It is the author’s opinion that the design curves provide a great deal of understanding about the problem at hand because they show how the span length, fundamental frequency, damping and number of spans influence the dynamic response.

Papers III present a probabilistic method for assessing a network of bridges. The efficiency of the method is demonstrated in a project initiated by the Swedish Government. More than 1000 bridges were analysed for the effects of high-speed train passages. The simulation results could be used for estimating the cost of upgrading the maximum allowed speed from the existing 200 km/h to 250 km/h.

Paper IV develop a closed-form solution to determine the response of a non-proportionally damped beam with general end conditions. Proper orthogonality conditions are derived. Moreover, modal impulse response functions are obtained. An analytical expression for a moving load is found by applying a convolution integral.

Paper V covers the topic of model updating of railway bridges using frequency response functions (FRF). Good agreement between theoretical and experimental results is obtained when the frequency response assurance criterion (FRAC) and frequency amplitude assurance criterion (FAAC) are used as the objective function and the soil–structure interaction is modelled. The computational time of the FE model (over 4.6 million DOFs) is reduced using model reduction techniques.
Chapter 2

Linear time-invariant dynamic system

This chapter gives some basic theoretical background on linear time-invariant dynamic systems. Even though there is nothing new in this chapter, it is included in the text in order to clarify the notations and keep the presentation as self-contained as possible. The chapter starts by discussing the fundamental differences between discrete and continuous systems (Section 2.1). This is followed by a section on computationally efficient models (Section 2.2). The governing equation of motion is derived in the context of continuous systems (Subsection 2.2.1). For simplicity, the derivation is limited to transverse vibration of beams. However, more complicated problems involving coupled bending, longitudinal and torsional vibration may be solved using discrete models. The computational time for this class of system can be reduced by applying model reduction techniques, as discussed in Subsection 2.2.2. The final part of this chapter (Section 2.3) is devoted to the concept of proportionally and non-proportionally damped systems, which applies to both discrete and continuous systems.

2.1 Discrete and continuous systems

The kinematic quantities necessary to describe the motion of a dynamical system are commonly known as degrees of freedom (DOF). The systems that can be described with a finite number of DOFs are referred to as discrete. For example, the elastically-supported beam shown in Fig. 2.1 can be simplified into a 2-DOF system.

The mathematical description of this system is provided by the following coupled, second-order ordinary differential equation:

\[ M\ddot{u}(t) + C\dot{u}(t) + Ku(t) = P(t) \]  

(2.1)
CHAPTER 2. LINEAR TIME-ININVARIANT DYNAMIC SYSTEM

Figure 2.1: Discrete system of an elastically-supported beam.

where \( \mathbf{M} \), \( \mathbf{C} \) and \( \mathbf{K} \) denote the mass, viscous damping and stiffness matrices, respectively. The column vectors \( \mathbf{P} \) and \( \mathbf{u} \) denote the force and displacement, respectively. This dynamical system will provide two distinct natural frequencies and corresponding mode shapes. The most popular procedures for calculating the time response for this system are direct time integration and modal superposition.

Most of the analytical work in this thesis, however, is based on a continuous formulation, instead of a discrete one. For beam structures, this corresponds to using a model based on an infinite number of DOFs. Accordingly, the solution provides an infinite number of natural frequencies and mode shapes. The governing equation of motion for this system is obtained from the free-body diagram of an infinite small segment \( dx \) (Fig. 2.2).

This results in the following equation of motion:

\[
\begin{align*}
  m \frac{\partial^2 u(x,t)}{\partial t^2} + c \frac{\partial u(x,t)}{\partial t} + EI \frac{\partial^4 u(x,t)}{\partial x^4} &= -p(x,t) \\
\end{align*}
\]  

(2.2)

Note that the above partial differential equation is far more difficult to solve than Eq. (2.1). This explains why closed-form solutions only exist for problems with sim-
ple geometry and boundary conditions. However, as stated by Rao (2007), ‘closed-form solutions that are available will often provide insight into the behaviour of more complex systems for which closed-form solutions cannot be found’. Moreover, this type of solution is also computationally efficient and provides accurate results for high frequencies that would otherwise require very fine discretisation.

2.2 Computationally efficient models

2.2.1 Continuous systems

The forthcoming pages present a complete derivation of the equation of motion of a continuous beam under moving loads. Further details on the theories underlying this section can be found in Rao (2007), Fryba (1972), Humar (2012) and Karoumi (1998).

As the starting point, consider a viscously damped Euler–Bernoulli beam with $j$ spans, as shown in Fig. 2.3. The beam is subjected to a distributed load $p(x, t)$. Each span $i$ is assigned an individual length $L$. In order to simplify the derivation, it is assumed that the mass per unit length $m$, Young’s modulus $E$, and moment of inertia $I$ are constant within each span and only varied at intermediate supports.

From the free-body diagram in Fig. 2.4, it follows that the sum of vertical forces (see Eq. (2.3)) and the sum of moments about the left-hand face (see Eq. (2.4)) should be zero:

\[ \downarrow: m \frac{\partial^2 u(x, t)}{\partial t^2} + c \frac{\partial u(x, t)}{\partial t} - \frac{\partial V}{\partial x} = p(x, t) \quad (2.3) \]

\[ \bigcirc: V + \frac{\partial V}{\partial x} \, dx + \frac{\partial M}{\partial x} \, dx + p(x, t) \, dx \left( \frac{m \frac{\partial^2 u(x, t)}{\partial t^2} \, dx}{2} - \frac{m \frac{\partial^2 u(x, t)}{\partial t^2} \, dx}{2} \right) + c \frac{\partial u(x, t)}{\partial t} \, dx = 0 \quad (2.4) \]

Note that the equations are only valid for Euler–Bernoulli beams, because shear deformation and rotational inertia are not taken into account. By neglecting higher-order terms, Eq. (2.4) becomes:

\[ V = -\frac{\partial M}{\partial x} \quad (2.5) \]
Elementary beam theory further gives:

\[
\frac{\partial M}{\partial x} = \frac{\partial}{\partial x} \left( EI \frac{\partial^2 u(x, t)}{\partial x^2} \right) \tag{2.6}
\]

The governing differential equation is obtained by substituting Eq. (2.6) into Eq. (2.5), differentiating and then substituting the result into Eq. (2.3):

\[
m \frac{\partial^2 u(x, t)}{\partial t^2} + c \frac{\partial u(x, t)}{\partial t} + \frac{\partial^2}{\partial x^2} \left( EI \frac{\partial^2 u(x, t)}{\partial x^2} \right) = p(x, t) \tag{2.7}
\]

The transverse displacement \(u(x, t)\) within a segment is obtained by solving Eq. (2.7).

The expansion theorem is then used to separate the general differential equation into a linear combination of normal modes \(\phi_n(x)\). In Eq. (2.8) \(q_n(t)\) is the generalised coordinate of the \(n\)th mode.

\[
u(x, t) = \sum_{n=1}^{\infty} \phi_n(x)q_n(t) \tag{2.8}
\]
2.2. COMPUTATIONALLY EFFICIENT MODELS

Substituting $u(x,t)$ into Eq. (2.7) and describing the moving concentrated load by means of the Dirac delta function $\delta (p = P\delta[x- vt])$ results in:

$$\sum_{n=1}^{\infty} \left( m \frac{d^2 q_n(t)}{dt^2} \phi_n(x) + c \frac{dq_n(t)}{dt} \phi_n(x) + \frac{d^2}{dx^2} \left( EI \frac{d^2 \phi_n(x)}{dx^2} \right) q_n(t) \right) = P\delta(x - vt) \quad (2.9)$$

In Eq. (2.9) $t$ is the time elapsed from the instant at which the moving concentrated load $P$ entered the span. The load is traveling at a speed $v$. Eq. (2.9) includes a fourth-order term that can be simplified by applying undamped and free vibration conditions to Eq. (2.7) and using the method of separation of variables to obtain an ordinary differential equation (ODE):

$$\frac{d^2}{dx^2} \left( EI \frac{d^2 \phi_n(x)}{dx^2} \right) = m \omega_n^2 \phi_n(x) \quad (2.10)$$

Inserting Eq. (2.10) into Eq. (2.9) gives:

$$\sum_{n=1}^{\infty} \left( \frac{d^2 q_n(t)}{dt^2} - m\phi_n(x) + c \frac{dq_n(t)}{dt} \phi_n(x) + m\omega_n^2 \phi_n(x) q_n(t) \right) = P\delta(x - vt) \quad (2.11)$$

Multiplying by an arbitrary normal mode $\phi_r(x)$ and integrating over $x$ gives:

$$\sum_{n=1}^{\infty} \left( \frac{d^2 q_n(t)}{dt^2} \int_0^L m\phi_n(x) \phi_r(x) dx + \frac{dq_n(t)}{dt} \int_0^L c\phi_n(x) \phi_r(x) dx + \omega_n^2 q_n(t) \int_0^L m\phi_n(x) \phi_r(x) dx \right) = \int_0^L P\delta(x - vt) \phi_r(x) dx \quad (2.12)$$

Since the normal modes are orthogonal Eq. (2.12) reduces to $n$ equations:

$$\frac{d^2 q_n(t)}{dt^2} \int_0^L m\phi_n^2(x) dx + \frac{dq_n(t)}{dt} \int_0^L c\phi_n^2(x) dx + \omega_n^2 q_n(t) \int_0^L m\phi_n^2(x) dx = \int_0^L P\delta(x - vt) \phi_n(x) dx \quad (2.13)$$

The modal damping $c_n$ is related to the modal damping ratio $\zeta_n$ by:

$$c_n = \int_0^L c\phi_n^2(x) dx = 2\omega_n \zeta_n \int_0^L m\phi_n^2(x) dx \quad (2.14)$$

In Eq. (2.14) $\omega_n$ is the natural angular frequency of vibration of the $n$th mode. Note that $\omega_n$ is related to the natural frequency $f_n$ by: $\omega_n = 2\pi f_n$. Inserting Eq. (2.14)
into Eq. (2.13) gives:

\[
\frac{d^2 q_n(t)}{dt^2} \int_0^L m\phi_n^2(x)dx + 2\omega_n \zeta_n \frac{dq_n(t)}{dt} \int_0^L m\phi_n^2(x)dx \\
+ \omega_n^2 q_n(t) \int_0^L m\phi_n^2(x)dx = \int_0^L P\delta(x - vt)\phi_n(x)dx
\]

(2.15)

Using the property of the Dirac delta function, see Eq. (2.16),

\[
\int_{-\infty}^{\infty} \delta(x - vt)f(x)dx = f(vt)
\]

and assuming that the normal modes are mass-normalised, Eq. (2.15) can finally be simplified to:

\[
\frac{d^2 q_n(t)}{dt^2} + 2\omega_n \zeta_n \frac{dq_n(t)}{dt} + \omega_n^2 q_n(t) = P\phi_n(x_v) \quad n = 1, \ldots, \infty.
\]

(2.17)

where \(x_v\) is the position of the concentrated load at time \(t\). Eq. (2.17) can be solved analytically in the Laplace domain and then inverted back to the time domain, resulting in an exact solution of the problem (for more details of this last step see Paper I).

### 2.2.2 Discrete systems

For certain applications, such as model updating of bridges using frequency response functions, it may be necessary to construct discrete models with more than 1 million DOFs. Solving such problems is computationally demanding and can be time-consuming. The model reduction techniques herein can be used to reduce the number of DOFs. The method starts by dividing the structure into several parts (often referred to as substructures) (Fig. 2.5). Each substructure is further divided into interior \(u_I\) and boundary \(u_B\) DOFs. Each substructure for the static problems is then described by the following linear equation system:

\[
\begin{bmatrix}
K_B & K_{BI} \\
K_{IB} & K_I
\end{bmatrix}
\begin{bmatrix}
u_B \\
u_I
\end{bmatrix}
= \begin{bmatrix}P_B \\0\end{bmatrix}
\]

(2.18)

The whole idea of model reduction is to eliminate the interior DOFs. Rewriting the second row of Eq. (2.18) gives Eq. (2.19):

\[
u_I = -K_I^{-1}K_{IB}u_B
\]

(2.19)

Note that Eq. (2.19) relates the boundary DOFs to the interior DOFs. The equation is then changed by substituting Eq. (2.19) into the first row of Eq. (2.18), which gives:

\[
K_{B,\text{red}}u_B = P_B \quad \text{where} \quad K_{B,\text{red}} = K_B - K_{BI}K_I^{-1}K_{IB}
\]

(2.20)
The abovementioned equation (Eq. (2.20)) is commonly known as the static reduction or Guyan reduction. The reduced stiffness matrix of the substructure is assembled into the global model to solve the original problem. The loads can only be applied at the boundary DOFs. Once the problem is solved and $u_B$ is known, the displacements within the substructure can be calculated using Eq. (2.19).

\[
\Phi_C \代表约束模式；\Phi_N \ 代表固定界面模式，
\]

and $q$ represents the generalised displacements for the interior DOFs. The constraint modes are equivalent to the static case (Eq. (2.19)) and are the displaced shapes caused by a unit displacement. Meanwhile, the fixed-interface modes can be obtained by restraining all the boundary DOFs and computing the eigenvectors. The eigenvectors corresponding to frequencies up to 1.5 times the excitation frequency should be kept (Young and Haile, 2000). Eq. (2.21), along with the fundamental equation $u_B = u_B$, can be written in a matrix form as follows:

\[
\begin{bmatrix}
    u_B \\
    u_I
\end{bmatrix} =
\begin{bmatrix}
    I & 0 \\
    \Phi_C & \Phi_N
\end{bmatrix}
\begin{bmatrix}
    u_B \\
    q
\end{bmatrix} =
T_{CB}
\begin{bmatrix}
    u_B \\
    q
\end{bmatrix}
\]

(2.22)
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Bampton equation of motion as follows:

\[
\begin{bmatrix}
\ddot{\mathbf{u}}_B \\
\ddot{\mathbf{q}}
\end{bmatrix} + \begin{bmatrix}
\mathbf{T}_{CB}^T \mathbf{M} \mathbf{T}_{CB} & \mathbf{T}_{CB}^T \mathbf{C} \mathbf{T}_{CB} \\
\mathbf{T}_{CB}^T \mathbf{C} \mathbf{T}_{CB} & \mathbf{T}_{CB}^T \mathbf{K} \mathbf{T}_{CB}
\end{bmatrix} \begin{bmatrix}
\dot{\mathbf{u}}_B \\
\dot{\mathbf{q}}
\end{bmatrix} + \begin{bmatrix}
\mathbf{T}_{CB}^T \mathbf{P}
\end{bmatrix} = \begin{bmatrix}
\mathbf{T}_{CB}^T \mathbf{P}
\end{bmatrix}
\] (2.23)

In order to illustrate the usefulness of the Craig–Bampton procedure, consider the model of the left embankment in Fig. 2.5. The original problem of the embankment alone contained over 300 thousand DOFs. A total of six DOFs are kept (boundary DOFs) for this problem at the interface between the embankment and the deck. Calculating the fixed-interface modes for frequencies of up to 45 Hz (1.5·30 Hz) provides an additional 258 DOFs. Thus, the reduced system is provided by Eq. (2.23) and comprises only 264 DOFs.

2.3 Proportionally and non-proportionally damped systems

According to the basic theory of dynamics, undamped linear dynamical systems possess classical normal modes that decouple the equations of motion. The eigenvalue problem associated with these systems is as follows:

\[
(K - \omega^2 M) \Phi = 0
\] (2.24)

where \(\omega\) and \(\Phi\) denote the undamped angular frequency and eigenvector, respectively. Consider, for example, the first bending mode of the free–free beam shown in Fig. 2.6.

\[\Phi_1\]

\[\text{Stationary node}\]

\[\text{0 0.2 0.4 0.6 0.8 1}\]

\[\text{x/L}\]

\[\Phi_1\]

\[\text{Stationary node}\]

\[\text{0 0.2 0.4 0.6 0.8 1}\]

\[\text{x/L}\]

Figure 2.6: 1st bending mode of a free–free beam (undamped or proportionally damped).

One might note herein that all points are either in-phase or out-of-phase with any other point, which consequently implies the existence of fixed stationary nodes that do not oscillate, at least for this mode. The remaining points pass through zero, maxima and minima at the same instance in time, thereby creating a standing wave pattern.
Decoupling the equation of motion for the damped linear dynamical systems is also possible. At the end of the 19th century, Lord Rayleigh (1896) showed that for a certain class of damped dynamical system (hereafter referred to as proportional damped) the equations of motion can be decoupled using the undamped normal modes. This work was later followed by Caughey and O'kelly (1965), who showed that a system is classically damped if and only if

\[ CM^{-1}K = KM^{-1}C \]  

(2.25)

Note that this condition is normally violated for real structures. However, structures are often assumed to be proportionally damped because of the many uncertainties of the damping parameters. By doing so, the energy dissipation is uniformly distributed over the structure. That said, it is necessary to recognise that such assumptions are not justified in all applications. This is especially true for problems where soil–structure interaction is important or for structures where discrete damping devices are present. These types of structures are hereafter referred to as non-proportionally damped. The eigenvalue problem for a non-proportionally damped structure is as follows:

\[ (MA^2 + CA + K)\Phi = 0 \]  

(2.26)

This equation is obtained by inserting \( u(t) = \Phi e^{\Lambda t} \) into Eq. (2.1) and neglecting the term on the right-hand side, i.e. the force. With this formulation, the equation of motion reduces to a quadratic eigenvalue problem with complex eigenvalues \( \Lambda \) as roots. A particular eigenvalue \( \Lambda_n \) is related to the undamped angular natural frequency \( \omega_n \) and the modal damping ratio \( \zeta_n \) by:

\[ \omega_n = |\Lambda_n| \]  

\[ \zeta_n = -\frac{\Re(\Lambda_n)}{\omega_n} \]  

(2.27a)

(2.27b)

If one is interested in the damped angular frequency \( \omega_d \) and damped natural frequency \( f_d \) these can be obtained as the absolute value of the imaginary part of the eigenvalue \( (\omega_d = |\Im(\Lambda)|) \) and \( f_d = \omega_d/2\pi \), respectively. The eigenvalues for the non-proportionally damped systems are either real-valued or complex-valued depending on the nature of the problem (Kawano, 2011). The mode is underdamped and the free response is a decaying oscillatory motion if an eigenvalue is complex. However, the response is a pure exponential decay if an eigenvalue is real. Based on the abovementioned definition of the undamped and damped natural frequencies, it is interesting to note that the damped natural frequency is always zero for a mode with a real eigenvalue, but the undamped natural frequency can be non-zero.

Furthermore, there is a fundamental difference between the eigenvectors of proportional and non-proportional damped systems. In order to illustrate this difference, consider again the free-free beam, but this time with discrete damping devices attached on both ends (Fig. 2.7).
Fig. 2.7: 1st bending mode of a viscously supported beam (non-proportionally damped).

The complex eigenvector describes a situation in which all points vibrate with their own phase, which implies that the fixed stationary nodes no longer exist. Consequently, the various points along the beam will not pass through zero, maxima and minima at the same instance in time, and therefore the motion is a combination of standing and traveling waves.
Chapter 3

Railway bridge dynamics

The dynamical analysis of railway bridges under moving loads has been an important research area ever since the collapse of the Chester railway bridge in 1847. The first reported work in this area was by Willis (1849), who undertook an experimental programme to study the effect of moving loads. Perhaps the most interesting finding of Willis’ work was that deflection induced by a moving load can exceed that of a static load. Today, this result is well understood and the focus has shifted more toward the resonant response of bridges under high-speed trains.

This chapter summarises the author’s view on railway bridge dynamics. The first section (Section 3.1) covers the design requirements. Section two (Section 3.2) discusses the resonance and cancellation conditions in detail. In the last section (Section 3.3), the author investigates and discusses how various modelling aspects, such as shear deformation, shear lag, train–bridge interaction and soil–structure interaction, can be incorporated in a 2D Euler–Bernoulli beam model.

3.1 Performance indicator

The Swedish code requirements for the design of bridges under high-speed trains were first introduced in BV-Bro (7th edn.) in 2004. In 2009, BV-Bro was replaced by EN1991-2 (2003) as a part of the implementation of the Eurocodes. However, the transition within this field was fairly straightforward as both BV-Bro and Eurocode adopted the procedure suggested by the European Rail Research Institute as part of the D214 project (Bucknall, 2003). The following criteria for traffic safety are provided in EN1990-A2 (2004):

- Vertical acceleration of the deck
- Deck twist
CHAPTER 3. RAILWAY BRIDGE DYNAMICS

- Vertical deformation of the deck
- Transverse deformation of the deck
- Longitudinal displacement of the deck

However, a number of studies have shown that the design is almost always governed by the first point. Therefore, in the remainder of this chapter, more focus will be placed on this design criterion than the other four listed above.

According to ERRI D-214 (1999b), the maximum acceleration for direct fastened tracks is limited to 1.0 g, while that for ballasted tracks is 0.7 g. Applying a safety factor of 2 reduces these values to 0.5 g and 0.35 g, respectively. The maximum acceleration should be less than 1.0 g for both direct-fastened and ballasted tracks in order to ensure contact between wheel and rail. However, an additional issue related to ballast instability exists for ballasted tracks.

Shake-table tests performed by Baeßler (2008) and SNCF (ERRI D-214, 1999a) (Fig. 3.1) showed that ballast began to show signs of instability at approximately 0.7–0.8 g. This phenomenon was explained by reduced interlocking at high acceleration amplitudes. However, one should be aware that no distinct value exists for ballast instability. Instead, the displacement rather gradually increases with the acceleration amplitude.

Figure 3.1: Lateral displacements of the sleeper $s_h$ after 500 cycles. Reproduced from (Zacher and Baeßler, 2008).
3.2. RESONANCE AND CANCELLATION

The present design code specifies that analyses should include frequencies up to a maximum of \((30 \text{ Hz}, 1.5f_0, f_3)\). The variable \(f_3\) is the third frequency of the structure. Unlike Eurocode, the old Swedish regulation states that considering only frequencies of up to 30 Hz is already sufficient. Both these limits were motivated by the finding that very high-frequency acceleration does not cause ballast instability (ERRI D-214, 1999b). However, these frequency limits should not be regarded as fixed. Moreover, these limits will likely be revised in the near future. In this context, there is an ongoing debate as to whether or not the frequency limit should be increased from the existing 30 Hz to 60 Hz (Zacher and Baessler, 2008).

### 3.2 Resonance and cancellation

#### Resonance

The maximum dynamic response of a bridge under the passage of high-speed trains occurs in most of the cases when resonance conditions are met. Such a condition is illustrated in Fig. 3.2 for the case of a simply supported beam.

\[
v_{cr} = \frac{f_n D}{l}, \quad l = 1, \ldots, \infty
\]  

Figure 3.2: Structural system for a simply supported beam under moving loads.

The beam is subjected to a series of loads at spacing \(D\) moving at speed \(v\). Resonance then occurs when the free vibration response of the first load vibrates in phase with the subsequent loads. A mathematical condition for this scenario can be obtained by making the time period of the moving loads equal to the time period of the beam:

\[
v_{cr} = \frac{f_n D}{l}, \quad l = 1, \ldots, \infty
\]
where \( f_n \) is the \( n \):th natural frequency and \( l \) is an integer multiple. A more theoretical derivation of this expression is described in detail by Xia et al. (2006). The typical values for \( D \) are within the range 18–27 m. The figure clearly shows that the amplitude of the free response for a single moving load plays an important role for the beam response. This result has already been recognised by Pesterev et al. (2003) and later by Museros et al. (2013) and Kumar et al. (2015) (with comments from Museros and Moliner (2016)), who found that the normalised amplitude of the free vibrations (normalised with respect to the static displacement for a concentrated load applied at the midpoint of the beam) just after a single moving load leaves the beam is given by:

\[
R_n = \frac{S_n \sqrt{2}}{1 - S_n^2} \sqrt{1 - \cos n\pi \cos \frac{n\pi}{S_n}}
\]  

(3.2)

This function is plotted in Fig. 3.3. The non-dimensional speed is defined as \( S_n = \frac{n\pi v}{\omega_n L} \). The figure illustrates that a number of local maxima seem to exist for the normalised amplitude of free vibration. These results are very important in the case of resonance \( (v = v_{cr}) \) because the resonance effect becomes more severe if the critical speed coincides with a local maxima (see case a).

**Figure 3.3:** Normalised amplitude of the free vibrations for simply supported beams.

**Cancellation**

Another important issue that needs some further clarification is the cancellation phenomenon. Looking again at Fig. 3.3 it is clear that there exist points of zero amplitude. Museros et al. (2013), who studied this problem in detail, derived the
following expression for the cancellation to occur:

\[
S_{\text{cancel}}^{nl} = \frac{n}{n \pm 2l} > 0, \quad l = 1, \ldots, \infty
\] (3.3)

The physical explanation of this is that the undamped beam starts from rest and returns to rest once the first load leaves the beam (see case b in Fig. 3.3). Thus, for relatively short bridges \((L < D)\), resonance will not be possible even though Eq. (3.1) is met. It should, however, be pointed out that for longer bridges resonance could still occur when two or more axles are on the bridge at the same time.

This result is very meaningful from a practical point of view. It is one reason why the design codes normally require that the bridge should be designed for a number of train configurations to overcome the problem with points of zero amplitude. This strategy ensures that at least some of the trains coincide with local maxima.

**Example 1**

In order to examine the way in which a certain train configuration affects the bridge response, consider a simply supported bridge with a span length of 36 m. The bridge has a bending stiffness of 172.2 GNm\(^2\), a mass of 17,000 kg/m and 0.5% damping. These values are obtained from a single-track ballasted composite railway bridge located in the northern part of Sweden. The bridge is loaded by the high-speed load model defined in EN1991-2 (2003) (see Fig. 3.4 and Table 3.1) for speeds ranging from 100 to 400 km/h. The natural frequencies of the first three modes for the bridge are 3.85 Hz, 15.43 Hz and 34.71 Hz. Used in this context, Eq. (3.1) implies that a resonance peak should be obtained for each of the train models for speeds ranging from 249.5 to 374.2 km/h.

![Figure 3.4: The load distribution for the high-speed load model (EN1991-2, 2003).](image-url)
CHAPTER 3. RAILWAY BRIDGE DYNAMICS

Table 3.1: Train configuration for the high-speed load model (EN1991-2, 2003).

<table>
<thead>
<tr>
<th>Train model</th>
<th>Number of intermediate coaches</th>
<th>Coach length</th>
<th>Bogie axle spacing</th>
<th>Point force</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>18</td>
<td>18</td>
<td>2.0</td>
<td>170</td>
</tr>
<tr>
<td>A2</td>
<td>17</td>
<td>19</td>
<td>3.5</td>
<td>200</td>
</tr>
<tr>
<td>A3</td>
<td>16</td>
<td>20</td>
<td>2.0</td>
<td>180</td>
</tr>
<tr>
<td>A4</td>
<td>15</td>
<td>21</td>
<td>3.0</td>
<td>190</td>
</tr>
<tr>
<td>A5</td>
<td>14</td>
<td>22</td>
<td>2.0</td>
<td>170</td>
</tr>
<tr>
<td>A6</td>
<td>13</td>
<td>23</td>
<td>2.0</td>
<td>180</td>
</tr>
<tr>
<td>A7</td>
<td>13</td>
<td>24</td>
<td>2.0</td>
<td>190</td>
</tr>
<tr>
<td>A8</td>
<td>12</td>
<td>25</td>
<td>2.5</td>
<td>190</td>
</tr>
<tr>
<td>A9</td>
<td>11</td>
<td>26</td>
<td>2.0</td>
<td>210</td>
</tr>
<tr>
<td>A10</td>
<td>11</td>
<td>27</td>
<td>2.0</td>
<td>210</td>
</tr>
</tbody>
</table>

However, this is not the case for the HSLM-A7 train model (Fig. 3.5a). This train model should induce a resonance peak for a speed of 332.6 km/h (= 3.6 × 3.85 × 24). Meanwhile, the HSLM-A2 train model seems to induce a peak of maximum acceleration at 263.3 km/h (= 3.6 × 3.85 × 19). The reason for this result becomes clear by examining the normalised amplitude of the free vibrations (Eq. (3.2) and plotted in Fig. 3.5b) for both speeds, where the former speed corresponds to a point of zero amplitude and the latter to a local maxima.

Figure 3.5: a) Envelope of the maximum vertical acceleration at midspan under a moving load and b) the normalised amplitude of the free vibrations.
3.3 Modelling aspects

Modern finite element softwares enable a very detailed description and analysis of structures. However, in the author’s opinion, this type of analysis tends to be time consuming and it is difficult to interpret the data. Instead, working with 2D beam models, where closed-form solutions sometimes exist, and adjusting the structural parameters to match the modal properties from the more complicated system is much more effective. The following aspects are typically relevant for the dynamical response (Fig. 3.6):

1. The shear deformation for the thick beams (bridges with low span-to-depth ratio) lowers the fundamental frequency.
2. The shear lag effect, which typically occurs in flanges or bridge decks with overhang, lowers the fundamental frequency.
3. Ballast distributes the axle loads over a greater area (Museros et al., 2002; Axelsson et al., 2014).
4. Eccentricity between the centre of rotation for the beam and the bridge bearings affects the frequency and the modal mass.
5. The train–bridge interaction lowers the natural frequencies, but increases the damping ratio. More details about this are given in Subsection 3.3.1.
6. The soil–structure interaction also affects the frequencies and damping ratios in a similar manner as the train–bridge interaction (Subsection 3.3.4).
7. The roller bearings behave differently depending on the contact force. The bearings are fixed for small amplitude, and roll for a larger amplitude, thereby changing the natural frequency of the system. Ülker-Kaustell and Karoumi (2013) also showed that the bearing can also introduce a significant amount of frictional damping.

Figure 3.6: Modelling aspects relevant for the dynamical response.
CHAPTER 3. RAILWAY BRIDGE DYNAMICS

3.3.1 Load models

The trains on high-speed lines normally comprise passenger carriages and a power car at one or both ends of the train. Each wagon includes a car body and bogies. The bogies have two levels of suspension (i.e. primary and secondary) to reduce the car body vibrations and wheel–rail forces. The complete train should be modelled if the purpose of the study is to predict the comfort of the passenger or design the rail–track system. However, a simplified approach can be adequate if the purpose is to study the bridge response. The three most common train models proposed in the literature are as follows: (a) moving load (ML), (b) simplified interaction model (SIM) and (c) two-layer suspension rigid beam (RB) (see Fig. 3.7).

![Train–bridge interaction models](image)

Figure 3.7: Train–bridge interaction models.

Track irregularities can be considered for the last two models. The ML model for a simply supported beam was first solved by Krylov (1905) using the eigenfunction expansion method. His work has been extended by a number of researchers to study various aspects of the bridge and the train. In 1922, Timoshenko (1922) studied the effects of a harmonic force on beams. In 1950, Ayre et al. (1950) looked at a continuous bridge with two equal spans. The PhD thesis by Hillerborg (1951) is another important contribution from the 1950s, which considered sprung masses on a beam similar to the SIM model but without damping. Both the SIM and RB models are most often solved by a numerical scheme based on the Newmark method. Interested readers are referred to the work of Arvidsson (2014); Arvidsson and Karoumi (2014); Arvidsson et al. (2014) for more historical details of these models. However, one significant difference between the SIM and RB models is worth noting. The RB model includes the inertial effects of the car body, whereas this is replaced by a constant force in the SIM model. This approximation is justified because the two-level suspension system partly decouples the car body from the dynamic interaction (ERRI D-214, 1999d; Arvidsson, 2014).
Example 2

Re-consider the single-track ballasted composite railway bridge discussed in Example 1 (Section 3.2), but this time with 0% damping and loaded with an ICE 2 train with mechanical properties as defined in ERRI D-214 (1999d). A series of numerical simulations were performed using the train–bridge interaction toolbox presented in Cantero et al. (2016). The train and the track irregularities are modelled by the RB model (see Fig. 3.7) and German track spectrum, respectively, for speeds ranging from 100 to 400 km/h. For the sake of simplicity, no track model is considered in the simulations. Fig. 3.8 shows the results of these simulations (red curve). From the figure, it is obvious that resonance occurs at approximately 370 km/h. The natural frequencies of the first three modes, without the train, are the same as that obtained in Example 1.

![Figure 3.8: Maximum vertical acceleration at midspan under an ICE 2.](image)

Performing a complex eigenvalue analysis (Section 2.3) for all the train locations on the beam is useful in providing more insights into the train–bridge interaction system. Determining how the natural frequencies and the damping ratios fluctuate during train passages is also possible by using Eqs. (2.27a) and (2.27b). Fig. 3.9 shows the first three bending modes. Examining the results depicted in Fig. 3.9 leads to the following observations:

1. The first bending mode is affected more than the higher-order modes.
2. The train mass lowers the natural frequency of the first bending mode by 1–2%.
3. The average damping ratio of the first bending mode is 0.37%.
4. Power cars provide more damping to the system than passenger carriages.

The analysis can now be repeated, but this time using the ML model and 0.37% damping. These results are also plotted in Fig. 3.8 (black line). The two sets of results show good agreement, except at speeds of 297 km/h and 327 km/h where the higher-order modes govern the response. Better agreement could be achieved by using different damping ratios for different modes. Nevertheless, these results clearly illustrate that the train–bridge interaction can be accounted for in a simplified manner by an additional damping ratio. This conclusion also agrees well with the results presented by ERRI D-214 (1999b); Liu et al. (2009); Arvidsson (2014). To close this section, Fig. 3.10 shows the contact force from the first axle in the power car. As one would expect, the contact force is constant for the ML model and oscillates around this constant value for the RB model.

Figure 3.9: Natural frequencies and damping ratios for the three first fundamental modes under an ICE 2.

Figure 3.10: Contact force for the first axle in the power car.
3.3.2 Structural system

A good dynamical model of a bridge should be simple, yet able to capture the true dynamical behaviour. This work is focused on bending vibrations, so issues related to longitudinal and torsional vibration are not dealt with here. However, for practical applications, this is a reasonable assumption since ERRI D-214 (1999b) showed that torsion can be neglected if $f_T > 1.2f_0$, where $f_T$ and $f_0$ are the first torsional and bending natural frequencies, respectively. This recommendation is also adopted in EN1991-2 (2003).

In this context, it is important to emphasise that the model should be able to capture relevant effects related to the bending modes, such as shear deformation and shear lag. One way to achieve this is to use solid elements that can account for the three-dimensional stress flow in the continuum. Another way is to use shell elements with a Mindlin–Reissner formulation to include shear deformations. With this approach the shear lag is accounted for by membrane actions in the shells. Shear deformation and shear lag are most pronounced for bridge decks with relatively low span-to-height ratio and wide flanges, respectively.

On the other hand, the models presented in this work are based on Euler–Bernoulli beam theory. Note that this formulation assumes that the deformation is governed by flexural vibrations, and therefore does not include effects due to shear deformations or shear lag. This is a reasonable assumption for long-span bridges where the abovementioned effects are small. Despite this, the models can still be used for short-span bridges if appropriate measures are taken. The basic idea is to adjust the structural parameters in the Euler–Bernoulli beam model so that the modal properties include all relevant three-dimensional effects. The procedure can be outlined as follows:

1. A detailed FE model is built, and the natural frequency and corresponding modal mass of the fundamental mode are calculated. Checking that the frequency of the fundamental mode is well separated from other modes is important (a commonly accepted value is at least 20%).

2. The mass and stiffness in the Euler–Bernoulli beam model must be updated, such that the frequency and modal mass of the fundamental mode match the more detailed model.

3. Dynamical analyses are performed on the updated beam model.

Below is an illustrative example of how the procedure can be used in practice.
Example 3
Consider a simply supported bridge (10 m long, 12 m wide) constructed of reinforced concrete that has a Young’s modulus \( E \) of 34 GPa, a Poisson’s ratio \( \nu \) of 0.2, a density \( \rho \) of 2500 kg/m\(^3\), a moment of inertia \( I \) of 0.91 m\(^4\), an area \( A \) of 9.42 m\(^2\) and a damping ratio of 2.2%. The bridge deck accommodates two slab track systems, which are not included in the present model. Dynamical analyses are performed for a HSLM-A2 train moving at speeds of 100 to 400 km/h. For this purpose three different models were developed: (a) 2D Euler–Bernoulli beam model, (b) Mindlin shell model and (3) solid model. Fig. 3.11 and Table 3.2 show an overview of the models together with the first three natural frequencies.

![Idealisation of overhang](image)

Figure 3.11: Three alternatives for modelling a simply supported reinforced concrete bridge.

<table>
<thead>
<tr>
<th></th>
<th>Beam model</th>
<th>Shell model</th>
<th>Solid model</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st longitudinal bending</td>
<td>18.03</td>
<td>14.41</td>
<td>14.98</td>
</tr>
<tr>
<td>1st torsion</td>
<td>-</td>
<td>18.14</td>
<td>19.16</td>
</tr>
<tr>
<td>1st transverse bending</td>
<td>-</td>
<td>23.52</td>
<td>29.11</td>
</tr>
</tbody>
</table>

Table 3.2: Undamped natural frequencies [Hz].

The table shows that all models include a bending mode, even though the corresponding eigenfrequency as predicted by the beam model is some 20% higher than for the other two models. The reason for this stiff behaviour is the shear deformation and shear lag effects being neglected in the beam
model. Next, while there is a good match for the first torsion between the shell and the solid models, some difference still exist for the first transverse bending. The origin of this error is probably the idealisation of the overhang in the shell model (Fig. 3.11). Finally it should also be noted that the last two frequencies are only obtained for the shell and solid models.

Consider now the envelopes of the maximum vertical displacements and accelerations shown in Fig. 3.12 for different train speeds. The results clearly show that the beam model provides poor results for all speeds. One manner of overcoming this poor behaviour is to update the moment of inertia in the beam model to account for shear deformation and shear lag. Using for the calibration the results from the solid model gives:

$$I_{\text{modified}} = \rho A \left( \frac{2L^2 f_0}{\pi} \right)^2 = \frac{2500 \times 9.42}{34 \times 10^9} \left( \frac{2 \times 10^2 \times 14.98}{\pi} \right)^2 = 0.62 \text{ m}^4$$

(3.4)

The results obtained using this modified beam model are presented in Figs. 3.12. The figures clearly show that the modified beam model performs much better for both displacements and accelerations.

Figure 3.12: Envelope of the maximum vertical response at midspan under a moving train. a) displacements, and b) accelerations.
CHAPTER 3. RAILWAY BRIDGE DYNAMICS

3.3.3 Damping

The energy dissipated in a dynamical system can be divided into viscous damping; Coulomb damping; structural damping, also known as hysteretic damping or rate-independent damping; and inelastic deformations. These can be either discrete or distributed along the member. Viscous damping, which is widely used in practical applications, is characterised by damping that is proportional to the velocity \( f_d = c \frac{du}{dt} \), which is also the reason why the damping force is frequency-dependent. Structural damping, on the other hand, is proportional to the displacement \( f_d = hui \), and therefore frequency-independent. If the analysis is performed in the frequency domain, it is often used to describe energy dissipation of materials. According to ERRI D-214 (1999b) and Neild (2001), damping in railway bridges is caused by:

- Bending of materials
- Opening and closing of cracks
- Friction at supports and bearings
- Ballast
- Train–structure interaction (Subsection 3.3.1)
- Soil–structure interaction (Subsection 3.3.4)

However, modelling the energy dissipation by distributed viscous dampers is more convenient because of the complex nature of the damping mechanisms. In doing so, the damping ratio \( \zeta \) can be determined from the free-vibration part of the signal as follows:

\[
\zeta = \frac{1}{2\pi j} \ln\left(\frac{\ddot{u}_1}{\ddot{u}_{1+j}}\right)
\]

where \( \ddot{u} \) is the acceleration amplitude, and \( j \) is the number of cycles (Chopra, 1995). The above equation can only be used for existing structures and is therefore not suitable for design purposes. Instead, the design codes, such as EN1991-2 (2003), are based on empirical damping ratios obtained from measurements on different bridges (ERRI D-214, 1999c). The measurements show that the structural material, span length and fundamental frequency are strongly correlated with the damping. Despite this fact, the influence of the fundamental frequency in the final equations was disregarded. Fig. 3.13 shows the damping ratios obtained from the free-vibration tests for the first bending mode. The lines represent the recommended damping values (Table 3.3).

Fig. 3.13 clearly shows that short-span bridges \( (L < 20m) \) display greater damping irrespective of the bridge type. This result is consistent with the results obtained
3.3. MODELLING ASPECTS

Figure 3.13: Measured damping values. Reproduced from ERRI D-214 (1999c).

Table 3.3: Recommended lower-bound damping values $\zeta$ according to EN1991-2 (2003).

<table>
<thead>
<tr>
<th></th>
<th>$L &lt; 20,\text{m}$</th>
<th>$L \geq 20,\text{m}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel and composite</td>
<td>$0.5 + 0.125(20 - L)$</td>
<td>0.5</td>
</tr>
<tr>
<td>Prestressed concrete</td>
<td>$1.0 + 0.07(20 - L)$</td>
<td>1.0</td>
</tr>
<tr>
<td>Reinforced concrete</td>
<td>$1.5 + 0.07(20 - L)$</td>
<td>1.5</td>
</tr>
</tbody>
</table>

for the soil–structure interaction (Subsection 3.3.4), where the same tendency is observed. Furthermore, concrete bridges are readily seen to have higher damping than steel bridges. It is important to recognise that the empirical damping values proposed by EN1991-2 (2003) represent the sum of all mechanisms that adsorb energy after the train leaves the bridge. Therefore, the damping values in Table 3.3 can not directly be used, especially for $L < 20\,\text{m}$, if SSI is accounted for in the model. A conservative approach would then be to also use the values for $L \geq 20\,\text{m}$ for short-span bridges.

3.3.4 Soil–structure interaction

Recent studies of soil–structure interactions (SSI) for bridges (Ülker-Kaustell et al., 2010; Romero et al., 2013; Doménech et al., 2015) showed that the soil medium and foundation type affected the dynamical behaviour of the system. Based on these studies, two general conclusions can be drawn for simply supported beam bridges: (1) elastic deformation in the soil lowers the resonance speed, and (2) geometric damping in the soil, also known as radiation damping, reduces the amplitude of the
vibrations at resonance. The work of Ülker-Kaustell et al. (2010) modelled the soil using a single homogeneous elastic layer, while the work of Romero et al. (2013); Doménech et al. (2015) used a homogeneous elastic half-space. All the authors assumed constant soil properties with depth.

Before further discussing the various aspects of SSI, the following presents a brief explanation of the mechanical properties of granular materials. For such materials the shear modulus for small strains $G_0$ is provided by (TK Geo 11, 2011):

$$G_0 = K_1 \cdot \sqrt{\sigma_m'}$$  \hspace{1cm} (3.6)

where the mean effective stress $\sigma_m' = (1 + 2K_0)\sigma_v'/3$ [kPa] depends on the vertical effective stress $\sigma_v'$, and the ‘at-rest’ coefficient of lateral earth pressure $K_0$. $K_1$ is a constant that varies between 15 000 and 30 000, where the lower value corresponds to sand and the higher value to gravel. Hardin (1978) published a more accurate equation to calculate $G_0$, but herein Eq. (3.6) will be used because it captures the essential features of a granular material. The shear wave speed can then be calculated as follows:

$$v_s = \sqrt{\frac{G_0}{\rho}}$$  \hspace{1cm} (3.7)

In contrast to the shear wave (s-wave) that travels through the soil skeleton, the compression wave (p-wave) travels either through the water phase or in the soil skeleton depending on which medium has the highest compression wave speed. A typical value for the compression wave speed in water is 1450 m/s. The compression wave speed of the soil skeleton is:

$$v_p = \sqrt{\frac{E(1-\nu)}{\rho(1+\nu)(1-2\nu)}}$$  \hspace{1cm} (3.8)

with $E = 2G_0(1+\nu)$. For small wave amplitudes, Sas et al. (2013) showed that the Poisson’s ratio was typically within the range 0.2–0.4. As seen from the abovementioned equation, the wave speeds seemed to increase with depth for an unsaturated granular soil (Fig. 3.14b). Meanwhile, Möller et al. (2000) proposed that $G_0$ can be estimated from the following equation for normally consolidated clay (cohesive soil):

$$G_0 = 504 \frac{c_u}{w_L}$$  \hspace{1cm} (3.9)

The corrected undrained shear strength $c_u$ and liquid limit $w_L$ are normally determined from laboratory tests. The wave speeds are also constant if $c_u$ and $w_L$ can be assumed constant with depth (Fig. 3.14b).
3.3. MODELLING ASPECTS

Figure 3.14: (a) Rigid circular footing loaded by a vertical harmonic concentrated force and (b) idealisation of the p- and s-wave profiles for granular and cohesive soils.

Several methods are available to model soil–structure interaction. The most straightforward approach is to model the bridge and surrounding soil medium using commercially available finite element software. The geometric damping resulting from waves traveling away from the structure could be accounted for with non-reflecting boundaries. However, these models are computationally expensive because of the large number of solid elements required for the soil.

To overcome this, researchers working with soil dynamics have derived impedance (dynamic stiffness) curves that can be applied to the structure boundary. The impedance curves $Z(\omega)$ are normally obtained by substituting $u(t) = U e^{i\omega t}$ and $P(t) = P e^{i\omega t}$ into Eq. (2.1) and rearranging the matrices as follows:

$$
\begin{pmatrix}
-\omega^2 & \mathbf{M}_B & \mathbf{M}_{BI} \\
\mathbf{M}_{IB} & \mathbf{M}_I \\
\end{pmatrix}
+ i\omega
\begin{pmatrix}
\mathbf{C}_B & \mathbf{C}_{BI} \\
\mathbf{C}_{IB} & \mathbf{C}_I \\
\end{pmatrix}
+ \begin{pmatrix}
\mathbf{K}_B & \mathbf{K}_{BI} \\
\mathbf{K}_{IB} & \mathbf{K}_I \\
\end{pmatrix}
\end{pmatrix}
\begin{pmatrix}
\mathbf{U} \\
\end{pmatrix}
= \mathbf{A}
$$

(3.10)

Note that $\mathbf{P} = [\mathbf{ZU}_B, \mathbf{0}]^T$ and $B$ are the degrees of freedom connected to the structure. The impedance is then obtained by applying a Guyan reduction (Subsection 2.2.2) to Eq. (3.10):

$$
\mathbf{Z}(\omega) = \mathbf{A}_B(\omega) - \mathbf{A}_{BI}(\omega)\mathbf{A}_I^{-1}(\omega)\mathbf{A}_{IB}(\omega)
$$

(3.11)

If a spring–dashpot model is used to reproduce the ‘exact’ impedance, then the stiffness and damping coefficients are frequency-dependent and calculated as follows: $k(\omega) = \Re(Z(\omega))$ and $c(\omega) = \Im(Z(\omega))/\omega$. The procedure is outlined in detail in Cottereau (2007).
Example 4

Consider now a circular footing supported by a homogeneous elastic half-space of radius $r$ and loaded by a vertical harmonic concentrated force $P$ (Fig. 3.14a). For this system (Fig. 3.15) the solution to Eq. (3.11) can be found in the Handbook of Impedance Functions (Siefert and Cevaer, 1992). An examination of the figure clearly shows that the stiffness and damping coefficients become frequency-dependent because of the model size reduction. One way to avoid frequency-dependent variables, which, otherwise, must be solved in the frequency domain, is to calibrate a lumped-parameter model to match the ‘exact’ impedance (Gazetas, 1991; Wolf, 1994).

The three models introduced in Fig. 3.15 are analysed by means of this example. For each model the parameters were determined by non-linear optimisation. The best fit for each model is also shown in the figure. For this particular example, Model C is the only model that can accurately capture the impedance in the entire frequency range. However, as shown in Section 3.2, in many instances the response at resonance is governed by the sum of the free-vibration responses generated by each axle load. Thus, in many cases, using Model A with parameters $k(\omega_0)$, $c(\omega_0)$ and $m = 0$, where $\omega_0$ is the angular frequency of the fundamental mode, is an adequate approximation since the free-vibration response of each axle load is often governed by the fundamental mode.
Example 5

Numerical simulations were performed on bridges with a span of 5 m (short) to 60 m (long) supported by a granular material with a Young’s modulus $E_{\text{soil}}$ of 50 MPa (loose) to 800 MPa (dense). The Poisson’s ratio and density of the soil were assumed constant at $\nu = 0.4$ and $\rho = 1800 \text{ kg/m}^3$, respectively. The size of the foundation was fixed as $8 \times 8 \text{ m}$. Both the mass per unit length $m$ and the bending stiffness $EI$ were assumed to vary with span $L$. This relationship was previously derived in Johansson et al. (2010), where Eqs. 3.12a and 3.12b are proposed for the single-track ballasted concrete bridges by collecting data from ‘real’ bridges.

$$m = 580L + 5350 \text{ [kg/m]} \quad (3.12a)$$

$$EI = 4(265L^{-1.28})^2mL^4/\pi^2 \text{ [Nm}^2] \quad (3.12b)$$

Numerical simulations were performed based on this information. In addition, the calculated modal damping ratios of the first bending mode are presented in Fig. 3.16. The measured data points in Fig. 3.16 are reproduced from a plot in ERRI D-214 (1999b), see also Fig. 3.13. The figure shows good consistency between the experimental and numerical data. The modal damping ratio is of correct magnitude and tends to decrease with the span. From a practical point of view, a typical soil in Sweden lies between 100 MPa and 400 MPa. Therefore, the damping ratios for the soils outside of this range should be considered as a theoretical exercise. With this in mind, the expected damping ratio from the soil alone is between 0.1% (long span & dense soil) and 6.6% (short span & loose soil). These findings clearly indicate that geometric soil damping is one of the main contributors to damping in railway bridges.

![Figure 3.16: Damping of the first bending mode (Svedholm et al., 2015). The solid lines are the simulated results.](image-url)
Chapter 4

The research work

The research work is divided into three parts. The first part of the work focused on the solution of proportionally damped continuous beams under moving loads (Fig. 4.1). A closed-form solution is derived herein (Paper I) instead of using the finite element method for dynamical analysis. This method is later used to produce design charts (Paper II) and assess the existing railway lines (Paper III). The second part deals with non-proportionally damped beams. A closed-form solution is derived for this case also (Paper IV). However, this time, due to the complexity involved in finding orthogonality conditions, the solution is limited to single-span beams. This solution is then used to study the effect of external damping mechanisms (Paper IV) and the soil–structure interaction (Svedholm et al., 2015). The abovementioned work focused on the design of new structures, for which the response is governed by the resonant behaviour of the first few bending modes. The last part deals with the issue of model updating of bridges (Paper V). The focus here is on understanding the structural system independent of the load. Therefore, the system must be able to capture not only longitudinal bending but also transverse bending and torsion. The dynamical analysis is performed using numerical techniques because of the complex nature of these models. Nevertheless, the computational time could be significantly reduced with model reduction techniques.

Section 1.2 stated that the main objectives of this thesis are as follows: (1) provide practical tools, and (2) give recommendations regarding the design of bridges for high-speed trains. In this light, it should be noted that Papers I, IV and V deal with the first objective and Papers II and III, along with Chapter 3 of the thesis, contribute to the second objective. More details on the appended papers are provided in the sections hereafter.
4.1 Proportionally damped beams

4.1.1 Closed-form solution of continuous beams (Paper I)

Robust and fast tools for analysing railway bridges are needed at an early design stage. Therefore, the paper derived a set of equations to calculate the dynamical response of general multi-span Bernoulli–Euler beam under moving loads. Previous publications suffered from limitations, which mainly fell into two categories: (1) the model was oversimplified (e.g. simply supported, equal span lengths or neglecting damping); and (2) the governing differential equation was solved in a recursive or approximate manner. However, this was not the case for the solution presented in Paper I. A closed-form solution was obtained herein by: (1) expanding the solution as a linear combination of normal modes; (2) solving the differential equation for each mode in the frequency domain after a Laplace transformation; and (3) determining the natural frequencies and mode shapes by applying the boundary conditions to the characteristic function of a beam.

The highlights of the paper are as follows:

- A closed-form solution for evaluating the dynamical behaviour of a beam is derived.
- The model can consider stepped sections, several spans and elastic supports.
4.1. PROPORTIONALLY DAMPED BEAMS

- The equations are exact because the problem has been solved analytically using a Laplace transform.
- The model has been verified with several numerical examples.

Effects such as shear lag, shear deformation and eccentricity can be considered by adjusting the structural properties (Section 3.3). Moreover, accounting for the train–bridge interaction with the methodology outlined in Subsection 3.3.1 is also possible.

4.1.2 Design charts (Paper II)

A closed-form solution for the moving load problem was previously developed in Paper I. The method is fast and exact, but does not provide any new insights for deciding which bridges are suitable for a high-speed line. Therefore, performing several parametric studies on ‘fictitious’ bridges to investigate the topic is of interest. The study covered multi-span beams with a uniform span length of 8 to 60 m, fundamental frequencies from 1.5 to 30 Hz and damping ratios of 0.5 to 3.0%. The beams were analysed for trains HSLM A1–A10 running at speeds of 100–300 km/h. To the author’s knowledge, this is the first study to present design curves for the high-speed load models in EN1991-2 (2003). Similar design curves have previously been obtained by ERRI D-214 (1999b). Unfortunately, these were produced before the high-speed load model was introduced.

The highlights of the paper are as follows:

- Design curves are developed for calculating maximum displacement, angular rotation and acceleration.
- Two case studies are presented to validate the model and show possible applications.
- Increased span length or fundamental frequency reduce the maximum acceleration.
- Multi-span bridges tend to have lower maximum acceleration compared with a similar simply supported bridge.

4.1.3 Assessment of existing railway lines (Paper III)

Globalisation has accelerated the development of new high-speed lines over the last several years. However, from an economic perspective, it would perhaps be better to upgrade an existing railway line for high-speed trains than build new lines. Therefore, the author, along with several other colleagues in the department, developed a statistical method to assess a network of bridges. This collaboration can be summarised in the three following parts: (1) The bridges are categorised into groups(blocks with similar properties (e.g. bridge type). Data from a number
CHAPTER 4. THE RESEARCH WORK

of drawings must then be collected to be able to perform any type of statistical evaluation and construct prediction bounds. (2) Information about all known deterministic variables (e.g. span lengths, number of spans and structural material) are collected. A sample for the unknown variables (e.g. fundamental frequency, mass and support stiffness) is then generated based on part 1. (3) Dynamic analysis is performed, and the probability of failure is calculated (formulated as the probability of exceeding the design limit criterion for acceleration). The method was used in 2011 when the Divisions of Structural Engineering and Bridges at the KTH Royal Institute of Technology and the ELU Konsult AB were asked to estimate the number of bridges that would need to be strengthened or replaced if the maximum allowed speed was increased to 250 km/h. The study covered more than 1000 bridges along the Western Mainline (Stockholm to Gothenburg), the Southern Mainline (Stockholm to Malmö) and The West Coast Line (Gothenburg to Malmö).

The highlights of the paper are as follows:

▷ Bridges on existing railway lines are analysed for high-speed trains.
▷ The structural properties are determined from the statistical analysis of a sample of 117 railway bridges.
▷ A simplified approach to analyse slab-frame bridges is proposed, neglecting the influence of soil-structure interaction.
▷ The results revealed that a large number of bridges required more detailed studies.

4.2 Non-proportionally damped beams (Paper IV)

Paper I is limited to proportionally damped beams that possess classical normal modes, which are then used to decouple the equations of motion. However, this is not the case for systems with discrete damping devices (e.g. soil-structure interaction and dampers; see Sections 2.3 and 3.3.4). For such systems, under moving loads an analytical solution has been derived for general end conditions. In these cases the equations of motion cannot be simply decomposed using real-valued eigen-modes. Instead, it is necessary to derive a set of orthogonality conditions valid for the problem at hand.

The highlights of the paper are as follows:

▷ Analytical solution for a non-proportionally damped beam under moving loads.
▷ The solution is useful in studying various types of damping mechanisms in bridges.
▷ Examples are included to provide insight into the problem of damped systems.
4.3. MODEL REDUCTION OF LINEAR SYSTEMS (PAPER V)

▶ Interesting results are presented for closely spaced modes.

The shear lag, shear deformation, eccentricity and train–bridge interaction can be accounted as in Paper I, and will not be repeated here. In addition to these effects, including the soil–structure interaction with the methodology outlined in Subsection 3.3.4 is also possible.

4.3 Model reduction of linear systems (Paper V)

A series of measurements was performed on an end-frame bridge located in Pershagen along the Western Mainline. The measurements primarily aimed to test the hydraulic exciter developed at the department. However, the structural response was also recorded. Some interesting results were obtained during signal post-processing, particularly for the torsional and second bending modes, where high damping ratios were obtained. A FE model was created and calibrated using the measured frequency response functions to obtain a better understanding of the dynamical system. Good agreement was achieved between the measured and predicted results, using a combination of a genetic algorithm and pattern search techniques. The computational time of the FE model (more than 4.6 million DOFs) was reduced by model reduction techniques (see Subsection 2.2.2 for further details). Dynamical simulations were carried out on the calibrated model to show the effect of SSI on the dynamical response. Based on the calculations, it was evident that the SSI plays an important role and must be included (for these types of bridges) to obtain realistic acceleration levels.

The highlights of the paper are as follows:

▶ Model updating of an end-frame bridge using the frequency response function.
▶ Computational time reduced with model reduction techniques.
▶ Considering the effects from soil–structure interaction were essential in reproducing the measured results.
▶ The FE model showed that soil–structure interaction reduces peak deck accelerations.
Chapter 5

Discussion and conclusions

This research focused on simple and effective modelling techniques for the dynamical analysis of railway bridges under high-speed trains. Previous experiences from the industry showed that engineers, even in the early design stage, often over-complicate the problem with detailed FE models that take days and weeks to solve. In the author’s opinion, it is preferable to begin examination of the bridge using a two-dimensional beam model that captures the fundamental vibration modes.

The project began with a literature review, which revealed that either most of the methods were oversimplified or the equation of motion was solved in a recursive or approximate manner. Consequently, the subsequent research focused on developing analytical procedures for analysing railway bridges under moving loads. Two models, for proportional and non-proportional damped systems, were analysed and closed-form solutions were presented. Both models were formulated as continuous systems, and general boundary conditions were assumed throughout the derivation. However, the proposed models are not applicable to all circumstances. First, the models are linear and, therefore, cannot be used to study any non-linear dynamical problem. Another assumption in the model is that the beam is described using Bernoulli–Euler theory, which is something that can only be motivated if the movement is dominated by flexural deformations. However if these assumptions are satisfied the model may be used to calculate the dynamic response with high accuracy.

From a practical point of view, the models may be applied for evaluating the dynamical response at resonance, which is often the governing factor for a railway bridge under high-speed trains. From a theoretical point of view, the models can be used to qualitatively study the effects of various parameters on dynamic response. However, for more complicated systems, where higher-order longitudinal bending, transverse bending and twisting are important factors, it is recommended to use a shell or solid model. This could be combined with model reduction techniques to avoid unnecessarily long computation time.
CHAPTER 5. DISCUSSION AND CONCLUSIONS

5.1 Theoretical conclusions

The theoretical part of this work was mainly devoted to the development of simple beam models to study the bridge responses under moving loads. Papers I and IV generalised some of the closed-form solutions for the moving load problem. This includes both proportionally damped (Paper I) and non-proportionally damped (Paper IV) systems. The following conclusions are drawn as a part of this process:

- For short bridges the free vibration response induced by the individual axles drives the resonance effect. Therefore, for design purposes, it is possible to argue that frequency-dependent parameters (e.g. soil–structure interaction) can be assumed to be constant. This should be performed by picking the value at the fundamental frequency of the structure.

- The effects of train–bridge and soil–structure interactions can be studied in detail using complex eigenvalue analyses. The changes in the natural frequencies and the modal damping ratios are typically obtained from the calculations.

- Non-proportionally damped systems often contain non-classical modes. These modes, which were investigated in Paper IV, were found to be highly damped. For the structures studied in the cited paper, the bending modes were found to attain maximum damping when their undamped natural frequency was similar to that of a non-classical mode.

- Methods based on statistical techniques are very efficient for the assessment of bridges on existing railway lines. However, such methods are only possible if they are coupled with rapid and robust analysis algorithms. Closed-form solutions of the type presented in this thesis are very efficient in this context.

- Updating methods based on frequency response functions are very reliable and efficient for non-proportionally damped structures. Moreover, these types of methods allow in-depth study of the damping mechanisms.

5.2 Practical conclusions

Extensive dynamic analyses were performed on the bridges located on several railway lines in Sweden. The results of this comprehensive parametric study provide several practical conclusions of relevance for designers:

- Increased span length, fundamental frequency, mass or damping ratio lowers the acceleration in the bridge deck. This further implies that concrete bridges are preferable to steel bridges because of the larger mass of the deck and the higher damping ratio. Short-span bridges are also more problematic than long-span bridges if the combined effect of the span length and the fundamental frequency is considered.
5.3. FURTHER RESEARCH

- Important effects, such as shear deformation, shear lag and eccentricity, can be accounted for in a simplified manner by adjusting the structural parameters.

- From a dynamical point of view, constructing multi-span bridges that are continuous over the supports is preferable to solutions consisting of series of simply supported bridges. Compared to the latter, continuous bridges show up to 60% reduction in maximum vertical deck acceleration (see Paper II).

- Axle load spreading lowers the maximum acceleration of the bridge deck, especially for short-span bridges (<30 m).

- Considering the effects of soil–structure interaction is crucial for end-frame and integral bridges. It is the author’s opinion that if this aspect is neglected, the model will be to conservative and will not reflect the true behaviour of the structure.

- The soil–structure interaction should not be modelled with simple springs, but with springs and dampers. If this is not possible, neglecting the soil–structure interaction is a better alternative (i.e. simply supported boundary conditions).

5.3 Further research

At the beginning of this section it should be emphasised that most of the results and subsequent conclusions presented in the thesis are based on numerical simulations. The output is thus strongly dependent on the underlying assumptions, and one could argue that there are key elements missing from the models. Therefore, improving the dynamical models by performing more measurements on real bridges is important; This would also provide data for modelling calibration and – if required – suggest a retrofitting strategy.

Within this line of thought the author believes that complex modal analysis can be used to develop better prediction models for railway bridge dynamics. By doing so, it is possible to replace empirical relations, especially with the view to damping mechanisms, soil–structure and train–bridge interaction with sound mechanical models. The following work is suggested within this scope:

- The thesis showed that the soil–structure interaction is important because it lowered the frequencies and increased the structural damping. Therefore, it is interesting to study this in a systematic manner to provide recommendations and guidance for the design process. For example, it would be of interest to derive approximate equations to calculate directly the fundamental frequency and the damping ratio of a beam supported on springs and dashpots. These equations, along with the impedance functions for the foundation, then provide a tool that can be used for practical problems.
- The train–bridge interaction topic has been extensively studied during the last decade. Most studies looked at the response and did not bother to investigate the modal parameters. Subsection 3.3.1 showed that the interaction effect can be accounted for by an added damping ratio, which is the procedure adopted in ERRI D-214 (1999b). In the ERRI documents the numerical data were used to fit a regression model. However, an approach based on complex modal analysis will be able to provide a sound theoretical background and will open for researchers to derive more precise expressions.

- During the review of Paper IV, two of the reviewers asked for further proof and physical explanation of the non-classical modes. These questions could not be addressed in the paper, and in the authors opinion it would be interesting to further examine these issues. It would also be of interest to find the physical explanation as to why a bending mode attained maximum damping when its undamped natural frequency it’s similar to that of a non-classical mode.
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Part B

Appended papers
Paper I

Closed-form solution for the mode superposition analysis of the vibration in multi-span beam bridges caused by concentrated moving loads

Paper II

Development of design curves for preliminary dynamic assessment of railway bridges to higher speeds

Paper III

A methodology for the preliminary assessment of existing railway bridges for high-speed traffic

Paper IV

Vibration of damped uniform beams with general end conditions under moving loads

Paper V

Model updating of a railway bridge using frequency response functions and a reduced order model

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