Performance Analysis of a Cooperative Communication Network Over $\kappa - \mu$ Shadowed Fading for Different Relaying Protocols

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Abstract

With the fast development of today’s multimedia services, engineers face a huge hurdle that is, the overwhelming need of highly reliable communication over long distances. Cooperative communication is a novel concept which tackles this problem effectively. The direct link is assisted by nodes called relays, which also reduce shadowing and pathloss effects in wireless networks. An added advantage of such a cooperative communication network is that when combined with multiple-input multiple-output (MIMO) antenna systems and cognitive radio networks (CRN), the system performance in terms of spectral efficiency and reliability, can be extremely enhanced without any extra power and spectrum.

The concept of cooperative communications in MIMO and CRN systems has gained immense interest in the literature. Most of the research works have assumed Rayleigh fading conditions. In this thesis, the performance of cooperative communications with practical constraints of shadowing is studied. Analytical expressions for the outage probability of cooperative networks under different relaying protocols with selection combining are presented under the assumption of $\kappa - \mu$ shadowing fading. Specifically, the relaying protocols that are investigated are incremental relaying, opportunistic relaying, adaptive amplify-and-forward and decode-and-forward. Furthermore, this system model is simulated and the simulation results are compared with the analytical results. Mathematica, a technical computing tool, is used for numerical estimations using stochastic processes and probability theory. Simulation is done in MATLAB.

In this thesis, along with the analytical framework for evaluating outage probability for the system is presented. Simulations are performed for various fading parameters and the results closely match with analytical results which validate the derivations.

Keywords: Cooperative communications, $\kappa - \mu$ shadowed fading, relaying protocols, outage probability
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Chapter 1

Introduction

1.1 Motivation

Wireless communications along with its applications is one of today’s most interesting fields of research. In every generation of wireless devices, we witness improvements in terms of device sizes, communication reliability, battery life, data rates and network connectivity. Several factors have been an influence for this technological development which has eventuated in some of the promising research areas like cooperative communication networks, cognitive radio networks (CRNs) and multiple-input multiple-output (MIMO) systems.

The Shannon expression for capacity $C$, of a channel of a communication system can be used to identify how the performance of the system can be increased by modifying the number of channels, the bandwidth and the power:

$$C = \sum_{\text{channels}} B \log_2 \left(1 + \frac{P}{N}\right)$$

(1.1)

where $B$ is the bandwidth, $P/N$ is the signal to noise ratio of the signal. A MIMO system is obtained with many channels, and increase in bandwidth gives a CRN and increase in power leads to cooperative systems. Cooperative communications is one such technological breakthrough and one of the fast growing areas of research due to the exciting and new advantages it provides as discussed in [1]. The idea behind cooperative communication networks is to allow progression of signals in a framework of nodes, called relays. The processing at nodes differs with the relay protocol used. Fixed relaying and adaptive relaying are major protocols that are currently in use [2].

Fading also has a significant impact on communication networks. Several distributions have been adopted to understand the statistics of a radio channel. However, the performance of a network is often analysed over Rayleigh and Rician fading channels in the literature, because Rayleigh and Rician fading are most suitable for modelling the propagation environment when there is no dominant propagation path or only one dominant propagation path exists, respectively.
On the other hand, when a direct communication path exists between the source and the destination, the $\kappa - \mu$ shadowed fading distribution gives an approximate model to understand the multipath fading as described in [3].

To the best of my discernment, analysis of a $\kappa - \mu$ shadowed fading is an interesting topic which is currently being explored. The main goal of this thesis is the study of the outage probability of a system using diversified relaying protocols over $\kappa - \mu$ shadowed fading channel.

\section{1.2 Related Work}

Cooperative communication technology as described in [4], is an efficient means to use several single antennas to create a multiuser network and has gained attention in recent years. Furthermore, [4] presents an overview of techniques which help the user to utilize mobiles with several single antennas to obtain the advantages of MIMO systems. In [1], the benefits of cooperative communications have been described along with an overview of the relaying strategies in different networks. The advantages and applications of cooperative mesh networks, ad hoc networks and wireless sensor networks are also discussed.

The error performance of opportunistic cooperation is studied in [5] for decode-and-forward (DF) transmission with maximal ratio combining (MRC) scheme in a Rayleigh fading environment. They have introduced the cooperative on demand protocol. Here, the threshold value of signal-to-noise ratio (SNR) is the main criterion which the user considers if he chooses to cooperate. The fundamental relay protocols and their performance metrics are addressed in [6] and the outage performance of the opportunistic amplify-and-forward (AF) relaying strategy over Nakagami-$m$ fading channels has been analysed.

The observations reported in [7] provide useful insights to the cognitive network designer about the system performance in realistic channel environments. The outage probability is calculated in $\kappa - \mu$ shadowed fading channels by considering a two-hop cognitive DF relay network. In [8], Paris has proposed a $\kappa - \mu$ shadowed fading model and derived the statistics in terms of the bivariate hypergeometric functions or the multivariate functions. The author in [9] gives an approximated and a closed-form expression of outage probability for the $\kappa - \mu$ shadowed fading. In [10], an efficient relaying scheme has been suggested for incremental relaying which incurs better throughput, with the help of minimum feedback from the destination. Two general fading distributions, the $\kappa - \mu$ distribution and the $\eta - \mu$ distribution, have been studied in [3], since these distributions are more flexible than the other fading distributions and they can yield better fits to experimental data.
Chapter 1. Introduction

1.3 Research Questions

As mentioned, many studies have been extensively done on the performance of cooperative communication systems under Rayleigh and Rician fading for different relay topologies. Hence, the scope of this thesis work is to extend the study to $\kappa - \mu$ shadowed fading for cooperative communication systems. The following research questions have been addressed:

- Does an analytical expression exist for the probability density function (PDF) of the SNR of the multiple relay cooperative communication system over $\kappa - \mu$ shadowed fading?
- Does an analytical expression exist for the cumulative distribution function (CDF) of the SNR of the system?
- Does an analytical expression exist for outage probability of this system over $\kappa - \mu$ shadowed fading?
- What is the performance of the system over $\kappa - \mu$ shadowed fading?
- What is the influence of relaying protocols on the performance of the system?

1.4 Main Contributions

The main contributions of this thesis include the analysis of the performance of cooperative communication systems with AF and DF relaying protocols over $\kappa - \mu$ shadowed fading channels. A study of the outage performance with change in relaying protocols and the number of relays along with the verification of these theoretical results by simulations is accomplished. The contributions are listed as follows:

- Derivation of analytical expressions for PDF and CDF of SNR for AF protocol over $\kappa - \mu$ shadowed fading channel.
- Derivation of analytical expressions for PDF and CDF of SNR for DF protocol over $\kappa - \mu$ shadowed fading channel.
- Derivation of analytical expression for outage probability of the system for AF protocol over $\kappa - \mu$ shadowed fading channel.
- Derivation of analytical expression for outage probability of the system for DF protocol over $\kappa - \mu$ shadowed fading channel.
- Simulation of the system model.
• Investigation of the effect of shadowing over a system model using the $\kappa - \mu$ shadowed fading distribution.

• Comparison of numerical analysis and simulated results.

The rest of this thesis is organized as follows. Chapter 2 provides an overview of the basics. The system model and the performance analysis are given in Chapter 3. In Chapter 4, the numerical and simulation results are compared for different cases by changing parameters. Lastly, the observation from results, conclusions and some engaging future work are presented in Chapter 5.
Chapter 2

Fundamentals

Wireless communications for commercial services has been a point of study since the 1960s. The past decade has seen a surge of advancement in this field. The major reasons for this being, ever increasing demand for tetherless networks and the evolution of very large scale integration (VLSI) technology for the circuits, which empowered the execution of complex signal processing algorithms in small space utilizing low power [2]. In this chapter, the features, advantages and applications of cooperative communication networks are discussed. An overview of the relaying protocols and the types of fading is also presented.

2.1 Cooperative Communications

2.1.1 Motivation for cooperative communications

One of the main reasons for the increasing utilization of cooperative communication networks is the higher spatial diversity, i.e. the resistance to both small scale and shadow fading effects. Another key factor is the achievable larger coverage with the use of the virtual network created by linking the relays. A few other important aspects that are added as bonus with the use of cooperative communication networks are the reduced interference or lower transmit power, accessibility to reuse frequency in a cellular and adaptability of the system to network conditions like opportunistic use and redistribution of network energy and bandwidth.

Generally, a wireless channel is broadcast in nature. Hence, most of the applications of wireless communications involve a single transmitter and many destinations. Even direct transmission can be considered as a kind of broadcast communication but with fewer recipients. In most cases, broadcasting a signal is useful, for example it is essential to broadcast a television signal or a radio signal at a particular frequency. While keeping this in mind, it is essential to understand that the resources are limited but the demand is growing. Broadcasting of signals also causes interference to other communication devices. This significant use of resources like power, has to be under control. In order to put some constraints
on power and to get a better performance with minimum power, a cooperative communication strategy can be very advantageous. In situations where a source signal cannot reach its destination due to severe fading, retransmission of the signal will not help instead we use a relay to assist the destination.

2.1.2 Cooperative communication network features

Cooperative communication networks utilize the broadcast nature of the wireless channel to generate a virtual MIMO system through randomly located nodes [4]. To understand the concept of cooperative communications, imagine a source $S$, that is broadcasting information as shown in Fig. 2.1. Here $D_1, D_2$ and $D_3$ receive the information directly from $S$. The nodes $R_1, R_2$ and $R_3$ are called relays and also receive data directly from the source. They are located within the coverage area of $S$ as shown using a hexagon and thus a direct link can be established. The relays assist $S$ to transmit information to $D_4, D_5, D_6$. Such networks can have various topologies and the function at the relays differs with the relaying protocols.

In a cooperative communication network, the users share their resources and coordinate with each other to get better information transmission quality. The concept of cooperative communications dates back to the day when the authors of [4], Cover and El Gamal studied the properties of the relay channel and analyzed the capacity of a three node network consisting of a source, destination and relay.

In Fig. 2.1, the source $S$ having a single antenna broadcasts information. Destinations $D_1, D_2, D_3, D_4, D_5$ and $D_6$ along with relays $R_1, R_2$ and $R_3$ act as a vir-
tually interconnected network and transmit information. In a cooperative radio network, or mobile network, each node is considered to transmit to the destination as well as assist the other nodes. In such systems, the source first transmits information to the relays. At the relay, this data is processed and forwarded to the destination. A cooperative communication network user has to address a few tradeoffs relating to code rates and transmit power [4]. For instance, when all the nodes are transmitting data to all other nodes and to the destination, arguably more power is needed. At the same time, because of the spatial diversity the required threshold power decreases thereby reducing the required transmit power, hence calling for a trade-off.

2.1.3 Applications and advantages of cooperative communication networks

In most practical situations, transmit diversity although useful, may not be achieved in a cellular base station due to limitations on size, space or cost. To use several single antennas in the wireless devices of a multi-user environment by creating a virtual multiple antenna environment and achieving transmit diversity is the advantage of a cooperative communication network.

Research has shown that cooperative communications can offer significant performance enhancements in terms of increased coverage, improved transmission reliability and spatial diversity [1]. The effect of fading can significantly be reduced using a cooperative communication network by transmitting independent copies of the signal thus achieving spatial diversity.

In convergent networks, where the nodes closer to the center of the network have to keep forwarding data, cooperative communications could help in balancing the load by providing more than one path for transmission. In a scenario where there is a moving destination that is currently receiving data, a set of virtual nodes can be helpful. In cases like this, group handoff is done, i.e, the signal is sent to the destination through many nodes to ensure that the signal received at the destination is reliable.

Cognitive radio is an exciting emerging technology that deals with the stringent requirement and scarcity of the radio spectrum. The radio cells sense, adapt and dynamically share the spectrum which results in the efficient use of the radio spectrum. Cooperative communications and networking allows distributed radio cells to collaborate and achieve spatial diversity to combat fading. In [11], the application of these technologies to spectrum sensing and spectrum sharing is studied. Cognitive space-time-frequency coding technique has been used which can adjust and adapt to dynamic spectrum opportunistically.
Cooperative communications improves network connectivity and communication. The superiority of cooperative communications in terms of deployment flexibility and hardware feasibility when compared to techniques like MIMO system, is an added advantage of cooperative communications that make it one of the promising techniques for future wireless communication systems. Most recently, the application of cooperative communications in Long Term Evolution (LTE) Release 10 as a key technology for future generation commercial wireless communication systems has been investigated and discussed in [12]. The performance of the cooperative communication LTE-Advanced system in terms of network capacity has been evaluated by considering two intra-cell coordinated multipoint schemes as examples. The incorporation of cooperative communications supports high speed communication.

Also for future safety applications, Vehicular Ad-Hoc networks (VANETs) cooperative communications could be adopted. VANETs enable us to efficiently track vehicles and predict a collision. An application planned for deployment over VANETs is Cooperative Active Safety (CAS) which uses cooperative communications. Here, the vehicles will send information like global positioning system (GPS) position and speed to neighbouring vehicles over a wireless channel which is used to monitor threats as mentioned in [1].

### 2.2 Relays

In the above section, we have learned the need for cooperative communications and understood that the main element in cooperative communication networks is the relay. Here, we will discuss the choice of the number of relays and their placement in the network.

#### 2.2.1 Single relay networks

A relay is placed between the source and the destination as suggested by the name. The distances between the source to the relay and the relay to the destination may vary.

A single relay cooperative communication network is illustrated in Fig. 2.2. In this network, $S$, $R$ and $D$ are the source, relay and destination, respectively. A direct link exists between the source and the destination. Here, $h_{s,d}$, $h_{s,r}$ and $h_{r,d}$ are the channel coefficients between the source-destination, source-relay and relay-destination, respectively.
2.2.2 Multiple relay networks

In multiple relay networks, we have two major topologies. In the first case, we arrange the relays such that we deploy multiple single relay links between the source and the destination. This topology can be used when severe fading exists on the communication channel. At the destination, a signal from each relay is received and these signals can be combined using several combining techniques to obtain a single output signal. In the second case, we arrange the relays such that we have multiple hops for a signal to pass from the source over multiple relays to the destination. This kind of transmission is useful for long distance communication.

In this thesis a multiple parallel relay network is used. An example of such a network is shown in Fig. 2.3. There are multiple links between the source and the destination. Each link is independent of the properties of the other and hence has its own channel coefficient and parameters. This is called a dual hop multiple relay network.
Unlike the single relay network with only 2 hops between the source and the relay illustrated in Fig. 2.2, in a multihop multiple relay network any number of relays can exist between a source and destination and this is demonstrated in Fig. 2.4. Here, we notice that the source and the destination are not only directly connected but also connected by several hops through the multiple relays. In the network shown in Fig. 2.4, the source $S$ broadcasts a signal, every relay in the network receives, processes and forwards this signal. All relays are interconnected which helps in maintaining the signal quality even in the case of very long distances. Hence, the destination receives the signal with maximum possible signal strength along with the weak direct signal from the source.

\begin{figure}[h]
\centering
\includegraphics[width=0.7\textwidth]{figure2_4.png}
\caption{Cooperative communication system with multiple sequential relays.}
\end{figure}

### 2.3 Relay Protocols

An important aspect in cooperative communications is the processing of the source signals at the relays. Among the many possible strategies, in this thesis, we consider simple protocols in which two users accurately estimate the realized outage probability between them. The users choose between continuing their own transmission, or relaying the other users transmission. The processing at the relay is to either simply amplify their received signals depending to their power constraint, or to fully decode, re-encode, and forward the messages which is elaborated in [13]. The single relay network having a direct path and a relay path between the source and destination shown in Fig. 2.2, is used in the further sections to describe relaying protocols.
2.3.1 Amplify-and-forward protocol

The AF relaying protocol is a fixed cooperation strategy. At the relay, the signal received is amplified and sent to the destination. In Fig. 2.2, for AF relaying, the transmitted signal $x$ is amplified by an amplification factor $G$ at the relay and sent to the destination $D$. The input signal $y_{s,r}$ to the relay and output signal $y_{r,d}$ to the destination can be expressed as

$$y_{s,r} = h_{s,r}x + n_{s,r}$$  \hspace{1cm} (2.1)  

$$y_{r,d} = Gh_{r,d}x + n_{r,d}$$  \hspace{1cm} (2.2)  

where $h_{s,r}$ is the channel coefficient between the source and the relay, $h_{r,d}$ is the channel coefficient between the relay and the destination, $n_{s,r}$ and $n_{r,d}$ represent adaptive white Gaussian noise (AWGN) with zero mean and variance $N_0$ at the relay and the destination, respectively. Here, $G$ is the amplification factor.

The amplification factor also known as relay gain is selected based on whether the relay is aware of the instantaneous channel power and the channel state information (CSI). Here, it is related to the transmit power of the source $P_s$ and transmit power of the relay $P_r$ as follows:

$$G = \frac{P_r}{P_s|h_{s,r}|^2}$$  \hspace{1cm} (2.3)  

An advantage of the AF protocol is that it incurs less delay compared to the DF protocol and other similar protocols. On the other hand, when processing the signal at the relay, it also amplifies noise which is clearly a drawback.

2.3.2 Decode-and-forward protocol

Decode-and-forward is a fixed relaying protocol which decodes the signal transmitted by the source, re-encodes the signal at the relay and transmits the signal to the destination. This processing of the signal at the relay introduces a delay but unlike in AF, the noise is removed. In view of Fig. 2.2, the source transmit signal $x$ is decoded at the relay and the estimated signal $\hat{x}$ is transmitted to the destination. This can be mathematical expressed as follows:

$$y_{s,r} = h_{s,r}x + n_{s,r}$$  \hspace{1cm} (2.4)  

$$y_{r,d} = h_{r,d}\hat{x} + n_{r,d}$$  \hspace{1cm} (2.5)  

where $h_{s,r}$ is the channel coefficient between the source and the relay, $h_{r,d}$ is the channel coefficient between the relay and the destination, $n_{s,r}$ and $n_{r,d}$ represent AWGN with zero mean and variance $N_0$ at the relay and destination, respectively. Here, $\hat{x}$ is the decoded and encoded signal which is transmitted to the destination.
In a fixed DF scheme, the relay has to fully decode and re-encode the signal to transmit to the destination, a fairly significant amount of time is needed to be correctly done as suggested in [2]. In this method, the destination assumes that the relay could not decode the signal \( x \) properly when the signal strength of the signal received at the relay is less than a given threshold. The cooperation is initiated only when the direct link fails, which depends on the instantaneous received SNR. When the SNR exceeds a threshold \( \gamma_{th} \), the direct link is successful. Otherwise, the relay link is needed. In such cases, the destination requests the source and relays retransmit. In contrast to this, if the source keeps retransmitting the signal through the direct link to the destination, this is adaptive DF relaying.

### 2.3.3 Opportunistic relaying

In a multiple relay network, to choose the best relay, an opportunistic relaying scheme has been introduced in [14]. This scheme selects a single relay from the relays based on which relay provides the best end-to-end path between source and destination, i.e., the relay with higher SNR.

![Figure 2.5: Cooperative communication system with multiple relays.](image)

For a network shown in Fig. 2.5, the end-to-end instantaneous SNR with the \( i^{th} \) relay is given by

\[
\gamma_i = \min\{\gamma_{s,r_i}, \gamma_{r_i,d}\} \tag{2.6}
\]

where \( \gamma_{s,r_i} \) is the instantaneous SNR of the source-and-relay link and \( \gamma_{r_i,d} \) is the instantaneous SNR of the relay-and-destination link. The opportunistic relay-node selection is implemented before the source transmission or after the source
transmission as suggested in [6]. The performance and outage analysis of this relay selection scheme can also be derived.

### 2.3.4 Incremental relaying

In a multiple relay scenario, the signal received from the destination from all the different paths is basically a shifted or scaled version of the same signal. To avoid redundancy and to improve the spectral efficiency, incremental relaying is introduced in [10]. It allows the relay to transmit to the destination only when the direct transmission from the source fails. In particular, incremental relaying performs full duplex transmission when the direct link between the source and destination exists, and half-duplex transmission when the direct link fails [15]. The transmission from the relay stops only when an acknowledgement from the destination is given or when a maximum number of transmissions is reached, i.e., the transmission is divided into first phase and second phase. In the first phase, the source directly transmits to the destination and if it is unsuccessful, then, in the next phase the relay transmits the data by employing relaying protocols like AF or DF.

### 2.4 Selection Combining

In the previous sections, we have discussed a few relaying protocols. The idea of using multiple relays in a wireless communication network has gained significant attention in the past few years. The above mentioned relaying protocols can be employed and the relays can forward the signal to the destination. Hence, the destination receives a signal from every relay present in the wireless network. These signals have to be processed somehow to obtain a final output signal which is briefly dealt with here. In this section, a technique based on the relay-selection at the source and the relays is discussed. Retaining the same diversity and achieving higher bandwidth efficiency is considered essential. In other words, in selection combining, only the maximum received signal is chosen [16]. In the single relay scenario, is it important to understand when to cooperate as explained in [2]. The source chooses to cooperate only when the ratio of the channel gains between the source-and-destination and source-and-relay is greater than or equal to a cooperation threshold. This clarifies why we have to use a relay when the direct link already exists.

In the multiple relay scenario, in addition to when to cooperate, we also focus on who to cooperate with. The user selects only one relay to cooperate with, [2] elaborates on how the user makes this decision. Diversity combining is helpful in multiple relay networks. Combining techniques have proven to be efficient in reducing multipath fading because the combined SNR is greater compared with the
Chapter 2. Fundamentals

SNR of each diversity branch [17]. The conventional selection combiner (CSC) identifies the signal from a diversity branch which has the highest instantaneous SNR.

A few other combining techniques include equal gain combining (EGC) and maximal ratio combining (MRC). In EGC, all the received signals are multiplied by a weight. In MRC, each individual received signal is multiplied with weight to maximise SNR according to the fading conditions [16].

2.5 \( \kappa - \mu \) Shadowed Fading

With the current demand for mobile multimedia services of wireless communications, the need for higher data rates will definitely raise in the near future. Two important issues that have to be carefully addressed to improve performance in wireless communication systems are intersymbol interference (ISI) and co-channel interference (CCI), as they effect the quality of communication. Path loss, large-scale fading and small-scale fading are the three major factors that have an impact on the strength of the received signal. The loss of signal power with increase in distance is named path loss [18].

Fading phenomenon is a fundamental aspect that makes wireless communication challenging. The large-scale fading arises due to shadowing and is characterized as a log-normal distributed stochastic process around the mean of path loss according to [18]. The small-scale fading is fast varying as it is initiated as a consequence of the multipath propagation. The multipath propagation cannot be completely avoided because it is caused when the transmitted signal is reflected, refracted, scattered and diffracted from objects of various sizes.

In this thesis, we have analysed the system over \( \kappa - \mu \) shadowed fading since other classical fading distributions are included in the \( \kappa - \mu \) distribution as special cases, e.g., one-sided Gaussian when \( \xi = 0.5 \), Rayleigh when \( \xi = 1 \), Nakagami-\( m \) and Rician. The \( \kappa - \mu \) fading is considerably more flexible and this has been observed and carried out by the author of [3] and by other researchers. Line-of-sight multipath propagation scenarios can also be modelled by controlling the two shape parameters \( \kappa \) and \( \mu \) as shown in [8]. According to [8], shadowing can be introduced in a line-of-sight multipath fading model by either assuming the total power is random or domain components are random. From [3], we can represent \( \mu > 0 \) as

\[
\mu = \frac{E^2(R^2)}{V(R^2)} \frac{1 + 2\kappa}{(1 + \kappa)^2}
\] (2.7)
where $R$ is the envelope of the fading signal, $\kappa > 0$ is the ratio between the total power of the dominant components and the total power of the scattered waves. The expectation and variance are denoted by $E$ and $V$, respectively.

In practice, the path losses of any two links are not the same. This is, in part, because the transmitted signal is obstructed by different objects as it travels to the receive antennas. Consequently, this type of impairment has been named shadowed fading. Since the nature and location of the obstructions causing shadowed losses cannot be known in advance, the path loss introduced by this effect is a random variable.

This fading distribution has wide applications, for example, it can be used to model land mobile satellite communication links and the access link between a user and its serving relay node [9]. In this work, our focus will be on the derivation of outage probability (OP) over $\kappa - \mu$ shadowed fading.

### 2.6 Outage Probability

The most consistently considered parameters for analysing the performance of a system are outage probability, symbol error rate and channel capacity. However, due to time limitation, in this thesis, we have evaluated the system performance in terms of outage probability.

Outage probability, in [16], is defined as the probability that the instantaneous SNR is lower than or equal to a predefined threshold $\gamma_{th}$. Outage probability is commonly preferred in slow fading conditions. A threshold is predefined and if the instantaneous SNR is less than this threshold, then it implies that the signal decoding fails. Accordingly, the definition of outage probability is given as follows:

$$P_{out} = Pr\{\gamma_D \leq \gamma_{th}\} = \int_0^{\gamma_{th}} f_{\gamma_D}(\gamma) d\gamma$$

(2.8)

Here, $f_{\gamma_D}(\gamma)$ is the probability density function (PDF) of the instantaneous SNR $\gamma_D$, and $\gamma_{th}$ is the threshold.
To comprehend the channel behaviour, it is important to find the statistics (in terms of PDF and CDF) of the channel coefficients. In this thesis, the magnitude of the channel coefficients is a random variable with a $\kappa - \mu$ distribution. The selection combining technique is used, in this technique the signal with the best SNR is chosen as the output. In this chapter, the system model is presented and outage probability is mathematically derived for different relaying protocols.

### 3.1 System Model

As discussed in Chapter 2, a cooperative communication network consists of a source, a destination and $N$ relays. Each node shall be equipped with a single antenna.

![Figure 3.1: Cooperative communication system with multiple relays.](image)

Fig. 3.1 shows the model used in this thesis. The nodes $S$ and $D$ are the source and destination, respectively. The nodes $R_1, R_2, ..., R_N$ are the relays between $S$ and $D$. There exists a direct link between $S$ and $D$ along with $N$ relay paths.
3.2 Outage Probability

Let the signal transmitted by the source be $x$. Then, the signal received at the relay $R_i$, $i = 1, 2, ..., N$ can be written as

$$
y_{s,r_1} = h_{s,r_1}x + n_{s,r_1}
$$
$$
y_{s,r_2} = h_{s,r_2}x + n_{s,r_2}
$$
$$
\vdots
$$
$$
y_{s,r_N} = h_{s,r_N}x + n_{s,r_N}
$$

where $h_{s,r_1}, h_{s,r_2}, ...$ and $h_{s,r_N}$ are the channel coefficients between the source and the relays. Further, $n_{s,r_1}, n_{s,r_2}, ...$ and $n_{s,r_N}$ represent the AWGN at the relays with zero mean and variance $N_o$.

3.2.1 Instantaneous SNR for AF

In case the AF protocol is used at the relays given in (3.1), the received signals at the destination can be written as

$$
y_{AF,r_1,d} = G_{r_1}h_{r_1,d}(h_{s,r_1}x + n_{s,r_1}) + n_{r_1,d}
$$
$$
y_{AF,r_2,d} = G_{r_2}h_{r_2,d}(h_{s,r_2}x + n_{s,r_2}) + n_{r_2,d}
$$
$$
\vdots
$$
$$
y_{AF,r_N,d} = G_{r_N}h_{r_N,d}(h_{s,r_N}x + n_{s,r_N}) + n_{r_N,d}
$$

where $h_{r_1,d}, h_{r_2,d}, ...$ and $h_{r_N,d}$ are the channel coefficients between the relays and destination. Moreover, $n_{r_1,d}, n_{r_2,d}, ...$ and $n_{r_N,d}$ represent the AWGN with zero mean and variance $N_o$. Further, $G_i$ is the amplification factor and can be found by scaling the received signal by a factor that is inversely proportional to the received power.

Let $P_s$ and $P_r$ be the power of the transmit signals at the source and the relay $R_i$, respectively, i.e,

$$
P_s = E\{x^2\}
$$
$$
P_r = E\{|G_i h_{s,r}|^2\}
$$

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Then, the amplification factor for the $i^{th}$ relay can be written as

$$ G_i = \frac{P_r}{P_s|h_{s,r_i}|^2} \quad (3.5) $$

The signal is amplified at relay $R_i$ and then sent to the destination. In this case, the SNR at the destination from relay $R_i$ can be written as

$$ \gamma_{AF}^{D_i} = \frac{E\{|h_{s,r_i}G_i|h_{r_i,d}|^2\}}{E\{|h_{r_i,d}\alpha n + n_d|^2\}} $$

$$ = \frac{E\{|h_{s,r_i}|^2G_i^2E\{|h_{r_i,d}|^2\}E\{x^2\}}{E\{|h_{r_i,d}|^2\}G_i^2E\{|n_r|^2\} + E\{|n_d|^2\}} $$

$$ = \frac{\alpha^2|h_{r_i,d}|^2|h_{s,r_i}|^2P_s}{\frac{P_r}{N_o}|h_{s,r_i}|^2|h_{r_i,d}|^2} $$

$$ = \frac{|h_{r_i,d}|^2 + \frac{1}{\alpha^2}}{\frac{P_r}{N_o}|h_{s,r_i}|^2|h_{r_i,d}|^2} $$

$$ = \frac{|h_{r_i,d}|^2 + \frac{P_s}{P_r}|h_{s,r_i}|^2}{\frac{P_r}{N_o}|h_{s,r_i}|^2|\alpha N_o + N_o|} $$

$$ = \frac{P_r|h_{r_i,d}|^2 + P_s|h_{s,r_i}|^2}{\frac{P_r}{N_o}|h_{s,r_i}|^2|\alpha N_o + N_o|} $$

$$ = \gamma_i^1 \gamma_i^2 $$

(3.6)

Substituting (3.3) into (3.6), we have

$$ \gamma_{AF}^{D_i} = \frac{G_i^2|h_{r_i,d}|^2|h_{s,r_i}|^2P_s}{G_i^2|h_{r_i,d}|^2N_o + N_o} \quad (3.7) $$

Substituting (3.5) into (3.6), we have

$$ \gamma_{AF}^{D_i} = \frac{\frac{P_s}{N_o}|h_{s,r_i}|^2|h_{r_i,d}|^2}{|h_{r_i,d}|^2 + \frac{1}{\gamma_i^2}} \quad (3.8) $$

Finally, we obtain the instantaneous SNR at the destination for AF protocol as

$$ \gamma_{AF}^{D_i} = \frac{\gamma_i^1 \gamma_i^2}{\gamma_i^1 + \gamma_i^2} \quad (3.9) $$

where

$$ \gamma_i^1 = \frac{P_s|h_{s,r_i}|^2}{N_o} = a_{X,s,r_i} \quad (3.10) $$
\[ \gamma_{i2} = \frac{P_r |h_{r_i,d}|^2}{N_0} = b X_{r_i,d} \] (3.11)

and

\[ a = \frac{P_s}{N_0} \] (3.12)

\[ b = \frac{P_r}{N_0} \] (3.13)

Here, \( a \) and \( b \) are the transmit SNR at the source and the relays. The symbols \( X_{s,r_i} \) and \( X_{r_i,d} \) denote the channel gains of the links between the source \( S \) and relay \( R_i \), and relay \( R_i \) and the destination \( D \), respectively.

### 3.2.2 Instantaneous SNR for DF

When the relay uses the DF protocol, first the \( i^{th} \) relay tries to decode the received signal as \( x'_i \) and then forwards the obtained signal to the destination. As a result, the received signals at the destination for the DF relaying protocol is written as

\[
\begin{align*}
\gamma_{DF}^D &= \min\{\gamma_{i1}, \gamma_{i2}\} \\
\end{align*}
\] (3.15)

where \( x'_i \) is the decoded signal at the relay. For the DF protocol, the instantaneous SNR at the destination is written as in [7]

\[
\begin{align*}
y_{r_{1,d}}^{DF} &= h_{r_{1,d}} x'_{1} + n_{r_{1}} \\
y_{r_{2,d}}^{DF} &= h_{r_{2,d}} x'_{2} + n_{r_{2}} \\
&\vdots \\
y_{r_{N,d}}^{DF} &= h_{r_{N,d}} x'_{N} + n_{r_{N}} \\
\end{align*}
\] (3.14)

where \( \gamma_{i1} \) is the instantaneous SNR for the first hop from the source to the relay \( R_i \), and \( \gamma_{i2} \) is the instantaneous SNR for the second hop from the relay \( R_i \) to the destination.
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3.2.3 Instantaneous SNR of the direct link

For the direct link between the source and the destination, the instantaneous SNR can be written as

$$\gamma_D^{(0)} = \frac{P_s}{N_0} |h_{s,d}|^2 = a X_{s,d} \quad (3.16)$$

where $X_{s,d}$ represents the channel power gain of the direct link from the source to the destination.

3.2.4 Total instantaneous SNR

Using the selection combining technique, the total SNRs at the destination of the cooperative system with AF and DF protocols, respectively, are given by

$$\gamma_{AF}^T = \max\{\gamma_{D1}^{AF}, \gamma_{D2}^{AF}, \ldots, \gamma_{DN}^{AF}, \gamma_D^{(0)}\} \quad (3.17)$$

$$\gamma_{DF}^T = \max\{\gamma_{D1}^{DF}, \gamma_{D2}^{DF}, \ldots, \gamma_{DN}^{DF}, \gamma_D^{(0)}\} \quad (3.18)$$

3.2.5 Expressions for PDF and CDF

Let $Z$ be a $\kappa - \mu$ shadowed random variable with mean $\bar{\gamma}$, having non-negative shaping parameters $\kappa, \mu$ and $\xi$. Then, the PDF $f_Z(z)$ of $z$ is given in [7] as

$$f_Z(z) = \frac{\mu^\mu (1 + \kappa)^\mu}{\Gamma(\mu) \bar{\gamma}(\mu \kappa + \xi)^\xi} \left(\frac{z}{\bar{\gamma}}\right)^{\mu-1} e^{-\frac{\mu (1 + \kappa) z}{\mu \kappa + \xi}} {}_1F_1 \left(\xi; \frac{\mu^2(1 + \kappa) z}{\mu \kappa + \xi}; \frac{\bar{\gamma}}{\bar{\gamma}}\right) \quad (3.19)$$

where ${}_1F_1(.,.,.)$ is the Kummer confluent hypergeometric function and $\bar{\gamma}$ is the channel mean power of the respective link.

The CDF of $Z$ can also be represented as

$$F_Z(z) = \frac{z^m \alpha^m}{\Gamma(m + 1)} {}_1F_1 \left(m; m + 1; -z \alpha\right) \quad (3.20)$$

where

$$m = \frac{\xi \mu (1 + \kappa)^2}{\xi + \mu \kappa^2 + 2 \kappa \xi} \quad (3.21)$$

$$\alpha = \frac{m}{\bar{\gamma}} \quad (3.22)$$
For simplification of the calculations, $\kappa - \mu$ shadowed fading is approximated by Nakagami-$m$ fading for integer $m$. Then, we approximate the $\kappa - \mu$ shadowed random variable $Z$ by a gamma random variable with parameter set $(m, 1/\alpha)$. As a result, the PDF $f_Z(z)$ of $Z$ can be approximated by the Gamma distribution as

$$f_Z(z) = \frac{\alpha_i^{m_i} z^{m_i - 1}}{\Gamma(m_i)} e^{-\alpha_i z} \quad (3.23)$$

The approximation for the CDF of $Z$ as Gamma distribution can be given by

$$F_Z(z) = 1 - \sum_{l=0}^{m_i-1} \frac{\alpha_i^l}{l!} z^l e^{-\alpha_i z} \quad (3.24)$$

### 3.2.6 CDF of total instantaneous SNR for AF

The CDF of the total SNR $F_{\gamma_{AF}}(\gamma)$ can be calculated from (3.17) as

$$F_{\gamma_{AF}}(\gamma) = Pr(\gamma_{AF} \leq \gamma) = Pr(\max\{\gamma_{D_1}, \gamma_{D_2}, ..., \gamma_{D_N}, \gamma_{(0)}\} \leq \gamma) \quad (3.25)$$

Since $\gamma_{D_1}, \gamma_{D_2}, ..., \gamma_{D_N}$ are independent, we can write $F_{\gamma_{AF}}(\gamma)$ as

$$F_{\gamma_{AF}}(\gamma) = F_{\gamma_{D_1}}(\gamma) \left[ F_{\gamma_{AF}}(\gamma) \right]^N \quad (3.26)$$

or

$$F_{\gamma_{AF}}(\gamma) = F_{\gamma_{D_1}}(\gamma) \left[ F_{\gamma_{AF}}(\gamma) \right]^N \quad (3.27)$$

where the CDF $F_{\gamma_{D_i}}(\gamma)$ of the SNR through relay $R_i$ can be calculated as

$$F_{\gamma_{D_i}}(\gamma) = \int_0^\infty F_{\gamma_A} \left( \gamma_{2i} = q \right) f_{\gamma_{2i}}(q) \, dq = \int_0^{\gamma} F_{\gamma_{AF}} \left( \frac{\gamma q}{\gamma - q} \right) f_{\gamma_{AF}}(q) \, dq \quad (3.28)$$

Since $\gamma_{i1} = aX_{s,r_i}$ and $\gamma_{i2} = bX_{r_i,d}$, we have $F_{\gamma_{i1}}(\gamma) = F_{X_{s,r_i}}(\frac{\gamma}{a})$, $F_{\gamma_{i2}}(\gamma) = F_{X_{r_i,d}}(\frac{\gamma}{b})$. Thus, we can rewrite (3.28) as

$$F_{\gamma_{D_i}}(\gamma) = F_{X_{r_i,d}} \left( \frac{\gamma}{b} \right) + \frac{1}{b} \int_\gamma^\infty F_{X_{r_i,d}} \left( \frac{q \gamma}{a(q - \gamma)} \right) f_{X_{r_i,d}} \left( \frac{q}{b} \right) \, dq \quad (3.29)$$
Using the approximations of $\kappa - \mu$ fading in (3.23) and (3.24), we obtain

$$F_{\gamma |d_i}(\gamma) = F_{X_{r_i,d}}\left(\frac{\gamma}{b}\right) + \frac{1}{b} \int_{\gamma}^{\infty} f_{X_{r_i,d}}\left(\frac{q}{b}\right) \, dq - \int_{\gamma}^{\infty} \sum_{l=0}^{m_{r_i,d}-1} \frac{1}{b} \frac{\alpha_{s,r_i}^l}{l!} \left(\frac{q}{a(q-\gamma)}\right)^l \times e^{-\frac{\alpha_{s,r_i}^l q}{a}} \frac{\alpha_{r_i,d}^{m_{r_i,d}-1}}{\Gamma(m_{r_i,d})} \left(\frac{q}{b}\right)^{m_{r_i,d}-1} e^{-\frac{\alpha_{r_i,d}^l q}{b}} \, dq$$

$$= F_{X_{r_i,d}}\left(\frac{\gamma}{b}\right) + 1 - F_{X_{r_i,d}}\left(\frac{\gamma}{b}\right) - \sum_{l=0}^{m_{s,r_i}-1} \frac{\gamma^l \alpha_{s,r_i}^l \alpha_{r_i,d}^{m_{r_i,d}-1}}{l! a^l b^{m_{r_i,d}} \Gamma(m_{r_i,d})} \int_{\gamma}^{\infty} q^{m_{r_i,d}-1+l} e^{-\frac{\alpha_{s,r_i}^l q}{a} - \frac{\alpha_{r_i,d}^l q}{b}} \, dq$$

$$= 1 - \sum_{l=0}^{m_{s,r_i}-1} \frac{\gamma^l \alpha_{s,r_i}^l \alpha_{r_i,d}^{m_{r_i,d}-1}}{l! a^l b^{m_{r_i,d}} \Gamma(m_{r_i,d})} \int_{\gamma}^{\infty} q^{m_{r_i,d}-1+l} e^{-\frac{\alpha_{s,r_i}^l q}{a} - \frac{\alpha_{r_i,d}^l q}{b}} \, dq$$

(3.30)

where $(m_{s,r_i}, \frac{1}{\alpha_{s,r_i}})$ and $(m_{r_i,d}, \frac{1}{\alpha_{r_i,d}})$ are the parameter sets of the approximated $\kappa - \mu$ shadowed fading from the source to the relay $R_i$ and from the relay $R_i$ to the destination, respectively.

Let $u = q - \gamma$, i.e., $du = dq$. Then, using the binomial expansion for $(x+y)^n$ and simplifying, we yield

$$F_{\gamma |d_i}(\gamma) = 1 - \sum_{l=0}^{m_{s,r_i}-1} \frac{\gamma^l \alpha_{s,r_i}^l \alpha_{r_i,d}^{m_{r_i,d}-1}}{l! a^l b^{m_{r_i,d}} \Gamma(m_{r_i,d})} \int_{0}^{\infty} u^{j-1} e^{-\frac{\alpha_{s,r_i}^l \gamma}{a} - \frac{\alpha_{r_i,d}^l \gamma}{b}} \, du$$

(3.31)

Applying Eq (3.471.9) from [19], $\int_{0}^{\infty} x^{v-1} e^{-\frac{a}{b} px} \, dx = 2 \left(\frac{2}{p}\right)^{\frac{v}{2}} K_v(2\sqrt{bp})$ to solve the integral in (3.31), we get

$$F_{\gamma |d_i}(\gamma) = 1 - \sum_{l=0}^{m_{s,r_i}-1} \frac{\gamma^l \alpha_{s,r_i}^l \alpha_{r_i,d}^{m_{r_i,d}-1}}{l! a^l b^{m_{r_i,d}} \Gamma(m_{r_i,d})} \int_{0}^{\infty} u^{j-1} e^{-\frac{\alpha_{s,r_i}^l \gamma}{a} - \frac{\alpha_{r_i,d}^l \gamma}{b}} \, du$$

(3.32)
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The CDF of the SNR from the direct link can be calculated from (3.32) as

$$F_{\gamma}^{(0)}(\gamma) = F_{X_{s,d}} \left( \frac{\gamma}{a} \right)$$

(3.33)

Substituting (3.49) in (3.27) gives the approximation

$$F_{\gamma_{AF}}(\gamma) \approx F_{X_{s,d}} \left( \frac{\gamma}{a} \right) \left[ F_{\gamma_{AF}}(\gamma) \right]^N$$

(3.34)

Let \((m_{s,d}, \frac{1}{a_{s,d}})\) be the parameter set of the approximated \(\kappa - \mu\) shadowed fading from source to destination. Then, \(F_{X_{s,d}} \left( \frac{\gamma}{a} \right)\) can be approximated as follows:

$$F_{X_{s,d}} \left( \frac{\gamma}{a} \right) \approx \left\{1 - \sum_{c=0}^{m_{s,d} - 1} \frac{\alpha_{s,d}^c}{c!} \left( \frac{\gamma}{a} \right)^c e^{-\frac{a_{s,d} \gamma}{a}} \right\}$$

(3.35)

Substituting (3.35) into (3.34), we have

$$F_{\gamma_{AF}}(\gamma) \approx \left\{1 - \sum_{c=0}^{m_{s,d} - 1} \frac{\alpha_{s,d}^c}{c!} \left( \frac{\gamma}{a} \right)^c e^{-\frac{a_{s,d} \gamma}{a}} \right\}$$

$$\times \left[ 1 - 2 \sum_{l=0}^{m_{s,r_i} - 1} \frac{\gamma}{a} \frac{\alpha_{s,ri}^l \alpha_{r_i,d}^{m_{r_i,d} - l - 1}}{l!a^{l}b^{m_{r_i,d}} \Gamma(m_{r_i,d})} \sum_{j=0}^{m_{r_i,d} + l - 1} \frac{m_{r_i,d} + l - 1}{j} (\gamma)^{m_{r_i,d} + l - 1 - j} \right]$$

$$\times \left( \frac{\alpha_{s,ri} \gamma^2 b}{\alpha_{r_i,d}} \right)^{j-l+1} K_{j-l+1} \left( 2 \sqrt{\frac{\alpha_{s,ri} \alpha_{r_i,d} \gamma^2}{b}} e^{-\frac{a_{s,ri} \gamma}{a}} e^{-\frac{a_{r_i,d} \gamma}{b}} \right)^N$$

(3.36)

Using the exact expressions (3.19) and (3.20) for \(\kappa - \mu\) fading, the CDF of the SNR for the AF protocol can be written as

$$F_{\gamma_{AF}}(\gamma) = F_{X_{r_i,d}} \left( \frac{\gamma}{b} \right) + \int_{\gamma}^{\infty} F_{X_{s,ri}} \left( \frac{q}{a(q-\gamma)} \right) \frac{1}{b} f_{X_{r_i,d}} \left( \frac{q}{b} \right) dq$$

(3.37)

where

$$F_{X_{r_i,d}} \left( \frac{\gamma}{b} \right) = \frac{\alpha_{r_i,d}^{m_{r_i,d}}}{\Gamma(m_{r_i,d} + 1)} \left( \frac{\gamma}{b} \right)^{m_{r_i,d}} 1_F \left( m_{r_i,d}, m_{r_i,d} + 1, -\frac{\gamma}{b \alpha_{r_i,d}} \right)$$

(3.38)

$$F_{X_{s,ri}} \left( \frac{q}{a(q-\gamma)} \right) = \frac{\alpha_{s,ri}^{m_{s,ri}}}{\Gamma(m_{s,ri} + 1)} \left( \frac{q}{a(q-\gamma)} \right)^{a_{r_i,d}}$$

$$\times 1_F \left( m_{s,ri}, m_{s,ri} + 1, -\frac{\alpha_{s,ri} q}{a(q-\gamma)} \right)$$

(3.39)
\[
F_{\gamma_{th}}(\gamma) = \frac{\alpha_{ri,d}^{m_{ri,d}} (1 + \kappa_{ri,d})^{m_{ri,d}}}{\Gamma(m_{ri,d} + 1)} \sum_{d} \left\{ \begin{array}{c}
\binom{\gamma}{\alpha_{ri,d}} \left( \frac{q \gamma}{b} \right)^{m_{ri,d} - 1} e^{-\frac{(1 + \kappa_{ri,d}) q \gamma}{b^{\gamma_{ri,d}}}} \\
\int_{b}^{\infty} \frac{\alpha_{ri,d}^{m_{ri,d}} (1 + \kappa_{ri,d})^{m_{ri,d}}}{\Gamma(m_{ri,d} + 1)} \left( \frac{q \gamma}{a(q - \gamma)} \right)^{\alpha_{ri,d}} 1F_{1} \left( m_{s,r,i}, m_{s,r,i} + 1, -\frac{\alpha_{s,r,i} q \gamma}{a(q - \gamma)} \right) \\
\left[ \frac{\alpha_{ri,d}^{m_{ri,d}} (1 + \kappa_{ri,d})^{m_{ri,d}}}{\Gamma(m_{ri,d} + 1)} \left( \frac{q \gamma}{b} \right)^{m_{ri,d} - 1} e^{-\frac{(1 + \kappa_{ri,d}) q \gamma}{b^{\gamma_{ri,d}}}} \right]^{N} \end{array} \right. 
\]

Thus, an exact expression for \( F_{\gamma_{th}}(\gamma) \) is obtained as

\[
F_{\gamma_{th}}(\gamma) = \frac{\alpha_{ri,d}^{m_{ri,d}} (1 + \kappa_{ri,d})^{m_{ri,d}}}{\Gamma(m_{ri,d} + 1)} \sum_{d} \left\{ \begin{array}{c}
\binom{\gamma}{\alpha_{ri,d}} \left( \frac{q \gamma}{b} \right)^{m_{ri,d} - 1} e^{-\frac{(1 + \kappa_{ri,d}) q \gamma}{b^{\gamma_{ri,d}}}} \\
\int_{b}^{\infty} \frac{\alpha_{ri,d}^{m_{ri,d}} (1 + \kappa_{ri,d})^{m_{ri,d}}}{\Gamma(m_{ri,d} + 1)} \left( \frac{q \gamma}{a(q - \gamma)} \right)^{\alpha_{ri,d}} 1F_{1} \left( m_{s,r,i}, m_{s,r,i} + 1, -\frac{\alpha_{s,r,i} q \gamma}{a(q - \gamma)} \right) \\
\left[ \frac{\alpha_{ri,d}^{m_{ri,d}} (1 + \kappa_{ri,d})^{m_{ri,d}}}{\Gamma(m_{ri,d} + 1)} \left( \frac{q \gamma}{b} \right)^{m_{ri,d} - 1} e^{-\frac{(1 + \kappa_{ri,d}) q \gamma}{b^{\gamma_{ri,d}}}} \right]^{N} \end{array} \right. 
\]

### 3.2.7 Outage probability for AF

The mathematical expression for the exact outage probability with outage threshold \( \gamma_{th} \) of the system over \( \kappa - \mu \) shadowed fading with \( N \) relays is given as

\[
P_{\text{out}} = F_{\gamma_{th}}(\gamma = \gamma_{th})
\]
where

\[ F_{\gamma_{AF}}(\gamma_{th}) = \left\{ \left( \frac{\gamma_{th}}{a} \right)^{m_{s,d}} \alpha_{s,d} \right\} \frac{1}{\Gamma(m_{s,d}+1)} \left[ \prod_{l=0}^{m_{r_i,d}+l-1} \left( \begin{array}{c} m_{r_i,d} + l - 1 \end{array} \right) (\gamma_{th})^{m_{r_i,d}+l-1-j} \right] \]

\[ \times \left( \frac{\alpha_{s,r_i} \gamma_{th}^2}{\alpha_{r_i,d}} \right)^{l+1} - \left( \frac{\alpha_{s,r_i} \alpha_{r_i,d} \gamma_{th}^2}{b} \right) \left[ e^{-\alpha_{s,r_i} \gamma_{th}^2} \right] \left( \frac{\alpha_{r_i,d}}{\gamma_{th}} \right)^{l+1} \]
where the CDF of the SNR through the relaying link $\gamma_{DF}^{i}$ can be derived from (3.15) as

$$F_{\gamma_{DF}^{i}}(\gamma) = Pr\{\gamma_{DF}^{i} \leq \gamma\} = Pr\{\min\{\gamma_{i1}, \gamma_{i2}\} \leq \gamma\} = 1 - Pr\{\min\{\gamma_{i1}, \gamma_{i2}\} \geq \gamma\}$$

(3.47)

Since $\gamma_{i1}$ and $\gamma_{i2}$ are independent, we have

$$F_{\gamma_{DF}^{i}}(\gamma) = 1 - Pr(\gamma_{i1} \geq \gamma)Pr(\gamma_{i2} \geq \gamma) = 1 - [1 - F_{\gamma_{i1}}(\gamma)][1 - F_{\gamma_{i2}}(\gamma)] = 1 - [1 - F_{X_{s,r}}(\frac{\gamma}{a})][1 - F_{X_{r,d}}(\frac{\gamma}{b})]$$

(3.48)

From (3.16), the CDF of the SNR through the direct link can be derived as

$$F_{\gamma_{DF}^{i}}(\gamma) = F_{X_{s,d}}(\frac{\gamma}{a})$$

(3.49)

Then, the CDF $F_{\gamma_{DF}^{i}}(\gamma)$ of total SNR $\gamma_{DF}$ for DF becomes

$$F_{\gamma_{DF}^{i}}(\gamma) = F_{\gamma_{DF}^{i,0}}(\gamma)\left[F_{\gamma_{DF}^{i}}(\gamma)\right]^{N} = F_{X_{s,d}}(\frac{\gamma}{a})\left[1 - [1 - F_{X_{s,r}}(\frac{\gamma}{a})][1 - F_{X_{r,d}}(\frac{\gamma}{b})]\right]^{N}$$

(3.50)

Exact expressions for $F_{X_{s,d}}(\frac{\gamma}{a})$, $F_{X_{DF}^{r,i}}(\frac{\gamma}{a})$ and $F_{X_{DF}^{r,i,d}}(\frac{\gamma}{b})$ are given by

$$F_{X_{s,d}}(\frac{\gamma}{a}) = \left(\frac{\gamma}{a}\right)^{m_{s,d}}\frac{\alpha_{s,d}}{\Gamma(m_{s,d} + 1)}I_{1}\left(m_{s,d}; m_{s,d} + 1; -\frac{\alpha_{s,d}\gamma}{a}\right)$$

(3.51)

$$F_{X_{DF}^{r,i}}(\frac{\gamma}{a}) = \left(\frac{\gamma}{a}\right)^{m_{r,i}}\frac{\alpha_{r,i}}{\Gamma(m_{r,i} + 1)}I_{1}\left(m_{r,i}; m_{r,i} + 1; -\frac{\alpha_{r,i}\gamma}{a}\right)$$

(3.52)

$$F_{X_{DF}^{r,i,d}}(\frac{\gamma}{b}) = \left(\frac{\gamma}{b}\right)^{m_{r,i,d}}\frac{\alpha_{r,i,d}}{\Gamma(m_{r,i,d} + 1)}I_{1}\left(m_{r,i,d}; m_{r,i,d} + 1; -\frac{\alpha_{r,i,d}\gamma}{b}\right)$$

(3.53)

Thus, the exact expression for the CDF of the total SNR $\gamma_{DF}$ in (3.50) can be written as

$$F_{\gamma_{DF}^{i}}(\gamma) = \left\{\left(\frac{\gamma}{a}\right)^{m_{s,d}}\frac{\alpha_{s,d}}{\Gamma(m_{s,d} + 1)}I_{1}\left(m_{s,d}; m_{s,d} + 1; -\frac{\alpha_{s,d}\gamma}{a}\right)\right\}$$

$$\times \left\{1 - \left[1 - \left(\frac{\gamma}{a}\right)^{m_{r,i}}\frac{\alpha_{r,i}}{\Gamma(m_{r,i} + 1)}I_{1}\left(m_{r,i}; m_{r,i} + 1; -\frac{\alpha_{r,i}\gamma}{a}\right)\right]\right\}$$

$$\times \left\{1 - \left(\frac{\gamma}{b}\right)^{m_{r,i,d}}\frac{\alpha_{r,i,d}}{\Gamma(m_{r,i,d} + 1)}I_{1}\left(m_{r,i,d}; m_{r,i,d} + 1; -\frac{\alpha_{r,i,d}\gamma}{b}\right)\right\}^{N}$$

(3.54)
Using the approximation of the CDF of the total SNR of the system in (3.24) to approximate (3.54) gives

\[ F_{\gamma}^{DF}(\gamma) \approx \left[ 1 - \sum_{c=0}^{m_{s,d}-1} \frac{\alpha_{s,d}^c}{c!} \left( \frac{\gamma}{a} \right)^c e^{-\alpha_{s,d} \frac{\gamma}{a}} \right] \times \left[ 1 - \left\{ 1 - \sum_{t=0}^{m_{s,ri}-1} \frac{\alpha_{s,ri}^t}{t!} \left( \frac{\gamma}{b} \right)^t e^{-\alpha_{s,ri} \frac{\gamma}{b}} \right\} \right] \times \left\{ 1 - \sum_{r=0}^{m_{r,d}-1} \frac{\alpha_{r,d}^r}{r!} \left( \frac{\gamma}{b} \right)^r e^{-\alpha_{r,d} \frac{\gamma}{b}} \right\} \right]^N \]

\[
F_{\gamma}^{DF}(\gamma) = \left\{ \frac{\gamma}{a} \right\}^m_{s,d} \frac{\alpha_{s,d}^m_{s,d}}{\Gamma(m_{s,d} + 1)} F_1 \left( m_{s,d}; m_{s,d} + 1; -\frac{\alpha_{s,d} \gamma}{a} \right) \right\} \times \left\{ 1 - \left\{ \frac{\gamma}{a} \right\}^m_{s,ri} \frac{\alpha_{s,ri}^m_{s,ri}}{\Gamma(m_{s,ri} + 1)} F_1 \left( m_{s,ri}; m_{s,ri} + 1; -\frac{\alpha_{s,ri} \gamma}{a} \right) \right\} \times \left\{ 1 - \left\{ \frac{\gamma}{b} \right\}^m_{r,d} \frac{\alpha_{r,d}^m_{r,d}}{\Gamma(m_{r,d} + 1)} F_1 \left( m_{r,d}; m_{r,d} + 1; -\frac{\alpha_{r,d} \gamma}{b} \right) \right\} \right]^N
\]

(3.55)

### 3.2.9 Outage probability for DF

The expressions for outage probability for the system model employing DF protocol over the \( \kappa - \mu \) shadowed fading is exactly given by

\[ P_{out} = F_{\gamma T}^{DF}(\gamma = \gamma_{th}) \]

(3.56)

where

\[
F_{\gamma T}^{DF}(\gamma_{th}) = \left\{ \frac{\gamma_{th}}{a} \right\}^m_{s,d} \frac{\alpha_{s,d}^m_{s,d}}{\Gamma(m_{s,d} + 1)} F_1 \left( m_{s,d}; m_{s,d} + 1; -\frac{\alpha_{s,d} \gamma_{th}}{a} \right) \right\} \times \left\{ 1 - \left\{ \frac{\gamma_{th}}{a} \right\}^m_{s,ri} \frac{\alpha_{s,ri}^m_{s,ri}}{\Gamma(m_{s,ri} + 1)} F_1 \left( m_{s,ri}; m_{s,ri} + 1; -\frac{\alpha_{s,ri} \gamma_{th}}{a} \right) \right\} \times \left\{ 1 - \left\{ \frac{\gamma_{th}}{b} \right\}^m_{r,d} \frac{\alpha_{r,d}^m_{r,d}}{\Gamma(m_{r,d} + 1)} F_1 \left( m_{r,d}; m_{r,d} + 1; -\frac{\alpha_{r,d} \gamma_{th}}{b} \right) \right\} \right]^N
\]

(3.57)
Similarly, the outage probability for the system employing DF protocol over $\kappa - \mu$ shadowed fading is approximated as

$$F_{\gamma}^{DF}(\gamma_{th}) \approx \left[ 1 - \sum_{c=0}^{m_{s,d}-1} \frac{\alpha_{s,d}^{c}}{c!} \left( \frac{\gamma_{th}}{a} \right)^{c} e^{-\alpha_{s,d} \gamma_{th}} \right]$$

$$\times \left[ 1 - \sum_{t=0}^{m_{s,r_i}-1} \frac{\alpha_{s,r_i}^{t}}{t!} \left( \frac{\gamma_{th}}{a} \right)^{t} e^{-\alpha_{s,r_i} \gamma_{th}} \sum_{r=0}^{m_{r_i,d}-1} \frac{\alpha_{r_i,d}^{r}}{r!} \left( \frac{\gamma_{th}}{b} \right)^{r} e^{-\alpha_{r_i,d} \gamma_{th}} \right]^N$$

(3.58)
In this chapter, the results obtained from numerical analysis using Mathematica and simulation using MATLAB on the proposed system model are presented. An investigation of the outage probability of the system model is carried out with the parameters being $N, \kappa_{s,d}, \kappa_{s,r_1}, \kappa_{r_1,d}, \mu_{s,d}, \mu_{s,r_1}, \mu_{r_1,d}, \xi_{s,d}, \xi_{s,r_1}, \xi_{r_1,d}$. Comparisons have been made by changing a few parameters and the results were observed for $P_s/N_0 = 20 \, \text{dB}$ at the source and SNR ranging from 0 to 20 dB. Later the cases are analysed and simulated using the AF and DF relaying protocols.

### 4.1 Comparison of AF and DF

The quality of the source-relay channel affects the choice of the relaying. The relay may opt DF, AF, or Direct-Transmission (DT) to forward signals. Here, performance of the system is examined through an outage probability analysis over $\kappa - \mu$ shadowed fading channels where the AF and the DF strategies are compared.

The importance of employing relaying protocols can be observed from the plot shown in Fig. 4.1. For the direct link the outage probability is higher than for AF and DF. For 2 relays, with outage threshold $\gamma_{th} = 3 \, \text{dB}$, 10 dB SNR at the relays, and the parameter set is shown in the Table 4.1, the plot in Fig. 4.1 is obtained. The outage initially is the same for the direct link, the AF protocol and the DF protocol curves. With increase in SNR, it decreases in all three cases but the rate of decrease is different. The simulation shows that the direct link proves to be expensive in terms of outage probability in high SNR scenarios. The AF and DF relaying protocols can be employed for transmission in such cases.

For the complete comparison of the AF and DF protocols, the simulated results are plotted along with the expressions for the outage probability of AF and DF protocols in (3.44), (3.45),(3.57) and (3.58) which are obtained after the detailed analysis. The simulated results from MATLAB and the analytical results of the exact and approximated expressions from Mathematica are plotted in Fig. 4.2. Table 4.2 shows the values of the parameters considered for this comparison.
Table 4.1: Parameter set for AF and DF and direct link comparison

<table>
<thead>
<tr>
<th>Name</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>SNR threshold</td>
<td>$\gamma_{th}$</td>
<td>3 dB</td>
</tr>
<tr>
<td>Transmit SNR at the relay</td>
<td>$P_r/N_0$</td>
<td>10 dB</td>
</tr>
<tr>
<td>Number of relays</td>
<td>$N$</td>
<td>2</td>
</tr>
<tr>
<td>Shaping parameter $\kappa$ for direct link</td>
<td>$\kappa_{s,d}$</td>
<td>0.34</td>
</tr>
<tr>
<td>Shaping parameter $\kappa$ for first hop</td>
<td>$\kappa_{s,r_i}$</td>
<td>2</td>
</tr>
<tr>
<td>Shaping parameter $\kappa$ for second hop</td>
<td>$\kappa_{r_i,d}$</td>
<td>0.7</td>
</tr>
<tr>
<td>Shaping parameter $\mu$ for direct link</td>
<td>$\mu_{s,d}$</td>
<td>3.107</td>
</tr>
<tr>
<td>Shaping parameter $\mu$ for first hop</td>
<td>$\mu_{s,r_i}$</td>
<td>5</td>
</tr>
<tr>
<td>Shaping parameter $\mu$ for second hop</td>
<td>$\mu_{r_i,d}$</td>
<td>2.5</td>
</tr>
<tr>
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<tr>
<td>Fading severity for first hop</td>
<td>$\xi_{s,r_i}$</td>
<td>2</td>
</tr>
<tr>
<td>Fading severity for second hop</td>
<td>$\xi_{r_i,d}$</td>
<td>2</td>
</tr>
<tr>
<td>Noise power</td>
<td>$N_0$</td>
<td>1</td>
</tr>
<tr>
<td>Normalised distance between source and destination</td>
<td>$d_{s,d}$</td>
<td>0.8</td>
</tr>
<tr>
<td>Normalised distance between source and relay</td>
<td>$d_{s,r_i}$</td>
<td>1.2</td>
</tr>
<tr>
<td>Normalised distance between relay and destination</td>
<td>$d_{r_i,d}$</td>
<td>1.2</td>
</tr>
</tbody>
</table>

Figure 4.1: Comparison of AF and DF and direct link for $N=2$. 
Table 4.2: Parameter set for AF and DF comparison

<table>
<thead>
<tr>
<th>Name</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>SNR threshold</td>
<td>$\gamma_{th}$</td>
<td>3 dB</td>
</tr>
<tr>
<td>Transmit SNR at relay</td>
<td>$P_r/N_0$</td>
<td>10 dB</td>
</tr>
<tr>
<td>Number of relays</td>
<td>$N$</td>
<td>2</td>
</tr>
<tr>
<td>Shaping parameter $\kappa$ for first hop</td>
<td>$\kappa_s,r_i$</td>
<td>2</td>
</tr>
<tr>
<td>Shaping parameter $\kappa$ for second hop</td>
<td>$\kappa_{r_i,d}$</td>
<td>0.7</td>
</tr>
<tr>
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<td>3.107</td>
</tr>
<tr>
<td>Shaping parameter $\mu$ for first hop</td>
<td>$\mu_s,r_i$</td>
<td>5</td>
</tr>
<tr>
<td>Shaping parameter $\mu$ for second hop</td>
<td>$\mu_{r_i,d}$</td>
<td>2.5</td>
</tr>
<tr>
<td>Fading severity for direct link</td>
<td>$\xi_{s,d}$</td>
<td>2</td>
</tr>
<tr>
<td>Fading severity for first hop</td>
<td>$\xi_{s,r_i}$</td>
<td>2</td>
</tr>
<tr>
<td>Fading severity for second hop</td>
<td>$\xi_{r_i,d}$</td>
<td>2</td>
</tr>
<tr>
<td>Noise power</td>
<td>$N_0$</td>
<td>1</td>
</tr>
</tbody>
</table>

Figure 4.2: Comparison of AF and DF for $N=2$. 
It is worth noting that, both amplify-and-forward and decode-and-forward are not very effective at low SNR. However, given the parameters in Table 4.2, we see that the outage decreases with the increase in SNR in both the relaying protocols. Fig. 4.2 illustrates that outage is greater for AF than DF for a given SNR, i.e., when equal power is allotted, the system with DF performances better than AF in terms of outage.

4.2 Impact of number of relays

In Section 4.1, the AF and DF relaying protocols are compared for 2 relays. In this section, the effect of the number of relays is investigated. The other parameters are selected as shown in Table 4.3.

<table>
<thead>
<tr>
<th>Name</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>SNR threshold</td>
<td>$\gamma_{th}$</td>
<td>3 dB</td>
</tr>
<tr>
<td>Transmit SNR at relay</td>
<td>$P_t/N_0$</td>
<td>10 dB</td>
</tr>
<tr>
<td>Shaping parameter $\kappa$ for direct link</td>
<td>$\kappa_{s,d}$</td>
<td>0.34</td>
</tr>
<tr>
<td>Shaping parameter $\kappa$ for first hop</td>
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<td>2</td>
</tr>
<tr>
<td>Shaping parameter $\kappa$ for second hop</td>
<td>$\kappa_{r_i,d}$</td>
<td>0.7</td>
</tr>
<tr>
<td>Shaping parameter $\mu$ for direct link</td>
<td>$\mu_{s,d}$</td>
<td>3.107</td>
</tr>
<tr>
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<td>$\mu_{s,r_i}$</td>
<td>5</td>
</tr>
<tr>
<td>Shaping parameter $\mu$ for second hop</td>
<td>$\mu_{r_i,d}$</td>
<td>2.5</td>
</tr>
<tr>
<td>Fading severity for direct link</td>
<td>$\xi_{s,d}$</td>
<td>2</td>
</tr>
<tr>
<td>Fading severity for first hop</td>
<td>$\xi_{s,r_i}$</td>
<td>2</td>
</tr>
<tr>
<td>Fading severity for second hop</td>
<td>$\xi_{r_i,d}$</td>
<td>2</td>
</tr>
<tr>
<td>Normalised distance between S and D</td>
<td>$d_{s,d}$</td>
<td>0.8</td>
</tr>
<tr>
<td>Normalised distance between S and $R_i$</td>
<td>$d_{s,r_i}$</td>
<td>1.2</td>
</tr>
<tr>
<td>Normalised distance between $R_i$ and D</td>
<td>$d_{r_i,d}$</td>
<td>1.2</td>
</tr>
</tbody>
</table>

The value of $N$ is varied to understand the relaying strategy better. In this context, the outage probability expressions obtained after the calculation from (3.54) and (3.55) for the DF relaying protocol are analysed in Mathematica and simulated in MATLAB using the values from Table 4.3. The plot shown in Fig. 4.4 illustrates the analytical and simulated results for $N = 2$, $N = 4$ and $N = 6$.

The outage probability for AF in (3.42) was approximated by (3.41) in order to simplify the calculations. Fig. 4.3 shows the results of both the exact and the approximated expression by (3.42) and (3.41) for AF. The values of the parameters including $\bar{N}, \kappa_{s,d}, \kappa_{s,r_i}, \kappa_{r_i,d}, \mu_{s,d}, \mu_{s,r_i}, \mu_{r_i,d}, \xi_{s,d}, \xi_{s,r_i}, \xi_{r_i,d}$ are chosen as in
Figure 4.3: Outage performance of AF for $N = 2$, $N = 4$ and $N = 6$.

Figure 4.4: Outage performance of DF for $N = 2$, $N = 4$ and $N = 6$. 
We observe from Fig. 4.3 and Fig. 4.4 that the exact and approximated results match the simulation results closely. Also, with the increase in SNR, the outage probability decreases as expected. Interestingly, when the number of relays are increased, for a particular value of SNR the outage is reduced. It can be hence understood that the increase in the number of relays gives a better performance.

### 4.3 Fading parameters

In wireless transmission scenarios, multipath channels are represented by statistical models [16]. An important parameter which affects the performance of the proposed scheme is the fading severity $\xi$.

By setting $\xi = 1$, we can directly reach a Rayleigh fading channel scenario. The parameter set considered for this observation is shown in Table 4.4.

<table>
<thead>
<tr>
<th>Name</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>SNR threshold</td>
<td>$\gamma_{th}$</td>
<td>3 dB</td>
</tr>
<tr>
<td>Transmit SNR at relay</td>
<td>$P_r/N_0$</td>
<td>10 dB</td>
</tr>
<tr>
<td>Shaping parameter $\kappa$ for direct link</td>
<td>$\kappa_{s,d}$</td>
<td>0.5</td>
</tr>
<tr>
<td>Shaping parameter $\kappa$ for first hop</td>
<td>$\kappa_{s,r_i}$</td>
<td>0.8</td>
</tr>
<tr>
<td>Shaping parameter $\kappa$ for second hop</td>
<td>$\kappa_{r_i,d}$</td>
<td>0.4</td>
</tr>
<tr>
<td>Shaping parameter $\mu$ for direct link</td>
<td>$\mu_{s,d}$</td>
<td>3.5</td>
</tr>
<tr>
<td>Shaping parameter $\mu$ for first hop</td>
<td>$\mu_{s,r_i}$</td>
<td>3</td>
</tr>
<tr>
<td>Shaping parameter $\mu$ for second hop</td>
<td>$\mu_{r_i,d}$</td>
<td>4.5</td>
</tr>
<tr>
<td>Fading severity for direct link</td>
<td>$\xi_{s,d}$</td>
<td>1</td>
</tr>
<tr>
<td>Fading severity for first hop</td>
<td>$\xi_{s,r_i}$</td>
<td>1</td>
</tr>
<tr>
<td>Fading severity for second hop</td>
<td>$\xi_{r_i,d}$</td>
<td>1</td>
</tr>
<tr>
<td>Normalised distance between source and destination</td>
<td>$d_{s,d}$</td>
<td>0.5</td>
</tr>
<tr>
<td>Normalised distance between source and relay</td>
<td>$d_{s,r_i}$</td>
<td>0.8</td>
</tr>
<tr>
<td>Normalised distance between relay and destination</td>
<td>$d_{r_i,d}$</td>
<td>0.8</td>
</tr>
</tbody>
</table>

From the outage versus SNR curves in Fig. 4.5 and 4.6, we note that the outage probability decreases rapidly with increase in SNR. DF outperforms AF for the above considered channel conditions and parameter set, i.e. the outage for AF is
Figure 4.5: Outage probability of AF when $\xi=1$.

Figure 4.6: Outage probability of DF when $\xi=1$. 
greater than DF protocol in Rayleigh fading.

A comparison is being made in Fig. 4.7 and 4.8 between the outage probability obtained for AF and DF when the value of fading severity parameter $\xi$ is changed from 1 to 2 while keeping the other parameters as shown in Table. 4.5. An observation can be made from Fig. 4.7 and 4.8 that when fading severity parameter $\xi = 1$, the outage probability is comparatively smaller for DF than AF implying that the performance of DF is better. Also, when the value of $\xi$ is increased to 2, the system performance improves further in both the cases as the outage decreases.

An extensive study has been done to compare the performance by varying $\kappa$ and $\mu$ and understand the complete effect on the system. The values of $\kappa_{s,d}, \kappa_{s,r_i}, \kappa_{r_i,d}$ are varied as $\kappa = 4, 7$ in the case of AF and $\kappa = 0.7, 2$ in the case of DF whereas the remaining parameters are set to the values in Table. 4.6. Here any comparison cannot be made between the AF and DF plots as the values of $\kappa$ considered are different. The motivation behind choosing these values of $\kappa$ is to obtain curves which do not overlap. The plot of outage probability against SNR for AF and DF protocols are shown in Fig. 4.9 and 4.10, respectively. We learn from Fig. 4.9 and 4.10 (which shows simulated outage probability results for the AF and DF by changing different $\kappa$ values) the lower the value of $\kappa$, the better the performance of the system in DF while it has little impact in the case of AF. We also study the probability of outage of this system when the shaping parameter $\mu$ is varied. For the plots shown in Fig. 4.11 and 4.12, the parameters assumed are as in Table 4.7. An improvement is observed in the system performance as $\mu$ is increased from 1.5 to 3 when both AF and DF are employed.

<table>
<thead>
<tr>
<th>Name</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>SNR threshold value</td>
<td>$\gamma_{th}$</td>
<td>3 dB</td>
</tr>
<tr>
<td>Transmit SNR at relay</td>
<td>$P_r/N_0$</td>
<td>10 dB</td>
</tr>
<tr>
<td>Shaping parameter $\kappa$ for direct link</td>
<td>$\kappa_{s,d}$</td>
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<tr>
<td>Shaping parameter $\kappa$ for first hop</td>
<td>$\kappa_{s,r_i}$</td>
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<tr>
<td>Shaping parameter $\kappa$ for second hop</td>
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<td>$\mu_{s,r_i}$</td>
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<tr>
<td>Shaping parameter $\mu$ for second hop</td>
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</tr>
<tr>
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Table 4.5: Parameter set for understanding effect of $\xi$
Chapter 4. Numerical Results and Simulation

Figure 4.7: Outage probability of AF protocol for $\xi = 1, \xi = 2$.

Figure 4.8: Outage probability of DF protocol for $\xi = 1, \xi = 2$. 
### Table 4.6: Parameter set for studying effect of $\kappa$

<table>
<thead>
<tr>
<th>Name</th>
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<tr>
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<td>$P_r/N_0$</td>
<td>10 dB</td>
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<tr>
<td>Normalised distance between source and relay</td>
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</tr>
<tr>
<td>Normalised distance between relay and destination</td>
<td>$d_{r_i,d}$</td>
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</tr>
</tbody>
</table>

### Table 4.7: Parameter set for studying effect of $\mu$

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</tr>
</thead>
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</tr>
<tr>
<td>Transmit power at relay</td>
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</tr>
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</tr>
<tr>
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</tr>
<tr>
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<td>$\kappa_{r_i,d}$</td>
<td>2.4</td>
</tr>
<tr>
<td>Fading severity for direct link</td>
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<tr>
<td>Normalised distance between relay and destination</td>
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</tr>
<tr>
<td>Number of relays</td>
<td>$N$</td>
<td>2</td>
</tr>
</tbody>
</table>
Chapter 4. Numerical Results and Simulation

Figure 4.9: Outage probability for different $\kappa$ values.

Figure 4.10: Outage probability for different $\kappa$ values.
Figure 4.11: Outage probability for different $\mu$ values.

Figure 4.12: Outage probability for different $\mu$ values.
4.4 Impact of transmission distances

Every parameter has its own importance and varying each parameter would result in a different output. On this note, we proceed to analyse the impact of change in the distance between the relays and the source.

Fig. 4.13 and 4.14, depict the outage characteristics for a set of cases in which the distances are changed as shown in Table 4.9. The considered cases have the parameter sets as in Table 4.8.

Table 4.8: Parameter set for observing impact of transmission distance

<table>
<thead>
<tr>
<th>Name</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>SNR threshold</td>
<td>$\gamma_{th}$</td>
<td>3 dB</td>
</tr>
<tr>
<td>Transmit power at relay</td>
<td>$P_r$</td>
<td>10 dB</td>
</tr>
<tr>
<td>Shaping parameter $\kappa$ for direct link</td>
<td>$\kappa_{s,d}$</td>
<td>0.5</td>
</tr>
<tr>
<td>Shaping parameter $\kappa$ for first hop</td>
<td>$\kappa_{s,r_i}$</td>
<td>0.8</td>
</tr>
<tr>
<td>Shaping parameter $\kappa$ for second hop</td>
<td>$\kappa_{r_i,d}$</td>
<td>0.4</td>
</tr>
<tr>
<td>Shaping parameter $\mu$ for direct link</td>
<td>$\mu_{s,d}$</td>
<td>3.5</td>
</tr>
<tr>
<td>Shaping parameter $\mu$ for first hop</td>
<td>$\mu_{s,r_i}$</td>
<td>3</td>
</tr>
<tr>
<td>Shaping parameter $\mu$ for second hop</td>
<td>$\mu_{r_i,d}$</td>
<td>4.5</td>
</tr>
<tr>
<td>Fading severity for direct link</td>
<td>$\xi_{s,d}$</td>
<td>1</td>
</tr>
<tr>
<td>Fading severity for first hop</td>
<td>$\xi_{s,r_i}$</td>
<td>1</td>
</tr>
<tr>
<td>Fading severity for second hop</td>
<td>$\xi_{r_i,d}$</td>
<td>1</td>
</tr>
<tr>
<td>Number of relays</td>
<td>$N$</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 4.9: Variation in transmission distance

<table>
<thead>
<tr>
<th></th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_{s,d}$</td>
<td>0.5</td>
<td>0.7</td>
<td>0.7</td>
<td>0.7</td>
</tr>
<tr>
<td>$d_{s,r_i}$</td>
<td>0.8</td>
<td>1.2</td>
<td>1.2</td>
<td>1.9</td>
</tr>
<tr>
<td>$d_{r_i,d}$</td>
<td>0.8</td>
<td>1.2</td>
<td>1.7</td>
<td>1.7</td>
</tr>
</tbody>
</table>

In Case 1, the distances are considered to be small and $d_{s,r_i}$ is set equal to $d_{r_i,d}$. In Case 2, while keeping $d_{s,r_i}$ equal to $d_{r_i,d}$, all the distances have been increased. In Case 3, $d_{r_i,d}$ is further increased while retaining the previous values for $d_{s,d}$ and $d_{s,r_i}$. In Case 4, the $d_{r_i,d}$ is increased, and the values of $d_{s,d}$ and $d_{r_i,d}$ are left
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Figure 4.13: Outage probability of AF for different distances.

Figure 4.14: Outage probability of DF for different distances.
unchanged.

For both AF and DF, the outage probability has increased when all the relay distances were increased from Case 1 to Case 2. When the distances are varied as mentioned, the outage kept rising in Cases 3 and 4 as well. This brings us to a conclusion that the distance from the source to the relay should be kept within a limit and multiple hops can be used for long distance transmissions.
Chapter 5

Conclusions and Future Work

In this thesis, the performance of a cooperative communications system over $\kappa - \mu$ shadowed fading channels has been studied by employing AF and DF relaying strategies along with selection combining. The analytical expressions for the PDF and CDF of SNR have been derived for both AF and DF protocols. Exact and approximated expressions for the outage probability have been calculated. The system model has been simulated in MATLAB. The obtained exact, approximated and simulated results have been compared.

On the basis of the study in Chapter 4, we can thus deduce that the performance of the system can be enhanced by employing relaying protocols like AF and DF. The performance of the system varies with change in different parameters. When AF and DF protocols are compared with a common parameter set, it was observed that the DF protocol had a better outage performance for that particular set. To further compare and understand the effects of relaying protocols on a system, the number of relays has been varied. For values of $N = 2$, $N = 4$ and $N = 6$, the outage probability has been plotted which indicated that the increase in the number of relays has a positive effect on the performance of the system in both the instances. In the next stage of analysis, the fading severity parameter $\xi$ has been varied. The impact on the system performance was major and it was studied that good results were obtained when higher values of $\xi$ are selected. Similarly, the shaping parameters $\kappa$ and $\mu$ have been varied. For AF protocol, the value of $\kappa$ was changed from 4 to 7 and for DF protocol, $\kappa$ has been changed from 0.7 to 2. We notice that lower value of $\kappa$ gives better performance for DF and has little impact in the case of AF. A boost is seen in the system performance as the shaping parameter $\mu$ is altered from 1.5 to 3, when both AF and DF were utilized. Lastly, the distances between the relays has been changed. We can conclude from the AF and DF plots that the distance between the source and the relays should not be too large. In all these instances, the analytical deductions coincide closely with the simulated results.

An extension of this thesis may include the investigation of other performance metrics such as symbol error rate, channel capacity and ergodic capacity. Differ-
ent combinations of relaying protocols can be employed. An interesting scenario would be to design criteria specifically for Alamouti codes and orthogonal space time block codes to optimize the performance of coded cooperation.
References


References


