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Short Term Scheduling of Hydrothermal Power Systems With Integer Hydro Constraints

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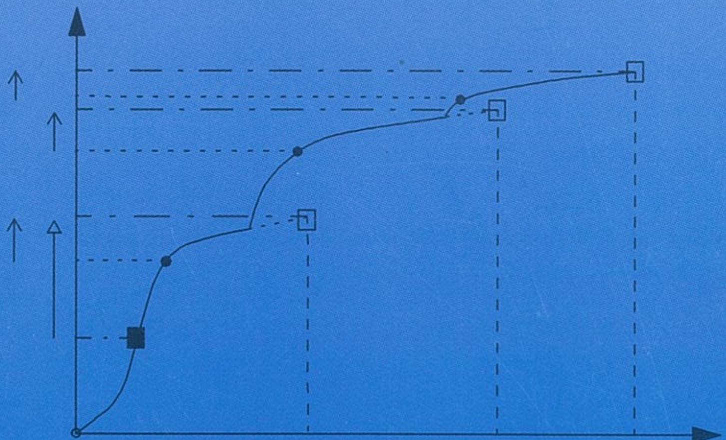

KUNGL TEKNISKA HÖGSKOLAN
Royal Institute of Technology

Short Term Scheduling of Hydrothermal Power Systems With Integer Hydro Constraints

by

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Abstract

This thesis presents models for short term planning (24 hours) of a hydro dominated hydrothermal power system. The purpose of the models is to minimize the system operation cost to provide a forecasted load and keep enough spinning reserve.

The thesis focuses on two issues in hydro power modelling. The first issue is the relationship between water discharged and power generated. This relationship is a non-linear and non-convex function. If the plant has several units, the efficiency of the plant will have local maximums, so called local best-efficiency points. The second issue is to take into account the cost of start-ups of hydro units in the planning.

The hydro model is mixed-integer. Discharges are allowed at zero flow, the local best-efficiency points and on the continuous part between the local best-efficiency point with the highest flow and the point with maximum flow. This last continuous part is modelled as a linear function. In order to get data for the start-up cost a survey among the largest power producers in Sweden has been made, where three questions about start-ups of hydro power units has been asked: What causes the costs in the start-up?, How much does a start-up cost? and How do start-ups effect the short-term scheduling strategies of power producers in Sweden? The results show that a fair estimate of the start-up cost is about $3/MW nominal output. For the thermal plants a standard model with polynomial operation cost, start-up costs and ramp-rate constraints has been used. The model also includes the possibilities of purchasing and selling power to forecasted prices.

The planning problem is formulated as a mathematical programming problem. The solution technique uses Lagrange relaxation to decompose the problem into subproblems. There will be one subproblem for each hydro and thermal plant. In order to find good feasible solutions a heuristic technique to change the integer variables in the hydro system has been developed. The Lagrange multipliers are updated with the subgradient method.

The models are tested in three different load situations; a winter day (heavy load), an autumn day (medium load) and a summer day (light load). The result shows that the method gives near optimal schedules in reasonable computation time in cases with a normal part of the thermal units committed. The assumed start-up cost results in that hydro units almost never are started or stopped for one hour only.

Acknowledgements

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I owe the staff at the division many thanks for the friendly atmosphere. I specially would like to thank the secretary of the division Mrs. Lillemor Hyllengren for her help in many practical matters, as well as checking the English in this thesis. I also owe many thanks to my former colleague at the department, Mr. Mattias Sandqvist, for our daily discussions concerning hydro power planning methods. A person who has my deepest respect is professor Janis Bubenko, whom I also have had the opportunity to cooperate with in a course, given at the division.

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Last but not least I would like to thank Cecilia for her support during this time.

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Notation

General

F	=	the objective function
t	=	index for hours
j	=	index for plants and contracts
D	=	load
R	=	reserve requirement
T	=	number of hours in the planning period
$\bar{\Diamond}$	=	upper bound for $\Diamond$
$\underline{\Diamond}$	=	lower bound for $\Diamond$
$\Diamond(t)$	=	$\Diamond$ for hour t
$\Diamond(j)$ and $\Diamond_j$	=	$\Diamond$ for plant/contract j
$\Diamond(j, t)$	=	$\Diamond$ for plant/contract j and hour t
$\Diamond^{\max}$	=	maximum value in vector $\Diamond$
$\Diamond^{\min}$	=	minimum value in vector $\Diamond$

Hydro generation

F_h	=	the cost of hydro generation
p_h	=	hydro generation
P_h	=	the set of constraints of the hydro system
J_h	=	the set of hydro plants
M	=	the set of upstream reservoirs
m	=	index for upstream plants
τ_{mj}	=	the delay time between plants m and j
u	=	discharge
x	=	reservoir contents
s	=	spillage
w	=	natural inflow between the plant and its upstream plant(s)
ρ	=	water storage value
ν	=	penalty term
X_{T+1}	=	allowed domain for final reservoir contents
i	=	index in piecewise linear models
I	=	number of segment in the piecewise linear model
$\Diamond_i$	=	$\Diamond$ for segment i in the piecewise linear model
σ	=	water storage value including penalty term
$f(\ldots)$	=	the generation as function of discharge, contents in upstream and downstream reservoirs

d	=	downstream reservoir
U	=	allowed discharge domain
$\Diamond_{\mathrm{pwl}}$	=	$\Diamond$ from a piecewise linear model
$\Diamond_{\mathrm{mi}}$	=	$\Diamond$ from a mixed integer model
p_0	=	generation without compensation for head dependence
α	=	head correction factor for upstream reservoir
β	=	head correction factor for downstream reservoir
Γ	=	generation conversion factor
k	=	index for mixed integer model
K	=	number of local best efficiency points
z	=	state variables
Z	=	set of state variables
χ	=	start-up cost
c_{start}	=	start-up cost for moving from one state to another
ups	=	index for a plant upstream a run of river plant
ror	=	index for a run of river plant
Q	=	discharge during a time period
η	=	relative efficinecy for a hydro plant
Θ_i	=	time of phase i during the start-up
Ψ	=	time variable of the start-up phase
α'	=	a function showing how much an increase of the reservoir contents for the upstream reservoir gives in increased head
β'	=	a function showing how much a change in the reservoir contents for the downstream reservoir affects the nominal head
α''	=	a function showing how much an increase of the contents of the upstream reservoir affects the generation
β''	=	a function showing how much an increase of the contents of the downstream reservoir affects the generation

Thermal generation

F_g	=	the cost of thermal generation
p_g	=	thermal generation
P_g	=	the set of constraints of the thermal system
J_g	=	the set of thermal plants
$g(\ldots)$	=	the cost as function of thermal generation
$a_0(j)$	=	coefficient in $g(\ldots)$
$a_1(j)$	=	coefficient in $g(\ldots)$
$a_2(j)$	=	coefficient in $g(\ldots)$
y	=	on/off-line indicator for thermal plants
S	=	start-up cost

b_0	=	start-up cost if the boiler is warm
b_1	=	additional start-up cost if the boiler is cold
H	=	hours the plant has been off-line
Υ	=	time constant for cooling
ν	=	start-up indicator
δ_{down}	=	maximum down ramping
δ_{up}	=	maximum up ramping

Power exchanges

F_e	=	the cost for power exchanges
p_e	=	the energy exchange, positive if power is bought and negative if vice versa
P_e	=	the set of constraints of the power exchanges
J_e	=	the set of contracts for power exchanges
γ	=	the price for power exchange

Optimization

φ	=	dual objective function
φ_0	=	the constant part of the dual objective function
φ_h	=	the hydro part of the dual objective function
φ_g	=	the thermal part of the dual objective function
φ_e	=	the power exchanges part of the dual objective function
φ_{ge}	=	the thermal and power exchanges part of the dual objective function
φ_z	=	the integer hydro part of the dual objective function
φ_q	=	the continuous hydro part of the dual objective function
q	=	splitting variable for discharge
λ	=	dual variable/Lagrange multiplier for the load balance
ζ	=	dual variable/Lagrange multiplier for the ramp rate
μ	=	dual variable/Lagrange multiplier for the spinning reserve
κ	=	dual variable/Lagrange multiplier for the splitting constraint
κ'	=	dual variable for the network constraints
$\underset{\sim}{\Diamond}$	=	start value for $\Diamond$
qu	=	splitting constraint
F_{EDC}	=	objective function of economic dispatch
λ_{EDC}	=	marginal cost of generation in feasible solution
D_{EDC}	=	the load which should be covered by thermal generation and power exchanges in an EDC

C	=	the optimal value to F_{EDC}, as function of
C_h	=	the optimal value to F_{EDC}, as function of the hydro generation
$1(\Diamond > 0)$	=	1 if $\Diamond > 0$, otherwise 0.
t_1	=	the first hour for a cluster of active ramp rate constraints
t_2	=	the last hour for a cluster of active ramp rate constraints
$\Diamond_{\lambda, \mu, \kappa, \zeta}$	=	$\Diamond$ is optimal for the set of dual variables
$a_1'(j)$	=	$a_1(j)$, adjusted for the dual variables
$\Delta\Diamond$	=	subgradient/mismatch
n	=	iteration counter
θ_n	=	step length in subgradient method, iteration n
Λ	=	parameter in Polyak rule II
ξ	=	parameter in Polyak rule II
ψ	=	parameter in Polyak rule II
φ^*	=	optimal value of φ
φ_n	=	value of φ in iteration n
F_n	=	the best primal solution found until iteration n
c_u	=	linearisation of non-linear objective with respect to discharge
c_x	=	linearisation of non-linear objective with respect to reservoir contents
c_s	=	linearisation of non-linear objective with respect to spillage
ι	=	step length in the line search
c_g	=	state cost in thermal dynamic programming
c_k	=	state cost integer hydro subproblem
$\hat{c}_k$	=	the costs on the arcs in the hydro plant dynamic programming
$\tilde{w}$	=	the natural inflow adjusted for discharge given of the integer hydro subproblem
O	=	original problem
O'	=	reformulated original problem
c_O	=	cost vector in original problem
ω	=	vector of variables in original problem
ω'	=	vector of variables for the second set of constraints in reformulated problem
A_1	=	matrix for first set of constraints
A_2	=	matrix for second set of constraints
W_1	=	right hand side of first set of constraints
W_2	=	right hand side of second set of constraints
Ω	=	domain for ω
L'	=	relaxed problem
L_1'	=	first part of relaxed problem

L_2'	=	second part of relaxed problem
π	=	Lagrange multiplier

Monte Carlo Simulation

$E[\Diamond]$	=	the estimated expected value of $\Diamond$
$sd[\Diamond]$	=	the estimated standard deviation of $\Diamond$
var($\Diamond$)	=	the variance of $\Diamond$
cov($\Diamond$, $\Diamond'$)	=	the co-variance between $\Diamond$ and $\Diamond'$
ϑ	=	the mean price of the day
ϑ'	=	the mean price sampled without correlation to the reservoir contents
l	=	length of the outage time
$G(l)$	=	the distribution function for l
B	=	parameter of $G(l)$
Ξ	=	parameter of $G(l)$
N	=	number of samples
Y	=	an example of a random parameter

1
Introduction

1.1 Background

In an electric power system there are several kinds of generation sources with different conditions for energy generation. In order to use the sources in an efficient way it is essential to make good generation schedules. The generation schedules are made for different time horizons. In the long-term operation planning (2-3 years) and the seasonal planning (6 months-1 year) it is important to consider the stochastics in the system, for example the uncertainty in the forecasts of river inflow, loads and plant reliability. The long-term planning will give final reservoir levels for the seasonal planning and the output of the seasonal planning will be input to the short-term planning (1 day-1 week). For short-term planning it is important to have detailed representation of the plants in the system, since the output of the short-term planning will be applied to the operation of the system. In short-term planning as well, it is important to consider the forecast uncertainty. The common approach to consider the uncertainties in the short-term planning is to make schedules as if the forecasts were perfect and keep reserve margins since all forecasts are uncertain. The schedules are updated when new forecasts are available.

The generation sources in the Swedish power system can be divided into three parts, nuclear, hydro and conventional thermal power. According to [99], the generation in these parts in 1996 were 50, 36 and 10 percents respectively. The total generation was 136,0 TWh. The hydro generation was 20 percent lower than the 40 years average. Fig. 1.1 shows yearly generation in the different power sources.

The conditions and thereby the strategies for the power sources are different.

Nuclear power units are used as base load units, since they have low fuel costs and high start-up costs. These plants are normally operating at the maximum generation limits. They are only stopped or operated with generation

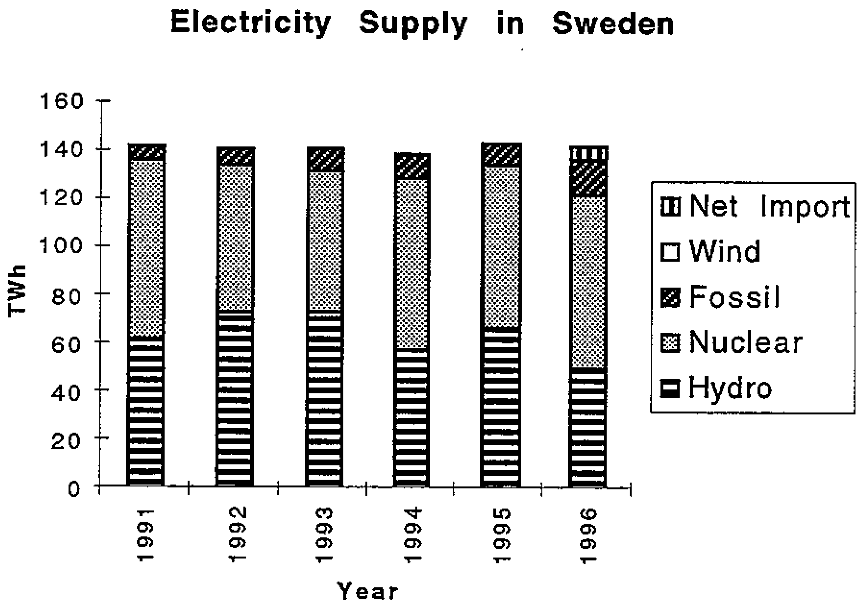


Fig. 1.1 The electricity supply in Sweden during 1991-1996. For 1991-1993 and 1995 there was a net export, which is not shown in the diagram.

lower than maximum during maintenance, forced outages and periods with low load.

Hydro power is dispatched to follow the load. The operation cost of the hydro system is low, but the system is energy limited. This means that the water should be used to minimize the cost of the thermal power. The reserve margins are normally kept in the hydro system. The Swedish hydro system is characterised by:

- Several rivers are included in the system. The rivers are often long and have many reservoirs. It is a complex system where the contents of many reservoirs are dependent on the operation of other plants.

- Large flows in the river system. Because of the large flows, several stations have two or more units. If a plant has several units the efficiency will have a local maximum for each combination of units, known as local best-efficiency points.

- Small heads (heights of fall). There are relatively large head variations during the day in small reservoirs.

The *conventional thermal power* consists of co-generation plants, fossil condensing power and gas turbines. In *co-generation plants* the generation of electricity is a part of heat generation or of an industrial process. The cost and the possibilities of generation in these plants are functions of the heat demand or operating conditions for the industrial process. *Fossil condensing power* is used at peak loads or during periods with low supply of water, since it is more expensive than nuclear power. *Gas turbines* are used as reserve units and only started if there are reserve problems. These units are never scheduled in the planning since the fuel cost is too high. In the Swedish system there is also some *wind power* with about 0, 1 TWh generation per year.

Besides the above mentioned power sources it may be possible to trade power on spot market or exchange with neighbour power systems. The conditions for power trading depend on the design of the contracts.

1.2 The purpose and contribution of the thesis

The advantages and possibilities of using mathematically based decision support systems in planning of electrical power generation have increased during the last years. There are several reasons for this. First, the deregulation of electricity markets all over the world increases the demand for decision support systems [19]. Secondly, the complexity of the energy systems increases [2]. Thirdly, the power system models and the mathematical methods have become better which means that they can handle more complex problems [39]. Fourth, but not the least important, the computers have become much faster.

This thesis deals with models for short term optimization of hydro-thermal generation. During the 90's models with integer representation of hydro plants have gained a lot of support. The reason is that these models are able to consider:

- the non-concave form of the efficiency curve. Depending on which units are committed the efficiency changes.
- the discharge domain where operation is not allowed. Owing to the risk of cavitation damages the units cannot be operated in certain intervals of the discharge domain.

- start-up cost and start and stop restriction for hydro units. Starting a hydro unit will, among other things, lead to wear and tear of the unit. To get an economically efficient operation of the system, these costs have to be considered.
- spinning reserve. Only units which are on-line will contribute to the reserve.

In my research I have developed methods which consider these requests. I have also performed a questionnaire investigation among the power producers in Sweden. This questionnaire investigation was about start-up costs and how these costs affect the short term scheduling strategies. As far as I know there is no such investigation somewhere else in the literature. I have also implemented the cost from the investigation in my models. My research has been focused on methods which are suited for a system with similar structure as the Swedish system. This means a system which is dominated by hydro and nuclear power but also contains some conventional power. Therefore the emphasis of the thesis is on the development of hydro power models and solution techniques, which fulfil the above mentioned requests. For the thermal power I have used established models in the field together with models I have developed for the hydro system.

In order to solve the planning problem I have used Lagrange relaxation. I decomposed the hydro part of the problem into subproblems for each hydro plant by using variable splitting. The advantage of this decomposition is that it makes it possible to use network programming and dynamic methods which are proved to work well for power system scheduling problems. The variable splitting keeps much of the original problem structure, which means that the dual objective function will yield as a strong bound on the primal objective function. The primal solution found by the algorithm will then be near optimal, which is proved by the small duality gap.

The planning model can be used to plan a national regulated power system with possibilities of power exchanges with neighbouring systems. The model can also be used to plan the generation and power exchanges on a spot market.

1.3 List of publications

Most of the work presented in this thesis has earlier been published in my licentiate thesis [64], two internal reports [65] and [66], and in transaction/conference papers [68]-[76]:

- O. Nilsson, *Hydro Power Models For Short Term Generation Planning,* Licentiate Thesis, TRITA-EES-9502, Department of Electric Power Engineering, Kungliga Tekniska Högskolan, Stockholm, Sweden, 1995

- O. Nilsson, *Startkostnader för vattenkraftaggregat*[1], Internal report, A-EES-9511, Department of Electric Power Engineering, Kungliga Tekniska Högskolan, Stockholm, Sweden, 1995

- O. Nilsson, *Vattenkraftmodeller för kortsiktig produktionsplanering*[2], Internal report, A-EES-9403, Department of Electric Power Engineering, Kungliga Tekniska Högskolan, Stockholm, Sweden, 1994

- O. Nilsson and D. Sjelvgren, "A Mixed-Integer Model for Daily Hydro Planning", *Proceedings of IEEE/KTH Stockholm Power Tech, Power Systems,* Stockholm, Sweden, June 18-22, 1995, pp. 99-104

- O. Nilsson and D. Sjelvgren, "A Planning Model With Mixed-Integer Representation of Both Hydro and Thermal Plants", *Proceedings of the 4th Conference on Power System Management and Control,* IEE Conference Publication No. 421, London, UK, April 16-18, 1996, pp. 280-285

- O. Nilsson and D. Sjelvgren, "Hydro Unit Start-Up Costs and Their Impact on Short Term Scheduling Strategies of Swedish Power Producers", *IEEE Transactions on Power Systems,* Vol. 12, No. 1, 1997, pp. 38-44

- O. Nilsson and D. Sjelvgren, "Improved Mixed Integer Programming in Short-Term Hydro Scheduling", *Proceedings of the 12th PSCC,* Dresden, Germany, August 19-23, 1996, pp. 382-389

- O. Nilsson and D. Sjelvgren, "Mixed-Integer Programming Applied to a Hydro-Thermal System", *Proceedings of PICA,* Salt Lake City, USA, May 7-12, 1995, pp. 158-163 and *IEEE Transactions on Power Systems,* Vol. 11, No. 1, 1996, pp. 281-286

- O. Nilsson and D. Sjelvgren, "Variable Splitting Applied to Modelling of Start-Up Costs in Short Term Hydro Scheduling", *IEEE Transactions on Power Systems,* Vol. 12, No. 2, 1997, pp. 770-775

1. In English: Start-up costs for hydro units
2. In English: Hydro power models for short term generation planning

- O. Nilsson and L. Söder, "Daily Generation Planning of a Hydro Dominated Hydrothermal System", *Proceedings of Hydropower 97,* Trondheim, Norway, June 30-July 2, 1997, pp. 293-300

- O. Nilsson and L. Söder, "Short Term Planning Methods Applied to Evaluation of Unavailability Costs in a River", *Proceedings of IPEC 97,* Singapore, May 22-24, 1997, pp. 36-41

- O. Nilsson, L. Söder and D. Sjelvgren, "Integer Modelling of Spinning Reserve Requirements in Short Term Scheduling of Hydro Systems", *IEEE paper PE-142-PWRS-0-06-1997, IEEE/PES Summer Meeting,* Berlin, Germany, July 21-25, 1997

1.4 Overview of the thesis

The outline of the thesis is as follows. Chapter 2 gives an introduction to power generation and its planning problem. In chapter 3 I formulate the short term planning problem mathematically. Chapter 4 contains the above mentioned investigation about start-up costs. Solution techniques for the planning problems are presented in chapter 5. In chapter 6 the models from chapter 3 and the solution techniques from chapter 5 are illustrated by numerical test cases. Finally, in chapter 7, the conclusions and ideas about future work are presented.

2
Power System Generation

This chapter describes the function of the power system, the different sources of generation in the system, how the deregulation of the system affects the planning and how the planning problem looks like from a producer's point of view. The main function of the electric power system is to transmit energy from a source to a place of demand. The sources of generation of electricity are normally based on conversion of potential energy in hydro reservoir or convention of chemical or nuclear energy through heating. One part of the chapter describes the different conditions for these sources of generation. The following part describes how the deregulated market works in systems where all producers and consumers are physically connected through the grid. The last part of the chapter gives an overview of the planning problem for different time horizons and specially for the short term perspective.

2.1 The function of the power system

Electric power is in most cases used as a means of energy transportation. The sources of electricity generation are for example hydro, fossil, nuclear, wind, sun and biomass. The electric energy is almost always converted into light, heat or mechanical energy when it is used. The reasons for not converting the energy of the sources directly into light, heat or mechanical energy are several. First of all, hydro resources are often located far away from the places of demand. Also for example nuclear resources are placed outside high urban density areas for security reasons. Another reason for having a distance between the consumer and the supplier is the economics of scale, since large facilities of generation often are more economical. This means that the large generation facilities will only be placed on a few locations. As a result the resources and the users of energy in many cases are placed far from each other. The consequence is that the energy has to be transported from the source to the demand area. Using electric energy as a means of transportation has proved to be an economically efficient way of transporting the energy. Another advantage of the interconnected electric power system is that the maximum total power demand of all users will be lower than the sum of the

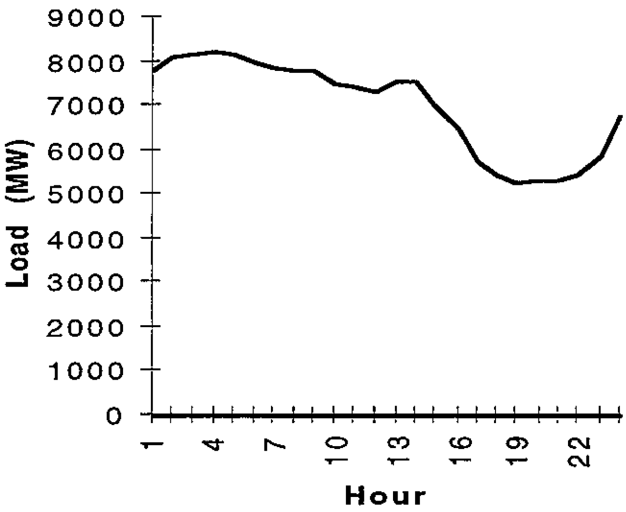


Fig. 2.1 Example of load variations during 24 hours, starting from 7 a.m.

maximum demand of each user since everybody does not use their maximum demand at the same time. This will lead to a lower maximum total capacity of the generation system and thereby lower costs. Furthermore, the integrated system with different types of energy sources connected to it, will open up possibilities of dispatching the different sources in such a way that the total cost of fulfilling the users' demand can be minimized. Methods for minimization of the total generation cost is the topic of the thesis and it will be focused on a short term perspective, about 24 hours. Fig. 2.1 shows an example of the load variations during 24 hours, from 7 a.m. to 7 a.m. The load also includes the losses in the transmission and distribution grids. In order to ensure safe operation of the system reserve margins are kept in the generation system.

There might be situations where 24 hours can be a bit too short for planning the operation of all the plants in the system. I have for example assumed that the operation av the nuclear plants are known in advance. I will come back to this assumption later in this chapter. Other situations when 24 hours horizon might be too short are very long rivers with large delay times and thermal plants with considerably large start-up costs. For a 200 MW thermal unit with around 100 000 SEK in start-up cost the planning horizon is between 24 and 48 hours. The same planning horizon is valid for some longer rivers in Sweden. However since I will use a detailed hydro model the computation time will rise fast when the planning horizon is extended. In order to avoid a long computation time but still be able to consider things that would need somewhat longer planning horizon, a simple model might be used to calculate final states of thermal plants and reservoirs in long rivers. I will come back to this discussion with a short remark in the last chapter.

2.2 Sources of generation

2.2.1 Hydro power

The hydro power plants are located in the rivers. Since the demand of electric energy does not match the natural flow in the rivers there are reservoirs located upstream of most power plants. Some reservoirs have large storage capacities in order to store the water from the spring flood to the winter when the demand is high. The large reservoirs are normally located in the upper part of the river. Other plants have reservoirs suited for matching the electric demand variations on weekly or daily basis. Some plants do not have any reservoirs and they are called run-of-the river plants. The discharge of these plants will be equal to the inflow. Since there often are several hydro plants in the same river these plants cannot be operated independently of each other. Water discharge from a plant will reach the downstream plants after some delay time.

Hydro power generation [108] utilises the difference in potential energy between the intake gate and the tailrace tunnel, see Fig. 2.2. As one knows from classic physics this means that electrical power will be proportional to the discharge of water and the head in the idealistic case. The head is the heights difference between the intake gate and the tailrace tunnel. However, as in most cases of energy conversion there will be losses. In the intake gate the head will decrease when the discharge increases and there will also be losses in the tunnel from the intake gate to the turbines. The turbine efficiency is also a function of the discharge with low efficiency for low and high discharge and a maximum somewhere between. The water level outside the tailrace tunnel will rise with increasing flow and there will also be losses in the tailrace tunnel. The losses will increase with the discharge. The water level at the intake gate will be a function of the contents of the upstream reservoir (if there is one). The water level outside the tailrace tunnel can be affected by the downstream reservoir level (if there is one). Since the losses are a function of the up- and downstream reservoir levels and the discharge, the generation in a hydro plant will be a function of up- and downstream reservoir levels and the discharge.

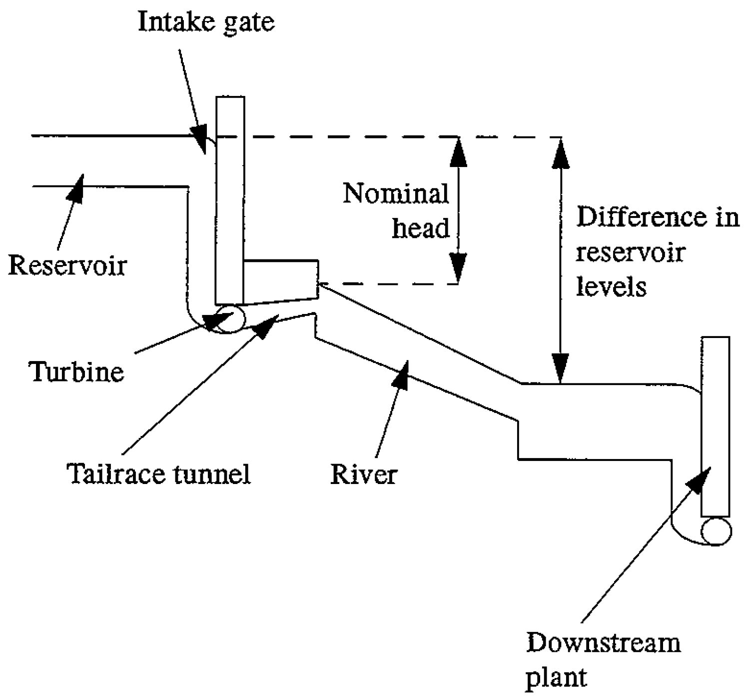


Fig. 2.2 Schematic figure of two hydro plants in cascade.

The potential energy minus losses divided with the potential energy is called the efficiency. Normally the efficiency is standardised by division of the highest efficiency for the plant. This is called relative efficiency. For a plant with more than one unit, it is desirable to share the flow between the units in such a way that the efficiency is maximized [87] and [107]. The losses could be different for the same discharge if one uses different unit combinations. From the optimization of sharing of the discharge, data for the total efficiency will be available. Fig. 2.3 shows an example of the relative efficiency for a plant with four units.

For each combination of units, there will be a local best-efficiency point. If a plant is scheduled between these points it is possible that the efficiency will be significantly lower than the efficiency at the local best-efficiency point. When the discharge increases above the local best-efficiency point with highest discharge, the efficiency will decrease monotonicly. When the consumption is high, plants are often operated above the local best-efficiency point

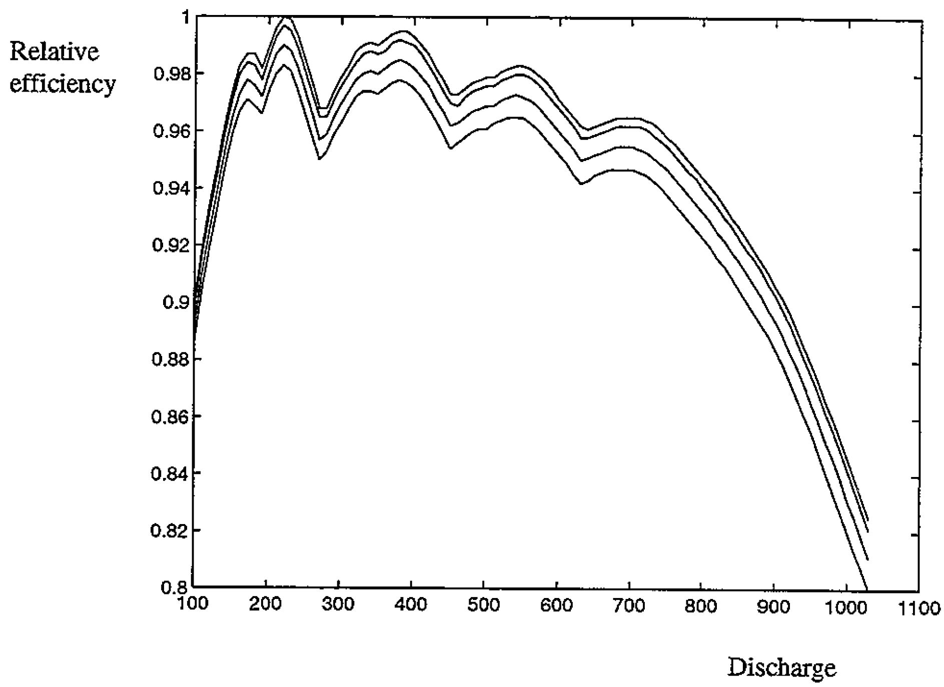


Fig. 2.3 An example of the relative efficiency for a hydro plant with four units. The different curves are for different heads.

with highest flow. Particular plants with only one unit are often operated in this domain.

Fig. 2.3 shows that the discharge point for the local best-efficiency changes slightly when the head changes. However, these changes are in most cases of the same magnitude as the uncertainties of the measurement of the discharge.

In some cases there are forbidden discharge intervals. Operation in these intervals can bring about cavitation damages owing to large vibrations in the turbine. The cavitation damages will be costly to repair and they will also lead to increased operation cost since the plant will not be available.

From Fig. 2.3 the generation can be calculated as the discharge multiplied by the efficiency and a constant. The constant represents the quotient of the generation and discharge at the point of the highest efficiency for current head. The generation as a function of the discharge and up- and downstream reservoir contents is called the generation characteristic. Fig. 2.4 shows an example of the generation characteristic for the plant shown in Fig. 2.3.

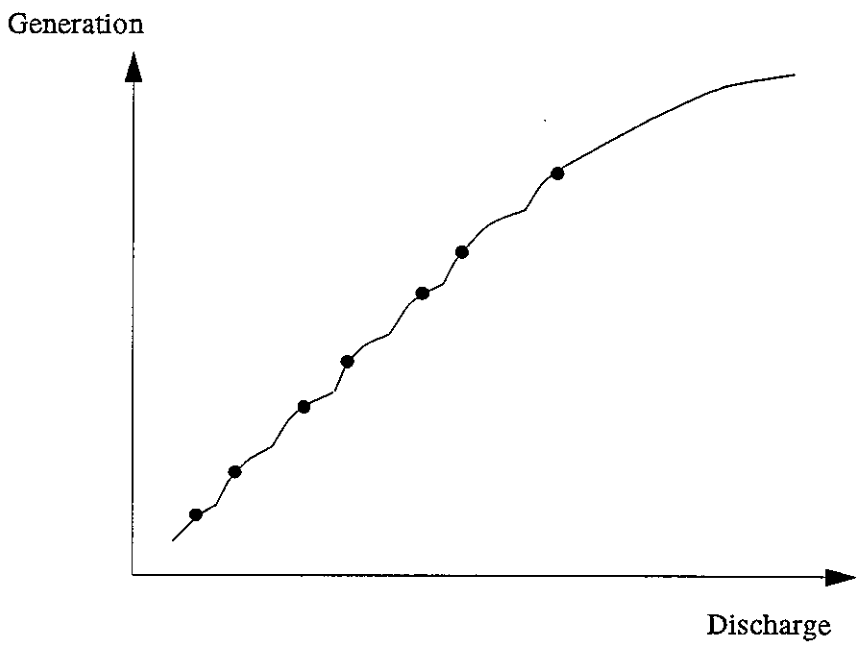


Fig. 2.4 Generation as function of discharge for a specific head. Same plant as in Fig. 2.3. Local best-efficiency points are market with a dot.

In order to fulfil the consumption on an instant basis most hydro units are equipped with frequency controllers to adjust their generation to the changes of operation conditions in frequency. The operation conditions in frequency can change owing to forced outages in power plants, demand forecasting errors and loss of consumption owing to outages in the grid. In order not to jeopardize the security of the system, reserve margins are kept in the plants.

In power systems with a large share of hydro, these reserve margins are kept in the hydro system, since the hydro plants are easy to re-dispatch. In cases where part of the consumption is lost owing to loss of consumption, hydro generation can be decreased (if its generation is higher than zero) to keep the balance between generation and consumption. In case of a plant outage, the on-line hydro units can increase their generation up to their maximum capacity. The difference between the maximum capacity of the on-line hydro units and their generation is the spinning reserve. If all the units in the hydro system are on-line, the spinning reserve will be equal to the difference between the total capacity of the hydro system and its current generation.

After an increase in the generation due to changed operation conditions, the spinning reserve will decrease. If the spinning reserve is lower than required, the spinning reserve needs to be restored. In order to restore the spinning reserve more hydro capacity can be started with a short start-up time if it is available. Otherwise we need to start gas turbines. The capacity which can be started in a short time is called secondary reserve. One alternative definition of secondary reserve is to add the surplus in spinning.

Reserve problems do seldom occur in the Swedish system. The circumstances in which reserve problems may occur are:

- very light load. If the nuclear power is producing on maximum, the share of the scheduled hydro power is too small to fill up the spinning reserve. There are two possibilities of achieving a sufficient reserve margin. The first is to dispatch the nuclear power to schedule enough hydro power. The second is to start up more hydro units without dispatching the nuclear power. In this approach we have to schedule the hydro plants at a lower efficiency compared to if we have not started up more units.

- very heavy load. If all hydro plants are producing on maximum or near to maximum the reserve in the hydro power is too small. We will have to set an upper limit for the hydro generation.

- spring flood. During spring flood it is possible that plants have spill since the inflow is larger than maximum discharge and the reservoir level is already at its upper bound. In this case reserve requirements will force plants to discharge on a lower level. The difference in discharge between the maximum and this lower level has to be spilled.

2.2.2 Thermal power

The cost of short term generation in thermal plants can be divided into three parts: start-up cost, fixed and variable generation costs. Fig. 2.5 shows an example of the generation cost. If the plant is not committed the generation cost will be equal to zero. When the plant is committed generation cost will be the sum of the fixed and variable generation costs. These generation costs consist of fuel costs, environmental fees and maintenance costs. Environmental fee should be paid if the plant is not equipped with an NO_x-cleaner.

The start-up cost will be a function of the time the plant has been off-line. The reason is that the boiler of the plant will cool off during the time the plant is not committed. This means that when the plant is committed again its

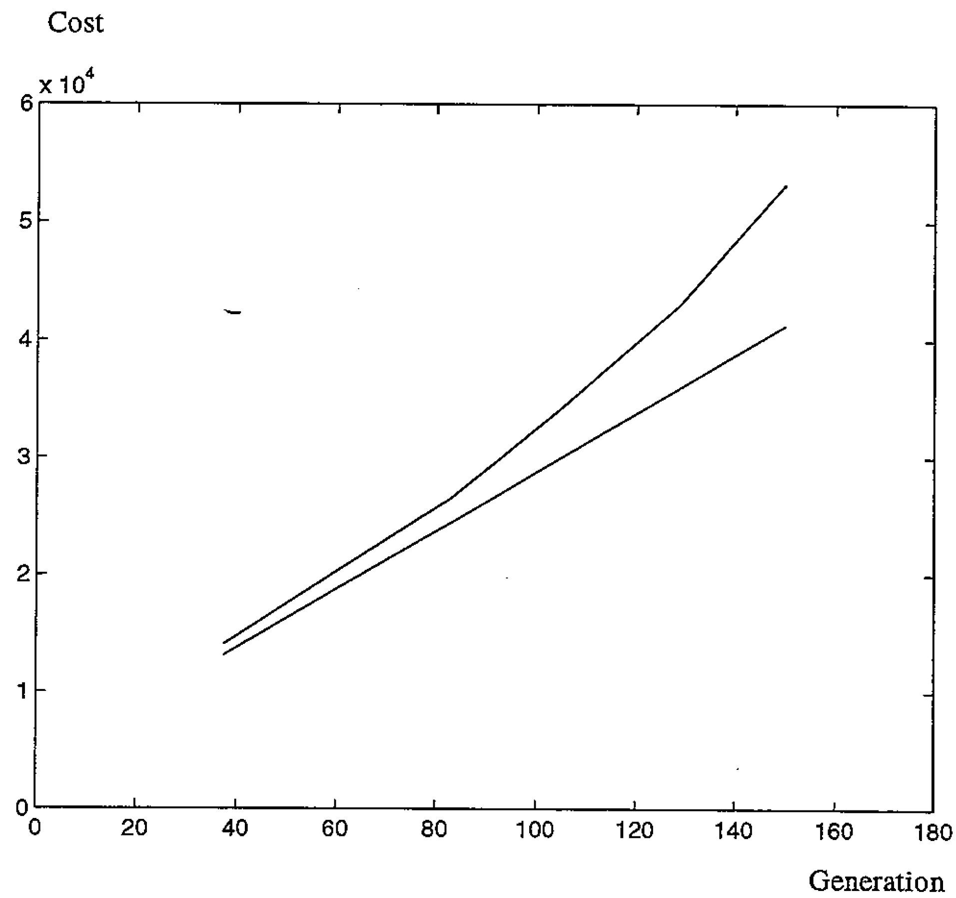


Fig. 2.5 The generation cost as function of the generation. The lower curve is the fuel cost and the upper curve is the fuel cost plus the NO_X-fee.

boiler will have a different temperature due to the off-line time. The cost to get the boiler to the generation temperature will then depend on how long the plant has been off-line, since more or less fuel will be needed to get the boiler to the generation temperature. Fig. 2.6 shows an example of the start-up cost as a function of the off-line time for a thermal plant. There are also limits on how much the generation in a thermal plant can be changed per minute. These constraints are called ramp-rate constraints. During the first hour after start-up, the allowed change of generation per minute is usually smaller, since the unit is cold. After this first hour the ramp rate constraints is about 1-6 per cents of maximum output. This means that after most units can operate at full capacity after two hours.

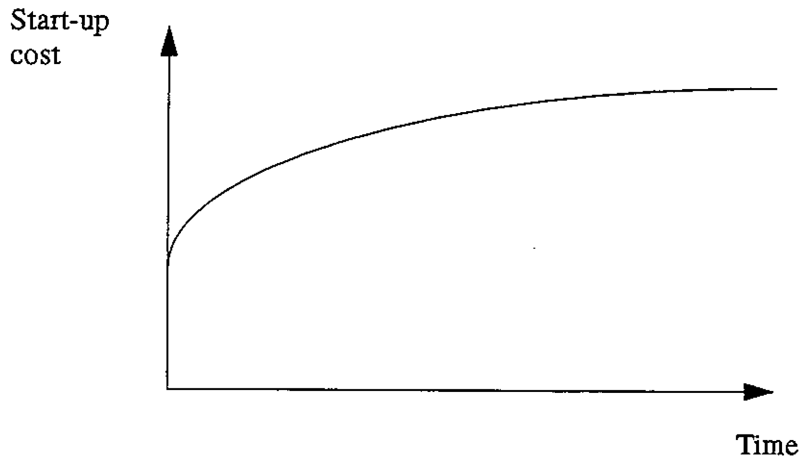


Fig. 2.6 The start-up cost in a thermal plant as a function of the off-line time.

The thermal power can basically be divided into four kinds: nuclear power, cogeneration, condensing power and gas turbine. The nuclear power plants have low generation costs and high start-up costs. Therefore the nuclear plants are normally used as base load units in systems with a large share of hydro power. This means that the nuclear plants are operating at their maximum capacity if the load is higher than the maximum capacity. The nuclear plants are only dispatched to lower generation than maximum if the load is low. Nuclear plants are sometimes shut down because of generation problems, so called forced outages. Nuclear power plants also have to be shut down for longer periods when the fuel in the plants is being exchanged and maintenance is being performed.

In cogeneration plants it is possible to produce both heat and electricity. This results in the fact that the capacity and the cost of electricity generation in cogeneration plants vary depending on the heat demand. Fig. 2.7 shows an example of that relationship. The lower curve shows the electricity generation as function of the heat demand when heat is the main product and electricity is the by-product. It is possible to increase the electricity output if it is equipped with a re-cooling system. This is shown by the middle line. The upper curve shows the case where electricity is the main product and heat is the by-product. In this case the plant is operating as a condensing plant. If cogeneration stands for a large part of the electricity generation, scheduling of heat and electricity has to be planned simultaneously. Since this is not the case for the kind of system I have studied I have assumed that both operation mode and heat demand are known.

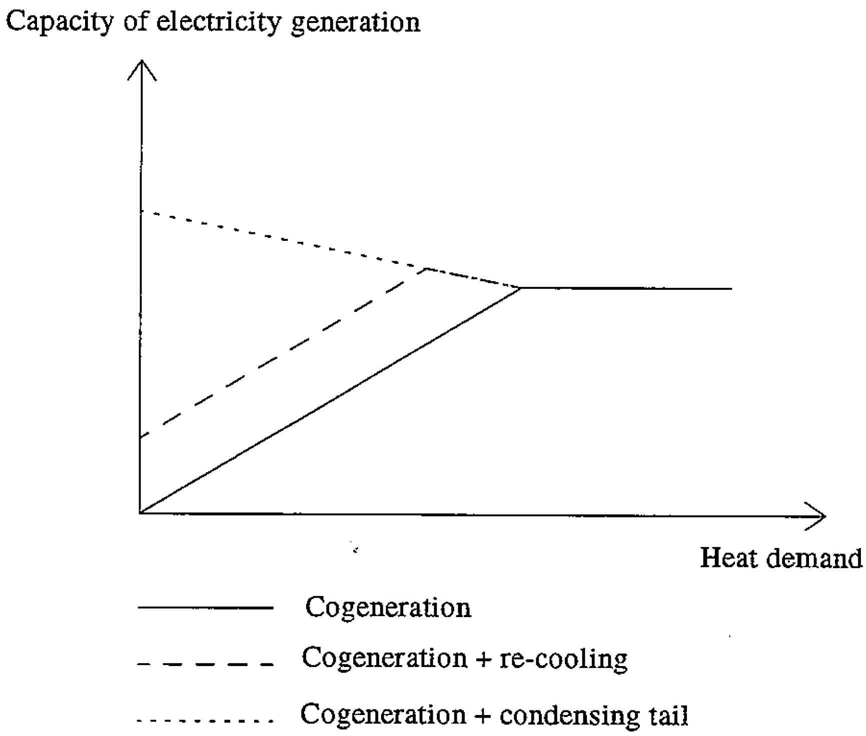


Fig. 2.7 Capacity electricity generation as function of the heat demand.

This means that I can treat cogeneration plants in the same way as condensing plants. The generation cost of cogeneration is higher than for nuclear power, but the start-up cost is lower.

The difference between cogeneration and condensing power plants is that the condensing power plants are used for electricity generation only. As a result the cooling water will not be used for heating. Since no heat generation will be sharing the cost of the condensing power it is more expensive than cogeneration.

In gas turbines heated gas is used to drive the turbine. This means that the gas is heated instead of water. Gas turbines have a low efficiency which results in high operation costs. On the other hand, since we do not need to heat a lot of water when we start the gas turbine, the start-up cost and the start-up time are low. Because of the high operation cost and the low start-up cost and start-up time, gas turbines are mainly used for reserve purposes.

2.3 Deregulation of the electricity market

From January 1, 1996 the electric power market in Sweden has been deregulated. In the new market each customer can choose from whom he or she wants to buy electricity. Before the deregulation each power company had monopoly in certain distribution areas. After the deregulation the transmission and distribution services are split from the generation and market. A state owned independent system operator, Svenska Kraftnät, will provide the transmission service and local distribution companies will provide the distribution service. Both transmission and distribution are regulated monopolies. Generation on the other hand is a free market where producers compete. This means that sellers and buyers can make bilateral contracts. The contracts can be designed in many different ways. One example are so called "take and pay"-contracts where the seller and buyer agree to a price but the amount of purchased power depends on the demand of the buyer at the time for consumption. The contract time can of course vary from contract to contract. Another type of contract can have both fixed price and fixed amount of energy during the contract time. A third kind of contract is a contract where the total amount of energy is fixed and the buyer can use it whenever he wants during the contract time. In these contracts there is normally an upper limit of how much power that could be used at the same time. The surplus of power which is not contracted will be traded at the spot market. At this market trade is based on hourly average values. There is also a future market based on weekly trade.

Svenska Kraftnät is not only responsible for the transmission of power from the producers to the consumption areas, but also responsible for the security of the system. All producers have to report their generation plans to Svenska Kraftnät. If it is not possible to transmit power according to the generation plans Svenska Kraftnät has to buy and sell power in such a way that the power flow in the grid becomes feasible.

As I mentioned above, the trade is based on hourly average values. Quite often there is a need for changes of the generation within smaller intervals than one hour. Still, Svenska Kraftnät is responsible for the balance of generation and consumption in the system. Svenska Kraftnät fulfils this responsibility by letting actors give bids on decrease or increase of generation within short notice.

In order to meet fast changes of operation conditions Svenska Kraftnät has concluded contracts with producers about keeping spinning reserves in their generation systems.

It should also be mentioned that the losses in the network now are treated as loads of the network owner. So, from the power producer's point of view there is no difference between network losses and other kinds of loads.

For a more detailed description of the deregulation in Sweden see [63], [97] and [98].

2.4 The planning problem from a producer's point of view

From a producer's point of view the goal is to create maximum profit at a reasonable risk, as for all other deregulated industries. In order to achieve this goal the power producer has to plan his activities. In the literature of planning methods the planning is often split into the following horizons [93]:

- **Expansion planning.** In this horizon the producer makes a strategy for building new generation facilities [94] and [110]. Before the deregulation the main uncertainties associated with this planning was the uncertainties in load growth and the development of investment and operational costs. After the deregulation the producer also has to consider how actual and potential competitors act. This will make the environment more uncertain for the producer.

- **Planning of several years.** The aim of this planning is to create a strategy for storage in the largest reservoirs and for scheduled maintenance of nuclear plants [110]. The main uncertainties in several years planning are the river inflow, availability of thermal plants and demand. The deregulation has added market uncertainties to planning problem.

- **Seasonal planning.** In the seasonal planning the goal is to find strategies for storage of the water in 6 to 12 months of time horizon [91], [93], [103] and [110]. The uncertainties are basically the same as for several years planning.

- **Short term planning.** The time horizon of this planning period is about 24 hours to one week [102] and [113]. The aim of the short term planning is to create schedules for when to commit and de-commit units and decide how much committed units should generate. The uncertainties are basically the same as for several years and seasonal planning. However, since planning is performed close to the real operation the magnitude of the uncertainties are not as large as in the three other planning horizons.

So, the goal of the short term planning from a producer's point of view is to schedule his plants to fulfil contracts and to trade at the spot market in such a way that the profit is maximized at a reasonable risk.

In a power system with a considerable amount of hydro generation there are some specific problems that do not occur in a thermal system. I have already mentioned that the hydro plants are coupled if they are located in the same river. In many cases there are different producers having different plants in the same river. This means that the producers having generation in the same river cannot plan their generation independently of each other. In Sweden a specific regulation company coordinates the suggestions of the different producers in each river. The producers suggest a generation schedule for their plants to the regulation company. The regulation company decides how much should be discharged from one specific plant in the river during certain periods. The owner of this specific plant has to obey this decision and the other producers will be notified of the decision and can plan their generation in the river based the decision.

Another problem in the hydro system is that the generation in hydro is energy limited. Since the water also could be used to produce electric power after the planning period the water should be given an alternative cost. The alternative cost of water for the short term planning is usually calculated from the seasonal planning. This alternative cost is here called the water storage value. In the seasonal this water storage value is usually calculated from the planning of several years.

I have created a small example to illustrate the short term planning problem from the producer's point of view. Suppose that we have a small power system with four generation units, one nuclear plant, one hydro plant, one cogeneration plant and one gas turbine, see table 2.1. The cost for the hydro generation is the water storage value.

Table 2.1: The power system in the example

Power plant	Capacity	Start-up cost	Variable cost	Fixed cost
Nuclear	1000 MW	base load	base load	base load
Hydro	1000 MW	-	120 kr/MWh	0 kr/h
Cogeneration	200 MW	100 000 kr	100 kr/MWh	1 000 kr/h
Gas turbine	50 MW	5 000 kr	400 kr/MWh	0 kr/h

The goal is now to minimize the generation cost for the coming 24 hours when the demand for the first eight coming hours are forecasted to be 1800 MWh/h, the next eight hours 2050 MWh/h and the last eight hours 1800

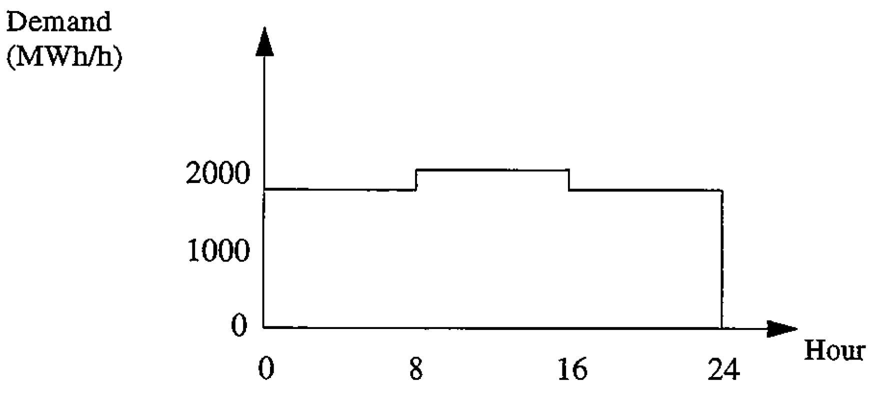


Fig. 2.8 The load in the small example.

MWh/h, see Fig. 2.8. It is obvious that generation in only nuclear and hydro will not cover the demand during the middle eight hours since the total capacity is too low. This means that either the gas turbine or the cogeneration plant has to be started to fulfil the demand during the peak load. Which of these two alternatives will give the lowest generation cost? A comparison will give the answer.

First, the case with the gas turbine. The load not fulfilled by nuclear generation will be the following:

Hour 1-8: 1800 MWh - 1000 MWh = 800 MWh
Hour 9-16: 2050 MWh - 1000 MWh = 1050 MWh

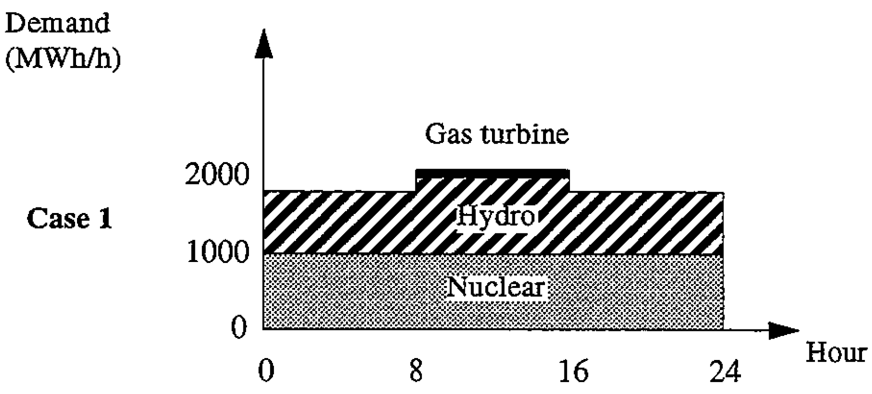


Fig. 2.9 Generation in the gas turbine case.

Hour 17-24: 1800 MWh - 1000 MWh = 800 MWh

Since hydro is cheaper than gas turbine, the hydro generation will be 800 MWh/h during the first and last eight hours and 1000 MWh/h during the eight middle hours, see Fig. 2.9. The total hydro generation will be

$$8 \cdot 800 \text{ MWh} + 8 \cdot 1000 \text{ MWh} + 8 \cdot 800 \text{ MWh} = 20800 \text{ MWh}$$

at a cost of

$$20800 \text{ MWh} \cdot 120 \text{ kr/MWh} = 2\,496\,000 \text{ kr}$$

The total nuclear and hydro generation during hours 9-16 will then be 2000 MWh/h and the load is 2050 MWh/h. The gas turbine will produce 50 MWh/h during these eight hours. The result is that the total generation in the gas turbine will be

$$8 \cdot 50 \text{ MWh} = 400 \text{ MWh}$$

The cost of the gas turbine generation will be the energy cost plus the start-up-cost

$$400 \text{ MWh} \cdot 400 \text{ kr/MWh} + 5\,000 \text{ kr} = 165\,000 \text{ kr}$$

The total cost of hydro and gas turbine generation for this case is

$$2\,496\,000 \text{ kr} + 165\,000 \text{ kr} = 2\,661\,000 \text{ kr}.$$

Now to the cogeneration case, see Fig. 2.10. The load not covered by nuclear generation is the same as in the gas turbine case. However, in this case it is more complicated to decide if it is best to run cogeneration during the whole period or just during hours 9-16. Since we need to start the cogeneration plant for hours 9-16 we can calculate the cost of generating 200 MWh/h in different source without considering the start-up cost. The cost of 200 MWh hydro generation is

$$200 \text{ MWh} \cdot 120 \text{ kr/MWh} = 24\,000 \text{ kr}$$

while the corresponding cost of cogeneration is

$$1\,000 \text{ kr} + 200 \text{ MWh/h} \cdot 100 \text{ kr/MWh} = 21\,000 \text{ kr}$$

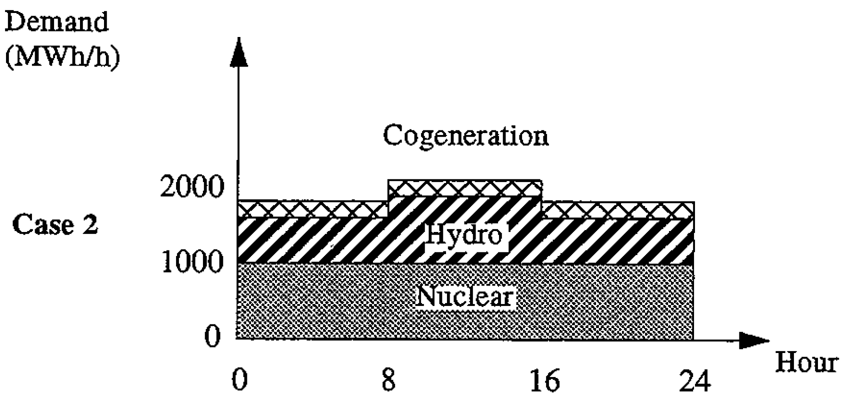


Fig. 2.10 Generation in the cogeneration case.

Given that we have to start the cogeneration plant in order to fulfil the demand for the middle eight hours, it is optimal to us it for the whole planning period. Note that it would not be optimal to start the cogeneration if the demand was not higher than 2000 MWh/h for any hour of the planning period. The cost, including the start-up cost, for cogeneration of 200 MWh/h for the whole planning period is

100 000 kr + 24 • 21 000 kr = 604 000 kr

The corresponding cost for hydro generation is

24 • 24 000 kr = 576 000 kr

In order to be able to calculate the total cost for case 2, we have to know the cost of the hydro generation. The hydro generation is 600 MWh/h during the first and last eight hours and 850 MWh/h during the peak period. Totally this will be

8 • 600 MWh + 8 • 850 MWh + 8 • 600 MWh = 16400 MWh

at the cost of

16400 MWh • 120 kr/MWh = 1 968 000 kr

Altogether with cogeneration the cost will be

604 000 kr + 1 968 000 kr = 2 572 000 kr

This means that the cost of the cogeneration case, 2 572 000 kr, is lower than the cost of the gas turbine case, 2 661 000 kr.

In a more complex power system the decision will be more difficult to survey. A tool for decision support could help the decision maker to make better decision. The first step in developing such a tool is to create some kind of model of the power system. The next chapter deals with mathematical modelling of the power system in such a tool.

The requirements on such a tool are that the modelling should be detailed enough to get realistic schedules and that the computation time should be low. This requirements can of course be in conflict.

3
Problem Formulation and Modelling

In this chapter I describe the mathematical models of the power system I have used. I also describe some other models from the literature. The planning problem will be formulated as an optimization problem. The objective function of the optimization problem will describe the cost of generation in the power system. The constraints and variable limits of the optimization problem will be the physical laws and the legal restrictions in the power system. The variables in this planning problem will be the discharge and spillage in the hydro power plants, the contents in the hydro power reservoirs, the generation in the thermal plants and power exchanges.

3.1 General mathematical problem formulation

Now I will mathematically formulate the power system planning problem from a producer's point of view. First, I will state a general formulation of the problem based on the description of the power system in the previous chapter. This problem formulation will include the so called high level constraints, which couple the power exchanges, the hydro and the thermal subsystems. The aim of power system planning is to maximize the expected benefit of the power system operation subject to several constraints. In the short term planning the problem will be to minimize the cost of generating the contracted load. Therefore the most obvious constraint is the power balance constraint. This constraint guarantees that the sum of produced and purchased energy is equal to the sum of sold energy and firm load. For a power system this is a physical constraint since the generation must be equal to the load according to the law of energy conservation. For single power producer, however, this is more an economic constraint since energy can be purchased or sold if the generation is not equal to the load.

In the following I assume that the spinning reserve is kept in the hydro system. This is a valid assumption if the share of hydro generation is large. In other cases reserve requirements are also kept in the thermal system. There will also be constraints associated with the generation conditions for the different power sources. For a planning period of 24 hours it is appropriate to divide the time into one hour time steps. With one hour time steps, the power system planning can be formulated as the following optimization problem:

$$\min\{F\} = \min_{p_h, p_g, p_e} \{F_h + F_g + F_e\} \tag{3.1}$$

subject to

$$p_h(t) + p_g(t) + p_e(t) = D(t) \tag{3.2}$$

$$r_h(t) \geq R(t) \tag{3.3}$$

$$p_h(t) \in P_h \tag{3.4}$$

$$p_g(t) \in P_g \tag{3.5}$$

$$p_e(t) \in P_e \tag{3.6}$$

$$t = 1, \ldots, T \tag{3.7}$$

where

- F is the objective function.
- F_h is the cost of hydro generation.
- F_g is the cost of thermal generation.
- F_e is the cost of power exchanges.
- $p_h(t)$ is the energy produced in the hydro system during hour t.
- $p_g(t)$ is the energy produced in the thermal system during hour t.
- $p_e(t)$ is the net power exchanges during hour t, positive if energy is purchased, negative otherwise.

- $D(t)$ is the load (including losses) during hour t.
- $r_h(t)$ is the total reserve in the hydro system during hour t.
- $R(t)$ is the minimum reserve requirement during hour t.
- P_h represents the constraints in the hydro system.
- P_g represents the constraints in the thermal system.
- P_e represents the constraints in the power exchanges.
- T is the last hour in the planning period.

As in most cases there is a conflict between request of short computation time and degree of detail in models. There always has to be a compromise between these two requests. In the following parts of this chapter I will present the models I have used in the thesis work together with some other models presented in the literature.

3.2 Hydro power models

In this section I am going to discuss F_h, the cost of hydro generation and P_h, the constraints in the hydro system. I will only give mathematical expressions for models, which I am going to use in the coming chapters.

The operational cost of the hydro system is very low, but the amount of water is limited. The water can be used during the planning period or stored for future use. There are several ways of treating the fact that water is a limited resource in the hydro system. One way is to give a value to the water stored at the end of the planning period. The value will indicate the benefit of using the water in the future instead of using it during the planning period. The value of water stored will then be included in F_h. Another way to model the limits of water is to introduce restrictions on the minimum amount of stored water energy at the end of the planning period. In this case the limits of the water will be included in the set of hydro constraints, P_h.

If there are start-up costs on hydro units and/or penalty functions on the changes of generation in the plants, these will also be included in F_h.

The constraints of the hydro system, P_h, can be divided into hydrological and generation constraints. The hydrological constraints will describe how operation of different plants in one river will affect each other. If water is discharged from one plant it will reach the downstream plant after some delay time. The generation constraints will describe the allowed discharge domain, generation as function of discharge and restrictions/costs for discharge changes.

3.2.1 Hydrological constraints

In the literature there is a broad span in degree of detail in the representation of the hydrological system. All from treating the hydro system as having only one aggregated reservoir [32] to having a detailed representation of the river between the power plants [44]. The models of the first case will only answer the question of how much hydro energy to produce and when to produce. However, they will not calculate the generation in the individual plants. In the case of detailed representation [44], the river is divided into river sections. Each section will be represented by its geometrical form and equations of fluid mechanics. This means that the time step in the model has to be significantly shorter than one hour.

A common way of representing the hydro system which is not as detailed as the second case is to treat the river between the power plants as a transport section with a certain travelling time. This leads to the following equation:

$$\begin{aligned} x(j, t+1) - x(j, t) + u(j, t) - \sum_{m \in M(j)} u(m, t - \tau_{mj}) + s(j, t) \\ - \sum_{m \in M(j)} s(m, t - \tau_{mj}) = w(j, t) \end{aligned} \tag{3.8}$$

where

- $M(j)$ is the set of upstream reservoirs for plant j.
- τ_{mj} is the delay time between plant m and plant j.
- $u(j, t)$ is the discharge from plant j during hour t.
- $x(j, t)$ is the contents of reservoir j in the beginning of hour t.
- $s(j, t)$ is the spillage from plant j during hour t.
- $w(j, t)$ is the natural inflow between plant j and its upstream plant during hour t.

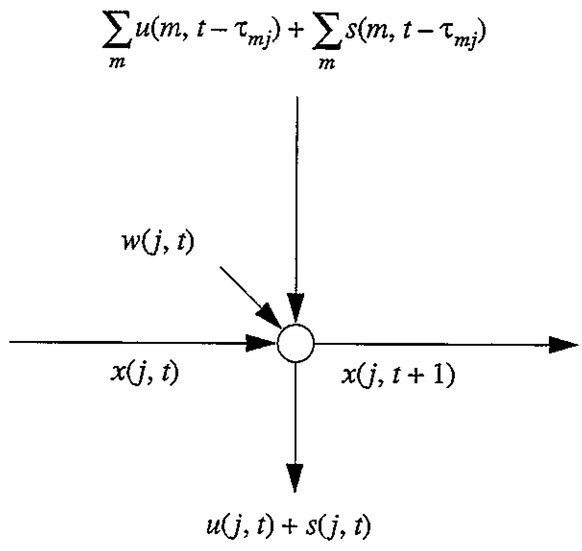


Fig. 3.1 The network structure of the equation describing the reservoir dynamics

If the delay time, τ_{mj}, is or can be approximated to a multiple of whole hours, (3.8) will have network structure, which is shown by Fig. 3.1. The horizontal arcs represent the reservoir contents while vertical arcs represent the discharge and spillage. The small arc into the node is the natural river inflow.

Equations with network structure have some nice characteristics which I will come back to in chapter 5. If the delay time between two plants is not a multiple of a whole hour there will be a problem, since the variables are only defined for whole hours. This can be solved in two ways. One way is to decrease the time step to the least common factor of the delay times. This will keep the network structure but the number of variables will increase. The other way is to let the water be transported to different nodes in the downstream reservoirs [67] and [101]. If the delay time is half an hour, half of the discharge will reach the downstream reservoir during the same hour and the other half will reach the downstream reservoir during the next hour. In this case the number of variables will not increase. However, (3.8) will loose its network structure. Fig. 3.2 shows the network structure in case of one hour delay time between two plants.

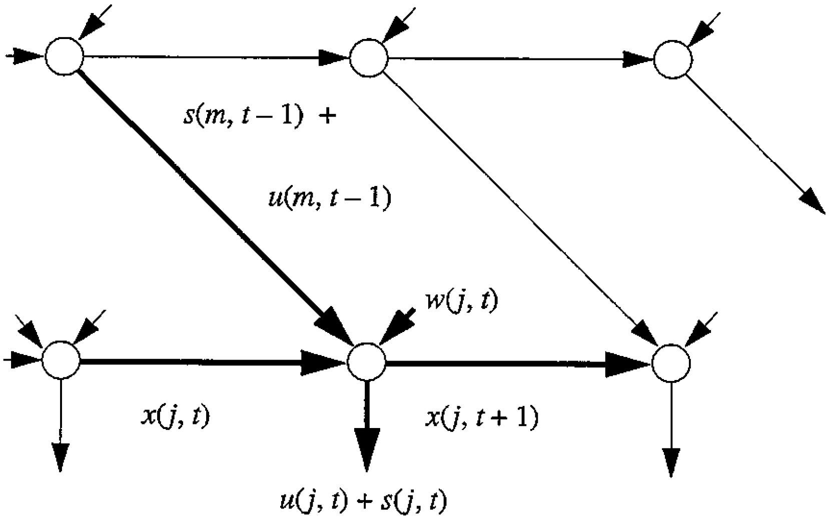


Fig. 3.2 The network structure with one hour delay time between plant m and plant j.

The delay time is in fact an increasing function of the discharge [67]. Taking this into account, (3.8) will be a non-linear constraint and the model will be almost as detailed as in the case with a model of the river section.

It should be mentioned that the river system treated in [44] only has run-of-the river plants. Therefore a model like (3.8) without any reservoir would not be suitable. The reason is that it is very important for a run-of-the river chain to know the exact arrival time of discharge to the next plant, since the water cannot be stored.

The reservoir contents have to be between some, by law, specified levels. Mathematical formulation of the limits of the reservoir contents:

$$0 \le x(j, t) \le \bar{x}(j) \tag{3.9}$$

Where $\bar{x}(j)$ is the maximum reservoir contents. Observe that if $x(j, t)$ is equal to zero this does not necessarily mean that the reservoir is empty, only that the reservoir level cannot be lower according to legal restrictions, i. e. $x(j, t)$ is the net reservoir contents.

There are several approaches to treat the final states of the reservoirs. One of the most common approaches, and the approach I will use, is to give a value to the water which is stored in the reservoirs after the planning period [11], [93] and [101]. The value can be determined by the seasonal planning. In this work the value is called *the water storage value,* ρ_j for reservoir j. This means that the total negative value of the stored water

$$-\sum_{j \in J_h} \rho_j x(j, T+1) \tag{3.10}$$

will be a part of the costs of hydro generation. J_h in (3.10) is the number of hydro plants. Owing to long term planning strategies, there can also be limits on the amount of stored water. This gives:

$$\underline{x}(j, T+1) \leq x(j, T+1) \leq \bar{x}(j, T+1) \tag{3.11}$$

where $\underline{x}(j, T+1)$ and $\bar{x}(j, T+1)$ are the lower and upper limits for the final reservoir contents respectively. Another approach is to *specify the final reservoir contents* [67] and [93]. If the final reservoir contents are specified or if the difference between the bounds in (3.11) is too small, it may not exist a feasible solution to the planning problem. The restrictions for the final reservoir contents can then be treated as a soft constraint [85] and [106], which means that if $x(j, T+1)$ deviates from the specified final reservoir contents and the interval in (3.11) respectively, a penalty cost multiplied by the deviations is added to the objective function. These constraints can be treated as soft constraints. The penalty function is equal to zero if the final reservoir fulfils (3.11). Otherwise the penalty function will be greater then zero, which means that the penalty function can be expressed as

$$\nu_j[\underline{x}(j, T+1) - x(j, T+1)] \text{ if } x(j, T+1) \leq \underline{x}(j, T+1) \tag{3.12}$$

$$0 \text{ if } \underline{x}(j, T+1) \leq x(j, T+1) \leq \bar{x}(j, T+1) \tag{3.13}$$

$$\nu_j[x(j, T+1) - \bar{x}(j, T+1)] \text{ if } x(j, T+1) \geq \bar{x}(j, T+1) \tag{3.14}$$

where ν_j is the penalty term for plant j.

I am going to test both approaches: with and without soft constraints for the final reservoir contents. In both cases the water stored will be given water storage values. In the case with soft constraints I will subtract the penalty cost as described above if the reservoir contents are outside the specified interval.

I will introduce X_{T+1} as the allowed domain for final reservoir contents. In the case without soft constraints X_{T+1} will be (3.11), to ensure that the final reservoir contents never are outside the specified interval. For the case with soft constraints (3.9) will be valid for $t = T+1$. This means that we will have the same bounds for the final reservoir contents as for the other hours. Then the total value of water stored at the end of the planning period will be expressed as a piecewise linear function:

$$-\sum_{j \in J_h} \sum_{i=1}^{3} \sigma_i(j) x_i(j, T+1) \tag{3.15}$$

where

$$x(j, T+1) = \sum_{i=1}^{3} x_i(j, T+1) \tag{3.16}$$

$$\sigma_1(j) = -(\rho_j + \nu_j) \tag{3.17}$$

$$\sigma_2(j) = -\rho_j \tag{3.18}$$

$$\sigma_3(j) = -\rho_j + \nu_j \tag{3.19}$$

$$0 \le x_1(j, T+1) \le \underline{x}(j, T+1) \tag{3.20}$$

$$0 \le x_2(j, T+1) \le [\bar{x}(j, T+1) - \underline{x}(j, T+1)] \tag{3.21}$$

$$0 \le x_3(j, T+1) \le [\bar{x}(j) - \bar{x}(j, T+1)] \tag{3.22}$$

In the case without soft constraints ν_j will be equal to zero, $x_1(j, T+1)$ will be equal to $\underline{x}(j, T+1)$ and $x_3(j, T+1)$ will be equal to zero.

A third approach which is seldom used in the short-term planning, but could be applied to seasonal planning is to *specify the stored energy in the hydro system* [93]. I will not use this criterion in this thesis.

If there is a delay time in water travel to the downstream reservoir some of the water will be in the river between the hydro plants at the end of the planning period. Since this water will reach the downstream reservoir in a few hours it is appropriate to assume that the water will have the same value as the water stored in the downstream reservoir. Then the following term should be added to the objective function:

$$-\sum_{j \in J_h} \sum_{t > T - \tau_{j,d_j}} \rho_{d_j}[u(j, t) + s(j, t)] \tag{3.23}$$

Discharge, spillage and reservoir contents are given as *hour equivalents (HE)*. One HE is the volume of 1 m^3/s water discharged during one hour, which means 3600 m^3.

As I mentioned in the previous chapter, there are often different power producers owning hydro plants in the same river. In Sweden there are so-called regulation companies, which are given the task of taking the suggestions of all producers into consideration. The regulation company determines a rough generation schedule for one specific plant in the river. This is done in the following way. First all producers determine how much they want to discharge in their plants. They can then calculate how their schedule would affect the discharge in the plant scheduled by the regulation company. This will be the desired discharge in this plant for each producer. Then the regulation company calculates the weighted average discharge in the specific plant. The weight for each company is equal to their percentage of the total generation capacity in the river. The schedule from the regulation company is given as average discharges for six hour periods. If the discharge calculated by the regulation company is lower than the desired discharge for one producer, he will receive power from a producer with higher discharge than desired. These exchanges of power are registered, in order to ensure that the net exchange for each producer is equal to zero on seasonal basis.

When the producers know the schedule of the specific plant, they can plan the generation of their own plants taking the known schedule into account. The producer owning the plant with the pre-defined schedule follows his 6 h-average values of schedule. The producer having the plant upstream from this plant can treat it as a reservoir with known outflow. This will prevent that the reservoir contents of the plant with the given schedule do not violate the bounds of reservoir contents. The owner of the plant downstream from the plant with specified schedule can treat the scheduled discharge as known inflow. However, it is still possible that the owners of the plants in the river do not have full information of the discharge from up- and downstream plants they do not own. In practice the producers schedule their plants based on the known information and then try to adapt their own generation according to other producers schedules.

This means that the model of the hydrological constraints above still hold. The power producer can use the model when making the proposed schedule, which will be submitted to the regulation company. In the same way as planning without support of an optimization tool, the power producer has to make some assumptions of how other producers in the river will operate their plants. In the next step, when the schedule from the regulation company is known, the producer can treat the plant scheduled by the regulation company as fixed discharge. Then it might be the case that the schedules have to be adjusted again. The company can use the model to create new schedules after receiving new information about the other producers schedules.

For completeness I should mention that a producer has the right to change his suggestions about the discharge in the plant scheduled by the regulation company with a notice three hour before. I should also mention that the owner of the plant scheduled by the regulation company does not have to follow the schedule strictly. The owner the of pre-scheduled plant can operate it with some marginal to the schedule.

After the discussion above one can ask two questions. First, is this the best way to coordinate the planning in the river? Secondly, given the rules above, could it be possible to determine the bids to the regulation company and the generation of different companies by game theory? I will not try to answer the first question since it is not in the scope of this thesis. However, the kind of optimization model I have described above will still be valid for generation planning even if the coordination of the planning in river is performed in some other way. Assume for example that the producers in the same river decide to start a scheduling company owned by the producers together. The company can then use a model of this kind to plan the generation in the river.

For the question about game theory, I am not sure that it is possible to find a stable equilibrium. It can also be the case that it is difficult model corporation. One example of corporation is when one producer may satisfy another producers suggestions in one part of the river if the other producer satisfies the firsts suggestions in another part of the river or even in another river. The game will be repeated each day, which means that if there are stable points of equilibrium for a single game, we might get other solutions for a repeated game.

3.2.2 Generation constraints

The generation in the hydro system will be the sum of the generation of each hydro power plant. As I described in the previous chapter the generation is a function of the discharge and the head. The head is a function of the upstream and downstream reservoir contents. An increase of the contents in the upper reservoir will increase the head, and in the lower reservoir it will decrease the head. This means:

$$p_h(t) = \sum_{j \in J_h} p_h(j, t) \tag{3.24}$$

$$p_h(j, t) = f(u(j, t), x(j, t), x(d_j, t)) \tag{3.25}$$

$$u(j, t) \in U(j) \tag{3.26}$$

where

- $p_h(j, t)$ is the generation in hydro plant j during hour t.
- $f(u(j, t), x(j, t), x(d_j, t))$ is the generation in a hydro plant as a function of discharge, upstream reservoir contents and downstream reservoir contents, the so-called generation characteristic.
- d_j is the downstream reservoir of plant j.
- $U(j)$ is the allowed discharge domain for plant j.

The generation as function of discharge, upstream and downstream reservoir levels is often available as tables. From these tables it is desirable to formulate a mathematical expression. I will now characterize models used in the literature of hydro scheduling in four aspects:

- allowed discharge domain,
- model of generation as function of discharge,
- treatment of head dependence and
- treatment of discharge changes.

First I will explain the meaning of the above mentioned characteristics and I will also give a short survey of the most common ways to model hydro power in the above mentioned aspects. After that I will give some examples of how different models in the literature combine different representations of the aspects. I have divided the models into four groups depending on their mathematical characteristics. These groups are: continuous linear, continuous non-linear, mixed integer-linear and mixed integer non-linear. When I go through the different models in the literature, I will also mathematically formulate the models I have used in this thesis.

Allowed discharge domain

The simplest and maybe the most common way is to allow operation in the whole domain from minimum to maximum allowed discharge. In this case there is no need for integer variables for the model of the allowed discharge domain. However, as I mentioned in the previous chapter there will be local best-efficiency points for each combination of units. Operation far from these points can lead to significantly lower efficiency and in some cases risk of cavitation. This means that it could be more accurate to only include parts of the interval from minimum to maximum discharge in the allowed discharge domain. Such a model requires some kind of integer representation, unless there is a legally restricted minimum discharge.

Model of generation as function of discharge

The generation as function of the discharge is a non-concave function, see Fig. 2.4. Several models are concave approximations of this non-concave relationship. The simplest model for the generation characteristic is a *linear model,* where the generation is proportional to the discharge. A common model of the generation characteristic is *piecewise linear approximation* of the discharge-generation curve, see Fig. 3.3 a. Chosen as breakpoints are zero flow, the best-efficiency point for the unit combination giving a concave function and the point of maximum discharge. Another model that has been tested in the literature is a *non-linear approximation* of the generation characteristic, see Fig. 3.3 b, with some kind of minimization of the average error compared with the real function. All linear, piecewise linear and non-linear models only need continuous variables. Therefore they can only be used together with a discharge domain which includes the whole interval between minimum and maximum discharge. Models with *integer representation* of the allowed discharge domain can be more accurate in modelling of the generation as function of the discharge in points where discharge is allowed. Fig. 3.3 c shows a model with mixed integer-linear representation of allowed discharge domain and the generation as function of the discharge. In this case operation is allowed at minimum discharge, at the local best-efficiency points and at the continuous part from local best-efficiency point with the highest discharge to maximum allowed discharge.

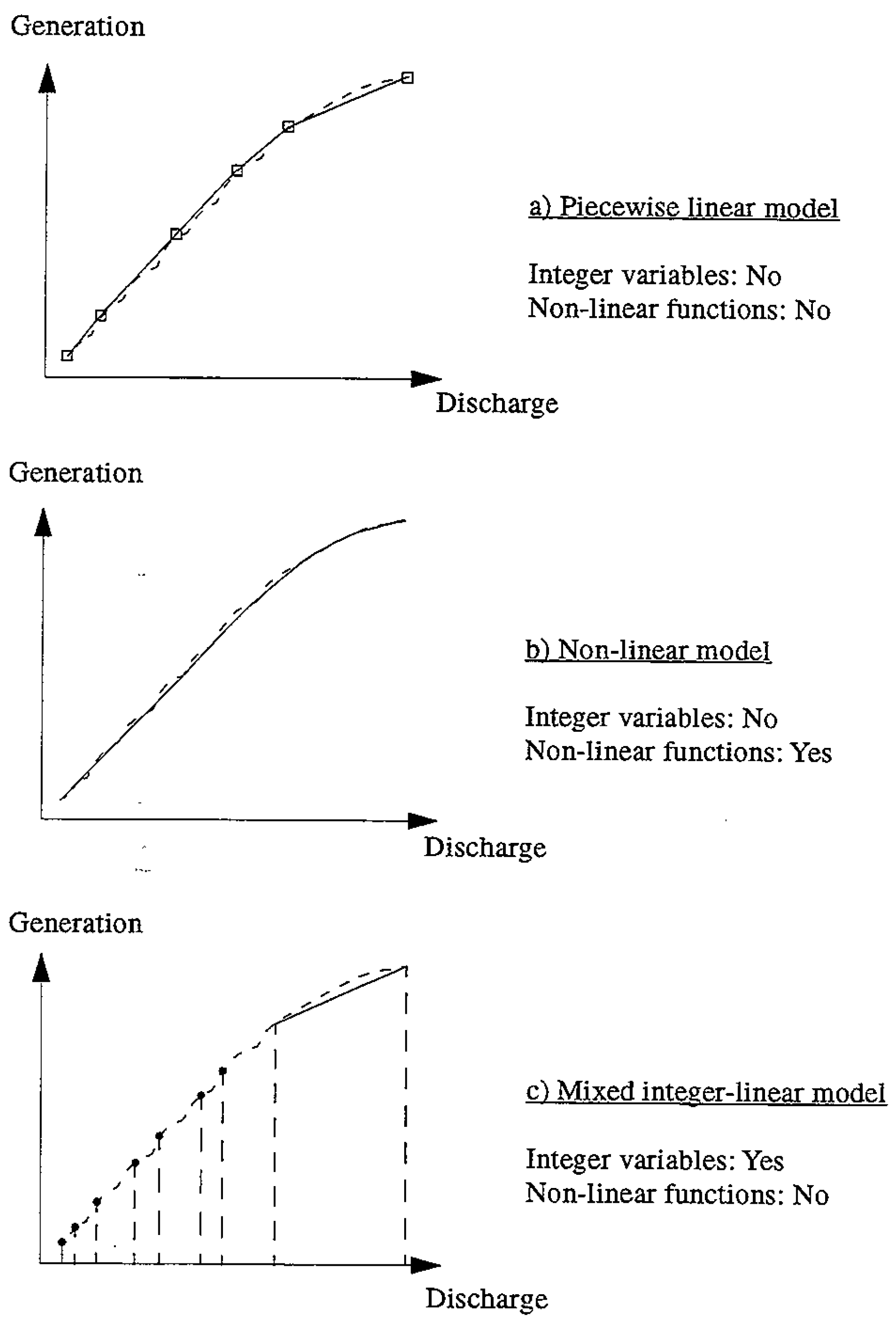


Fig. 3.3 Models of generation as function of discharge for the plant in Fig. 2.4.

Treatment of head dependence

When the hydro plants are dispatched, the reservoirs will be at different levels for different hours. This means that a plant will not have the same head during the planning period. For the treatment of head dependence I will only separate between taking the head dependence into consideration and not doing it. If the head dependence is considered the problem will be non-linear.

Treatment of discharge changes

The treatment of discharge changes can be divided into five groups: no consideration of discharge changes, restrictions on discharge changes, costs of discharge changes, restrictions on starts and stops of units and cost for start of units. The characteristics of the groups can of course be combined in various ways. The first three groups do not need integer representation whereas the last two ones do.

In the following I will describe a few examples of models with different combination of allowed discharge domain, model of generation as function of discharge, treatment of head dependence and treatment of discharge changes. This will lead to the fact that the models are either continuous or mixed integer and that the models include either non-linear functions or not. Table 3.1 shows a list of the different kinds of models in the literature. As I mentioned earlier I will mathematically formulate the models I have used in this thesis.

Continuous linear models

The simplest model is to approximate the generation to a linear function of the discharge. One advantage is that it is possible to use linear programming [53] which is a fast method. The disadvantages with a linear model are, however, several. First, it is possible that the schedules often contain operation points far away from the local best-efficiency points, which means that the plant operates on a point with low efficiency. Secondly, it is not possible to consider constraints or costs in start-ups and shutdowns accurately if one uses linear programming. The third disadvantage is that the linear programming cannot take the head dependence into consideration. So, if one wants to model head dependence it is necessary to do this outside the linear programming. The final disadvantage with the linear model is that it is not possible to consider forbidden discharge intervals. This means that the allowed discharge domain always will be between minimum and maximum allowed dis-

Table 3.1: Examples of models in the literature

Kind of models	Allowed discharge domain	Generation as function of discharge	Head dependence	Discharge changes
Continuous linear	From minimum to maximum discharge levels	Linear Piecewise linear	Not treated Outside the model	Not treated Cost of discharge changes
Continuous non-linear	From minimum to maximum discharge levels	Piecewise linear Non-linear	In the model	Not in the literature but possible
Mixed integer linear	Specific points or intervals	Linear or piecewise linear in the specific intervals	Not treated Outside the model	Not treated Start-up cost Start and stop restrictions
Mixed integer non-linear	Specific points or intervals	Non-linear in the specific intervals	In the model	Not treated Start-up cost Start and stop restrictions

charge. [32] and [115] use linear models and do not include any head dependence and no treatment of discharge changes.

A common model of the generation characteristic is *piecewise linear approximation* of the discharge-generation curve, see Fig. 3.3 a. The breakpoints for the piecewise linear function are the minimum discharge, the local best-efficiency points and the maximum discharge. Observe that points making the piecewise linear curve non-concave must be excluded. The advantage of the piecewise linear model compared with the linear model is that the plant more often will operate on the local best-efficiency points, since the solution of this linear programming problem will have most of the discharge variables at their minimum or maximum bounds. However, the model will still give schedules with some operation points far from the local best-efficiency points and maybe in forbidden intervals. If the discharge is scheduled between the breakpoints the generation will be overestimated if the discharge is less than the discharge of the local best-efficiency point with highest flow: Otherwise, if the discharge is above the local best-efficiency point with highest flow the generation will be underestimated. Without head dependence this model will be continuous and linear [36], [37] and [93]. In [85] the model is piecewise linear for each unit in the plant. The model also includes linearized equations for the losses in the tunnels. Normally these models do not have any restrictions on discharge changes. However, [18] and [59] uses the piecewise linear model with extra costs for discharge changes.

Now to the mathematical formulation of the model I have used. The generation in station j, hour t for the piecewise linear model can be written as:

$$p_{\text{pwl}}(j, t) = \sum_{i=1}^{I_j} \Gamma_i(j) u_i(j, t) \tag{3.27}$$

$$0 \le u_i(j, t) \le \bar{u}_i(j) \tag{3.28}$$

where

- $p_{\text{pwl}}(j, t)$ is the piecewise model of generation as function of the discharge for plant j.
- $\Gamma_i(j)$ is the slope of segment i in Fig. 3.3 a.
- u_i is the part of the discharge for segment i.
- I_j is the number of segments in Fig. 3.3 a.

The total discharge in station j during hour t will be

$$u(j, t) = \sum_{i=1}^{I_j} u_i(j, t) \tag{3.29}$$

Non-linear continuous models

The first kind of non-linear models is the piecewise linear models with head dependence [23], [27] and [67]. Another model that has been tried is a *non-linear approximation* of the generation characteristic see Fig. 3.3 b. The advantage of a non-linear model is that the average error compared with the real characteristic is less than in the piecewise linear model. However, the non-linear model does not agree with the characteristics in the important local best-efficiency points, which the piecewise linear model does. This means that the plant more often will operate far from the local best-efficiency points. If one uses a non-linear model one has to apply a non-linear optimization technique, which is more time consuming than linear programming. One advantage of non-linear programming is that it is possible to include the head dependence directly in the model. It is also possible to use penalty functions for the change of discharge. Still, there is no guarantee to obtain schedules with operation only on points with good efficiency. [13] is an example of a non-linear approximation without head dependence. [9], [10], [27], [28], [35], [77], [78] and [89] are examples of such models with head dependence. In continuous non-linear models the head dependence is considered either as some explicit function (linear or polynomial) of the head or reservoir contents or as interpolation in tables. The model for head dependence which I have used in this thesis is the same as in [9], [10], [23] and [93].

$$\begin{aligned} f(u(j,t), x(j,t), x(d_j,t)) &= p_0(u(j,t)) \\ &+ [\alpha(j)x(j,t) - \beta(j)x(d_j,t)]u(j,t) \end{aligned} \tag{3.30}$$

where

- $p_0(u(j,t))$ is the generation as function of the discharge for plant j. In (3.27) $p_0(u(j,t))$ is modelled as a piecewise linear function, $p_{\mathrm{pwl}}(j,t)$.
- $\alpha(j)$ is the head correction factor for the upstream reservoir.
- $\beta(j)$ is the head correction factor for the downstream reservoir. If there is no downstream reservoir $\beta(j)$ will be set to zero.

In (3.27) $\alpha(j)$ and $\beta(j)$ are assumed to be zero. If we want to use head dependence in (3.27) we have to update Γ according to the reservoir contents in the solution of the planning problem and run a new optimization.

How to calculate $p_0(u(j, t))$, $\alpha(j)$ and $\beta(j)$ based on data tables for generation as function of discharge for different reservoir contents is described in [64] and in Appendix A.

Mixed integer-linear models

To overcome the problem with operation points far away from the local best-efficiency points, mixed integer models have been suggested. The advantages of mixed integer models are first of all, that the plant will only allow operation on the local best-efficiency points and secondly, that it is straightforward to include start-up and shutdown costs or restrictions of hydro units. Models with integer representation also avoid operation in forbidden discharge intervals. In [7], [11] and [105] the discharge is allowed only at a finite number of points, mainly the local best-efficiency points. In [64], [68], [69], [70], [71] and [73] the operation is also allowed between the local best-efficiency point with the highest flow and the maximum generation point. These models have an advantage compared with models only allowing discharges at a finite number of values. The advantage is that the model does not force the discharge to a point with a lower efficiency than necessary, when the plants operate on the part between the local best-efficiency point with the highest flow and the maximum flow point. Assume for example that it is optimal to operate somewhere between the local best-efficiency point with highest flow and the maximum flow point. If one uses an integer model one has to choose one of these points. This means that the model chooses either the upper point with lower efficiency than necessary or the lower point. If the model chooses the lower point, the model will either force another plant to go up to a point with lower efficiency than necessary or we will have to purchase energy to fulfil (3.2). An example of the mixed integer-linear model with discharge allowed at all points between the local best-efficiency point with the highest flow and the maximum discharge is shown in Fig. 3.3 c. [15] has integer representation for the on/off-line status of the plant and piecewise linear approximation of the generation as function of discharge. The model in [106] has piecewise linear approximation of the generation characteristic for each unit as in [85] with the difference that [106] has an integer representation for commitment status of the unit. In [24] a mixed integer model for a pump storage plant is presented. Pumping is only allowed at discrete points whereas generation is modelled as a piecewise linear function of the discharge.

Now to the mathematical modelling of the mixed linear model in Fig. 3.3 c, which I have used:

$$p_{\mathrm{mi}}(j,t) = \sum_{k=1}^{K_j} p_k(j) z_k(j,t) + \Gamma_{K_j+1}(j)[u_{K_j+1}(j,t) - \underline{u}_{K_j+1}(j)] \quad (3.31)$$

the discharge can be written as

$$u(j,t) = \sum_{k=1}^{K_j} u_k(j) z_k(j,t) + u_{K_j+1}(j,t) \quad (3.32)$$

the following constraints for the binary variables must be fulfilled

$$z_k(j,t) \geq z_{k+1}(j,t) \quad (3.33)$$

$$0 \leq u_{K_j+1}(j,t) \leq \bar{u}_{K_j+1}(j) z_{K_j}(j,t) \quad (3.34)$$

$$z_k(j,t) \in \{0, 1\} \quad (3.35)$$

where

- $p_{\mathrm{mi}}(j,t)$ is the mixed integer model of the generation as function of the discharge for plant j during hour t.
- $\sum_{k=1}^{k^*} p_k(j)$ is the generation at the best-efficiency point k^*.
- $\sum_{k=1}^{k^*} u_k(j)$ is the discharge at the best-efficiency point k^*.
- $z_k(j,t) = 1$ if the plant operates at point k or higher, 0 otherwise.
- $\Gamma_{K_j+1}(j)$ is the slope on the continuous part.
- $\bar{u}_{K_j+1}(j)$ is the upper limit of the discharge on the continuous part.
- K_j is the number of local best-efficiency points.

There are many ways of treating discharge changes or start-ups/shut-downs in the mixed integer linear models. For example in [11] a minimum time between changes of the discharge is required. [24] has a cost for going from one state to another in the pump-storage plant. In [64] the minimum up and down times are two hours for hydro units. [64], [68], [69], [70], [71] and [73] assign a start-up cost to start-ups of hydro units. Another approach is to not allow start-ups in low load periods [105]. Finally, in [106] the number of start-ups for a unit is limited.

The start-up cost can be formulated as

$$\chi(j, t) = c_{\text{start}, j}(Z(j, t), Z(j, t-1)) \tag{3.36}$$

where

- $\chi(j, t)$ is the total start-up cost for plant j during hour t.
- $c_{\text{start}, j}(Z(j, t), Z(j, t-1))$ is the start-up cost for moving from state $Z(j, t-1)$ to state $Z(j, t)$
- $Z(j, t) = [z_1(j, t), \ldots, z_{K_j}(j, t)]$.

To use the mixed integer linear model (3.31)-(3.35) as for run-of-the river plants and set the upper bound of the reservoir equal to zero would not result in a suitable model. The reason is that the total inflow to the run-of-the river plant will be equal to the discharge from the upstream plant plus the natural inflow. If discharges only are allowed at discrete points for both plants, one have to spill from the run-of-the river plant if the total inflow does not match one of the allowed points. This is not a rational way of operating a hydro plant. In this case it is better to treat these two plants as one. The total generation from the plants will be a function of the discharge from the upstream plant:

$$p_{\text{tot}}(u(\text{ups}, t)) = p_{\text{ups}}(u(\text{ups}, t)) + p_{\text{ror}}(u(\text{ups}, t) + w(\text{ror}, t)) \tag{3.37}$$

where

- $p_{\text{ups}}(u(\text{ups}, t))$ is the generation of the upstream plant
- $p_{\text{ror}}(u(\text{ups}, t) + w(\text{ror}, t))$ is the generation of the run-of-the river plant.

$p_{\text{tot}}(u(\text{ups}, t))$ can then be plotted as a function of $u(\text{ups}, t)$ in the same way as the dashed graphs in Fig. 3.3. From this curve it is possible to calculate the points with best-efficiency and only allow discharge in these points. Note, that if there is a delay time between the upstream plant and the run-of-the river plant, $p_{\text{up}}(u(\text{ups}, t))$ and $p_{\text{ror}}(u(\text{ups}, t) + w(\text{ror}, t + \tau_{\text{ups, ror}}))$ will be produced at different hours.

Mixed integer-non-linear models

One example of a mixed integer-non-linear model is [101] which is the same model as the model in Fig. 3.3 c with head dependence by interpolation in tables. There is also a penalty cost for discharge changes. [72] is almost the same but with a polynomial approximation of the continuous part and the head dependence as in (3.30) and hydro unit start-up costs. [40] has a non-linear approximation of the generation characteristic for each unit. There is no head dependence in this case since the model only includes run-of-the river plants. [96] has a non-linear approximation with head dependence for each unit combination. This model also includes forbidden discharge intervals. [51] has the same generation characteristic with start-up costs and minimum up and down-times for hydro units.

In my work I use a mixed integer model with non-linear head dependence. However I will not solve a non-linear mixed-integer problem. Instead I will use constant heads and update these heads in different steps.

Section 3.2 dealt with hydro power models. I have discussed models for the costs of hydro generation F_h and the operation constraints for the hydro system P_h. I have also mathematically formulated the two different models I use in the thesis, one piecewise linear and one mixed-integer linear. In the piecewise linear model the cost is the value of used water. In the mixed integer model we add a start-up cost for hydro power units. In both models there are restrictions on water balance, discharge and reservoir levels. In the mixed integer model there is also restrictions that the discharge only is allowed at specific points. As I just mentioned above I gradually update the head dependence.

3.3 Thermal system models

In this section models for F_g, the cost of thermal generation and P_g, the constraints in the thermal system will be described. I will assume that nuclear plants are used as base load units and that they are scheduled at their maximum capacity or some other specified level. The total thermal generation is the sum of the generation in each plant.

$$p_g(t) = \sum_{j \in J_g} p_g(j, t) \tag{3.38}$$

where

- $p_g(j, t)$ is the generation of plant j during hour t.
- J_g is the number of thermal plants.

As in the hydro section I will describe the models I have used in the thesis together with some other models in the literature. I have characterized the thermal models in the following aspects:

- allowed generation domain.
- cost as function of generation.
- treatment of start-ups.
- restrictions for changing generation in committed plants.

Following the structure of the hydro section I will explain the aspects above and give examples of how they are treated in the literature. After that I will go through different models in the literature and their treatment of the above mentioned aspects. I will have the same group of models as in the hydro section: continuous linear, continuous non-linear, mixed integer-linear and mixed integer non-linear. I will also mathematically formulate the models I have used in the thesis.

Allowed generation domain

Normally there are lower limits for on-line generation in the thermal plants. I will separate between models with and without lower limits for on-line generation.

Cost as function of generation

The cost as function of the generation can either be linear or non-linear. Fig. 3.4 a and b show examples of linear and non-linear models of the cost curve respectively.

Treatment of start-up

In the literature there are three kinds of models for start-up costs in thermal plants: warm and cold start-up costs, piecewise linear, and exponential. The three models are shown in Fig. 3.5.

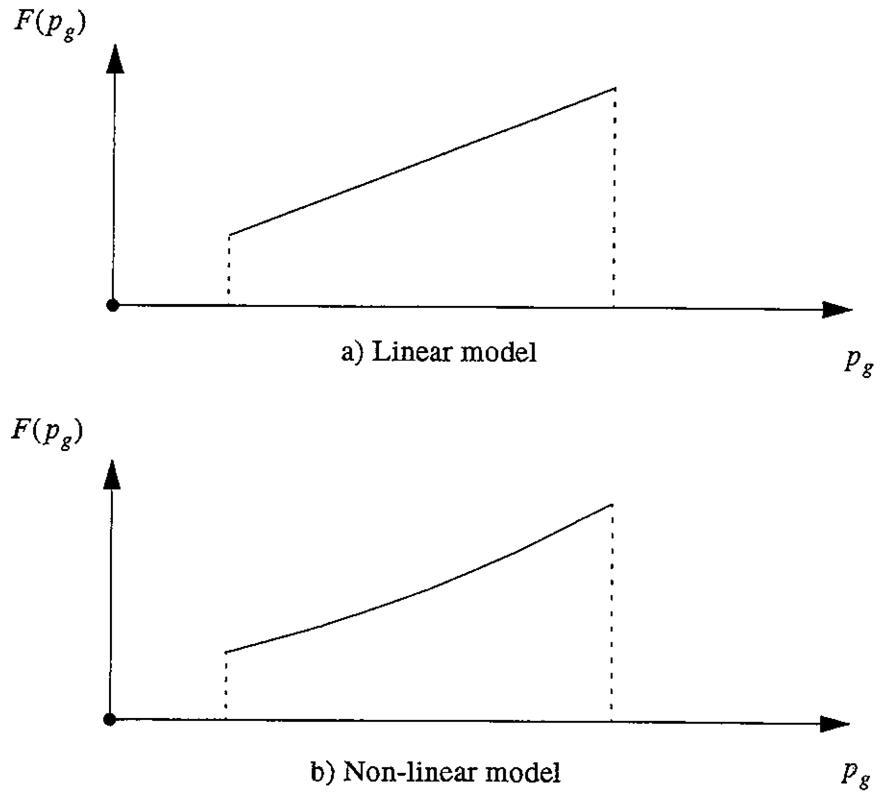


Fig. 3.4 The generation cost as function of the generation

There might also be minimum up and down times for the thermal plants. According to my experience from the test system I have been working on there is no need for such restrictions, since the start-up cost will prevent plants from starting and stopping for only a few hours. It should be mentioned that this conclusion is only valid for the kind of system I have been studying. This system is hydro dominated. For a pure thermal system or a thermal dominated system restrictions on minimum up and down times may be needed.

Restrictions for changing generation in committed plants

Because of technical constraints for the thermal units, there might be restrictions on how much the generation can be changed per minute. These restrictions are called ramp rate restrictions. The maximum change of generation per minute is usually lower just after the plant has been started. Here I will only separate between taking the ramp rate restrictions into account or not.

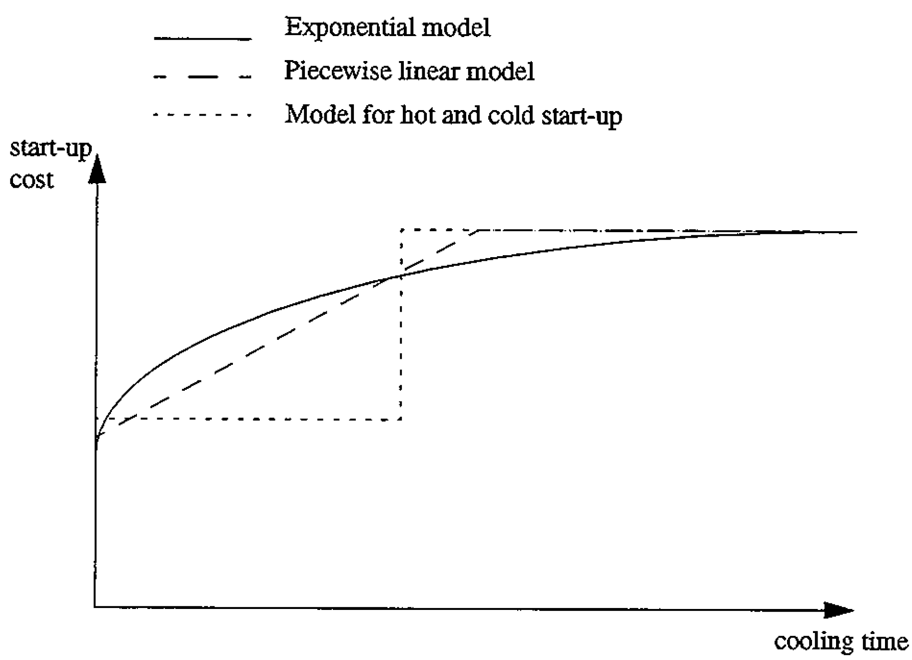


Fig. 3.5 The three different models depending on the cooling time

After this characterization of the thermal models I will go more into details and describe a few models in literature. In the same way as for the hydro models I will separate the thermal models into the four groups continuous linear, continuous non-linear, mixed integer-linear and mixed integer-non-linear. I will also mathematically formulate the models I use in the thesis. Table 3.2 shows a list of how the different models are treated in the literature.

Continuous linear models

The simplest thermal system model is a cost function [9]. Each plant will be represented by upper and lower limits of generation and a linear or piecewise linear cost function. In this case F_g will be a convex, not necessarily everywhere differentiable, function, see Fig. 3.6.

This kind of models can for example be used if the commitment of the thermal system is known. This is normally called EDC (economic dispatch). The slope of the segments in Fig. 3.6 will be equal to the variable cost of the corresponding plant. The minimum generation can then be subtracted from the load and only the difference between maximum and minimum generation is included in the curve. If we want to have restrictions on ramp rates, the cost

Table 3.2: Examples of thermal models in the literature

Kind of models	Allowed operation domain	Cost as function of generation	Treatment of start-up	Generation changes
Continuous linear	Commitment known: From minimum to maximum generation levels Otherwise: From zero to maximum generation	Linear	Not treated	Not treated Treated in dynamic EDC
Continuous non-linear	Commitment known: From minimum to maximum generation levels Otherwise: From zero to maximum generation	Non-linear	Not treated	Not treated Treated in dynamic EDC
Mixed integer linear	From minimum to maximum generation levels and zero	Linear	Start-up cost Minimum up and down times	Not treated Ramp rate restrictions
Mixed integer non-linear	Specific points or intervals	Non-linear	Start-up cost Minimum up and down times	Not treated Ramp rate restrictions

curves will be dependent on each other, and most of the simplicity of the cost curve approach is lost. If the commitment is known and we have constraints coupling the thermal generation from one hour to the next, it is called dynamic EDC.

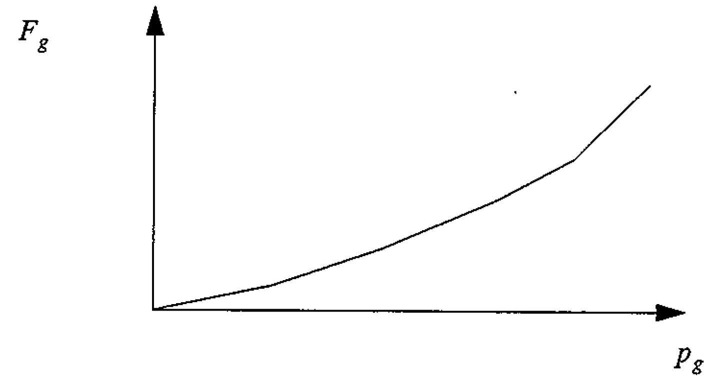


Fig. 3.6 Thermal system modelled as a cost function

If this model is used for the commitment of the thermal plants it not possible to have a lower limit for the generation, and the cost function for each plant must be simplified so that the cost is directly proportional to the generation.

Continuous non-linear models

In the continuous non-linear case the cost function is non-linear instead of linear. [78] for example uses a quadratic cost function for each thermal plant. In this case it is assumed that the commitment of the plants is already known.

Mixed integer linear models

There are also more detailed models which consider the on/off-line characteristic of the thermal plants. Modelling the on/off-line characteristic needs integer variables. Mixed integer linear models mean linear models of the variable cost as function of the generation [36], [60] and [69]. The upper graph in Fig. 3.4 shows the linear model of the cost function. The linear model of the cost function can be written as:

$$g(p_g(j,t)) = a_0(j) + a_1(j)p_g(j,t) \tag{3.39}$$

$$\underline{p}(j)y(j,t) \le p_g(j,t) \le \bar{p}(j)y(j,t) \tag{3.40}$$

$$y(j,t) \in \{0, 1\} \tag{3.41}$$

where

- $g(p_g(j, t))$ is the generation cost for plant j during hour t.
- $a_0(j)$ is the fixed cost for operation of plant j.
- $a_1(j)$ is the marginal cost for operation of plant j.
- $\underline{p}(j)$ is the minimum generation of plant j.
- $\bar{p}(j)$ is the maximum generation of plant j.
- $y(j, t)$ is equal to one if plant j is on-line during hour t and otherwise equal to zero.

The exponential model of the start-up cost can be formulated as:

$$S(j, t) = b_0(j) + b_1(j)(1 - e^{-H/\Upsilon}) \tag{3.42}$$

$$y(j, t) - y(j, t-1) - v(j, t) \leq 0 \tag{3.43}$$

$$v(j, t) \in \{0, 1\} \tag{3.44}$$

- $v(j, t)$ is equal to one if plant j is started in the beginning of hour t and otherwise equal to zero.
- $S(j, t)$ is the start-up cost if plant j is started in the beginning of hour t.
- $b_0(j)$ is the cost of a warm start-up for plant j.
- $b_0(j) + b_1(j)$ is the cost of a cold start-up for plant j.
- H is the time the plant has been off-line.
- Υ is the time constant for cooling.

It can be noted that (3.42) is a non-linear function of the cooling time. This is not a problem since the model is time discrete and the start-up cost will only be calculated for discrete time values. The total start-up cost will be:

$$\sum_{j \in J_g} \sum_{t=1}^{T} S(j, t) v(j, t) \tag{3.45}$$

Mixed integer-non-linear models

For mixed integer-non-linear models the cost as function of the generation is a non-linear function [17], [46], [55], [89] and [100], see Fig. 3.4 b. With a quadratic approximation of the cost curve (3.39) is changed to

$$g(p_g(j,t)) = a_0(j) + a_1(j)p_g(j,t) + a_2(j)p_g(j,t)^2 \tag{3.46}$$

where $a_0(j)$, $a_1(j)$ and $a_2(j)$ now are the coefficients in polynomial approximation of the generation cost for plant j.

In [100], for example, there are also constraints for ramp-rates, which describe how fast the generation can be changed in a thermal plant. These can be formulated as:

$$-\delta_{\text{down}}(j) \le p_g(j,t) - p_g(j,t-1) \le \delta_{\text{up}}(j) \tag{3.47}$$

I will sum up this section about thermal power models with the models I have used in the thesis. The cost function, F_g, has to include start-up cost and generation cost. For the start-up cost I have used the exponential model (3.42). For the generation cost I have used the quadratic model (3.46) if the plant is not equipped with a NO_X-cleaner, see Fig. 2.5. If the plant is equipped with a NO_X-cleaner, the cost function will be very close to (3.39). However, there can be problems in the computation of the solution of the problem with this model, owing to oscillations. In order do avoid these oscillations in the solution procedure, I will use a non-linear approximation of the curve [32]. This means that the cost curve is quadratic (3.46) but still matches very close to the linear curve.

The feasible domain, P_g, for the thermal generation is restricted by the minimum and maximum generation limits (3.40) and the ramp rate constraints (3.47). The maximum generation limit is lower during the first hour after the start-up since the generator is cold. The generation limits can be formulated as follows:

$$\underline{p}(j)y(j,t) \le p_g(j,t) \le \bar{p}(j)y(j,t) + [\bar{p}_{\text{cold}}(j) - \bar{p}(j)]v(j,t) \tag{3.48}$$

where $\bar{p}_{\text{cold}}(j)$ is the maximum generation of plant j during the first hour after the start-up.

3.4 Models for power exchanges

This section gives the mathematical expressions for the cost of power exchanges, F_e, and the restrictions, P_e. The models for power exchanges are depending on the contract. If the contract is designed as a fixed amount of power, which will be purchased and sold, the power exchanged will be subtracted from or added to the load. If the contract is designed as prices for different amounts of purchase and sale the power exchanges can be modelled as a price function. This means that the exchanges can be treated in the same way as the thermal power but without consideration of start-up/shut-down costs/restrictions, see Fig. 3.7. The piecewise linear model of such a contract will be

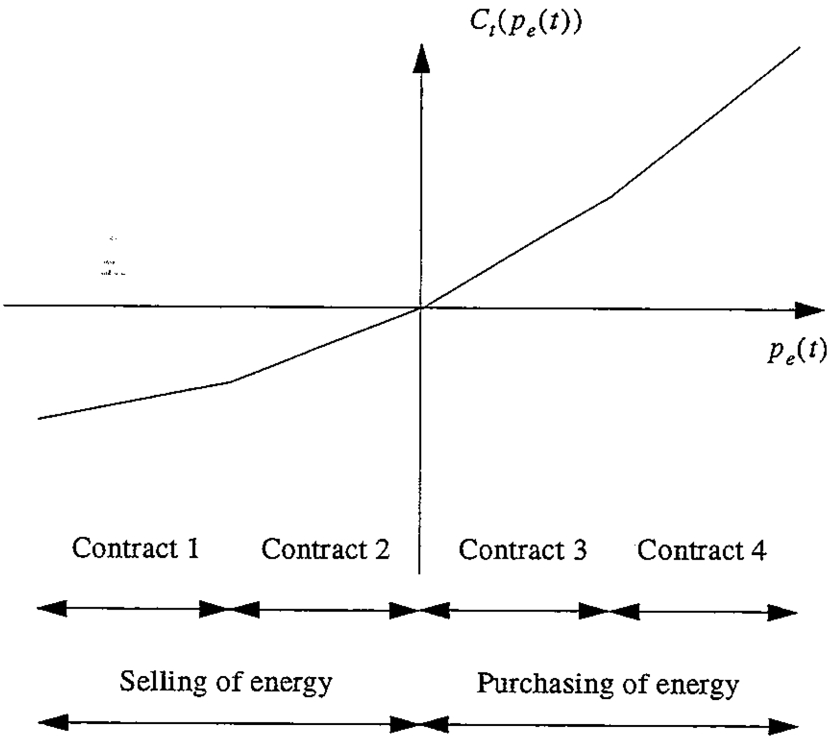


Fig. 3.7 The power exchange cost function. The price for each contract is equal to the slope of the cost function.

$$C_t(p_e(t)) = \sum_{j \in J_e} \gamma_j(t) p_e(j, t) \tag{3.49}$$

$$p_e(t) = \sum_{j \in J_e} p_e(j, t) \tag{3.50}$$

$$\underline{p}_e(j, t) \le p_e(j, t) \le \bar{p}_e(j, t) \tag{3.51}$$

where

- $C_t(p_e(t))$ is the cost of power exchanges during hour t.
- $\gamma_j(t)$ is the price j during hour t.
- $p_e(j, t)$ is the power exchanges according to price j during hour t.
- $\bar{p}_e(j, t)$ is the upper bound for $p_e(j, t)$. $\bar{p}_e(j, t) = 0$ if $\gamma_j(t)$ is a selling price and otherwise positive.
- $\underline{p}_e(j, t)$ is the lower bound for $p_e(j, t)$. $\underline{p}_e(j, t) = 0$ if $\gamma_j(t)$ is a purchase price and otherwise negative.
- J_e is the number of exchange prices.

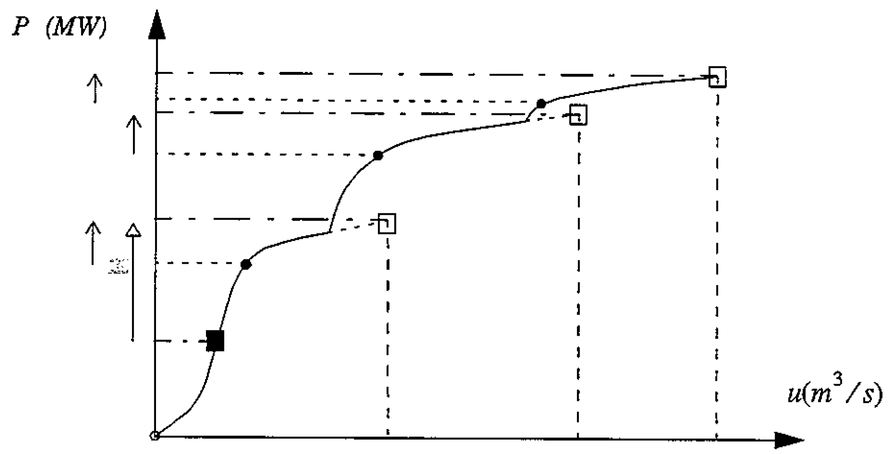
The price curve in Fig. 3.7 can be a model of the spot price if the producer is large enough to affect the spot price. If the producer wants to buy more the price will rise. Otherwise, if the producer wants to sell more the price will fall.

There are no limitations in the design of contracts and there are many types of contracts treated in the literature [39] and [46].

3.5 Reserve requirements modelling

In power systems with a large share of hydro, these reserve margins are kept in the hydro system, since the hydro plants are easy to re-dispatch, as I described in chapter 2. In cases where part of the load is lost owing to loss of load, hydro plants can be shut down to keep the balance between generation and load. In case of a load increase or a plant outage, the on-line hydro units can increase their generation up to their maximum capacity and more hydro capacity can be started with a short start-up time. The difference between the maximum capacity of the on-line hydro units and their generation is the spinning reserve. If all the units in the hydro system are on-line, the spinning reserve will be equal to the difference between the total capacity of the hydro system and its current generation. Modelling and optimization in this case are covered by the literature [8], [14], [32], [40], [46], [89] and [115]. In [29] the spinning reserve requirement is modelled as a fuzzy constraint. A model for EDC with the security constraint expressed as a minimum acceptable frequency after an outage is presented in [80].

However, when hydro plants are scheduled not to have all their units committed, the spinning reserve will be depending on which units that are on-line. Fig. 3.8 shows an example of the generation characteristic of a hydro plant with three units. The spinning reserve is shown for minimum generation and the local best-efficiency points. The plant is operated close to the points most of the time, since operation far from these points will lead to a lower effi-



○	Minimum generation if it is equal to zero (no spinning reserve at this point)
■	Minimum generation if it is greater than zero
•	Local best-efficiency point
□	Maximum generation point for on-lined units
⇧	Spinning reserve at minimum generation, if it is greater than zero
↑	Spinning reserve at local best-efficiency

Fig. 3.8 An example of the generation characteristic of a hydro plant with three local best-efficiency points.

ciency and in some cases a risk of cavitation damages as described earlier. For short periods the generation can be increased to preserve system security. Since the generation and the spinning reserve are depending on which units that are on-line, integer variables are needed for representation of unit status. In section 3.2.1 I described a mixed integer model for the generation (3.31)-(3.35). Mathematical formulation of the spinning reserve in this model will be:

$$r_h(t) = \sum_{j \in J_h} r(j, t) \tag{3.52}$$

$$\begin{aligned} r_{\text{mi}}(j, t) = r_1(j)z_1(j, t) + \sum_{k=2}^{K_j} [r_k(j) - r_{k-1}(j)]z_k(j, t) \\ -r_{K_j}(j)\frac{u_{K_j+1}(j, t)}{\bar{u}_{K_j+1}(j)} \end{aligned} \tag{3.53}$$

where

- $r(j, t)$ is the spinning reserve in hydro plant j during hour t.
- $r_{\text{mi}}(j, t)$ is $r(j, t)$ for the mixed integer model.
- $r_k(j)$ is the spinning reserve at the local best-efficiency point for unit combination k. Observe that $r_{K_j}(j) = \Gamma_{K_j+1}\bar{u}_{K_j+1}(j)$.

Earlier hydro power plants have been scheduled to on-line but with the generation equal to zero, in order to provide spinning reserve and as reactive source, when the load is low. However, nowadays this is not so usual in Sweden since the equipment needed is considered to be too expensive.

3.6 Problem formulation

To sum up, the whole planning problem is formulated as an optimization problem. As already mentioned, the objective (3.1) is to minimize the cost of power exchanges, maximize the value of stored water and minimize penalty costs, if any. With the expressions I have presented earlier in this chapter this will be (3.54), each term corresponds to F_h, F_g and F_e respectively. The constraints (3.2)-(3.6) will be:

- The sum of produced power and power exchanges has to be equal to the load (3.2). With expressions from the chapter it will be (3.55).
- The requirements of spinning reserve (3.3), which will be (3.56).
- The constraints for the hydro system (3.4): model of the generation characteristics (3.58), reservoir balance equation (3.59), allowed discharge domain (3.60), final reservoir contents (3.61), and bound for reservoir contents (3.62)-(3.63).
- The constraints for the thermal system (3.5): cost as function of generation (3.64), limits of generation (3.65), model of start-up cost (3.66), ramp rate constraints (3.67), and relation between feasible domain for variables for commitment and start-ups (3.68)-(3.70).
- The constraint for the power exchanges (3.6): bounds for exchange (3.71).

This will give us the following minimization problem:

$$\begin{aligned}\min\{F\} = \min\Bigg\{ & \sum_{j \in J_h} \Bigg[\sum_{i=1}^{3} \sigma_i(j) x_i(j, T+1) + \sum_{t=1}^{T} \chi(j, t) \\ & - \sum_{t > T - \tau_{j, d_j}} \rho_{d_j}[u(j, t) + s(j, t)] \Bigg] \\ & + \sum_{j \in J_g} \sum_{t=1}^{T} [g(p_g(j, t)) + S(j, t) v(j, t)] + \sum_{t=1}^{T} \sum_{j \in J_e} \gamma_j(t) p_e(j, t) \Bigg\}\end{aligned} \tag{3.54}$$

Subject to

System constraints

$$\sum_{j \in J_h} p_h(j, t) + \sum_{j \in J_g} p_g(j, t) + \sum_{j \in J_e} p_e(j, t) = D(t) \tag{3.55}$$

$$\sum_{j \in J_h} r(j, t) \geq R(t) \tag{3.56}$$

$$t = 1, \ldots, T \tag{3.57}$$

Hydro system constraints, $j \in J_h$.

$$p_h(j, t) = f(u(j, t), x(j, t), x(d_j, t)) \tag{3.58}$$

$$\begin{aligned} x(j, t+1) - x(j, t) + u(j, t) - \sum_{m \in M(j)} u(m, t-\tau_{mj}) + s(j, t) \\ - \sum_{m \in M(j)} s(m, t-\tau_{mj}) = w(j, t) \end{aligned} \tag{3.59}$$

$$u(j, t) \in U(j) \tag{3.60}$$

$$x(j, T+1) = \sum_{i=1}^{3} x_i(j, T+1) \tag{3.61}$$

$$0 \le x(j, t) \le \bar{x}(j) \tag{3.62}$$

$$x(j, T+1) \in X_{T+1} \tag{3.63}$$

Thermal system constraints, $j \in J_g$

$$g(p_g(j, t)) = a_0(j) + a_1(j)p_g(j, t) + a_2(j)p_g(j, t)^2 \tag{3.64}$$

$$\underline{p}(j)y(j, t) \le p_g(j, t) \le \bar{p}(j)y(j, t) + [\bar{p}_{\text{cold}}(j) - \bar{p}(j)]v(j, t) \tag{3.65}$$

$$S(j, t) = b_0(j) + b_1(j)(1 - e^{-H/\Upsilon}) \tag{3.66}$$

$$-\delta_{\text{down}}(j) \le p_g(j, t) - p_g(j, t-1) \le \delta_{\text{up}}(j) \tag{3.67}$$

$$y(j, t) - y(j, t-1) - v(j, t) \le 0 \tag{3.68}$$

$$v(j, t) \in \{0, 1\} \tag{3.69}$$

$$y(j, t) \in \{0, 1\} \tag{3.70}$$

Power exchange constraints, $j \in J_e$

$$\underline{p}_e(j, t) \le p_e(j, t) \le \bar{p}_e(j, t) \tag{3.71}$$

where

- $\sigma_i(j)$ in (3.54) and bounds for stored water at the end of the planning period (3.63) are

$$\sigma_1(j) = \sigma_2(j) = \sigma_3(j) = -\rho_j \tag{3.72}$$

$$x_1(j, T+1) = \underline{x}(j, T+1) \tag{3.73}$$

$$0 \le x_2(j, T+1) \le [\bar{x}(j, T+1) - \underline{x}(j, T+1)] \tag{3.74}$$

$$x_3(j, T+1) = 0 \tag{3.75}$$

if we do not use soft constraints. Otherwise $\sigma_i(j)$ in (3.54) and bounds for stored water at the end of the planning period (3.63) are

$$\sigma_1(j) = -(\rho_j + \nu_j) \tag{3.76}$$

$$\sigma_2(j) = -\rho_j \tag{3.77}$$

$$\sigma_3(j) = -\rho_j + \nu_j \tag{3.78}$$

$$0 \le x_1(j, T+1) \le \underline{x}(j, T+1) \tag{3.79}$$

$$0 \le x_2(j, T+1) \le [\bar{x}(j, T+1) - \underline{x}(j, T+1)] \tag{3.80}$$

$$0 \le x_3(j, T+1) \le [\bar{x}(j) - \bar{x}(j, T+1)] \tag{3.81}$$

- the start-up cost function in (3.54) is set to zero in the piecewise linear model:

$$\chi_{\text{pwl}}(j, t) = 0 \tag{3.82}$$

and set to the following expression in the mixed integer model:

$$\chi_{\text{mi}}(j, t) = c_{\text{start}, j}(Z(j, t), Z(j, t-1)) \tag{3.83}$$

- the spinning reserve (3.56) is not considered in the piecewise linear model, since correct representation requires integer variables. This means that

$$r_{\text{pwl}}(j, t) = \infty \tag{3.84}$$

while the spinning reserve (3.56) in the mixed integer model is

$$\begin{aligned} r_{\mathrm{mi}}(j,t) = r_1(j)z_1(j,t) + \sum_{k=2}^{K_j} [r_k(j) - r_{k-1}(j)]z_k(j,t) \\ -r_K(j)\frac{u_{K_j+1}(j,t)}{\bar{u}_{K_j+1}(j)} \end{aligned} \tag{3.85}$$

- $f(u(j,t), x(j,t), x(d_j,t))$ is the model of the generation characteristics (3.58).

$$\begin{aligned} f(u(j,t), x(j,t), x(d_j,t)) &= p_0(u(j,t)) \\ + [\alpha(j)x(j,t) - \beta(j)x(d_j,t)]u(j,t) \end{aligned} \tag{3.86}$$

The piecewise linear model has the following expression

$$p_{0,\mathrm{pwl}}(u(j,t)) = \sum_{i=1}^{I_j} \Gamma_i u_i(j,t) \tag{3.87}$$

and the mixed integer model the following

$$p_{0,\mathrm{mi}}(u(j,t)) = \sum_{k=1}^{K_j} p_k(j)z_k(j,t) + \Gamma_{K_j}[u_{K_j}(j,t) - \underline{u}_{K_j}(j)] \tag{3.88}$$

- $U(j)$ is the set of equations describing the restrictions on the discharge (3.60). For the piecewise linear model there is the sum of discharges for the different slopes

$$u_{\mathrm{pwl}}(j,t) = \sum_{i=1}^{I} u_i(j,t) \tag{3.89}$$

$$0 \le u_i(j,t) \le \bar{u}_i(j) \tag{3.90}$$

For the mixed integer model there are the restrictions that discharges only are permitted at zero flow, local best-efficiency points and the part between the local best-efficiency point and maximum flow.

$$u_{\mathrm{mi}}(j,t) = \sum_{k=1}^{K_j} u_k(j) z_k(j,t) + u_{K_j+1}(j,t) \tag{3.91}$$

$$z_k(j,t) \geq z_{k+1}(j,t) \tag{3.92}$$

$$0 \leq u_{K_j+1}(j,t) \leq \bar{u}_{K_j+1}(j) z_{K_j}(j,t) \tag{3.93}$$

$$z_k(j,t) \in \{0,1\} \tag{3.94}$$

4 Start-Up Costs in Hydro Power Units

This chapter deals with start-up costs of hydro power units. In the introduction we address three questions about start-ups of hydro power units: What causes the costs in the start-up? How much does a start-up cost? How do start-ups effect the short-term scheduling strategies of power producers in Sweden? In order to answer these questions, we have interviewed employees working with generation planning at the eight largest power producers in Sweden. We found five aspects causing start-up costs: Loss of water during maintenance. Wear and tear of the windings owing to temperature changes after the start-up and shut down. Wear and tear of mechanical equipment during the start-up and shut-down. Malfunctions in the control equipment during the start-up. Loss of water during the start-up. The aspects causing the largest costs are maintenance owing to the wear and tear and the unavailability and personnel costs owing to malfunctions in the control equipment. The cost of lost water is usually small. The conclusions are the following: The start cost will depend on the nominal power of the unit and the unit model. The majority of the power producers consider start-ups in their planning. There is a need for better knowledge about start-up cost. There is a need for planning software which considers start-up costs of hydro units.

4.1 Introduction

As I stated in chapter 2 the aim of short term power system planning is to minimize the operation cost of the system to fulfil a forecasted load. It is therefore important to consider all relevant costs related to the operation of the system. In the general problem formulation in the first section of chapter 3, I divided the operation costs into two parts; costs of power trading and costs of power system operation. The costs of the operation of a hydro system were of two kinds. First, the cost of the water used to produce electric power during the planning period (3.10). The second part of the cost is the cost of start-ups (3.36). These costs are not immediate costs. However, it is important to consider these costs in the short term planning since the operation of systems affects the maintenance cost. This means that a part of the maintenance cost can be assigned to the start-up of hydro units. In the start-up phase there is also a risk of malfunction in the control equipment. A malfunction in the control equipment can lead to personnel and unavailability costs. The start-up will also cost some water since the unit will not be operated in an optimal way during this phase.

Strictly speaking, some of the costs I have described above should be assigned to a cycle of start-ups and shut-downs. However, since a start-up sooner or later leads to a shut-down, it is accurate to assign the cost to the start-up.

As I showed in chapter 3 there are different proposals of how to treat start-ups in the literature of algorithms for short term generation planning. For example restriction on discharge changes, costs for discharge changes, restrictions for starts and stops of units and cost for start of units. However, in none of the papers describing the models the background of the chosen model is explained. This study addresses three questions about start-ups of hydro units:

- *What causes the start-up cost?* I have listed all relevant aspects contributing to the start-up cost.
- *How much does a start-up cost?* To answer this question I have to assign a value to all the aspects listed according to the first question. For some of the aspects it is possible to come up with a good estimate, for others it is only possible to make a good guess or render opinions of experienced operators.
- How do start-ups affect the short-term scheduling strategies of power producers in Sweden? I will show if and how power producers in Sweden consider hydro unit start-ups in the short term scheduling.

In order to answer these questions I have interviewed employees working on generation planning, during the summer of 1995. The eight largest power producers in Sweden at that time were Graningeverken, Gullspångs Kraft, Skandinaviska Elverk, Skellefte Kraft, Stockholm Energi, Stora Kraft, Sydkraft and Vattenfall. The result of the interviews is reported in this chapter[1].

In order to give a background to some of the questions I start with a short repetition of the history of the changed power system operation in Sweden up to the summer of 1995. Until 1993 most of the largest producers except Vattenfall cooperated in the KGS-pool, in which the members minimized the total production cost for the pool. This means that some of the producers bought power from the pool if their marginal cost of production was higher than the marginal cost of some other member of the pool. The price for these exchanges was the average marginal cost of the companies involved. If the pool marginal price was different from the marginal price of Vattenfall, the pool and Vattenfall exchanged power in the same way.

During 1995 the first step towards deregulation was taken. The way of performing short term exchanges described above was replaced by a pool for all producers. To this pool the producers provided bids for trading. The bids were supposed to be based on marginal costs.

Before I consider the results of the interviews I will just mention that in the end of this chapter I will give some references to works in this area which have been published after the work reported here.

4.2 Start-up Costs

As a result of the interviews I have identified the following aspects causing start-up costs:

- Wear and tear of the windings owing to temperature changes during the start-up.
- Loss of water during maintenance.
- Wear and tear of mechanical equipment during the start-up.
- Malfunctions in the control equipment during the start-up.
- Loss of water during the start-up.

In the following I will describe each of these aspects more in detail.

1. This chapter was first published in [70]. I have rewritten it somewhat to fit into the thesis.

The cost of increased maintenance of the windings: According to an internal report at one of the power producing companies the costs of increased maintenance of windings owing to start-ups will be about $125 per start-up. This is based on the assumption that a change of the windings will cost about $3,3 million. The calculations are based on the following assumptions:

- A plant with two 50-60 MW units.
- A start-up decreases the life-time of the windings by about 15 hours. Two cases are compared; a base case with 40 years´ lifetime and a case with 150 additional start-ups per year, which means that the lifetime will be 31,5 years. This shortening of the life-time is due to the change of the temperature in the windings which occurs in the start-up. Some units have equipment controlling the temperature of the windings and will not be affected in the same way.
- The maintenance is performed during low load season and will take about four months.
- The power price is about $13/MWh and the difference between the day price and the night price is about $5/MWh.
- The interest rate is 5 percent.

The cost of lost water during maintenance: According to the report mentioned above the cost of lost water during maintenance is about $10 per start-up. It is assumed that the start-ups shorten the life-time of the windings. Owing to this shortening of the life-time the maintenance will be performed more often. During the maintenance period the power station cannot be operated in the usual way, which means that water will be lost. The losses can be assigned to spillage and decreased possibility of utilizing the price difference between day and night. A case with a shorter lifetime was compared with a case with a longer lifetime. The difference in cost per year is divided by the difference in start-ups in the cases. Other assumptions are the same as above.

If a single unit plant was studied the cost would have been higher since all the water has to be spilled. The cost would also have been higher if the maintenance was performed according to a forced outage during high load season.

The cost of increased maintenance of mechanical equipment: The starts and stops also effect the lifetime of this equipment. If maintenance of mechanical equipment and windings is planned to be performed at the same time the cost of maintaining the mechanical equipment would not be so significant. However, if the maintenance is performed due to forced outages this cost can be more significant.

The cost of malfunction of the control equipment: The cost of a malfunction in the control equipment originates from personnel costs and unavailability costs. Personnel costs are the costs of sending a repairman to the plant in order to repair the malfunction. This kind of work is often overtime work and the costs must include wages, payroll taxes and transportation. A fair estimation of this cost is about $70 per hour. The cost of the unavailability is the difference of the power prices for the period of unavailability and the period when the water is used instead. In the worst case this water has to be spilled. The expected cost of a malfunction in the start-up is the probability of the malfunction multiplied by the total cost of the malfunction. From one company it was reported that this probability is high for pump-storage units, for some as high as 50 percent. If one assumes that the time of the repair for such a plant is about two hours including transportation, the expected personnel cost of the start-up is about $70. For most units this probability is lower. From a control centre I have been in contact with it was reported that the average probability of a malfunction in the start-up was about 20 percent.

With the purpose of estimating the cost of the unavailability one uses the same reasoning as for the cost of lost water during maintenance. One can for example assume that the interruption lasts two hours and the water can be stored and later discharged to a $5/MWh lower price. If there is a unit of 50 MW nominal power, this interruption will cost $500. If the probability of a malfunction is 20 percent, the expected cost of the unavailability is $100.

It should also be mentioned that the probability of a malfunction during the start-up partly depends on the standard of the equipment. It is therefore possible to decrease the probability of malfunctions by investments in more reliable equipment. Normally the probability of a malfunction in the start-up is essentially lower than 20 percent.

The cost of lost water during start-up: The cost of lost water during start-ups can be assigned to three phases of the start-up:

- Acceleration of the unit from zero speed to 90 percent of the nominal speed.
- Acceleration of the unit from 90 percent of the nominal speed to the nominal speed and connecting the unit to the network.
- Increasing the flow to the flow of the best operating point, see Fig. 4.1

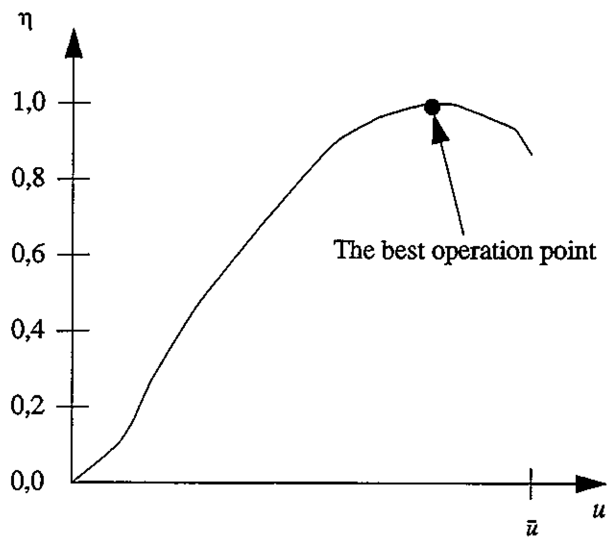


Fig. 4.1 An example of the relative efficiency as a function of the flow for a hydro unit.

This means that the lost water during the start up can be expressed as:

$$Q_{\text{lost}} = u_1\Theta_1 + u_2\Theta_2 + \int_0^{\Theta_3} [\eta_{\text{opt}} - \eta(\Psi)]u_3(\Psi)d\Psi \tag{4.1}$$

where

- Q_{lost} is the total lost water during the start-up in m^3.
- u_i is the discharge during phase i.
- Θ_i is the time of phase i.
- η_{opt} is the relative efficiency at the best operation point. Therefore $\eta_{\text{opt}} = 1$.
- $\eta(\Psi)$ is the efficiency as a function of the point of time when the unit is connected to the network until it is operating at the best point.
- Ψ is the time variable.

According to one of participants in this study, an estimate of the typical values of these parameters were: $\Theta_1 = 15$ s, $\Theta_2 = 60$ s, $\Theta_3 = 20$ s, $u_1 = 0{,}2\bar{u}$ and $q_2 = 0{,}08\bar{u}$, where $\bar{u}$ is the maximum discharge from the unit. It is realistic to assume that the flow of best operating point is about 75 per cent of the maximum flow and that efficiency is very low (approximately zero) at first when the unit is connected to the network. If one assumes that both the efficiency and the flow will increase linearly during the third phase the total lost water during the start-up will be about $11\bar{u}$. This means that a start-up is equivalent to spilling at maximum discharge for 11 s. For a 50 MW unit and assuming the power price is about $15/MWh, the cost of lost water will be about $2. The conclusion of these calculations is that the cost of lost water for an average unit is negligible. However, the parameters vary a lot and for some units the cost may be five times higher. Still, this cost will be low compared with the costs mentioned earlier.

4.3 Impact of Start-Up Costs on Short Term Scheduling Strategies

In this section I will report how the power producers treat hydro unit start-ups in their short term scheduling. As mentioned earlier I have carried out this investigation by asking questions to employees working on production planning. I have asked the following questions:

1. How is your short-term planning performed today?

2. Do you have a strategy for treatment of start-ups in the planning?

3. What is your opinion about how much a hydro unit start-up costs and have you made any investigations about these start-up costs?

4. Do you think that your way of planning would change if there was a planning software with representation of start-ups available?

5. Has the first step towards deregulation changed the way of treating start-ups?

Question 1: Today's planning. The first question I asked was how the short term planning was performed in the studied companies. No company used an optimization software to schedule their plants. There were several reasons why optimization was not used. From one of the smaller companies it was stated that the price of such a software was higher than the potential savings.

At another company they thought the potential savings of using an optimization software were small since the price forecasts are too uncertain. Several companies thought that the representation of the hydro plants in the software models was not detailed enough to give realistic schedules. One company reported that they had tested an optimization software afterwards to compare with the performed schedule. For some rivers it was shown that a use of optimization software should increase the profit. The experience from this comparison has partly led to a change of scheduling strategies for these rivers.

Question 2: Strategy for treatment of start-ups. The next question I asked was if the companies had a strategy for how to treat start-ups in the short term planning. One company said that they had no strategy. From another company I was told that they had no strategy but they thought that the start-ups cost some money. A third company reported that they had no strategy but they tried not to start and stop "too often". By "too often" they meant a few times during the day. They also tried to distribute the starts and stops uniformly among the units. The strategies at the other companies were either minimum up-time, maximum start-ups per day and unit or a start-up cost. The minimum up-time was set to 2-6 hours depending on company and unit model. The corresponding for maximum start-ups was 2-4 start-ups per day. One company used a three hours minimum up-time in the planning and in the replanning they used start-up costs. The start-up cost was about \$130 for smaller units up to \$330 for pump-storage units.

Question 3: Cost of start-ups. Then I asked about their opinion of how much start-ups cost and if their companies had made any investigation of start-up cost. Most of the companies had made no investigations. One company had made an investigation and it was reported earlier in this chapter. Two other companies reported that they were doing investigations for the time of the inquiry. At one of these companies they said that they thought knowledge about start-up costs for different units would be more important in the future. From another company it was reported that they have had quite much discussion about start-up costs during the last year. They are now planning to carry out an investigation of start-up costs.

The answers to the question about the estimate of the start-up cost was the following:

- "If you drive the matters to an extreme, you can say that we, who are working on production planning do not think that start-ups cost anything at all whereas the ones working on maintenance think that start-ups are very expensive."

- "About $70 for starting 20-50 MW units. The cost will also depend on the unit model and the number of units in the plant"
- "About $130 for a start-up of a 50-60 MW unit. This cost will probably depend linearly on the nominal output of the unit."
- "About $200-270 for 80-110 MW units. We compared quote of the maintenance cost and the number of start-ups for different units."
- "About $70-130 depending of the nominal output and unit model."
- "We know that they cost but not how much."
- "About $130 for smaller units up to $330 for pump-storage units."
- "We do not know how much a start-up costs. We think that a start-up of a unit with cold windings will cost more than a start-up of a unit with warm windings."

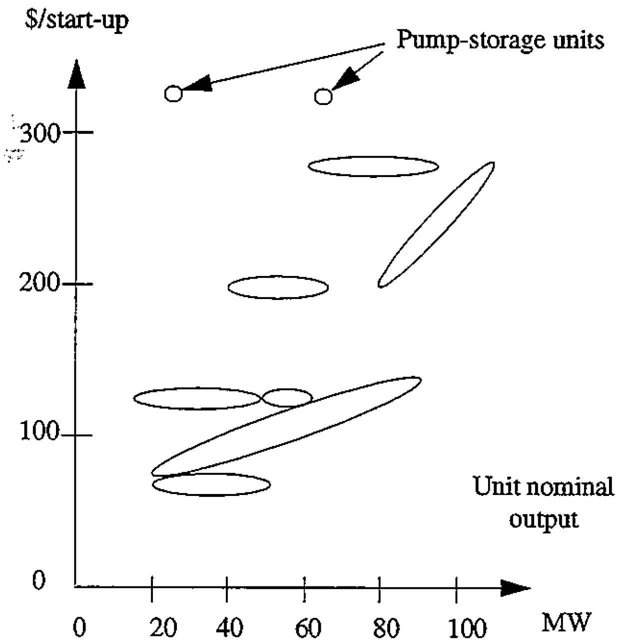


Fig. 4.2 Examples of the estimations of start-up costs made by the power producers.

Some of these answers are viewed in Fig. 4.2. The figure indicates that the start-up cost is about $3 per MW unit nominal output on average.

Almost all of the participants in this study thought that a unit with a lower nominal output should have a lower start-up cost.

Question 4: Impact of planning software. I also asked if they thought that their way of planning would change if there was a planning software with representation of start-ups available. About half of the companies answered that they were interested in such a software. One company reported that they are developing some kind of planning software where the start-up cost would be considered. Some other companies thought that it would be possible if there was a good estimate of the start-up cost and if the software offered potential savings. Others thought that it would not change the planning, since they thought that they are handling the start-ups in a correct manner or that the price forecast was too uncertain. One company said that the planning could be somewhat affected if it was shown that there will be a cost for a start-up of a unit with warm windings.

Question 5: Impact of deregulation on start-ups. Finally I asked if the first step towards deregulation had changed the way of treating start-ups. The largest company Vattenfall has experienced a decrease in the number of start-ups. The other companies have experienced no change or an increase of start-ups. Several companies reported that the number of start-ups increased quite a lot at first when the new pool started to operate. After some time, the number of start-ups decreased and most of the companies reported that the number now is almost back to normal. Some of the companies that experienced an increase in the number of start-ups began to dispatch their on-line units more to avoid start-ups. This means that the units will operate on a lower efficiency and more water will be used to produce the same amount of energy, see Fig. 4.1. It is also possible that operation far from the best-efficiency will increase the wear and tear of the unit since the units are designed to operate around the point of best-efficiency.

There are two possible reasons for the increase of start-ups for the smaller companies.

- The first is that the smaller companies cannot coordinate the planning in the KGS-pool in the same way as before. The companies instead tried to keep their generation equal to their contracted load, which led to more start-ups.

- The second reason is that the new pool leads to a new situation and new situations are often hard to treat in the beginning.

4.4 Summary of results

In this chapter I have presented how start-ups affect the cost of short term hydro operation and how these costs affect short term scheduling strategies of power producing companies in Sweden. From this work it is possible to draw the following conclusions:

- *A hydro unit start-up costs.* I have reported that there are several things which will result in costs at the start-up. The most important costs are the costs caused by increased maintenance of windings and mechanical equipment and by malfunctions in the control equipment. Since the increased maintenance will decrease the time between periods of maintenance the unavailability of the plants will increase. This increase of unavailability will be related to a cost. This cost is quite low if the maintenance can be planned to be performed at low load season. If the maintenance is performed owing to forced outages this cost could be much higher. During the start-up some water will be spilled. The total value of this water is normally low compared with the costs mentioned above. The results of interviews with the producers indicates that the start-up cost is about $3 per MW unit nominal output on average.

- *Swedish power producers consider start-ups in their planning.* According to our interviews most of the largest power producing companies in Sweden have some strategy for treatment of start-ups. These strategies are based on maximum start-ups per day and unit, minimum up-time and start-up costs. There are no common practice how to calculate start-up costs. A majority of the companies have some estimate of the start-up cost, but these estimates were seldom based on large investigations.

- *There is a need for better knowledge of start-up costs.* In this study I have shown examples of the start-up cost for given conditions. However, the start-up cost will differ for different conditions. In order to correctly consider start-up costs of hydro units in the short-term planning there is a need for future investigations on how much a start-up costs as a function of nominal output and the unit model. Our experience of this investigation is that there are almost as many opinions about the cost of start-ups and how to treat them in the scheduling as there are people working in this area. However, the estimates of the cost for different kinds of units were of the same magnitude.

- *There is a need for development of planning software which considers start-up costs or strategies.* About half of the companies answered that they were interested in such a software. To be able to consider the start-up costs or start-up strategies in software tools, these tools have to be developed.

I will end this chapter with some comments about other works in this area, which have been published after the work I presented above. In [109] it is stated: "Starting and stopping of a 60 MW Francis unit is calculated to be equivalent to about 10 minutes of full load (10 MWh), or a reduced service life between major overhauls of about 10 hours". To get the same estimated cost as I have calculated ($3 per MW unit nominal output), we have to assume that the power price is $18/MWh, which is not unreasonable. [21] reports that frequent starts and stops in air-cooled units will lead to a combination of mechanical stress, plastic deformations and sticking of the winding itself in the generator stator-windings. Water-cooled generators, on the other hand, are better suited to withstand frequent starts and stops. During starts and stops there will also be mechanical stress on all active parts in the circuit breaker: "Serious problems have been reported on these breakers used in pumped-storage plants where the operating mode is frequently changed". Finally, with more frequent starts and stops there might be a need for upgrading or changing equipment, such as excitation equipment, HV-switchgear and protection/monitoring/control system.

5 Methods for Solving the Short Term Planning Problem

In this chapter the solution techniques for the problems stated in chapter 3 are presented. First I will present a survey of methods in the literature. After that I will present the methods I have used to solve the short term planning problem. The solution procedure is divided into three steps. The first step uses a piecewise linear hydro power model. The aim of this step is to find a good initial solution for the step with a more detailed model. The second step updates the head dependence and the third step optimizes the system with a mixed integer model of hydro power generation.

5.1 Introduction

In chapter 3 I discussed different types of models for the short term planning problem. I also formulated the models used in the thesis mathematically. The first part of this chapter, section 5.1, gives an overview of the methods applied to the models from chapter 3. In sections 5.2-5.5 I will give a more detailed description of methods I have applied to solve the short term planning problem.

First I will describe some techniques used to decompose the power system planning problem into subproblems. After that I will describe solution methods for the continuous and mixed-integer models from chapter 3. The solution technique will depend on the model and the problem formulation. If the model for example contains integer constraints one has to use a method which is able to treat integer problems.

5.1.1 Decomposition

If the problem is formulated with a quite detailed representation of both the hydro and thermal systems it is common to use some decomposition techniques. The decomposition will split the problem into a master problem and smaller subproblems. The subproblems often have special structures where suitable optimization methods can be applied. For the power system planning problem it is usual to decompose it into a hydro and a thermal subproblem. Fig. 5.1 shows the iterative process in the hydro thermal coordination. In the following I will describe three examples of methods used for decomposition.

The primal decomposition technique [9] can be used only if the load balance equation is the only coupling constraint. In this approach the load balance equation (3.55) is reformulated in such a way that the thermal generation can be substituted into the cost function of the system. For a given thermal commitment, the problem is to solve the hydro system according to a cost function given by the committed thermal units. This cost function will be nonlinear. The thermal subproblem will be to cover the part of the load not served by hydro. The coordinating information from the hydro subproblem will be the hydro production for each time step. The corresponding information from the thermal subproblem the committed units.

In *Benders decomposition* [36] the problem is decomposed into an integer master problem and a continuous subproblem. In this case the hydro power model is piece-wise linear and the thermal model includes start-up costs, stop and start restrictions and linear fuel costs. This means that the integer problem is to determine when the thermal units should be started and stopped. The continuous problem will be to fulfil the load with hydro and thermal power at least cost for a given thermal commitment.

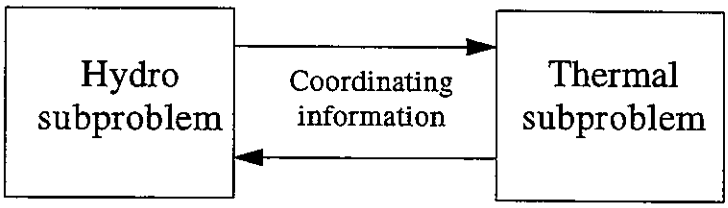


Fig. 5.1 Decomposition of the planning problem into one hydro and one thermal subproblem.

The most common decomposition technique in power system planning is probably *Lagrange relaxation* [7], [8], [9], [32], [46] [54], [57], [58], [89], [114] and [115]. The coupling constraints between the hydro and the thermal systems, for example load balance and reserve requirements, are relaxed by Lagrange multipliers. These multipliers carry price information about the right hand side in the relaxed constraints. If we relax the load balance equation, the corresponding multiplier will be the marginal cost of the energy in the system. The subproblems are optimized according to this marginal cost. If the sum of produced energy will be higher than the load we have to lower the price and vice versa. This is the principle of Lagrange relaxation. If the problem is decomposed into one hydro and one thermal subproblem, the hydro subproblem will be to maximize the benefits of the hydro system according to the dual prices achieved when the hydro-thermal master problem is decomposed into subproblems. In this maximizing process we will only have to consider the constraints of the hydro system. I will come back to this approach since the solution technique of the thesis is based on Lagrange relaxation.

Also in the case of pure hydro or pure thermal problems it can be useful to decompose the problem into subproblems. If the load balance and the reserve requirements are the only constraints coupling the thermal plants or different rivers, a decomposition can give subproblems for each thermal plant or river respectively.

Since original problems are either, linear continuous, non-linear continuous, mixed integer-linear or mixed integer-non-linear the subproblems can be divided into these four groups. If the representation of the hydro system is non-linear continuous and the representation of the thermal system is mixed integer-linear the total problem will be mixed integer-non-linear. A decomposition into a hydro and a thermal subproblem will then give a continuous non-linear hydro subproblem and a mixed integer-linear thermal subproblem. In the next section I will describe some solution techniques applied to the original or decomposed problems for the four groups of models.

5.1.2 Continuous models

Some techniques for continuous programming

First, a short overview of some of the most popular continuous techniques.

Linear programming [53] can be used when the objective function and the constraints are linear.

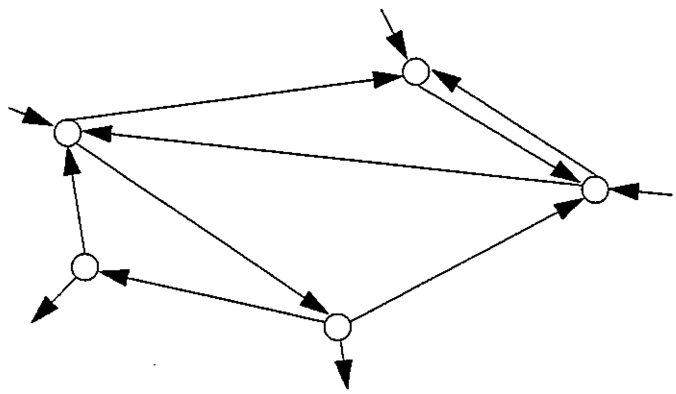


Fig. 5.2 Graphical representation of a problem with network problem.

A special case of linear programming is *linear network programming* [48] which can be used if all constraints have a network structure. This kind of problem can be viewed as a set of source and sink nodes connected in a network, see Fig. 5.2. Connections between nodes, called arcs, have maximum and minimum bounds and a cost for flow. The goal is then to find the cheapest way of transporting the flow from the sources to the sinks. To solve a pure network problem with linear network programming is much faster than solving it with ordinary linear programming since no matrices have to be inverted.

Non-linear programming [53] represents search techniques used for finding the optimum for a problem where the objective function and/or the constraints are non-linear. The techniques are normally based on using some kind of linearization. This can for example be calculation of gradient in some point and then search for a point with better objective function value. Again it is possible to utilize the network structure, if the problem has only network constraints, to decrease the computation time.

Dynamic programming [41] is suitable when we have a problem where we want to optimize something over several time steps and some resource can be stored from one time step to the next. The largest advantage of dynamic programming is that it can treat uncertainties in a suitable way.

There are also other techniques. *Decomposition* techniques can for example be used in the subproblems.

Hydro power

In the *continuous linear* case, problems can be solved by linear programming. In [85] the generation characteristic of the hydro power units is modelled as a piecewise linear function for each unit in the plant. The generation in a plant will depend on the discharge for each unit and the losses in the tunnel system of the plant. The losses in the tunnels are modelled as non-linear functions of the flow. Spillage is a non-linear function of the reservoir level above the spillway. The contents of minor and medium reservoirs are assigned to weekly targets based on the long-term storage strategy. To avoid infeasibility for the reservoir levels at the end of the planning period the reservoir levels are allowed to be within some bounds near the weekly target. If the reservoir levels are outside their bounds, a penalty cost multiplied by the deviation from the bound is added to the objective function. For long term reservoirs a water storage value is added to the objective. The generation has to fulfil a forecasted load. The problem will be non-linear and large scale. The problem is linearized and solved by linear programming.

If the constraints of the problem have pure network structure, as described in section 3.2.1, linear network programming can be used to solve the problem. Network programming has shown to be an efficient technique to solve hydro subproblems. Linear network programming is much faster than ordinary linear programming in solving problems with pure network structure.

If the model of the generation characteristic is *non-linear* and there only are constraints with pure network structure, this problem can be solved by non-linear network programming [9] and [10].

[23] shows how non-linear head dependence can be implemented in the network programming if the generation as function of the discharge is piecewise linear. This modified network programming model can find stationary points. Then it is possible to find the global optimum via branch and bound.

In [9], [14] and [40] reserve constraints are treated as side constraints in the network programming.

For the non-linear continuous case some kind of non-linear technique has to be applied. In [35] non-linear programming is used. Here the non-linearities are introduced by the head dependence, which is modelled as a correction factor. The model of the generation as function of the discharge is piecewise linear. Delay times in the hydro system are not approximated to whole hours. The operation of the hydro system is optimized according to an energy price for each hour.

Even though most short term models do not include uncertainties, continuous dynamic programming has been applied to the hydro planning problem. As I said earlier, dynamic programming is suitable when we have a problem where we want to optimize something with time couplings and stored resources. In the hydro problem this resource is water. In [62] it is shown how to use dynamic programming for solving hydro subproblems. The so-called one reservoir model is used. In this model all plants in the hydro system are treated as one single plant where energy can be generated or stored. This means that the solution will tell us how much hydro electric energy that should be generated at each time step, but nothing about the dispatch of the plants. This method could be used if the hydro generation is only a small part of the total generation and is only used at peak load hours.

In [90] the dynamic programming technique for hydro problems is further developed. First each hydro plant is optimized as if it does not have any hydrological couplings to the other plants in the system. The operation of the other plants is kept constant during the optimization of each plant. This will give a feasible initial solution. In the next step single plant optimization is performed, but now considering how changes of discharge will affect the downstream plant. This is an iterative process and it will continue until optimum is reached.

A decomposition of the hydro subproblem is used in [104]. In this decomposition the planning horizon is divided into periods. The planning of each period is solved separately. The periods are thereafter coordinated at a higher level. The aim of this decomposition is to decrease the computation time since the subproblems can be solved in parallel.

<u>Thermal power</u>

For thermal power there is no point in treating methods for linear and non-linear models separately. Since this section deals with models without integer representation, I will only treat the case where the commitment is known. If the commitment of the thermal units is known we solve the problem with economic dispatch (EDC). The EDC problem is to find the optimal dispatch of the committed units with the constraint that the total generation must be equal to the load. To this formulation several types of constraints can be added: spinning reserve requirements for the system or different areas [56], transmission losses [22], [45] and [112], non-convex cost functions [4],[12], [45] and [56], ramp rate [5], contingency [3] and security constraints [5] and [116].

Lagrange relaxation is maybe the most popular solution method in EDC. Usually the system constraints such as load balance and reserve requirements are relaxed [4], [5], [22] and [47]. Examples of other applied solution techniques are: genetic algorithms [12], [50] and [92], non-linear Danzig-Wolfe decomposition [3], interior point programming [116], dual programming [20] and the Newton method [45]. Most of these techniques are some kind of non-linear programming.

5.1.3 Mixed-integer models

Before I present the solution techniques, which have been used to solve mixed-integer power planning problem, I will shortly describe some techniques for integer programming. After that I will give some examples of methods applied to linear and non-linear hydro problems. Finally, I will discuss some methods for mixed integer thermal problem, e. g. thermal unit commitment.

Some techniques for integer programming

In the *branch and bound technique* [41] the integer constraints on the variables are relaxed. This gives a continuous problem. If the solution of this problem fulfils the integer constraints it is the solution of the integer problem. If not, choose one of the variables which is not fulfilling its integer constraint. From this two new continuous problems are defined. In the first problem the chosen variable is set to its lower integer value and in the second problem the variable is set to its upper integer value. These problems will be treated in the same way as the first problem. This means that each of the

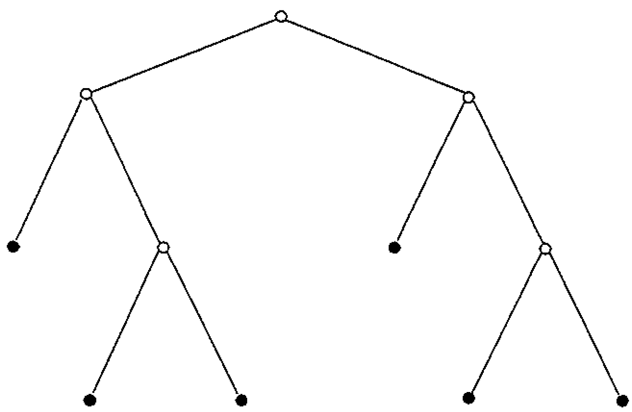


Fig. 5.3 The tree structure of the Branch and Bound method.

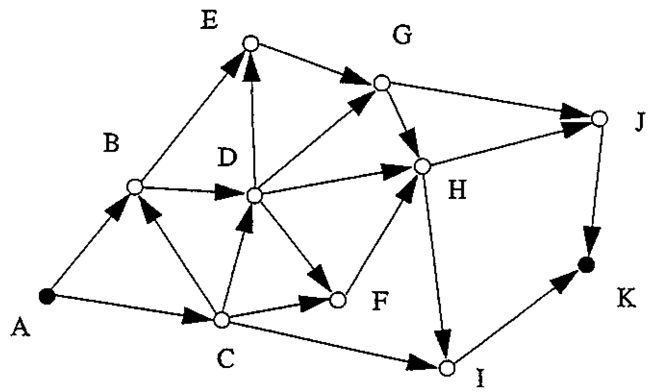


Fig. 5.4 Shortest path problem. Which is the shortest way from point A to point K?

problems can be split into two new problems. The splitting procedure will build a tree structure where each node will represent a continuous problem (Fig. 5.3). The splitting procedure goes on until a node is fathomed. There are three ways in which a node can be fathomed. The first one, if the solution is integer. The second one, if there are no feasible solutions. The third one, if the best found integer solution is better than the objective for the node. When the whole tree is fathomed the node with the best integer solution is the optimal one. The branch and bound technique can be applied to small systems but might be too time consuming when the system is large.

A shortest path problem can be solved with *dynamic programming* [41], see Fig. 5.4. In this example we want to find the shortest path from point A to point K. To solve this problem we can start in point K. From that point we go to the two points connected to K. In these points, I and J, we can calculate the shortest path to K as the distance represented by the arcs to K. Now we can go to point H, where the shortest path to K will be the distance to point I plus the shortest distance from I to K or the distance to J plus the shortest distance from J to K. Then we will move to points F and G and repeat this procedure until we have reached the starting point A.

The interest in evolutionary algorithms has been rising fast. *Genetic algorithms* are the most popular form of evolutionary algorithms [61]. They derive the behaviour from a metaphor of evolution in nature. Genetic algorithms are based on three operations: selection, crossover and mutation. These operations will be applied to a set of possible solutions of the problem.

This set of solution is called a generation. The quality of each solution is evaluated by some fitness function. In the selection process, solutions with high quality have a higher probability to be selected than the solutions with a low value in the fitness function. In the crossover process new solutions are created from the old solutions, like chromosomes of a child are created from its parents. Finally, in the mutation process some parts of the existing solutions are changed with a very low probability.

Hydro power

First the mixed integer linear models. In [7] a model with integer representation of the hydro generation characteristic is presented. At first the hydro characteristic is piecewise linear and the global constraints such as load balance and reserve requirements are relaxed. After that the hydro power model only includes the local best-efficiency points. Then the goal is to find the integer solution which is closest to the solution from the continuous problem. This is done by a dynamic programming method for one plant at the time, while the generation in the other plants are constant in the same way as in [90].

In recent years mixed integer-linear programming has been tried to solve problems with integer representation of the generation characteristic. In [15] hydro plants are modelled with a piecewise linear model for generation as function of the discharge. Integer variables are used to model the on-off behaviour of the hydro plants. In this model there are also constraints for the electric network.

Also [106] uses mixed integer-linear programming to solve a similar problem. In this case there are restrictions for the number of start-ups for a hydro unit and also minimum up and down times.

However, when the problem is large and coordination with thermal units is needed, computational requirements may come too large for practical implementation of mixed integer-linear programming. In these cases it can be useful to decompose the hydro problem into subproblems. [24] shows one application of Lagrange relaxation for a pump-storage plant. In this paper pumping can only be performed at discrete points, while generation is a continuous function of the discharge. There is also a cost for moving from one state to another. It is assumed that the scheduling of the pump-storage unit is a subproblem to a large problem and that energy prices are given as Lagrange multipliers. The scheduling of the pump-storage units is then solved by relaxing the load balance equations and applying dynamic programming to the subproblem. The Lagrange multipliers in the inner loop are updated with a line search method. Also in [31] Lagrange relaxation is used to solve a problem where the focus is on scheduling of pump-storage units. In this work both the generation and the pumping domain are divided into nominal and fringe operation regions. Load balance and reserve requirements are then relaxed. In the pump-storage unit subproblems the reservoir bounds are relaxed. The multipliers corresponding to the reservoir bounds are updated by the subgradient method at intermediate level. The subgradient method is also used at the higher level to update multipliers for load balance and reserve requirements.

Two different decompositions are proposed in [11]. The hydro power model is integer, where discharges are allowed at minimum flow, the local best-efficiency points and maximum flow. There are restrictions on how to change the discharge. The first approach is to relax the equations of reservoir balance. This gives one subproblem for each power plant. The value of generation in each plant should be maximized according to the energy price and the multiplier of the equation of reservoir balance. The second approach is to split the equation of reservoir balance into two equations. One for the total inflow to plant j during hour t and for reservoir contents, discharge and spillage of the plants. The first part couples the plant together with operation of other plants in the system. When this equation is relaxed the problem is decomposed into one subproblem for each hydro plant. The goal of these subproblems is to optimize each plant according to the energy price, the multiplier of relaxed equation, the part of the reservoir equation that was not relaxed and the local constraints. In both cases the multipliers are updated with augmented Lagrangian [53]. The interpretation of the multipliers in the first place will be the value of the water in the corresponding reservoir. In the second approach it will be the price that the downstream reservoir would pay the upstream reservoir to get water.

In [105] the objective is to maximize the benefits from hydro generation according to energy price and specified final reservoir storage. In this case the objective is linear and the hydro power model is integer, exactly the same as in [11]. There are also restrictions on how to change the discharge, however not exactly the same restrictions as in [11]. The solution technique is the following. First, the generation of each plant and hour is multiplied by a compensation coefficient. Each coefficients is initialized to 1,0. Then the problem is decomposed into a subproblem for each plant, all constraints except the constraint for the specified reservoir content at the end of the planning period are relaxed. The subproblem is to allocate the free amount of water in such a way that the value is maximized, according to the energy price and the compensation coefficient. The solutions from the optimization of each plant are then brought together and the feasibility is checked. If the solution is feasible and better than the best earlier found feasible solution the new solution is saved. The next step is to update the compensation coefficients. The coefficients are updated to get a better objective value and, if the constraints are violated, to become feasible in that constraint. The loop goes on until the limit of iterations is reached.

Now to the *mixed integer-non-linear* hydro power models. Linear programming has for example been a part of the solution technique in [101]. The objective function in this case is to maximize the value of stored water. The hydro power model is a mixed-integer model (lower graph in Fig. 3.3). Delay times in the hydro system are not approximated to be whole hours. The non-linearities is introduced by the head dependence. First a schedule is produced to fulfil the forecasted load by using linear programming and a piecewise linear hydro power model (upper graph in Fig. 3.3). If, during the first hour of the planning period, one or more operating points are non-feasible according to the mixed-integer model the plants are rescheduled with a rule based technique. In this rescheduling procedure a more realistic hydro power model is used taking into account head dependence and the efficiency curve. Then a new schedule is produced for the coming hours according to a load forecast. This goes on until the end of the planning period is reached.

A somewhat similar approach is presented in [49]. This is a three step method. First, a standard piecewise linear model of generation as function of discharge with head dependence. This problem is solved by successive linear programming and separable non-linear programming. The second step is a heuristic one, where the schedule is improved according to minimum up and down times and spinning reserve requirements. Finally, a non-linear problem is solved to optimize generation of the units committed in the second step. In the final step the generation as function of discharge uses a non-linear model.

[38] presents a method for maximizing the generation for given maximal discharges for each hour. The method considers start and stop costs/restrictions, forbidden discharge intervals and head variations.

In [96] hydro plant generation characteristic is represented by polynomial functions with corrections for head losses. There are different polynomials for all the possible combinations of units of the plant. The model also includes constraints for the electric network. The solution procedure is a heuristic one. At first only one unit is committed per plant. If it would be better to commit more units according to the discharge in the solution, the solution procedure is re-run with the new commitment. This procedure is repeated until the commitment does not change.

Lagrange relaxation has been used to decompose the hydro problem into subproblems in the mixed integer non-linear case. In [51] the generation is a non-linear function of the discharge for each combination of committed units. This is represented as integer variables for the commitment of the units and non-linear generation functions for each unit combination. The reservoir balance equation is relaxed. In order to decrease the computational effort to solve the master problem it is assumed that the Lagrange multipliers for the reservoir balance equation are the same during the whole planning period for each plant. Another application of Lagrange relaxation is used in [33] where the reservoir limits are relaxed. This means that it is possible to eliminate all variables for reservoir contents. Then the operation of different plants will be independent of each other and there will be a subproblem for each plant. Relaxing the reservoir limits will have an advantage compared with relaxing the reservoir balance. The advantage is that reservoirs seldom hit their limits and the corresponding dual variable will thereby be equal to zero. In [33] and [51] there are minimum up and down times for the hydro units. [51] also has a start-up cost for the hydro units. In both cases the subproblems are solved by dynamic programming.

Another type of methods which has gained more popularity during resent years is genetic algorithms. [78] applies genetic programming to a discrete model of the generation as function of the discharge. The non-linearities come from the head dependence and a non-linear cost function, which represents committed thermal plants. In [44] the genetic programming is applied after a first step with linear programming. The linear programming gives initial solutions for the genetic programming. In this case the model had detailed non-linear representation of the river between the plants. It had mixed integer non-linear mode of the generation as function of the discharge. [111] uses evolutionary strategies in solving the hydro subproblem for a hydro model with detail modelling of the efficiency and head dependence.

Thermal power

In this part I will only treat thermal problems/subproblems where the commitments are not known. This requires integer variables and is called unit commitment.

The simplest way of solving the thermal subproblem is *the priority list* approach [9], [17]. The thermal units are arranged in a list based on "average" generation cost. Local search rules can be used to improve the attended solution. The priority list approach is fast but not the best method to come close to the optimal solution.

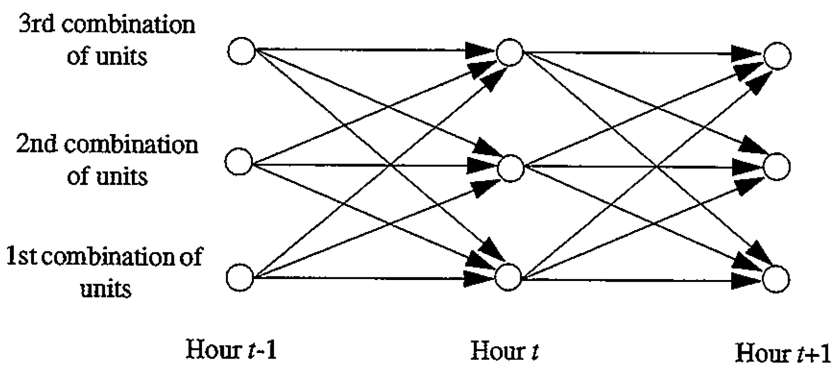


Fig. 5.5 Dynamic programming in scheduling of the thermal units.

Branch and bound methods can be used to solve the thermal subproblem [36].

The unit commitment problem can also be solved by *dynamic programming* [42], [81], [95] and [113]. Fig. 5.5 shows the dynamic programming model, where each state represents a combination of thermal units fulfilling the load constraint and other constraints. The arcs to the state represents the cost of generation in that state plus the cost of moving from previous state. In [1] the neural networks are used as the first step to find a good starting solution for the dynamic programming.

If there are no constraints coupling the plants together, or such constraints can be relaxed, the thermal plants can be optimized separately. This can be solved by dynamic programming [41]. The states are defined as how long the unit has been on-line, if positive or off-line, if negative. The cost on the arc going from a positive state will be the maximum benefit of having the unit on-line for this hour according to the dual price and the fuel cost. If the arc goes from a negative state to state 1 this cost will be added to the start-up cost. Arcs going to a negative state will have their cost equal to zero. In Fig. 5.6 the dyn-p net for a unit with minimum on-line time equal to three hours and minimum off-line time equal to two hours is shown. If the start-up cost is time dependent there have to be more negative states to know how long the unit has been off-line.

The dynamic programming model can be extended if there are several operation modes for the thermal plant. In [16] the operation modes are combined cycles, fuel switching/blending, constants/variable pressure, overfire and dual boiler. For each mode there are rules for if and how it is possible to go

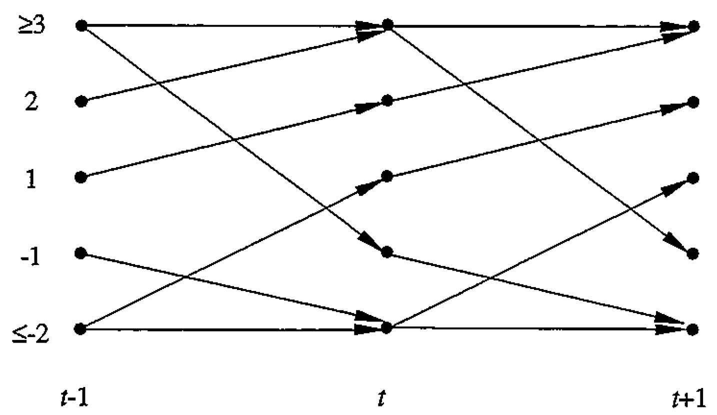


Fig. 5.6 Dyn-p net of the thermal subproblem

from one mode to another. The rules can be interpreted to the dynamic programming network by introducing new states for the different modes.

In [30], [84] and [100] ramp rates are treated. In these papers the ramp rate constraints are relaxed. [30] uses an inner loop where Lagrange multipliers for the ramp rate constraints are updated while the multipliers for the system wide constraints are fixed. [84] and [100] introduce extra states in the dynamic programming so that the generation of newly started units does not increase too much.

Lagrange relaxation often leads to a commitment of more units than necessary. [52] presents an algorithm for decommitment of units after the Lagrange relaxation and dynamic programming procedure. Different ways of decommitting units are compared and the most economically efficient is chosen. The procedure is repeated until there are no more possibilities of decreasing the objective function through decommitment. During the whole procedure the schedules are feasible.

Genetic algorithms have also been used to solve the thermal unit commitment problem. [92] applies genetic programming straight forward. However, genetic algorithms are often applied together with some other technique. [117] uses a parallel programming technique to speed up the calculation. [79] combines genetic programming and priority list. [43] first uses a genetic enhanced neural network and then improves the solution by dynamic programming.

5.2 Overview of the proposed solution method

The model in section 3.6 is mixed integer-non-linear. A common approach to deal with mixed-integer problems is to relax some coupling constraints, of which I have given examples in the previous section. This will decompose the problem into subproblems which are easier to solve than the original problem. The aim of relaxation in this thesis is to decompose the hydro part of the problem into one subproblem for each hydro plant. However, the non-linearities in hydro system introduced by the head dependence will cause some difficulties. First, the head dependence will couple the plant to the downstream plants. This means that after relaxation of other coupling constraints, the plants will still be coupled through the head dependence. Secondly, the head dependence is non convex which results in that it can be difficult to find the global optimum. Thirdly, the hydro subproblem will be non-linear and the solution technique will require more time than if the problem was linear. In my licentiate thesis [64] I presented a way to overcome these problems. However, this technique will have some other drawbacks which I will come back to later. Instead I will start with a model without head dependence and update the head dependence after a while.

Fig. 5.7 shows on overview of the solution technique. The success in solving the dual master problem will depend on the start values of dual variables. In order to get good estimates of the dual variables for the master problem I will start with the piecewise linear model of the generation characteristic (step 1, Fig. 5.7). Before going into the step with mixed integer hydro model I will update the head dependence (step 2, Fig. 5.7). In these steps there will be no constraints for spinning reserve since the modelling of spinning reserve requires an integer hydro power model, which is only used in step 3.

The third step (step 3, Fig. 5.7) does not change the Lagrange multipliers for the load balance equation significantly as long as the constraints for the spinning reserve requirement is inactive. If the Lagrange multipliers for the load balance equation is almost unchanged, it is probable that the thermal unit commitment is not going to change. The thermal unit commitment is, as I will show later on, only depending on the multipliers of the load balance equation and multipliers for the ramp rate constraints. As long as the multipliers for the load balance are unchanged, the multiplier for ramp rate constraints will not change. This means that it is possible to use the commitment from step 1 in step 3 if the spinning reserve requirements are inactive. In the numerical examples in the next chapter I will test both the case with unchanged commitment and the case with new unit commitment in step 3.

Step 1

Aim: Find start values for Lagrange multipliers and initial thermal unit commitment
Hydro model: Piecewise linear, no reserve constraints
Thermal model: Mixed integer
Solution technique: Relaxation of the load balance equation
Hydro subproblem: Linear network programming
Thermal subproblem: Dynamic programming
Feasible solution: Economic dispatch of committed thermal units with respect to ramp-rate constraints

Step 2

Aim: Compensate the hydro generation characteristics for head variations
Hydro model: Piecewise linear with head dependence, no reserve constraints
Thermal model: Cost function for the committed units
Solution technique: Franke-Wolfe with linear network programming

Step 3

Aim: Find commitment of hydro units and the optimal hydro and thermal generation
Hydro model: Mixed-integer, reserve constraints
Thermal model: Cost function for the committed units and ramp-rate constraints or mixed-integer model
Solution technique: Relaxation of the load balance and variable splitting for the discharge variables
Hydro subproblem: Dynamic programming and linear network programming
Feasible solution: Linear network programming and heuristic search

Fig. 5.7 Overview of the solution algorithm.

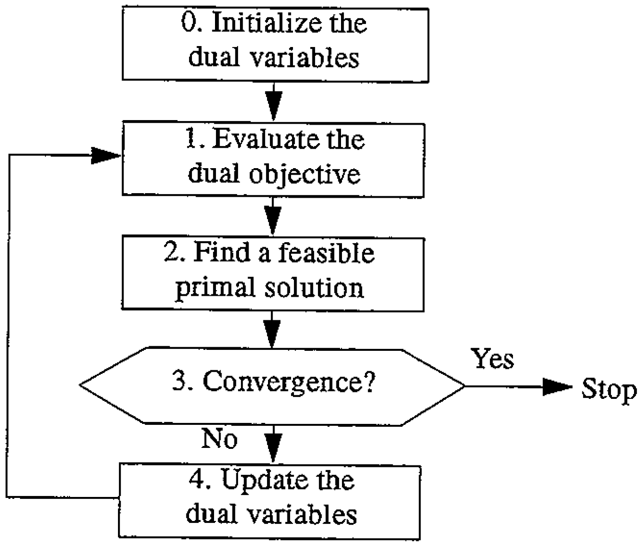


Fig. 5.8 General flow chart of dual optimization

In steps 1 and 3 I use a dual technique to solve the problem. The dual solutions are seldom feasible primarily in the relaxed constraints. This means that we have to find a feasible primal solution based on the dual solution. To sum up we need to

- formulate the dual problem by relaxing some coupling constraints.
- maximize the dual objective.
- find feasible primal solutions based on the dual solutions.

In the following three sections I will describe the steps in the algorithm.

5.3 Step 1: Piecewise linear model

The aim of the first step is to find start values of the dual variables for step 3. The problem in step 1 is solved by a dual technique. The dual objective function will be evaluated in each iteration, see Fig. 5.8 (box 1). To be able to evaluate the dual objective we have to formulate the dual problem. The dual problem is formulated for the models from section 3.6. I will come back to the initialization of the dual variables (Fig. 5.8, box 0) after I have described the dual problem.

5.3.1 Dual problem

The dual problem can now be formulated by introducing Lagrangian multipliers and lifting up the restrictions with non-network structure in the objective function. Let $\lambda(t)$ be Lagrangian multipliers for the load balance equation (3.55) and $\zeta_{\text{up}}(j, t)$ and $\zeta_{\text{down}}(j, t)$ Lagrangian multipliers for the upper and lower limit in (3.67) respectively. The dual objective function will then be

$$
\begin{aligned}
\varphi(\lambda, \zeta) = \min_{u, x, s, p_g, p_e, y, v} \Bigg\{ & F - \sum_t \lambda(t) \Big[\sum_{j \in J_h} p_h(j, t) \\
& + \sum_{j \in J_g} p_g(j, t) + \sum_{j \in J_e} p_e(j, t) - D(t) \Big] \\
& - \sum_{j \in J_g} \sum_{t=1}^{T} \zeta_{\text{up}}(j, t)[\delta_{\text{up}}(j) - p_g(j, t) + p_g(j, t-1)] \\
& - \sum_{j \in J_g} \sum_{t=1}^{T} \zeta_{\text{down}}(j, t)[\delta_{\text{down}}(j) + p_g(j, t) - p_g(j, t-1)] \Bigg\}
\end{aligned}
\tag{5.1}
$$

subject to (3.57), (3.59)-(3.66), (3.68)-(3.71)

Where

- (3.82) is substituted into (3.54) (no hydro unit start-up costs).
- (3.84) is substituted into (3.56) (no spinning reserve constraints).
- (3.86), (3.87) and (3.58) are substituted into (3.55). In (3.86) the reservoir contents are assumed not to be changed during the optimization. If the head dependence in (3.86) is changed when the actual reservoir contents change in the optimization, the model would have been non-linear.
- the allowed discharge domain (3.60) will be (3.89) and (3.90).
- (3.89) is substituted into (3.55)

The problem (5.1) can be decomposed into four parts:

$$\varphi(\lambda, \zeta) = \varphi_0(\lambda, \zeta) + \varphi_h(\lambda) + \varphi_g(\lambda, \zeta) + \varphi_e(\lambda) \tag{5.2}$$

where

- $\varphi_0(\lambda, \zeta)$ is the constant part.
- $\varphi_h(\lambda)$ is the hydro part, subject to (3.59), (3.61)-(3.63) and (3.90).
- $\varphi_g(\lambda, \zeta)$ is the thermal part, subject to (3.64)-(3.66) and (3.68)-(3.70).
- $\varphi_e(\lambda)$, is the exchange part, subject to (3.71).

In the following I will describe how to solve each subproblem.

The constant part

The constant part will be the right hand side multiplied by the dual variables.

$$\begin{aligned}\varphi_0(\lambda, \zeta) = \sum_{t=1}^{T} \Big[\lambda(t)D(t) - \sum_{j \in J_g} \zeta_{\mathrm{up}}(j, t)\delta_{\mathrm{up}}(j) \\ - \sum_{j \in J_g} \zeta_{\mathrm{down}}(j, t)\delta_{\mathrm{down}}(j) \Big]\end{aligned} \tag{5.3}$$

The hydro subproblem

The hydro subproblem will be to maximize the benefit of hydro generation for given energy shadow prices $\lambda(t)$ (or is as it stated here: minimize the negative benefit)

$$\begin{aligned}\varphi_h(\lambda) = \min_{u,\,x,\,s} \Bigg\{ - \sum_{j \in J_h} \Bigg[\sum_{t=1}^{T} \sum_{i=1}^{I_j} \lambda(t)\Gamma_i(j) u_i(j, t) \\ + \sum_{t > T - \tau_{j,d_j}} \rho_{d_j} \Bigg[\sum_{i=1}^{I_j} u_i(j, t) + s(j, t) \Bigg] - \sum_{i=1}^{3} \sigma_i(j) x_i(j, T+1) \Bigg] \Bigg\}\end{aligned} \tag{5.4}$$

subject to (3.59), (3.61)-(3.63) and (3.90).

where $\lambda(t)\Gamma_i(j)$ are the energy shadow prices multiplied by the generation equivalent. This will be the economic benefit of generation. There will be no costs on the reservoir variables except for the ones describing the final reservoir content. If we use soft constraints for the final reservoir contents, we will have three arcs representing the final reservoir contents. This results in a piecewise linear model of the value of water stored. For the arc representing stored water below $\underline{x}(j, T+1)$ the cost will be $\sigma_1(j) = -(\rho_j + \nu_j)$. The arc representing stored water between $\underline{x}(j, T+1)$ and $\bar{x}(j, T+1)$ will have the cost $\sigma_2(j) = -\rho_j$. The arc representing stored water above $\bar{x}(j, T+1)$ will have the cost $\sigma_3(j) = -\rho_j + \nu_j$. The cost on the spillage variables will be equal to zero. This is a linear programming problem with pure network structure, see Fig. 5.9. The horizontal arcs are the reservoir contents. The vertical arcs are the discharge and the spillage variables. The short arcs describe the natural inflows.

Thermal subproblem

The thermal subproblem can be divided into a subproblem for each thermal plant

$$\varphi_g(\lambda, \zeta) = \sum_{j \in J_g} \varphi_{g,j}(\lambda, \zeta) \tag{5.5}$$

The thermal plant subproblem will be to maximize the benefit of thermal generation subject to shadow prices for energy, $\lambda(t)$, and shadow prices for change of generation $\zeta_{\text{up}}(j, t)$ and $\zeta_{\text{down}}(j, t)$.

$$\begin{aligned}\varphi_{g,j}(\lambda, \zeta) = &\min_{p_g, y, v} \left\{ \sum_{t=1}^{T} [g(p_g(j, t)) + S(j, t)v(j, t)] \right.\\ &+ \sum_{j \in J_g} p_g(j, t)[-\lambda(t) + \zeta_{\text{up}}(j, t) - \zeta_{\text{down}}(j, t)\\ &\left. -\zeta_{\text{up}}(j, t+1) + \zeta_{\text{down}}(j, t+1)]\right\}\end{aligned} \tag{5.6}$$

subject to (3.64)-(3.66) and (3.68)-(3.70)

These subproblems will now be solved by dynamic programming as shown in Fig. 5.6. The cost of the states where the unit is off-line will be equal to zero. The number of states where the unit is off-line will be equal to the number of hours from the unit is stopped until the boiler is cold. The start-up

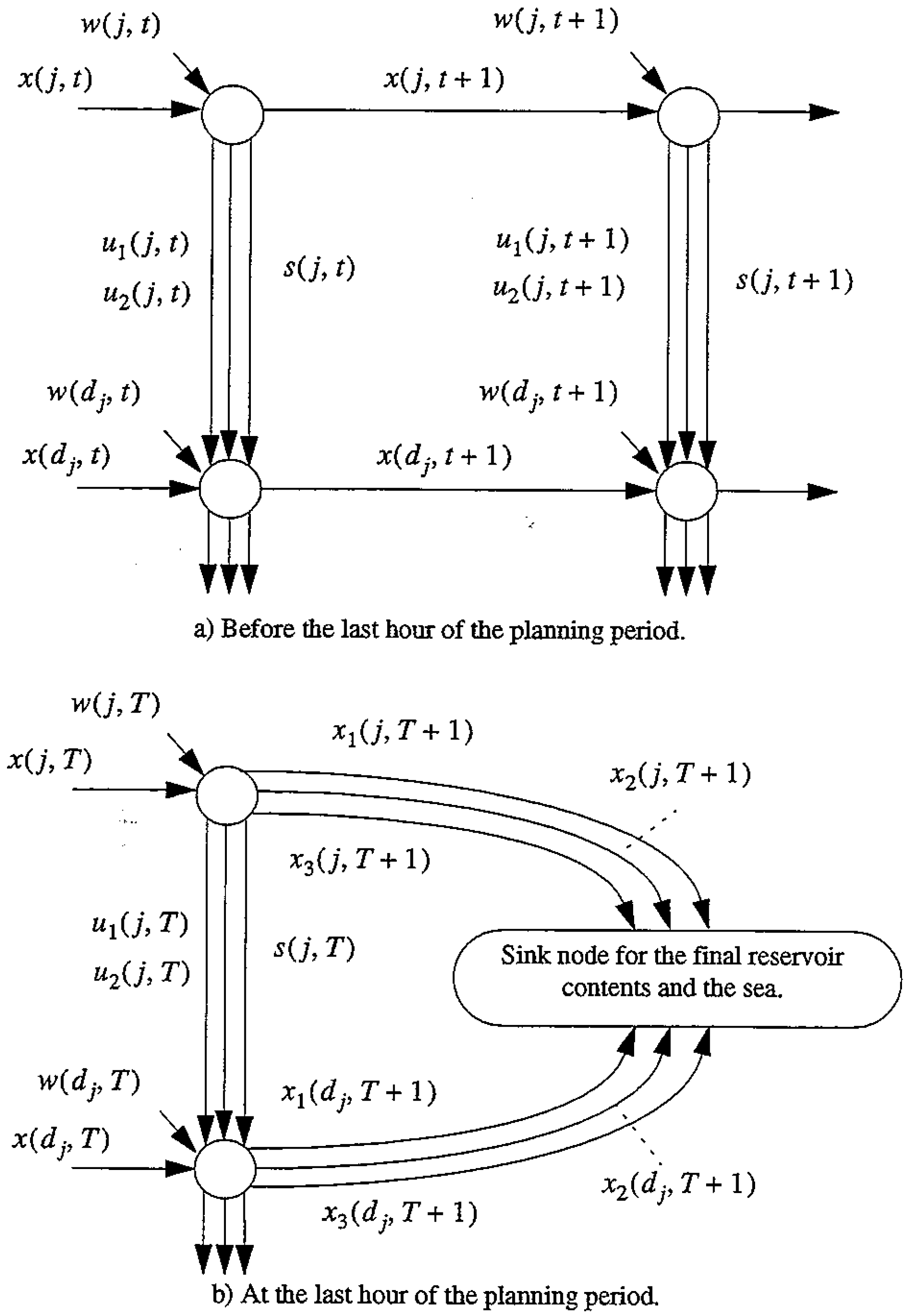


Fig. 5.9 The network structure of the hydro problem in step 1.

cost will be higher for a state with longer down time. When we go from an off-line state to an on-line state we have to add the start-up cost to the cost of generation. Since there are no minimum up times in this case, there will only be one on-line state and one off-line state. The cost of the on-line state is

$$c_g(j,t) = \min_{p_g} \{g(p_g(j,t)) + S(j,t)v(j,t) + p_g(j,t)[-\lambda(t) \\ \zeta_{\text{up}}(j,t) - \zeta_{\text{down}}(j,t) - \zeta_{\text{up}}(j,t+1) + \zeta_{\text{down}}(j,t+1)]\} \tag{5.7}$$

For the on-line states the optimal generation for the subproblem can be found by differentiating (5.7). This gives:

$$p_g(j,k)_{\lambda,\zeta} = \begin{cases} \underline{p}(j) & \text{if} \quad -\dfrac{a_1'(j)}{2a_2(j)} \le \underline{p}(j) \\ -\dfrac{a_1'(j)}{2a_2(j)} & \text{if } \underline{p}(j) \le -\dfrac{a_1'(j)}{2a_2(j)} \le \bar{p}(j) \\ \bar{p}(j) & \text{if} \quad -\dfrac{a_1'(j)}{2a_2(j)} \ge \bar{p}(j) \end{cases} \tag{5.8}$$

where

$$a_1'(j) = a_1(j) - \lambda(t) + \zeta_{\text{up}}(j,t) \\ -\zeta_{\text{ner}}(j,t) - \zeta_{\text{up}}(j,t+1) + \zeta_{\text{ner}}(j,t+1) \tag{5.9}$$

For the lowest state (if the unit starts during the hour) we have to change $\bar{p}(j)$ to $\bar{p}_{\text{cold}}(j)$.

<u>Subproblem for the power exchanges</u>

The subproblem for the power exchanges will find the optimal purchase and sale subject to the energy shadow price $\lambda(t)$

$$\varphi_e(\lambda) = \min_{p_e} \left\{ \sum_{t=1}^{T} \sum_{j \in J_e} [\gamma_j(t) - \lambda(t)] p_e(j,t) \right\} \tag{5.10}$$

The optimal power exchanges will then be

$$p_e(j,t)_\lambda = \begin{cases} \underline{p}_e(j) & \text{if} \quad \gamma_j(t) - \lambda(t) \geq 0 \\ \bar{p}_e(j) & \text{if} \quad \gamma_j(t) - \lambda(t) < 0 \end{cases} \tag{5.11}$$

Initialization of dual variables

In an on-line application the dual variables from yesterday's planning would normally be used as start values for the optimization of the coming day. Since there is no yesterday in the tests in chapter 6, I have estimated the maximum and minimum values of λ and calculated the other start values for λ by linear interpolation with the load as the independent variable:

$$\underset{\sim}{\lambda}(t) = \underset{\sim}{\lambda}^{\min} + \frac{\underset{\sim}{\lambda}^{\max} - \underset{\sim}{\lambda}^{\min}}{D^{\max} - D^{\min}}(D(t) - D^{\min}) \tag{5.12}$$

ζ have their start values equal to zero.

5.3.2 Feasible solution

If we now use the primal variable calculated in section 5.3.1 we will probably have mismatches in all load balance constraints (3.55) and maybe in some ramp rate constraints (3.67). This means that the solutions are not feasible. How do we find feasible solutions? We can find a feasible solution by first keeping $[u, x, s]$ unchanged. Therefore the power generated in the hydro system is the same in the primal and dual solutions. The difference between generation and load could then be eliminated with thermal generation and power exchanges. During the finding of a feasible solution the commitment of the thermal plants is unchanged.

The procedure is to first find a feasible solution. After that I fulfil the load at least cost for one hour at a time, without violating the ramp rate constraints. I repeat this until it is not possible to decrease the objective function in the way. Then I check if the current schedule is optimal. If it is, the search for a feasible solution terminates. Otherwise, I change the generation for several hours at the same time to improve the schedule. In the following I will describe the details in this procedure.

First, we have to find a solution which is feasible in the ramp rate constraints and in the load balance. One way to do this is to start with the generation given by (5.8) for the committed units. If we find that the change in generation from one hour to the next is too high, we check how much we change the generation for the first of these two hours. The maximum generation is the generation for the first on-line hour added with the maximum ramp rate summarized up to the hour when we want to change the generation. In the same way, the minimum generation is the generation for the first on-line hour minus the sum of maximum ramp rates up to the hour when we want to change the generation. In this case we may have to change the generation for earlier hours to not become infeasible. If this change is not enough we change the generation for the second of the two hours violating the ramp rate constraint. Then we go to the next hour. When we have done this for all plants we set power exchanges equal to the difference between the load and the total generation.

Next, we run an EDC with ramp-rate constraints. Fig. 5.10 shows a flow chart of the algorithm for sequential EDC dispatch with ramp rate constraints.

After starting in box 1 with a feasible schedule, we will check if the schedule is changed for the hour before or after the hour studied (box 2). If yes, we go to box 3 where we calculate upper and lower bounds for the generation with respect to generation and ramp rate limits. Otherwise, move to box 5. After box 3 we will run an Lagrange based EDC [93] in box 4 and move on to box 5. If we then have checked all hours without any new changes for the hour before and after, the algorithm will terminate. Otherwise we move to the next hour and start again.

However, this procedure will not guarantee an optimal solution. The only guarantee we have is that the schedule of each hour is optimal given the schedule of the hour before and after the hour studied. In Fig. 5.11 the ramp rate constraints are active from hours 18 and 19 to hours 22 and 23. Assume that this is the solution from the sequential EDC. In the sequential EDC we would not be able to decrease the generation for hours 19 and 21, or increase the generation for hours 20 and 22, owing to active ramp-rate constraints. Still, It might be possible that we can lower the cost by changing the generation for the hours where ramp rate constraints are active. This means that we have changed the generation for several hours simultaneously. If we for example want to increase the generation for hour 20 we have to have the same increase of generation for hours 19 and 21.

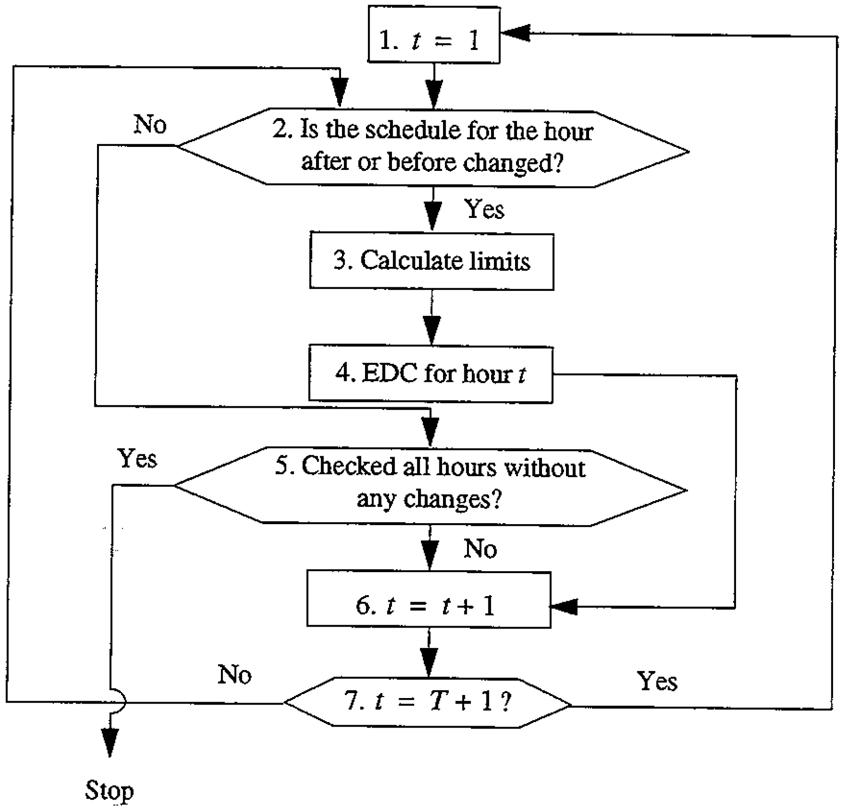


Fig. 5.10 Sequential EDC.

How do we calculate if it is possible to decrease the objective function for clusters of hours where the ramp rate constraints are active? The cost of generation in a plant is given by (3.64). The total cost for the current schedule will then be

$$\begin{aligned} F_{\text{EDC}} = & \sum_{t=1}^{T} \sum_{j \in J_g} [a_0(j) + a_1(j) p_g(j,t) + a_2(j) p_g(j,t)^2] \\ & + \sum_{t=1}^{T} \sum_{j \in J_e} [\gamma_j(t) - \lambda(t)] p_e(j,t) \end{aligned} \tag{5.13}$$

If we change the generation in this plant, we have to change the generation in other plants and/or the power exchanges to fulfil the load balance. An esti-

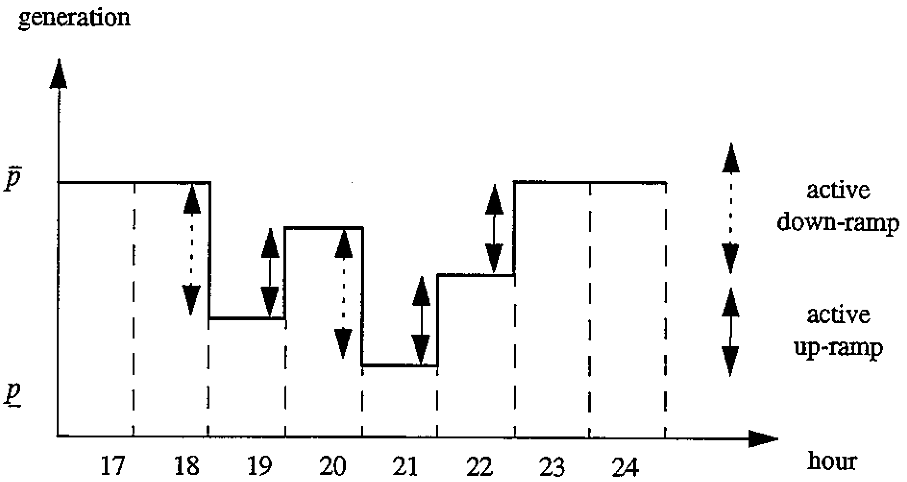


Fig. 5.11 Example of a schedule with active ramp rate constraints.

mate of this cost is the Lagrange multiplier for the load balance equation. We can call the multiplier $\lambda_{\mathrm{EDC}}(t)$ to distinguish it from $\lambda(t)$ in the dual problem. To show this mathematically we can relax the load balance equation

$$F^{\lambda}_{\mathrm{EDC}} = \sum_{t=1}^{T} \sum_{j \in J_g} [a_0(j) + a_1(j)p_g(j,t) + a_2(j)p_g(j,t)^2]$$
$$+ \sum_{t=1}^{T} \sum_{j \in J_e} [\gamma_j(t) - \lambda(t)]p_e(j,t) + \tag{5.14}$$
$$\sum_{t=1}^{T} \lambda_{\mathrm{EDC}}(t)\left[\sum_{j \in J_g} p_g(j,t) + \sum_{j \in J_e} p_e(j,t) - \left[D(t) - \sum_{j \in J_h} p_h(j,t)_\lambda\right]\right]$$

$F_{\mathrm{EDC}} = F^{\lambda}_{\mathrm{EDC}}$ for the feasible schedule since the load balance equation is fulfilled and thereby the second row in (5.14) will be equal to zero. If we differentiate (5.14) we will get:

$$\frac{\partial F^{\lambda}{}_{\mathrm{EDC}}}{\partial p_g(j,t)} = -\lambda_{\mathrm{EDC}}(t) + a_1(j) + 2a_2(j)p_g(j,t) \tag{5.15}$$

(5.15) shows how the cost will change if we increase the generation for unit j during hour t. Now we can calculate the derivative for changing the generation for clusters of hours:

$$\frac{dF_{\mathrm{EDC}}}{dp_g(j, t_1 \le t \le t_2)} = \sum_{t=t_1}^{t_2} \frac{\partial F^{\lambda}_{\mathrm{EDC}}}{\partial p_g(j,t)} \tag{5.16}$$

where t_1 and t_2 are the first and last hour of the cluster we are interested in. For the schedule in Fig. 5.11 there are seven inequality constraints which are active for hours 18 to 23. These are the five ramp-rate constraints, which are marked in the figure, and the maximum generation limit for hours 18 and 23. This means that we have six variables, the generation for hour 18-23, and seven active inequality constraints. In search of a direction to improve the objective function we can let some of these constraints be inactive. After that we calculate a decent direction for the change of generation. If this direction does not violate the constraints we just made inactive, we can decrease the objective function by changing the generation in that direction.

For the hour where the generation limit is active, we have two possibilities. First, let the generation be equal to the limit and let ramp-rate constraints before and after the hour studied be inactive. Secondly, let the active ramp-rate constraint be an equality constraint and let the limit be inactive. For an hour, like hour 19 or 21, with higher generation both before and after itself, it is no point in making the ramp-rate constraints inactive. The reason is that the only feasible direction in this case is when the generation is decreased for the hours before or after the hour studied. The fact is that the decent directions for hours 19 and 21 are to decrease the generation, otherwise the ramp-rate constraints before and after the these hours would be active in the first place. Therefore a small decrease in the generation for the hour before and after the hours above would still keep the ramp-rate constraints active. From the same reasoning for hours with higher generation before and after the itself, as hour 20, it is obvious that it is no point in making the corresponding ramp-rate constraints inactive to calculate possible decent directions. However, if an hour has lower generation before and higher generation after, or vice versa, it is possible to get decent direction by making the ramp-rates inactive. In this case the two options are to let the constraints be active or let the generation be constant and the constraints inactive.

Altogether there are three hours in the schedule (Fig. 5.11) where we must check the possible decent directions for constant generation or active ramp-rate constraints. Since we have either-or alternatives for these hours, the total number of combinations will be equal to the number of hours we have to check squared. This means that we will have eight combinations for the schedule from Fig. 5.11. To illustrate the calculation of a decent direction I will go through this combinations.

The first combination is when all ramp-rate constraints are active. We will get the total derivative from (5.16) with $t_1 = 18$ and $t_2 = 23$. If the derivative is greater than zero, we want to decrease the generation, which is possible. If the derivative is less than zero, we want to increase the generation, which is not possible.

In the second combination we let the generation for hour 18 be constant. This gives that $t_1 = 19$ and $t_2 = 23$. Also in this case it is only possible to decrease the generation.

In the third combination the generation in hour 22 is constant. We will now have two clusters of hours to check. In the first we will have $t_1 = 18$ and $t_2 = 21$. However, for this cluster it is neither possible to increase the generation, owing to upper limit hour 18, nor to decrease the generation, owing to ramp-rate constraint between hours 21 and 22. The second cluster contains only hour 23. For hour 23 it is only possible to decrease the generation. However, this will not result in a lower objective function, otherwise the generation would not be equal to the upper limit.

The fourth combination, with constant generation for hour 23, will be similar to the second combination. Combination number five will have constant generation for both hours 18 and 22. As for combination three, we will have two clusters. The first cluster contains hours 19-21 and the second hour 23. For the first cluster it is possible to increase the generation. As for combination three it is no point in changing the generation for the second cluster.

Combination number six will have constant generation for hours 18 and 23. There will be one cluster containing hours 19-22. For this cluster it is possible to increase the generation. The seventh combination, with constant generation for hours 22-23, will give cluster one in the third combination. The eighth combination, with constant generation for hours 18, 22 and 23, will give cluster one for combination number five.

After repeating this procedure for all thermal plant we have got either a decent direction or found out that the current schedule is optimal.

Fig. 5.12 shows an algorithm for treating the active set of ramp rate constraints. In box 1 we use the above described procedure to check if the cur-

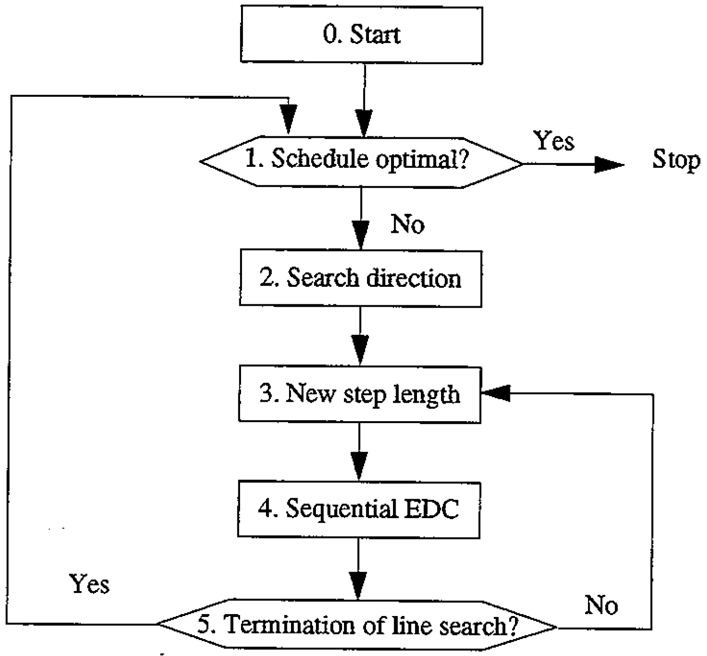


Fig. 5.12 Active set for ramp rate constrained EDC.

rent schedule is optimal. If it is, we stop. Otherwise we calculate search directions in box 2. If we are in this box for the first time we will use decent direction from box 1. Otherwise we use the conjugated gradient [53], which we calculate from the decent direction in this iteration and the search direction in the previous iteration. This will prevent the search from zig-zaging. Then we calculate a new step length in box 3. If it is the first time with a new search direction we will calculate the maximum step length without any violations of other constraints. Otherwise we will use the golden section rule [53] to determine the step length. The next step is to solve a sequential EDC (Fig. 5.10). Above, the start point for the sequential EDC was the first feasible solution. In box 4 (Fig. 5.12) the start point is the schedule from box 1 plus the search direction multiplied by the step length. Since several start points in the sequential EDC can lead to the same schedule, it might be possible that the solution emerges in box 4 during different iterations. Finally, in box 5 we check if we are close to optimal in the line search or if we should go back to box 3. If we terminate the line search and go back to box 1 it might be possible that the active set is changed.

5.3.3 Updating the dual variables

A common way to solve dual problems is to use the subgradient method [26]. The subgradient is the mismatch in the corresponding relaxed constraint. The subgradient in (3.55) can be calculated as

$$\Delta D(t) = D(t) - p_h(t)_\lambda - p_g(t)_{\lambda,\zeta} - p_e(t)_\lambda \tag{5.17}$$

and in (3.67) as

$$\Delta\delta_{\mathrm{up}}(t) = \delta_{\mathrm{up}}(j) - p_g(j,t)_{\lambda,\zeta} + p_g(j,t-1)_{\lambda,\zeta} \tag{5.18}$$

$$\Delta\delta_{\mathrm{down}}(t) = \delta_{\mathrm{down}}(j) + p_g(j,t)_{\lambda,\zeta} - p_g(j,t-1)_{\lambda,\zeta} \tag{5.19}$$

where

- $\Delta D(t)$ is the subgradient in (3.55).
- $\Delta\delta_{\mathrm{up}}(t)$ is the subgradient for the upper limit in (3.67).
- $\Delta\delta_{\mathrm{down}}(t)$ is the subgradient for the lower limit in (3.67).
- $p_h(t)_\lambda$ is the hydro generation during hour t, for a given set of multipliers λ.
- $p_g(t)_{\lambda,\zeta}$ is the thermal generation during hour t, for a given set of multipliers λ and ζ.
- $p_g(j,t)_{\lambda,\zeta}$ is the thermal generation for plant j during hour t, for a given set of multipliers λ and ζ.
- $p_e(t)_\lambda$ is the energy exchange during hour t, for a given set of multipliers λ.

The dual variables are then updated with a step in the subgradient direction

$$\lambda(t) \Leftarrow \lambda(t) + \theta_n \Delta D(t) \tag{5.20}$$

$$\zeta_{\mathrm{up}}(j,t) \Leftarrow \max\{0, \zeta_{\mathrm{up}}(j,t) + \theta_n \Delta\delta_{\mathrm{up}}(j,t)\} \tag{5.21}$$

$$\zeta_{\text{down}}(j, t) \Leftarrow \max\{0, \zeta_{\text{down}}(j, t) + \theta_n \Delta\delta_{\text{down}}(j, t)\} \tag{5.22}$$

where θ_n is the step length in iteration n.

Since the ramp rate constraints are inequality constraints, the Lagrange multipliers, δ, must be greater than zero. This means that if one of these multipliers is set to a value lower than zero in the updating procedure, we set it to zero.

To obtain convergence in the subgradient method there are two conditions on the step length

$$\sum_{n=1}^{\infty} \theta_n = \infty \tag{5.23}$$

$$\sum_{n=1}^{\infty} \theta_n^2 < \infty \tag{5.24}$$

If the optimal value is known, the step length could be determined by the Polyak rule II [86]:

$$\theta_n = \Lambda \cdot \frac{\varphi^* - \varphi_n}{\|[\Delta D, \Delta\delta]\|^2} \tag{5.25}$$

where

- $0 < \Lambda < 2$
- φ^* is the optimal value
- φ_n is the dual objective in iteration n

If the step length is determined as above we will have $\varphi_{n+1} > \varphi_n$.

However, it is not the common situation that φ^* is known. Instead we can use the following heuristic rules to determine the step length [25]:

$$\theta_n = \xi_n \cdot \frac{\hat{F}_n - \varphi_n}{\|[\Delta D, \Delta\delta]]\|^2} \tag{5.26}$$

where

- $\xi_n = \psi \cdot \xi_{n-1}$ if φ has not increased during the n_p latest iteration, otherwise $\xi_n = \xi_{n-1}$.
- $0 < \psi < 1$
- $\hat{F}_n$ is the until iteration n best primal solution found.

There are also other methods like augmented Lagrangian [11] and [46], and bundle methods [54], [57] and [58], which have been used to update the dual variables in power system planning problems. In augmented Lagrangian a penalty term is added to the objective function. The penalty term consists of penalty coefficients multiplied by the sum of the quadrate of the mismatches in the relaxed constraints. This penalty term is linearized in the decomposition. The problem with augmented Lagrangian is that the solution performance is very sensitive to the choice of the penalty term. If the term is small the penalty term will not affect the solution. On the other hand, if the term is too large the solution may get stuck if we have discrete variables. Bundle methods use a bundle of subgradients to compute a new update direction. These methods have gained popularity during recent years. In [83] a bundle method is used to find the optimum of the dual problem in thermal unit commitment. After that augmented Lagrangian is used to find a primal feasible solution from the dual optimum.

5.3.4 Criteria for Convergency

We need a criterion for convergency so we know when to stop the iterative process (Fig. 5.8, box 3). The simplest way is to set up a limit for the maximum number of iterations. If we have run the method several times we will approximately know how many iterations it takes to come close to the optimum. However, this will not be a robust criterion. If we for example want to use the method on another system or change something in the system we do not know how the method will work.

To our estimate how far we are from the optimal solution we can use the dual objective, which is a lower bound on the primal objective. We can stop the iterations when the difference between the primal and the dual objective function, the duality gap, is satisfactorily small.

In the planning problem in the thesis the dual gap will be different for different load situations. If we do not have a good idea of the size of the dual gap in the optimum it can be difficult to use the dual gap as convergency criterion we have to use different stop limits on the duality gap to get a fair comparison. Instead I will *stop when the duality gap has not decreased for a specified number of iterations*. The solution of the problem is then the best found primal solution. In step 1 the convergency limit is 20 iterations without any decrease in the duality gap.

5.4 Step 2: Piecewise linear model with head dependence

Step 1 gives a preliminary schedule. The aim of step 2 is to compensate for the head dependence in this schedule. In step 2 the unit commitment of the thermal plants will be unchanged.

In this step the model of hydro generation is piecewise linear and the model of the thermal generation is an aggregated cost function for the committed units and power exchanges. This function is the value of the optimal solution to a ramp rate constrained EDC with the load $D_{\mathrm{EDC}}(t)$ for hour t. It is now possible to write the objective function (3.54) as (remember that $\chi(j, t) = 0$ in the piecewise linear model):

$$F = C(D_{\mathrm{EDC}}) + \sum_{j \in J_h} \sum_{i=1}^{3} \sigma_i(j) x_i(j, T+1) \tag{5.27}$$

where $C(D_{\mathrm{EDC}})$ is the optimal value to F_{EDC} in a ramp rate constrained EDC. This means that it is necessary to solve a ramp rate constrained EDC for each calculation of $C(D_{\mathrm{EDC}})$. We can write $D_{\mathrm{EDC}}(t)$ as a function of the total hydro generation, $p_h(t)$, by rewriting (3.55) as:

$$D_{\mathrm{EDC}}(t) = D(t) - p_h(t) \tag{5.28}$$

The problem of finding an optimal solution can now be formulated as a nonlinear problem by substituting (5.28) into (5.27):

$$\min_{u,\,x,\,s}\ \{F\} = \min_{u,\,x,\,s}\left\{C_h(p_h) + \sum_{j \in J_h}\left[\sum_{i=1}^{3}\sigma_i(j)x_i(j, T+1) - \sum_{t > T-\tau_{j,d_j}}\rho_{d_j}\left[\sum_{i=1}^{I_j}u_i(j,t) + s(j,t)\right]\right]\right\} \tag{5.29}$$

subject to (3.59), (3.61)-(3.63) and (3.90).

where $C_h(p_h)$ is $C(D_{\text{EDC}})$ with the substitution (5.28). All costs and constraints related to the thermal system and power exchanges are implicitly included in $C_h(p_h)$.

The linearized objective function will be:

$$\min_{u,\,x,\,s}\left\{\sum_{j \in J_h}\left[\left[\sum_{t=1}^{T}\sum_{i=1}^{I_j}c_u(j,t,i)u_i(j,t) + c_x(j,t)x(j,t) + c_s(j,t)s(j,t)\right] + \sum_{i=1}^{3}\sigma_i(j)x_i(j,T+1)\right]\right\} \tag{5.30}$$

where

$$\begin{aligned} c_u(j,t,i) = {} & -\lambda'(t)\cdot[\Gamma_i(j) + \alpha(j)x(j,t) - \beta(j)x(d_j,t)] \\ & -\rho_{d_j}1(t > T-\tau_{j,d_j}) \end{aligned} \tag{5.31}$$

$$c_x(j,t) = -\lambda'(t)\left[\alpha(j)u(j,t) - \sum_m \beta(m)u(m,t)\right] \tag{5.32}$$

$$c_s(j,t) = -\rho_{d_j}1(t > T-\tau_{j,d_j}) \tag{5.33}$$

$$\lambda'(t) = \frac{\partial C_h(p_h)}{\partial p_h(t)} \tag{5.34}$$

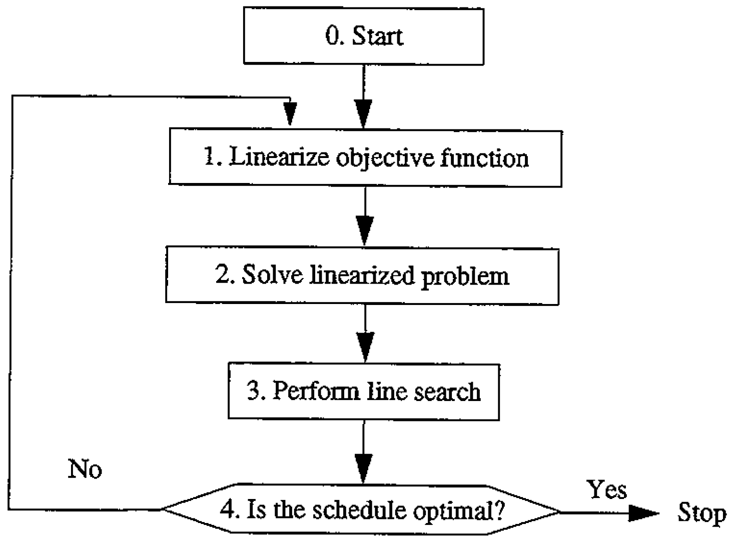


Fig. 5.13 Flow chart for step 2.

$$1(t > T - \tau_{j,d_j}) = \begin{cases} 1 \text{ if } t > T - \tau_{j,d_j} \\ 0 \text{ if } t \le T - \tau_{j,d_j} \end{cases} \tag{5.35}$$

$\lambda'(t)$ is equal to $\lambda_{EDC}(t)$ in the optimal solution to the ramp rate constrained EDC.

Assume that the objective function is linearized in the best found solution from step 1, call the point $[u, x, s]_0$. The solution of this linearized problem will give a new feasible point $[u, x, s]_1$. Since the constraints are linear and thereby convex all points between these two points will be feasible. Then it will be possible to minimize the objective function between these points:

$$\min_{0 \le \iota \le 1} \{F([u, x, s]_0 + \iota([u, x, s]_1 - [u, x, s]_0))\} \tag{5.36}$$

Call the optimal point $[u, x, s]_2$, The next step is to linearize the objective function in this new point and after that repeat the procedure until there is no decrease in the objective function. This is called the Franke-Wolfe method [41]. Fig. 5.13 shows a flow chart for the Franke-Wolfe method. After starting in box 0, the objective function is linearized as above, in box 1. Then box 2 calculates a new point by solving the linearized problem. This problem is the same problem as the hydro subproblem in step 1, where $\lambda'(t)$ replaces $\lambda(t)$. This means that the problem can be solved by linear network programming as in step 1. In the third box the line search is performed. As in the ramp rate constrained EDC the golden section rule is used. Since the objective function is calculated in each new point in the line search, we have to calculate $C_h(p_h)$ in each of these points. Therefore we have to do a ramp rate constrained EDC in each point in the line search. Finally, we check in box 4 if the new point is satisfactorily close to the previous point. If it is, then stop, otherwise go back to box 1.

In the next step I will use the heads from the solution in step 2.

It should be mentioned that the head dependence makes the objective function in (5.29) non-convex. This result in that the Franke-Wolfe method only guarantees a local optimum. However, in step 3 the schedule will be somewhat changed since the hydro model is more detailed in this step. Since the schedule is going to change, there is no need for finding the exact compensation for head dependence.

5.5 Step 3: Mixed integer model

The aim of the third step is to find the optimal schedule of the hydro and thermal plants. This step is, as step 1, based on Lagrange relaxation. In this step two more sets of constraints are relaxed and the head is assumed to be equal to the head in the solution from step 2.

Since the hydro model in this step includes integer variables, it can be difficult to find a feasible solution of high quality without changing the integer variables for the hydro systems. Thus I have two parts in the feasible solution method. The first part, which keeps the integer variables unchanged is based on linear network programming (LNP). In the second part we will change the integer variables without violating the feasible constraints: The aim is to get a feasible solution in the first place and a better solution if the solution already is feasible. However, the second step is much more time consuming than the first, so we cannot afford to go into this step in every iteration. If the objective function in the LNP is among a certain number of the best LNP-solutions up to this iteration, we will enter the heuristic search.

The solution method is summarized in Fig. 5.14. As in step 1, we start by initializing the dual variables (box 0). After that box 1 evaluates the dual objec-

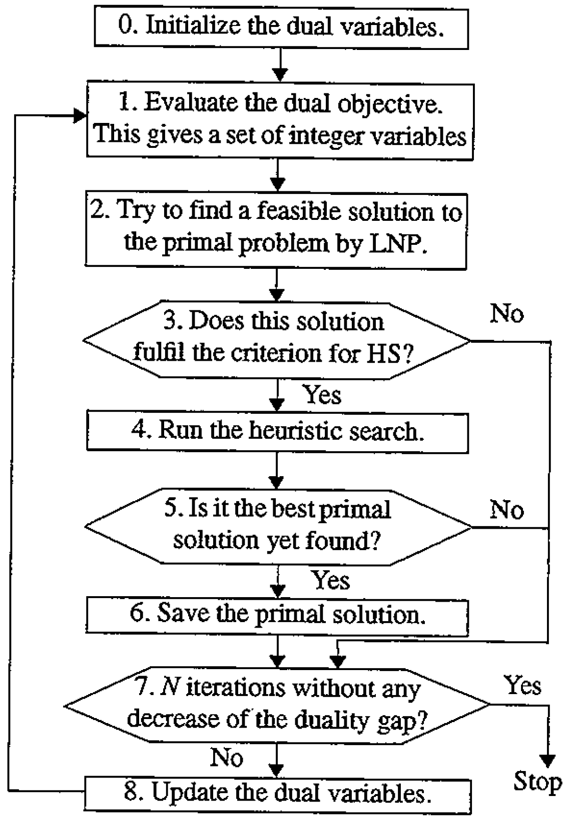


Fig. 5.14 Flow chart of the algorithm for step 3.

tive according to the formulas in the next section. Boxes 2-4 contain the feasible solution method. The first part of the feasible solution method is the LNP which keeps the integer variables unchanged (box 2). Then we apply the above mentioned criterion in box 3. If the answer is yes we will go further on to the second step of the feasible solution method (box 4). This step uses a heuristic search (HS), which will be described later in this chapter. In box 5 we check if it is the best solution yet found. If that is the case we save the solution (box 6). Box 7 checks the convergence and box 8 updates the dual variables.

Since the schedule from step 3 is somewhat changed compared with that of step 2, the compensation for head dependence will not exactly correspond to the actual head. If this difference is large, a possible extension could be to add a step with the Franke-Wolfe method after step 3. This time, the commitment of the hydro plants is unchanged. In [69] we have done something similar. The difference in [69] is that there are no ramp rate constraints for the thermal units and that the operation cost is a linear function. This means that there are no coupling between different hours in the scheduling of the thermal plants. Thus, the aggregated cost function thermal generation and power exchanges can be divided into a cost function for each hour. This cost function will be piecewise linear, since the operation cost is linear. That the cost function is separable and piecewise linear will decrease the computation time essentially. In [69] we also took the non-linearities of generation characteristic into consideration. In order to solve this problem we used non-linear network programming.

5.5.1 Dual problem

In my licentiate thesis [64] I compared two different kinds of relaxations for a problem with the same hydro power model. One was to relax the reservoir balance. The drawback with this relaxation is that we do not consider the network structure of the reservoir balance in the dual problem, since the reservoir balance is relaxed. As the network structure is lost the reservoir contents in the dual problem are determined only by difference in dual variables for the reservoir balance for the studied and the following hours. This will cause us some difficulties. First, the reservoir content will be at its upper bound or zero, depending only on these dual variables. The reservoir content from the dual solution will be of little help in finding primal feasible solutions. Secondly, when the water values are changed in the iterative process some of the reservoir contents will change from one bound to the other. The dual solution in reservoir contents will be very sensitive to changes in the dual variables. This will result in slow convergency.

Considering the network structure in the dual problem will probably lead to a better estimate of the primal optimum since more of the original structure of the primal problem is kept. The disadvantage considering the network structure in the dual problem is that the computation time for one dual iteration probably will be longer. I will come back to the discussion about the advantages and disadvantages of the different relaxations after the mathematical description of the relaxation where the network structure is kept.

To keep the network structure in formulation of the dual problem an extra constraint is introduced, so-called variable splitting (see [34] and Appendix B). The discharge will be represented by two sets of variables:

$$q_k(j,t) = \begin{cases} u_k(j)z_k(j,t) & \text{for } k = 1, \ldots, K_j \\ u_{K_j+1}(j,t) & \text{for } k = K_j + 1 \end{cases} \tag{5.37}$$

where

$$0 \le q_k(j,t) \le u_k(j) \tag{5.38}$$

$$0 \le q_{K_j+1}(j,t) \le \bar{u}_{K_j+1}(j) \tag{5.39}$$

$\sum_{k=1}^{K_j+1} q_k(j,t)$ is substituted for $u(j,t)$ in the reservoir dynamics equations (3.59). This means that there are one set of discharge variables, u, in the integer constraints ((3.88), (3.91) and (3.93)) and another one, q, in (3.59).

We can now rewrite (5.37) as $q_k(j,t) - u_k(j)z_k(j,t) = 0$ and $q_{K_j+1}(j,t) - u_{K_j+1}(j,t) = 0$. Then we can formulate the dual problem by relaxing the load balance equation (3.55), reserve requirement (3.56), splitting equation (5.37) and ramp-rate constraints (3.67):

$$\begin{aligned}
\varphi(\lambda, \mu, \kappa, \zeta) = \min_{u_K, q, x, s, z, p_e} \Bigg\{ & F - \sum_{t=1}^{T} \lambda(t)\Bigg[\sum_{j \in J_h} p_h(j,t) + \\
& \sum_{j \in J_g} p_g(j,t) + \sum_{j \in J_e} p_e(j,t) - D(t)\Bigg] - \sum_{t=1}^{T} \mu(t)\Bigg[\sum_{j \in J_h} r(j,t) - R(t)\Bigg] \\
& - \sum_{j \in J_h} \sum_{t=1}^{T} \Bigg[\sum_{k=1}^{K_j} \kappa_k(j,t)[q_k(j,t) - u_k(j)z_k(j,t)] \\
& \quad - \kappa_{K_j+1}(j,t)[q_{K_j+1}(j,t) - u_{K_j+1}(j,t)]\Bigg] \\
& - \sum_{j \in J_g} \sum_{t=1}^{T} \zeta_{\mathrm{up}}(j,t)[\delta_{\mathrm{up}}(j) - p_g(j,t) + p_g(j,t-1)] \\
& - \sum_{j \in J_g} \sum_{t=1}^{T} \zeta_{\mathrm{down}}(j,t)[\delta_{\mathrm{down}}(j) + p_g(j,t) - p_g(j,t-1)] \Bigg\}
\end{aligned} \tag{5.40}$$

subject to (3.57), (3.59)-(3.66), (3.68)-(3.71), (5.38)-(5.39)

Where

- (3.83) is substituted into (3.54).
- (3.85) is substituted into (3.56).
- (3.86), (3.88) and (3.58) are substituted into (3.55). As for step 1 the model does not change the head dependence (3.86) if the reservoir contents change.
- the allowed discharge domain (3.60) is (3.91)-(3.94).
- (3.91) is substituted into (3.55).

The problem (5.40) can be decomposed into four parts:

$$\varphi(\lambda, \mu, \kappa, \zeta) = \varphi_0(\lambda, \mu, \zeta) + \varphi_{ge}(\lambda, \zeta) + \varphi_q(\kappa) + \varphi_z(\lambda, \kappa, \mu) \qquad (5.41)$$

where

- $\varphi_0(\lambda, \mu, \zeta)$ is the constant part.
- $\varphi_{ge}(\lambda, \zeta)$ is the parts for thermal generation and power exchanges, subject to (3.64)-(3.66) and (3.68)-(3.71).
- φ_q, is the hydrological subproblem, subject to (3.59), (3.61)-(3.63), (5.38)-(5.39).
- φ_z is the commitment problem for the hydro units, subject to (3.92)-(3.94).

In the following I will go into details for each part.

<u>The constant part</u>

The constant part is, as in step 1, the right hand side multiplied by the dual variables

$$\varphi_0(\lambda, \mu, \zeta) = \sum_{t=1}^{T}\Bigg[\lambda(t)D(t) + \mu(t)R(t) - \sum_{j \in J_g} \zeta_{\text{up}}(j, t)\delta_{\text{up}}(j) - \sum_{j \in J_g} \zeta_{\text{down}}(j, t)\delta_{\text{down}}(j)\Bigg] \tag{5.42}$$

The parts for thermal power and power exchanges

The parts for thermal power and power exchanges are exactly the same as in step 1, according to (5.6) and (5.10) respectively. If it is a case where the commitment is unchanged, we will skip the dynamic programming part.

The hydrological subproblem

The hydrological subproblem is to find the optimal q-variables, reservoir contents and spillage according to given dual variables κ

$$\varphi_q(\kappa) = \min_{x, s, q}\left\{-\sum_{j \in J_h}\left[\sum_{t=1}^{T}\sum_{k=1}^{K_j+1}\kappa_k(j, t)q_k(j, t) - \sum_{i=1}^{3}\sigma_i(j)x_i(j, T+1) + \sum_{t > T-\tau_{j,d_j}} \rho_{d_j}\left[\sum_{k=1}^{K_j+1} q_k(j, t) + s(j, t)\right]\right]\right\} \tag{5.43}$$

subject to (3.59), (3.61)-(3.63) and (5.38)-(5.39).

This is exactly the same problem as φ_h in step 1 and can by solved by the same technique. $-\kappa_k(j, t)$ will replace $-\lambda(t)\Gamma_i(j)$ as the cost on the arcs representing the q-variables. $-\kappa_k(j, t)$ will be the benefit of discharging water through arc k.

The commitment problem for the hydro units

The hydro unit commitment subproblem can be divided into one subproblem for each hydro plant

$$\varphi_z(\lambda, \kappa, \mu) = \sum_{j \in J_g} \varphi_{z,j}(\lambda, \kappa, \mu) \tag{5.44}$$

The commitment problem for a single hydro plant will be the following:

$$\varphi_{z,j}(\lambda, \kappa, \mu) = \min_{u_{K_j}, z} \left\{ \sum_{t=1}^{T} \sum_{k=1}^{K_j} c_k(j,t) z_k(j,t) + \sum_{t=1}^{T} \chi(j,t) \right\} \tag{5.45}$$

where

$$c_1(j,t) = -\lambda(t) p_1(j) - \mu(t) r_1(j) + \kappa_1(j,t) u_1(j) \tag{5.46}$$

$$c_k(j,t) = -\lambda(t) p_k(j) - \mu(t)[r_k(j) - r_{k-1}(j)] + \kappa_k(j,t) u_k(j) \quad \text{for } k = 2, \ldots, K_j - 1 \tag{5.47}$$

$$\begin{aligned} c_{K_j}(j,t) = {} & -\lambda(t) p_{K_j}(j) - \mu(t) r_{K_j}(j) + \kappa_{K_j}(j,t) u_{K_j}(j) \\ & + \min\{0, -\lambda(t) \Gamma_{K_j+1}(j) u_{K_j+1}(j) \\ & + \mu(t) r_{K_j}(j) + \kappa_{K_j+1}(j,t) u_{K_j+1}(j)\} \end{aligned} \tag{5.48}$$

subject to (3.92)-(3.94).

$c_k(j, t)$ represents the cost of operation in state k, for plant j, during hour t If we for example have a plant with two units, the states will be

0) no unit on-line,

1) unit 1 operating on its local best-efficiency point,

2) unit 2 operating on its local best-efficiency point,

3) both units operating somewhere between their combined local best-efficiency point and the maximum generation-point.

This could be solved by dynamic programming. The costs on the arcs are the state cost according to (5.46)-(5.48) plus the cost of moving from one state to another, see Fig. 5.15. The arcs from states 1 and 2 are neglected to increase the readability in the figure. The costs in the dynamic programming can be expressed as

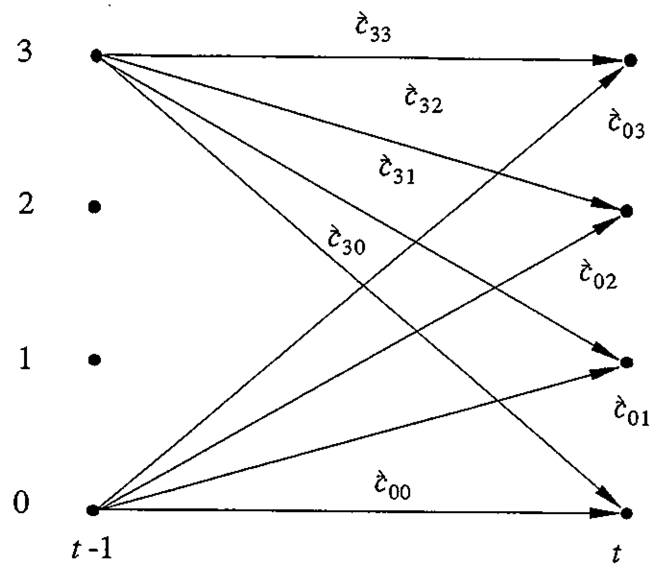


Fig. 5.15 The arcs from the lowest and the highest state in the dyn-p net of the hydro unit commitment problem in step 3.

$$\tilde{c}_{kk'} = c_{k'}(j, t) + c_{\text{start}}(k, k') \tag{5.49}$$

where k is the state for hour $t-1$ and k' is the state for hour t. It should be mentioned that $c_{\text{start}}(k, k') = 0$ if no units are started when we move from state k to state k'.

The result of the variable splitting is that the hydro subproblems can be solved by well known and frequently applied methods for power system planning problem. The hydro subproblems, φ_z and φ_q, are solved by dynamic programming and network programming respectively. In the literature dynamic programming has been proved to be a very efficient technique for solving problems similar to φ_z in thermal systems and network programming is standard for problems like φ_q.

We can interpret the relaxations as if the hydro system sells water to the generation at the price of $\kappa_k(j, t)$. In φ_q we want to determine the optimal use of water, when and how much to "sell" and how much to store at the end of the planning period. In φ_z we want to optimize the generation when the cost of water is $\kappa_k(j, t)$. There will probably be some difference between where and how much the generation wants to "purchase" and the hydro system wants to sell. This means that the solution will not be feasible primarily. In relaxation of the reservoir balance (3.59) we used the difference in the corresponding dual variables between the upstream and downstream reservoir as the cost of the water. What is the difference? In the variable splitting we will keep more of the original structure of the problem than in the relaxation of (3.59), since we in here have to fulfil the reservoir dynamic equation (3.59). As result the variable splitting probably will be a closer relaxation than the relaxation of (3.59). κ will then be a better estimate of the cost of discharging water.

The drawbacks of the variable splitting compared with the relaxation of (3.59) are, first, that we have to solve a linear network problem in each evaluation of the dual problem. This means that each iteration will be more time consuming with variable splitting than the relaxation of (3.59). Secondly, the number of dual variables will increase with relaxation of variable splitting equation compared with the relaxation of (3.59), since instead of one dual variable for each plant and hour we will have the number of κ equal to the number of operation states. The increased number of variables could result in an increase of the number of iterations. However, the variable splitting will cause a lower magnitude of oscillations in the solution procedure. For many κ the subgradient will be equal to zero since many discharge variables will be the same in the solutions of φ_q and φ_z. In my licentiate work [64] I had problems using the subgradient as search directions in the relaxation of (3.59) since the oscillations in variables from one iteration to the next were too large. Instead I used search directions based on the difference between the actual dual variable and the corresponding dual variable in the feasible solution.

5.5.2 Initialization of the dual variables

Before we start the iterative solution method in step 3, we have to initialize the dual variables. Step 1 gives sets of dual variables, λ and ζ. These can be used as start values in step 3. The spinning reserve constraints are often inactive, which means that a good guess for the corresponding dual variables, μ, is to set them equal to zero.

According to the discussion above $\kappa_k(j, t)$ is the price at which the water system "sells" water to the generation. The generation will then "earn" $-\lambda(t)p_k(j)/u_k(j)$ from converting the potential energy into electric power. The linear network programming in step 1 gives optimal dual variables for the load balance equation for each subproblem (5.10). Call these dual variables from the best solution found in step 1 for κ'. The decrease in value for water discharged from reservoir j to reservoir d_j is $\kappa'(j, t) - \kappa'(d_j, t + \tau_{j, d_j})$. If the potential energy should be converted into electric energy $\kappa_k(j, t)$ must be lower than $\lambda(t)p_k(j)/u_k(j)$, otherwise the "cost" is higher than the "earnings". On the other hand $\kappa_k(j, t)$ should be larger than $\kappa'(j, t) - \kappa'(d_j, t + \tau_{j, d_j})$, otherwise it would be uneconomical for the water system to "sell" water to the generation. A good estimate of $\kappa_k(j, t)$ is therefore

$$\underline{\kappa}_k(j, t) = \frac{\lambda(t)p_k(j)/u_k(j) + \kappa'(j, t) - \kappa'(d_j, t + \tau_{j, d_j})}{2} \tag{5.50}$$

5.5.3 Feasible solution

The solution achieved from dual calculations is seldom feasible primarily. There will be a mismatch in the relaxed constraints, (3.55), (3.56), (3.67) and (5.37). I have divided the feasible solution method (box 2-4, Fig. 5.14) into two parts:

Part I: (integer variables unchanged, box 2 in Fig. 5.14) The hydro unit commitment problem (5.44) will give us the integer variables $[z]$. If we keep these integer variables unchanged, we are free to change the discharges on the continuous part (if $z_{K_j}(j, t) = 1$), reservoir content and spillage to get a feasible solution. This will be done in the following way:

- First, fulfil (5.37) by changing the spillage, the final reservoir contents and, if possible, the discharge on the continuous part. This can be achieved by linear network programming (LNP). Then it is possible to calculate the generation in the hydro system.
- Secondly, (3.55) and (3.67) can be fulfilled in the same way as in step 1, by running a ramp rate constrained EDC for the load not covered by hydro.

This does not guarantee that (3.56) is fulfilled. However, during most operation conditions (3.56) will be automatically fulfilled, since spinning reserve problems seldom occur. If there is a mismatch in (3.56) after part I, part II will resolve that mismatch.

The fulfilling of (5.37) is the following network problem:

$$\min\left\{-\sum_{j\in J_h}\left[\sum_{t=1}^{T}[\lambda(t)-\mu(t)]\Gamma_{K_j+1}(j)u_{K_j+1}(j,t)\right.\right.$$
$$\left.\left.+\sum_{t>T-\tau_{j,d_j}}\rho_{d_j}\left[\sum_{k=1}^{K_j+1}q_k(j,t)+s(j,t)\right]-\sum_{i=1}^{3}\sigma_i(j)x_i(j,T+1)\right]\right\} \tag{5.51}$$

subject to (3.59) and (3.61)-(3.63)

$$0\le u_{K_j+1}(j,t)\le \bar{u}_{K_j+1}(j)z_{K_j}(j,t)_{\lambda,\mu,\kappa} \tag{5.52}$$

If a plant is scheduled to operate on the continuous part, then $z_{K_j}(j,t)_{\lambda,\mu,\kappa}=1$ in (5.52), otherwise $z_{K_j}(j,t)_{\lambda,\mu,\kappa}=0$. From the solution of this problem it is possible to calculate the total generation of the system. This problem has almost the same structure as φ_h for step 1, with the difference that the discharge is at one of the points with zero flow or local best-efficiency or at the continuous part. The problem has network structure (Fig. 5.16).

Arcs in the horizontal way correspond to reservoir contents. Where the discharge is scheduled to be at a specified point there is only one arc in the vertical way. This arc represents spillage. If the discharge is scheduled to be on the continuous part there will be one more arc in the vertical way representing this discharge. $\tilde{w}$ in the figure means that the river inflow is adjusted according to the integer variables in the dual problem.

$$\tilde{w}(j,t)=w(j,t)+\sum_{m\in M(j)}\left[\underline{u}(m)+\sum_{k=1}^{z(m,t-\tau_{mj})_{\lambda,\mu,\kappa}}u_k(m)\right]$$
$$-\underline{u}(j)+\sum_{k=1}^{z(j,t)_{\lambda,\mu,\kappa}}u_k(j) \tag{5.53}$$

The mismatch between the load and the generation can then be eliminated by ramp rate constrained EDC as in step 1. This means that the only constraints which still can be infeasible are the reserve constraints.

After this part we will test if the current solution fulfils the criterion for the heuristic search, box 3 Fig. 5.14. As I mentioned earlier the criterion is that if

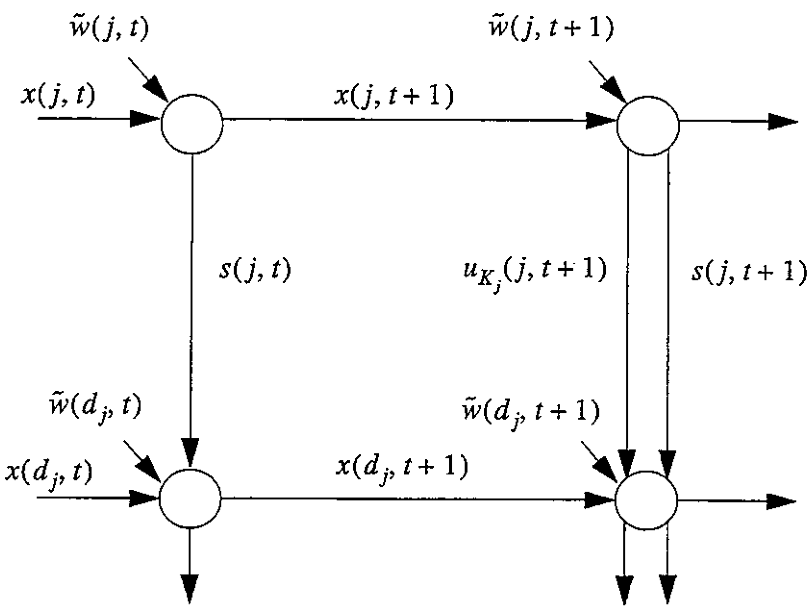


Fig. 5.16 The network structure of the feasible solution method in step 3.

the objective function in the LNP is among a certain number of the best LNP-solutions up to this iteration, we will enter the heuristic search.

Part II: (heuristic search for changing of integer variables in hydro system, box 4 Fig. 5.14) If we calculate a feasible primal solution without changing the integer variables we could get rather far from the optimum. In this case a heuristic method is used to try to change the integer variables to improve the solutions given by the method above.

This method has some similarities to the decommitment method for thermal units in [51]. The basic idea of the heuristic feasible method, which is presented in [71], is to change the discharge in the plants one at a time. The difference between [51] and this work is that we change the generation in the hydro system and thereby have to keep the reservoir balance equations feasible. This leads to that the generation of different plants and hours is tightly coupled.

The heuristic feasible solution method has two modes, see Fig. 5.17. Mode 1 is when there are not enough spinning reserve committed. This means that there is a mismatch in (3.56). In this case the goal is to find a feasible solu-

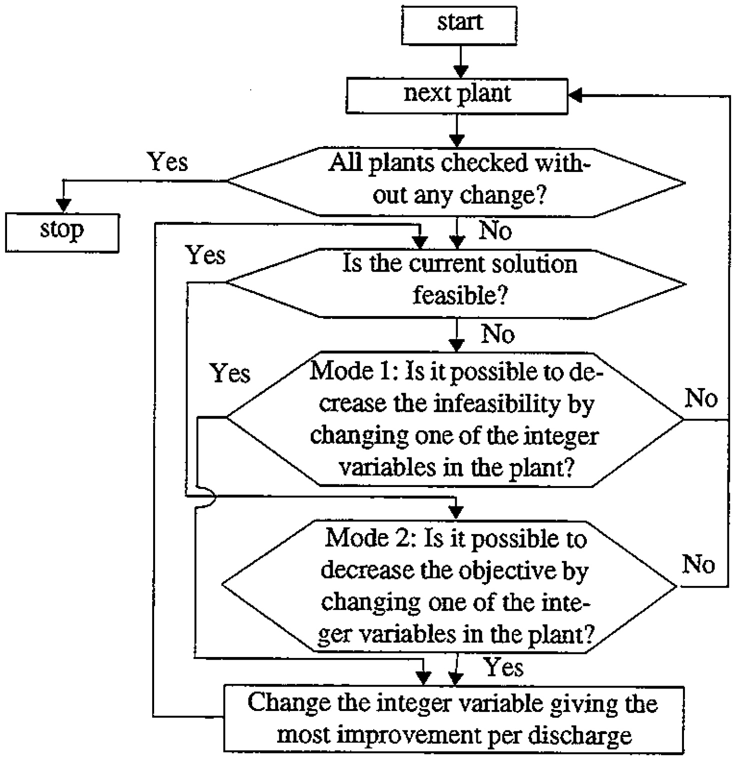


Fig. 5.17 Flowchart of the heuristic algorithm.

tion. Mode 2 is when we have a feasible solution and want to improve this solution without violating any constraints. For every plant we check if it is possible to get a better solution by changing the discharge for one hour. A better solution means a less infeasible solution for mode 1 and a decrease in the objective for mode 2. A better solution can be found in three ways, depending on the operation state of the plant:

- If the plant is operating on the continuous part we can change the discharge within the limits of the continuous part.

- If the plant is operating at a lower state, we can change the operation state. In this case we will only go to a state with one more or one less onlined unit compared with the previous state.

- If the plant is operating on the lower limit of the continuous part, we check decreases of the discharge in the same way as for an operation state lower than the continuous part. The possibilities of increasing the discharge will be checked in the same way as in the first point.

When we have changed one integer variable we check if we can decrease the objective function further by changing the discharge for another hour. This will continue until it is not possible to decrease the objective function by changing the discharge for one hour. In order to keep the hydrological balance of the system when an integer variable is changed for one hour, we have to change some other variable or variables for the plant. This can be done in three ways:

- change of the discharge for another hour.
- change of the spillage.
- change of the final reservoir contents.

For each hour we check if it is possible to change the discharge according to the reservoir limits and, if we are in mode 1, the requirement of the spinning reserve. Then we calculate how this change affects the system objective function (5.29) and, if we are in mode 1, how much the total infeasibility in (3.56) will change. If we are in mode 1 we change the discharge for the hour where the infeasibility decreases the most per change of discharge. If there are several alternatives giving the same decrease in infeasibility per change of discharge, we will choose the one having the most favourable affect on the objective function. Note that the change in the objective function or in the infeasibility is the sum of the change for the hour which is a candidate for a discharge change and the hours by which we compensate this discharge change. If we are in mode 2 we change the discharge for the hour where the cost decreases the most per change of the discharge. This will be an iterative process where we change the discharge for one plant at a time and then move on to change the discharge in the next plant.

As objective function in the heuristic algorithm we have the generation costs of the committed thermal units and the cost for the power exchanges. For each hour we create a piecewise linear cost function by loading the committed thermal units and power exchanges by merit order. With a piecewise linear cost function for each hour there is no need to do ramp-rate constrained EDC for each evaluation of the objective function. In order to construct the piecewise linear cost function I have approximated the quadratic cost function to a linear or piecewise linear cost function for each thermal unit. If the marginal cost of the unit is equal to an exchange price somewhere between the minimum and maximum outputs, I use a piecewise linear model. In this model the point with marginal cost equal to the exchange price will be a breakpoint. The other breakpoints are the points of minimum and maximum generation. If the marginal cost of the unit is not equal to any exchange price between minimum and maximum outputs, I use a linear model between minimum and maximum generation. For the plants equipped with a NO_x-cleaner (see section 3.3) this means that we go back to the cost function before the non-linear approximation.

When there are constraints for ramp rates the situation gets somewhat complicated, since the operation during one hour can affect the cost functions of the hours before and after the hour studied. To include this will counteract the goal of having a simple model of the thermal system in part II. One way to overcome this problem is to set the upper and lower bounds for thermal generation equal to the most restricting of the generation limits or the ramp rate constraints. Assume for example that the generation for one hour and the hour after are 100 and 140 MWh/h respectively, the maximum up-ramp rate is 60 MW/h and maximum generation is 150 MW. The lower limit for the first hour will be set to 140 - 60 = 80 MW. The upper limit for the second hour will be 150 MW, since the upper bound is less than 100 + 60 = 160 MW. However, it might be possible that the generation in thermal plants for the hours with active ramp rate constraints will change in such a way that it will be infeasible in the ramp rate constraint. In the example above the generation can be 80 MWh/h and 150 MWh/h for the first and second hour respectively. Another suggestion to get around the problem is to set the thermal generation limits to values which make it impossible to get infeasible solutions. In the example the ramp rate constraint becomes infeasible with maximum 10 MW. To avoid this we can change the limits from 80 and 150 MW to 85 and 145 MW respectively. Thereby we would not risk that the thermal generation becomes infeasible. A problem that could arise instead, is that it might have been right to change the thermal generation for one hour with the maximum allowed amount according to ramp rate constraints while keeping it unchanged for the other hour.

After the heuristic search the thermal schedule might be changed after the heuristic search and thereby also the cost function. If the cost function is changed I will perform a new heuristic search and continue to do this until the thermal schedule is unchanged. If we have used the first way of tackling the problem with ramp rate constraints, the thermal schedule becomes feasible after we have performed a new EDC. If we have used the second approach we will have new limits for the thermal generation in the next iteration in the heuristic search. The difference between the approaches is that the first approach might underestimate the cost of a change in the hydro system and that the second approach might overestimate the cost of making a change.

The following example will illustrate the heuristic method. Assume that we want to calculate the cost of increasing the discharge of hour $T-1$, this is the arc between node 2 and 5 in Fig. 5.18. What we will gain is that the cost of power exchanges will decrease for hour $T-1$, since we increase the generation for this hour. Assume that we have spillage in hour $T-2$ and that it is no problem to decrease this spillage in order to increase the discharge for hour $T-1$. This is change I in Fig. 5.18. If the spillage in hour $T-2$ was less than the discharge change of hour $T-1$, we have to change some other variable too. Assume that this will be the final reservoir contents. This is change II in Fig. 5.18. Node 7 is a dummy node for the final reservoir contents. The cost of this change is equal to the difference in water storage value between the reservoir of the plant and the downstream reservoir multiplied by the change of the reservoir contents. Assume that we need some more flow to complete the discharge change of hour $T-1$ and that we can get this flow from decreasing the discharge of hour T. This is change III in Fig. 5.18 and its cost is equal to the cost of changed power exchanges for hour T. In the total cost of changing the discharge of hour $T-1$ we must include the change of the start-up cost. If the unit, which now will be on-line for hour $T-1$, was on-line for the hours $T-2$ and T, we will prevent a start-up. As a result we can subtract the start-up cost from the total cost. If the unit was not on-line hours $T-2$ and T, we will cause a start-up and we have to add the start-up cost to the total cost. In other cases we will not change the cost of start-ups. In more generalized cases with more time periods than three, there might be

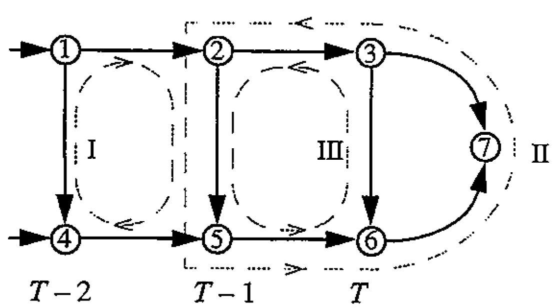


Fig. 5.18 An example of the method. The horizontal arcs represent reservoir contents and the vertical ones represent discharge and spillage.

more alternatives of how to compensate for the changed discharge in hour $T-1$. In this case it can be possible to compensate for the discharge change with changes of discharge and spill for earlier hours than $T-2$. If there are several alternatives of how to compensate for the discharge change in hour $T-1$, we should choose the one which improves the objective function most.

The idea is now to compare the change of the objective function per change of discharge for all possible changes of discharge in which we just move to points with one more on-lined or off-lined. The change of the discharge on the continuous part will be limited by two criteria. The first is the maximum change of the cheapest way of changing the discharge. The second is the maximum change of the discharge without changing the derivative of the power exchange cost function. A detailed description of the implementation of the heuristic algorithm is shown in Appendix C.

5.5.4 Updating the dual variables

As for the piecewise linear model, the dual variables will be updated with subgradient and Polyak rule II. The subgradient will be the mismatches in (3.55), (3.56), (3.67) and (5.37). The dual variables μ and ζ has to be greater than zero since they are Lagrange multipliers of inequality constraints. The mismatches in the relaxed constraints will now be

$$\Delta D(t) = D(t) - p_h(t)_{\lambda,\mu,\kappa} - p_g(t)_{\lambda,\zeta} - p_e(t)_\lambda \tag{5.54}$$

$$\Delta R(t) = R(t) - r_h(t)_{\lambda,\mu,\kappa} \tag{5.55}$$

$$\Delta qu_k(j,t) = \begin{cases} u_k(j)z_k(j,t)_{\lambda,\mu,\kappa} - q_k(j,t)_\kappa & \text{for } k = 1, \ldots, K_j \\ u_{K_j+1}(j,t)_{\lambda,\mu,\kappa} - q_{K_j+1}(j,t)_\kappa & \text{for } k = K_j + 1 \end{cases} \tag{5.56}$$

$$\Delta\delta_{\text{up}}(t) = \delta_{\text{up}}(j) - p_g(j,t)_{\lambda,\zeta} + p_g(j,t-1)_{\lambda,\zeta} \tag{5.57}$$

$$\Delta\delta_{\text{down}}(t) = \delta_{\text{down}}(j) + p_g(j,t)_{\lambda,\zeta} - p_g(j,t-1)_{\lambda,\zeta} \tag{5.58}$$

where

- $\Delta R(t)$ is the subgradient in (3.56).

- $\Delta qu_k(j, t)$ is the subgradient in (5.37).
- $p_h(t)_{\lambda, \mu, \kappa}$ is the hydro generation during hour t, for a given set of multipliers λ, μ and κ.
- $r_h(t)_{\lambda, \mu, \kappa}$ is the spinning reserve in the hydro system hour t, for a given set of multipliers λ, μ and κ.

The dual variables are updated with a step in the subgradient direction

$$\lambda(t) \Leftarrow \lambda(t) + \theta_n \Delta D(t) \tag{5.59}$$

$$\mu(t) \Leftarrow \max\{0, \mu(t) + \theta_n \Delta R(t)\} \tag{5.60}$$

$$\kappa_k(j, t) \Leftarrow \kappa_k(j, t) + \theta_n \Delta qu_k(j, t) \tag{5.61}$$

$$\zeta_{\mathrm{up}}(j, t) \Leftarrow \max\{0, \zeta_{\mathrm{up}}(j, t) + \theta_n \Delta\delta_{\mathrm{up}}(j, t)\} \tag{5.62}$$

$$\zeta_{\mathrm{down}}(j, t) \Leftarrow \max\{0, \zeta_{\mathrm{down}}(j, t) + \theta_n \Delta\delta_{\mathrm{down}}(j, t)\} \tag{5.63}$$

with the step length

$$\theta_n = \xi_n \cdot \frac{\hat{F}_n - \varphi_n}{\|[\Delta D, \Delta R, \Delta qu, \Delta\delta]\|^2} \tag{5.64}$$

where ξ_n is calculated in the same way as in step 1.

The criteria for convergency will be the same as in step 1, but the convergency limit is 50 iterations without any decrease in the duality gap.

6
Tests and Results

Theory developed in the previous chapter is here tested on a numerical example. There are three different test days: high load (winter), medium load (autumn) and low load (summer). Within the test days there are different load scenarios. The test shows that the method works well for cases with normal thermal commitment. At the end of the chapter a simplified model is used to calculate unavailability cost in a river through Monte Carlo simulation.

6.1 Test System

In this chapter I apply the models and methods to a test system.

The hydro test system consists of parts of the Ume, Skellefte and Lule rivers (see Fig. 6.1). In the test system there are 32 plants. The installed capacity of the system is 5850 MW. In this system there are 59 units. Data for the system is achieved from SEVAP[1]-tables, which are based on measurements of the plant efficiency. The start-up cost for hydro plants is set to 20 SEK per MW unit nominal output, which was the estimated average value from the survey in chapter 4.

The thermal test system is made up by 24 thermal plants, all located in Sweden. The marginal costs of the plants amount to between 100-300 SEK/MWh and the capacities are between 12-335 MW. The total capacity of the thermal plants is 3400 MW.

To get a fair estimation of the problem size I have minimized the number of variables and constraints by substitution. This means for example that there are no variables for the hydro generation, since its expression is substituted in the power balance equation. Now there are:

1. SEVAP=System för Effektivare VAttenkraftProduktion (In English: System for more efficient Hydro Power Generation).

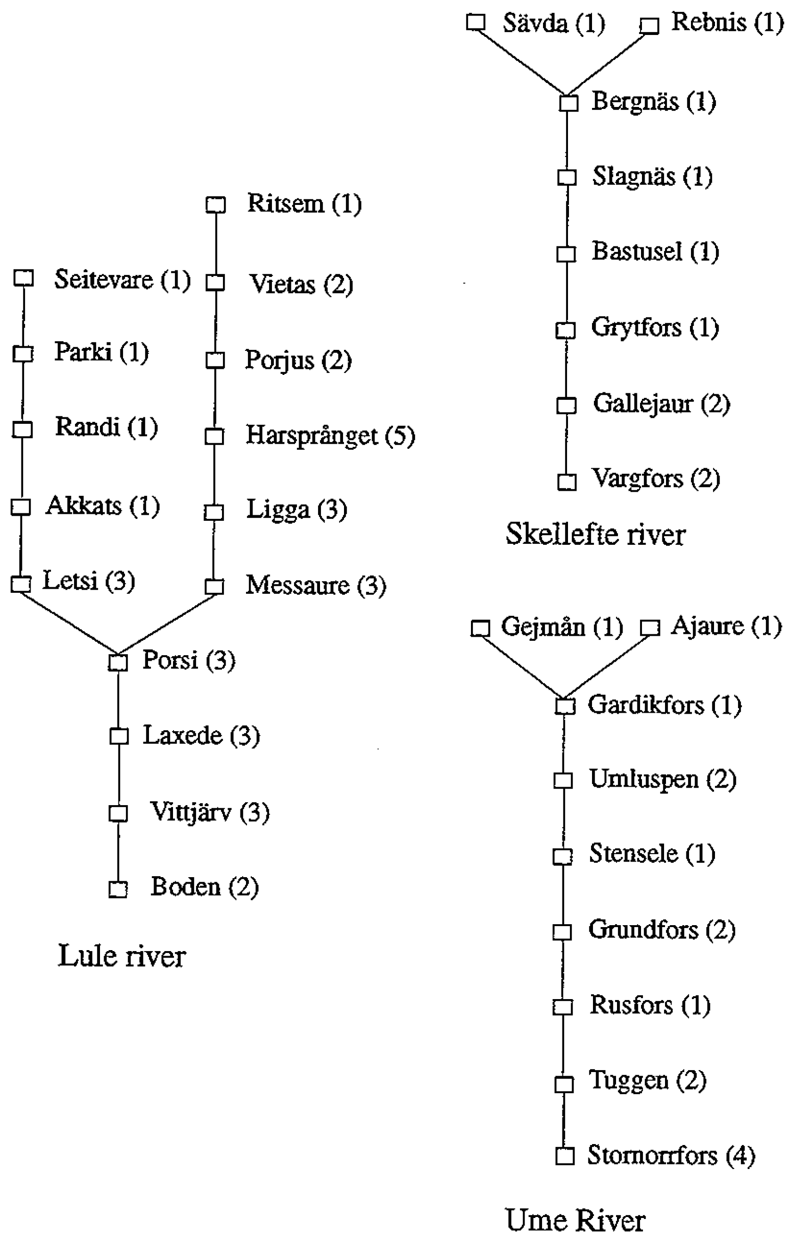


Fig. 6.1 Test system, number of units in brackets.

- one load balance equation (3.55) for each hour.
- one equation for spinning reserve requirements (3.56) for each hour.
- one reservoir dynamics equation (3.59) for each hour and plant.
- one equation for the lower and one for the upper bounds for the thermal generation (3.65) for each hour and plant.
- one equation for the lower and one for the upper bounds for the ramp rate constraints in the thermal generation (3.67) for each hour and plant.
- one equation for the relationship between state and start variables for the thermal generation (3.68) for each hour and plant.
- one equation for the relationship between state and transition variables for the hydro generation (implicitly expressed by (3.83)) for each hour, plant and local best-efficiency point.
- the number of local best-efficiency points minus one, number of equations for the relationship between state variables in the hydro plants (3.92) for each hour and plant.
- one equation for the upper bound of the discharge on the continuous part (3.93).

The total number of constraints will be about 7100. There will be continuous variables for:

- discharge on the continuous part, one for each hour and hydro plant.
- spillage, one for each hour and hydro plant.
- reservoir contents, one for each hour and hydro plant, plus two per plant for the soft constraints for the final contents.
- thermal generation, one for each hour and plant.
- purchase and sale, one for each hour and price.

This will result in a total number of continuous variables of about 2900. There will be integer variables for:

- states in the model of generation as functions of the discharge, one for each local best-efficiency point, hour and plant.
- transition between states in the hydro generation, one for each local best-efficiency point, hour and plant.
- states in the thermal generation, one for each hour and plant.
- start-ups of the thermal plants, one for each hour and plant.

The number of integer variables in the model is about 4600.

6.2 Numerical examples

The test cases will be tested for three different loads;

- heavy load, winter day.
- medium load, autumn day.
- light load, summer day.

The load in these examples is obtained as real load of Vattenfall minus the generation assumed to be produced by nuclear plants. The remaining load is

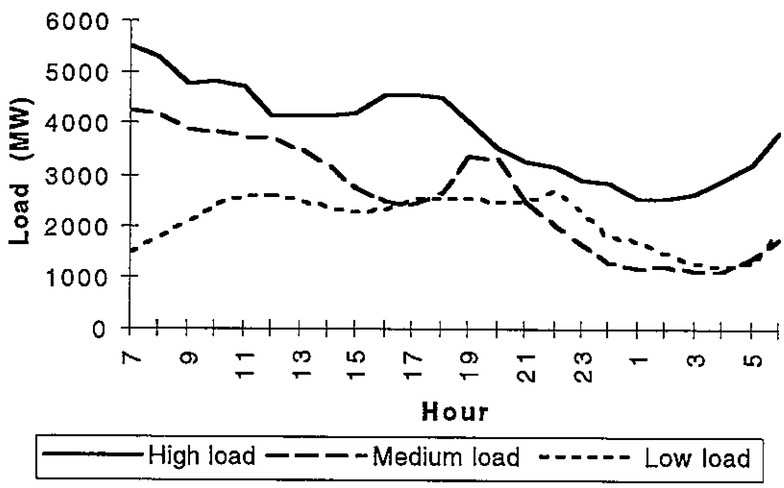


Fig. 6.2 The load in the base cases.

thereafter scaled to the size of the test system. The load profiles of the base cases are shown by Fig. 6.2. The required spinning reserve is set to 300 MW for all hours. For each test day I have one base case and two cases with extra load and changed exchange prices and in one of the cases changed water storage value. The aim of the cases with extra load is to test the model for situations with a substantial part of thermal generation. These test cases are maybe somewhat unrealistic, but the purpose is to see how the model works for these extreme circumstances.

The prices for purchased and sold energy for the base case are shown in Table 1. The interval of 100 MWh purchase and sale will help the finding of feasible solutions if only hydro units are committed and operating on discrete points.

Energy (MWh)	Purchase price (SEK)	Selling price (SEK)
0-100	110	90
>100	125	80

Table 6.1: Prices in the base case.

The value of stored water is 100 SEK/MWh. To get the value of stored water in a specified reservoir, we have to sum the forecasted generation equivalents in MWh/HE for the plant corresponding to the reservoir and all downstream plants. When we multiply the value of stored water by the aggregated forecasted generation equivalent, we will get the water storage value in SEK/MWh.

The load is 2000 MWh higher for each hour in the cases with extra load. In the first case with extra load I will assume that the water storage value is unchanged. I have changed the purchase prices from table 6.1 to 150 and 200 SEK/MWh. The increased load and purchase prices will lead to more thermal generation. I have not changed the initial values for the thermal plants. This means that the thermal plants have to be started at the beginning of the planning period.

In the second case with extra load I have changed the water storage value in such a way that the amount hydro used is approximately the same as in the base case. In this case I have changed the purchase prices from table 6.1 to 1,5 and 2,0 times the water storage value and the selling price to 0,5 times the water storage value. This will force the system to operate more as a closed system. I have also changed the initial states of the thermal plants.

Plants with average variable cost above the water storage value are committed when the planning period starts.

In the numerical examples I will test which are the settings of some parameters. These are:

- treatment of final reservoir contents: if soft constraints for the final reservoir contents should be used. I have set the penalty cost in the soft constraints to 10 per cent of the difference in water storage value between the plants and their downstream plants.

- treatment of heuristic search: when the heuristic search should be performed. If the objective function in the LNP is among a certain number of the best LNP-solutions up to the current iteration, we will enter the heuristic search, see section 5.5. In the numerical example I will test different values for this number of best LNP-solutions. I have tested numbers between 2 and 10.

- treatment of ramp rate constraints in the heuristic search, see section 5.5.3. I will test the following alternatives for the bounds in thermal generation:

  1) *the most restricting of the generation limits or the ramp rate constraints.*

  2) the mean value of *the most restricting of the generation limits or the ramp rate constraints* and *limits making it impossible to get infeasible solutions.*

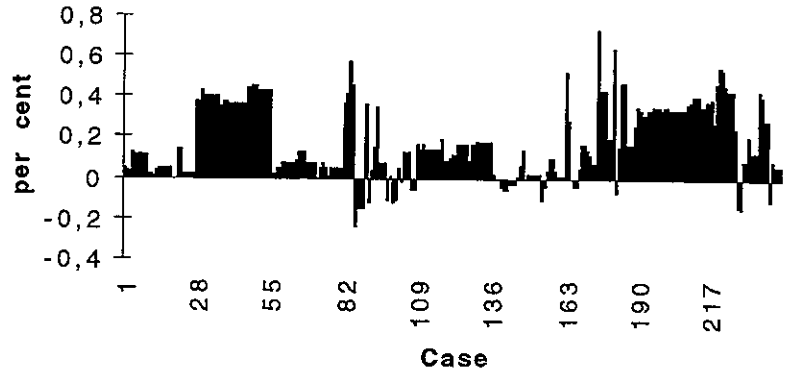
  3) *the limits making it impossible to get infeasible solutions.*

I have used a Power Macintosh (120 MHz) in the tests. First I will test the settings of the parameters and then I will use one set of parameters to evaluate the method for the different test cases.

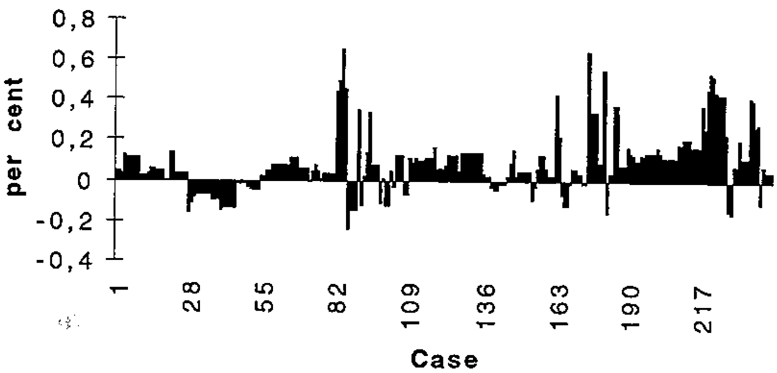
6.2.1 Settings of parameters

<u>Soft constraints for the final reservoir contents</u>

In a convex problem solved with an exact method soft constraints always will lead to an unchanged or better value of the objective function. In this case when the problem is non-convex and solved with an approximative method it



a) Difference in objective function



b) Difference in duality gap

Fig. 6.3 Comparison of using versus not using soft constraints for the final reservoir contents. The difference is shown in per cent of the mean of objective function with and without soft constraints. Note that the same per cent does not correspond the same value in SEK for different load situations.

is not guaranteed that the objective function will decrease, even though it is very likely. The results of the test are shown in Fig. 6.3. The figure shows the difference in objective value and duality gap between the cases with and without soft constraints while other parameters are unchanged. The first 27 cases are from the high load test day with no extra load. The first 9 of these

27 cases use the first way to treat ramp rate constraints in the heuristic search, the second 9 the second way and the last 9 the third way. In these groups of nine cases the first case has the limit of 2 for heuristic search, the second the limit 3 and so on. The second and third group of 27 cases are from the high load test day with extra load. The same cases are then shown for the medium and low load test days.

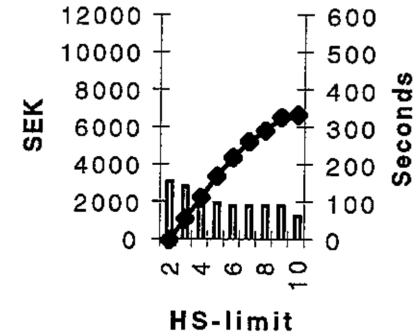
As expected the soft constraints lead to a lower objective function in most cases. It also often leads to a lower duality gap. This arises from the fact that the optimal value of the difference between the dual objective function with and without soft constraints is lower than the same difference for the primal objective function. One reason for decrease in the primal objective function is that there sometimes are small amounts of spillage and starts and stops for one hour in the cases without soft constraints. It can be cheaper to go outside the specified reservoir interval than to spill smaller amounts of water or start and stop units for one hour only. Since there seldom are the same problems in the dual solutions, the introduction of soft constraints will not decrease the dual objective function as much as the primal.

Another advantage of the soft constraints is that it is possible to temporarily go outside the bounds for the final reservoir contents in the heuristic search. Then to that it is easier to find good feasible solutions.

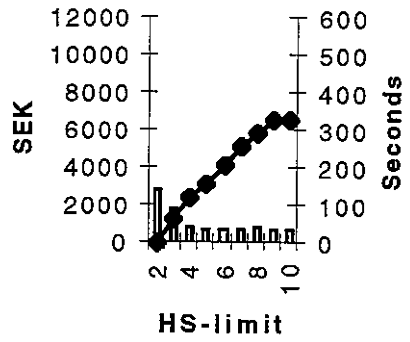
The question is if the soft constraints correspond to the reality. How much can the reservoir contents deviate from the specified interval? If more water is discharged from one plant, there will probably be less discharge from another plant. This means that it does not matter so much where the water is stored. However, if a plant has an almost empty reservoir at the beginning of the day and a long delay time from the upper reservoir it may have difficulties in producing at maximum capacity at the morning peak hours. Still, I think the advantages mentioned above outweigh this disadvantage. So my recommendation is that soft constraints should be used.

When to perform heuristic search

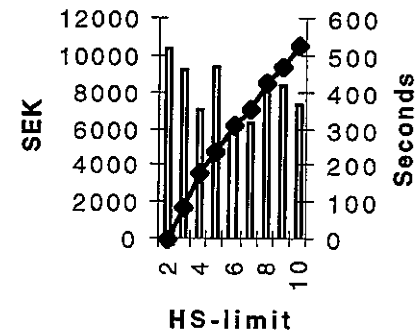
The result is shown in Fig. 6.4. First I have divided the cases into sets. A set consists of all cases for one load situation where all parameters except the limit for heuristic search are the same. There are 9 cases in each set. Then I have calculated the lowest objective function for each set. After that I have subtracted the lowest objective function in the set from the objective functions in the same set. I divided these differences into groups for each heuristic search limit. For each test day there are 18 cases with the same heuristic search limit, since there are three different load situations, two ways of treat-



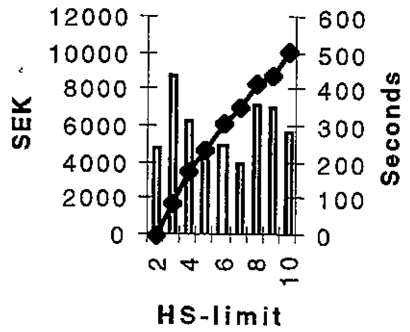
a) Winter day: all cases.



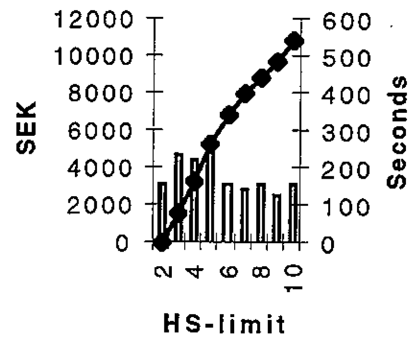
d) Winter day: only soft constraints.



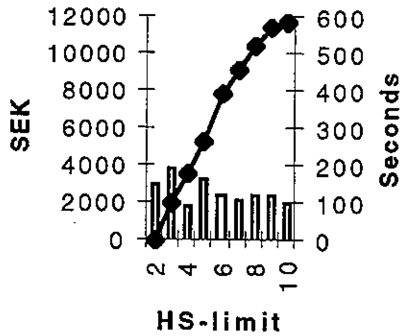
b) Autumn day: all cases.



e) Autumn day: only soft constraints.



c) Summer day: all cases.



f) Summer day: only soft constraints.

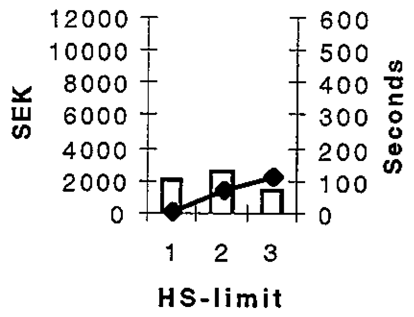
Fig. 6.4 The results for different limits when to performed heuristic search. Bars [SEK] and dots [seconds].

ing final reservoir contents and three ways of treating ramp rate constraints. Fig. 6.4 shows the average of differences for each group. The dots are in the same way the computation time. Note that the minimal average is not necessary equal to zero, since the result is the average of the objective function or computation time subtracted with the minimum of the set for three different load situations. Not surprisingly the computation time on average increases with the limit for heuristic search. The computation time for the heuristic search limit equal to 10 is on average about twice the computation time when the limit is equal to 2. The computation time is about the double for the second case with extra load compared with the two first cases. This leads to the increase in the computation time in Fig. 6.4 is higher for the second case with extra load and lower for the other two. The results for the high load test day show that the first increase of the limit gives the most decrease in the objective function. For the other two test days it is harder to draw any conclusions of which is the best limit for heuristic search. Since the step length in the updating of the dual variables depends on the best solution found, the chosen limit for the heuristic search will also affect the whole optimization process. For example the limit 5 gives the best objective function for the base cases for the medium and the low load test day. The same limit gives one of the highest values of the objective function for the second case with extra load for the summer day.

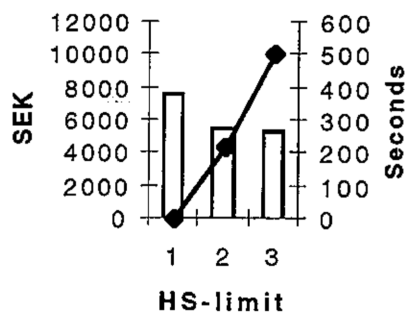
The difference in the objective function between the various limits for heuristic search is in most cases lower than one tenth of a per cent of the objective function. In all cases but the second with extra load for the low load day, the limit 5 gives one of the best solutions without increasing the computation time too much. Table 6.2 shows the benefit of using heuristic search for three test days without extra load and the limit for heuristic search equal to 5.

Test day	**Heuristic search**	**Cost (SEK)**	**Duality gap (per cent)**
High load	Yes	9455046	0,08
High load	No	9479426	0,34
Medium load	Yes	6304936	0,57
Medium load	No	6480342	3,26
Low load	Yes	5110840	0,58
Low load	No	5324084	4,53

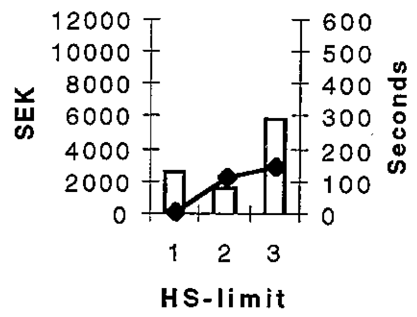
Table 6.2: Benefit of the heuristic search.



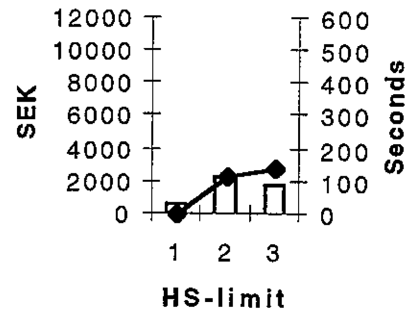
a) Winter day: all cases.



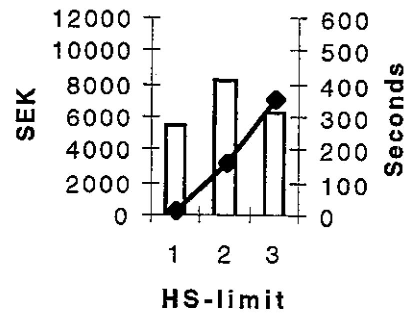
b) Autumn day: all cases.



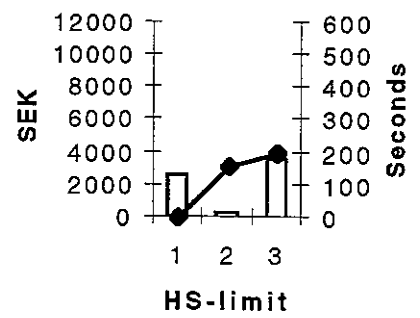
c) Summer day: all cases.



d) Winter day: only soft constraints and HS-limit equal to 5



e) Autumn day: only soft constraints and HS-limit equal to 5



f) Summer day: only soft constraints and HS-limit equal to 5.

Fig. 6.5 The results for different ways of treating ramp rate constraints in the heuristic search. Bars [SEK] and dots [seconds].

Treatment of ramp rate constraints in the heuristic search

The results for the test related to this parameter are shown by Fig. 6.5. The comparison between the different cases is done in the same way as for the previous parameter. Also for this parameter it is hard to draw any general conclusion of the best parameter setting. With the limit for heuristic search equal to 5 the second way of treating the ramp rate constraints gives one of the best solutions for all cases except for the second case with extra load for the autumn day.

Also the treatment of ramp-rate constraints will affect the optimization process. This is illustrated by the low load day without any extra load, for which there will be no thermal units committed in the solution. In spite of this there will be different solutions depending on the way of treating ramp rate constraint. The reason is that there will be thermal units committed in some iterations and if the corresponding solutions are the best so far this will change the updating of dual variables.

The average computation time is lowest for the first way and highest for the third way. The reason is that the heuristic search takes longer time when the step length in thermal generation is more limited. Then there has to be more reruns of the heuristic search for new limits in the thermal system.

6.2.2 Overall performance of the method

In the following I will show the results more in detail when soft constraints are used, the limit for heuristic search is set to 5 and the ramp rate constraints in the heuristic search are treated in the second way.

Before I present the detailed results we can take a look at the convergence in the dual optimization. Fig. 6.6 shows the primal and dual objective function for each iteration in step 1 and step 3 and the objective function in step 2 for the high load test day without extra load. In this case 4,9 per cent of the computation time was spent in step 1, 1,5 per cent in step 2 and the remaining 93,6 per cent in step 3.

The high load test day

For the case with no extra load almost all power will be produced by hydro power. The reserve constraints are not active for any hour. The computation time is 372 seconds and the duality gap is 0,08 per cent. Since the heads have changed between step 2 and step 3 there will be some error in the calculated

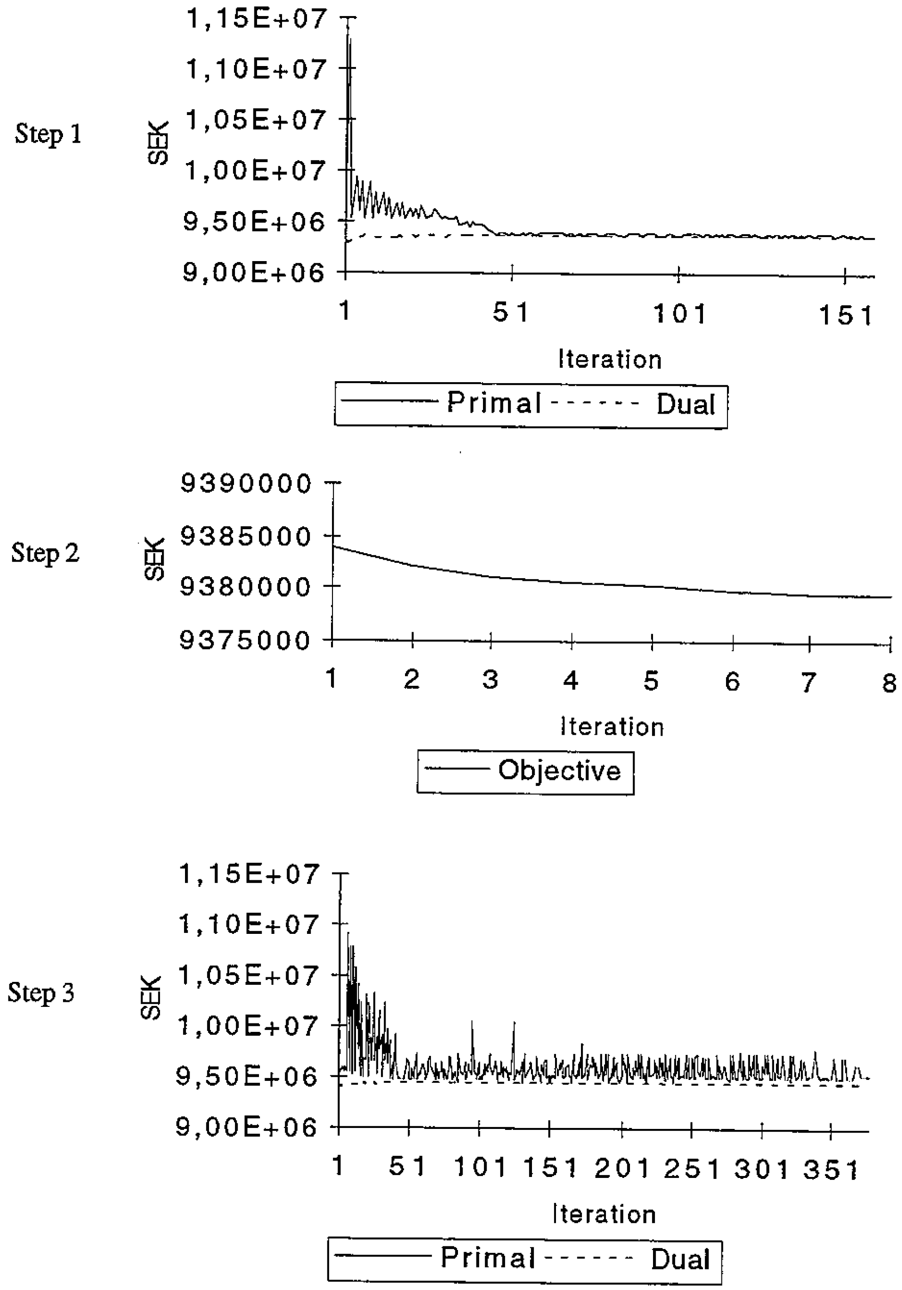


Fig. 6.6 The primal and dual objective function for each iteration in step 1 and 3 and the objective function for step 2.

generation in step 3. I have calculated the errors as the absolute value of difference in the generation with the heads after step 3 and step 2 respectively. The average error for this case, due to changed heads, is 0,17 per cent (0,30 per cent after step 1). In step 1 the heads are assumed to be constant during the planning period. The schedule from step 1 includes some points where the units are scheduled at non-efficient points. Fig. 6.7 shows an example where the discharge for hours 23 and 24 are between zero flow and the first local best-efficiency point. After step 3 the discharge for these hours increased to the first local best-efficiency point. The start-up cost leads to that units are never started or stopped for one hour only. For the plant in Fig. 6.7 the change of the schedule between step 1 and step 2 is small. For this specific plant largest changes are less than 4 HE. For four plants the final reservoir contents are outside the specified interval with up to some hundred TE. Fig. 6.8 shows the discharge schedule for ten plants in the Lule river. The figure shows that the plants are operated in a smooth way. Vittjärv is all the time operating on the continuous part and the increase during the last but one hour does not lead to a start-up.

Fig. 6.9 and Fig. 6.10 show the generation and spinning reserve respectively for the first case when the load is 2000 MW above normal. We will have to start thermal generation or purchase energy to a relatively high price to fulfil the load during the peak hours. During the first hours the started thermal plants can not produce at their maximum capacity owing to the ramp-rate

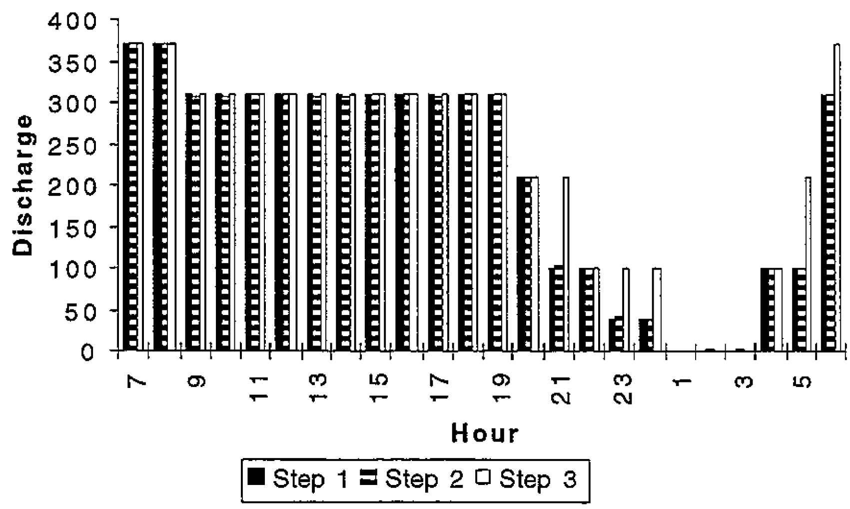


Fig. 6.7 Discharge schedule for a plant after the different steps for the high load test day with no extra load.

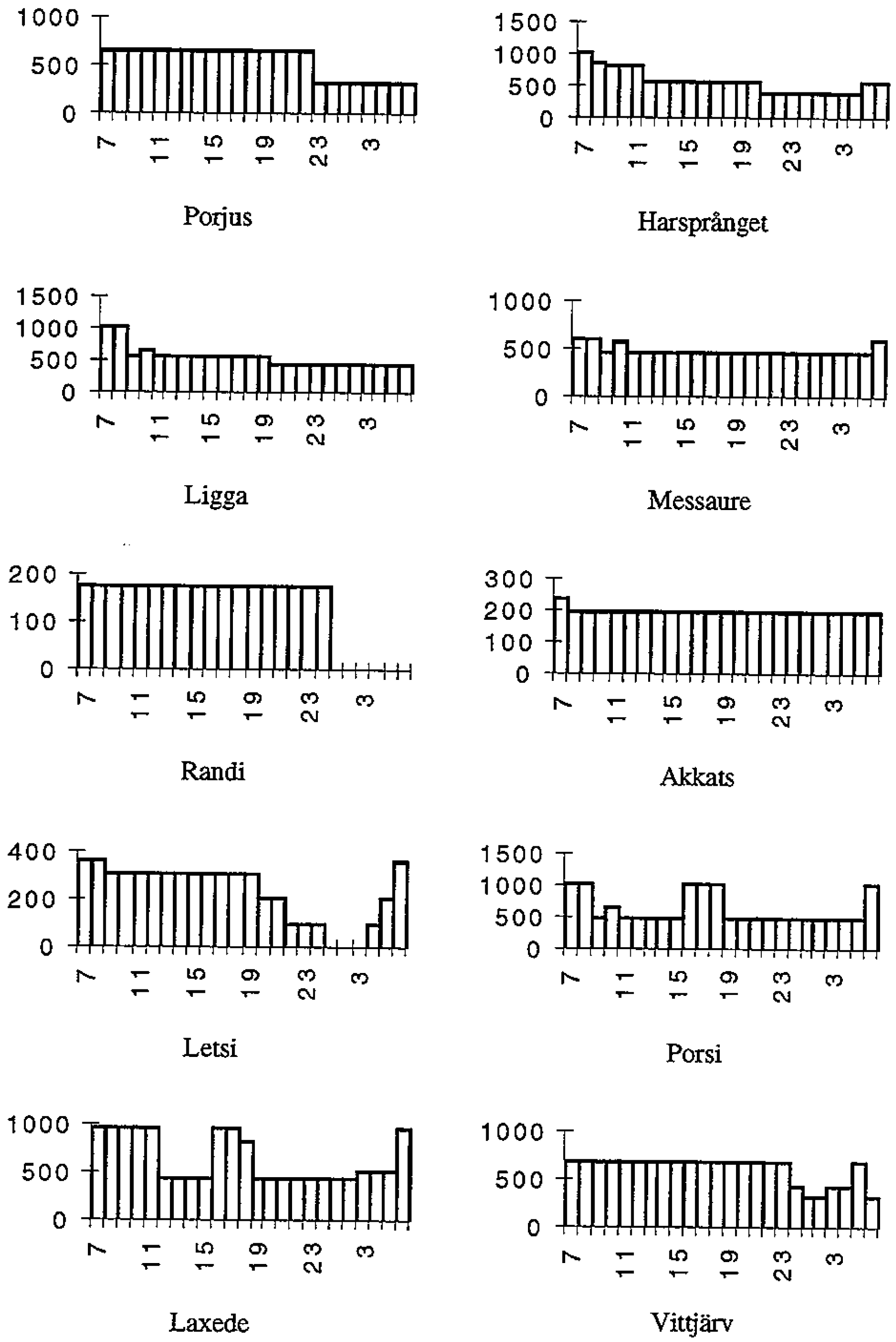


Fig. 6.8 Discharge schedule for ten of the plants in the Lule River for the high load case without any extra load.

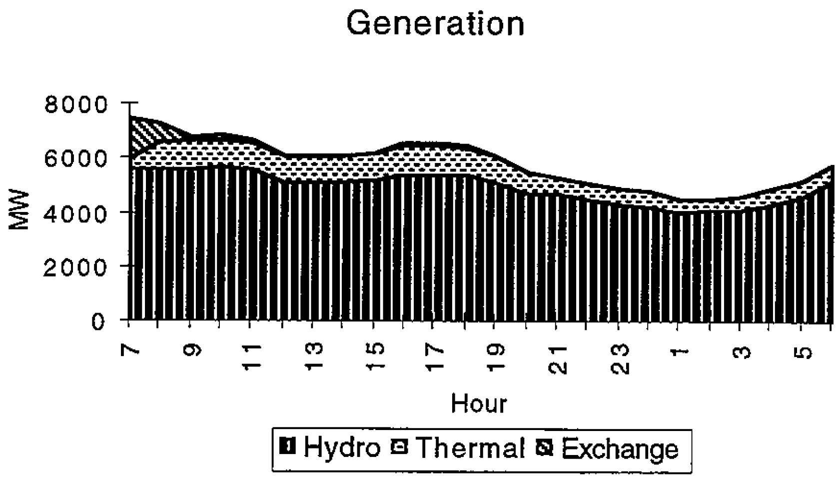


Fig. 6.9 Generation for the high load test day in the first case with extra load.

constraints. Neither can all hydro plants produce at their maximum capacity because of the spinning reserve requirements, Fig. 6.10. This means that we need to purchase energy to fulfil the load. Most of the started thermal plants will therefore be used at off-peak hours since start-up cost is a sunk cost. Fig. 6.10 also shows that the shadow prices (dual variables) for spinning reserve

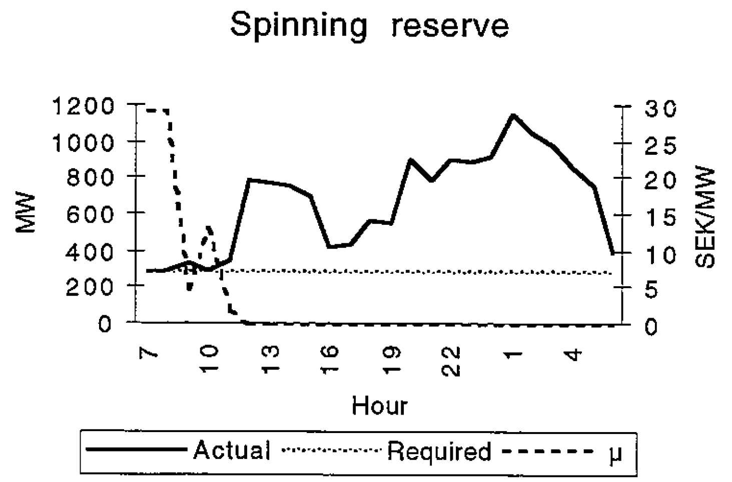


Fig. 6.10 Actual and required spinning reserve in the first case with extra load for the high load test day. The shadow prices are greater than zero if there are problems with the spinning reserve.

constraints are greater than zero for the hours when there might be a problem with the spinning reserve. Note that the dual variable can be greater than zero even if the constraints are not active in the best found solution. This is due to the fact that the problem is solved by the subgradient method. The dual variables are equal to their start values added with the step length multiplied by the mismatch summarized over all iterations.

The computation time is 322 seconds and the duality gap is 0,37 per cent, the average error in calculation, due to changed heads, is 0,15 per cent (0,39 per cent after step 1). Also for this case the start-up cost leads to that no units are started or stopped for one hour only. Also for this case there are four plants where the final reservoir contents are outside the specified level. Some small reservoirs are almost emptied in order to produce more power in this case with higher hydro generation. There is spillage in two plants in the solution. In one of the plants the reason is that it has not enough capacity to take care of the high flows coming from the upstream plant. The reason for the spillage from the other plant is that the heuristic search gets caught in a point where both the upstream and downstream reservoirs are at their upper levels, because of the high flows in this example. This means that it is not possible for the heuristic search to change the discharge and spillage for these hours. In practice this will not be a problem since the hour before the first hour having spillage, and the hour after the second hour with spillage, have higher discharges. The spillage is much smaller than the difference between the local best-efficiency points. Thus it is possible to go from the higher generation to the lower a few minutes into the first hour with scheduled spillage, instead of changing the generation exactly when the new hour starts. Then there will be no reason to spill water. In the same way it is possible to avoid spillage for the hour with higher generation after, by increasing the generation somewhat earlier. It could be mentioned that if the ramp rate constraints were treated in the first way and the other parameters were unchanged for this case, this kind of small spillage was almost avoided.

The generation for the second case with extra load is shown in Fig. 6.11. The results for hydro system is similar to the case without extra load since the thermal system produces approximately the extra load. The main difference is the computation time which is 1110 seconds since the time for EDC increases with the number of hours and plants with committed thermal generation. The duality gap is 0,075 per cent and the error in calculation of generation, due to changed heads, is 0,11 per cent (0,31 per cent after step 1). No hydro units are started and stopped for one hour only and the final reservoir contents are slightly outside the specified interval for two plants.

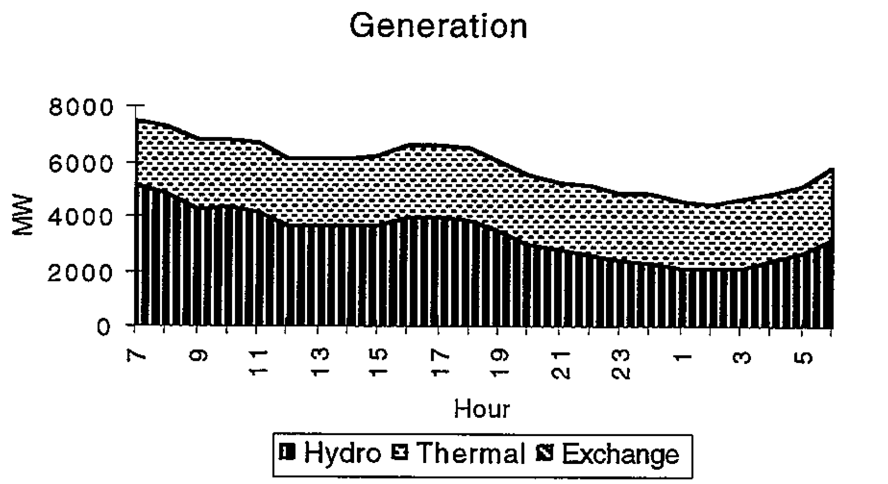


Fig. 6.11 The generation for the second case with extra load for the winter day.

The medium load test day

The results for the medium load test day with no extra load are quite similar to the same situation for the high load test day. Almost all load is fulfilled by hydro generation and hydro units are never started and stopped for one hour only. The computation time is 441 seconds and the duality gap 0,57 per cent. The error in calculation of the generation is average 0,20 percent (0,33 per cent after step 1). As for the high load test case there are plants scheduled between the local best-efficiency points in step 1. There are five plants with final reservoir contents outside the specified interval. After step 1 there are plants where units are started and stopped for one hour. This is then corrected by step 3, see Fig. 6.12. The figure also shows how step 3 changes the schedule so that operation between local best-efficiency point is avoided. For the plants in the figure the schedule does not change so much between step 1 and step 2.

The first case with extra load has also some similarities with the corresponding case for the high load test day. There will be purchase for the first hours since hydro power cannot produce at maximum level owing to the reserve constraints and thermal plants cannot produce at maximum level owing to the ramp rate constraints. The total purchase during the first hours will be much lower than for the high load day since the load is lower and thereby the hydro capacity can cover more of the load. Also the total thermal generation

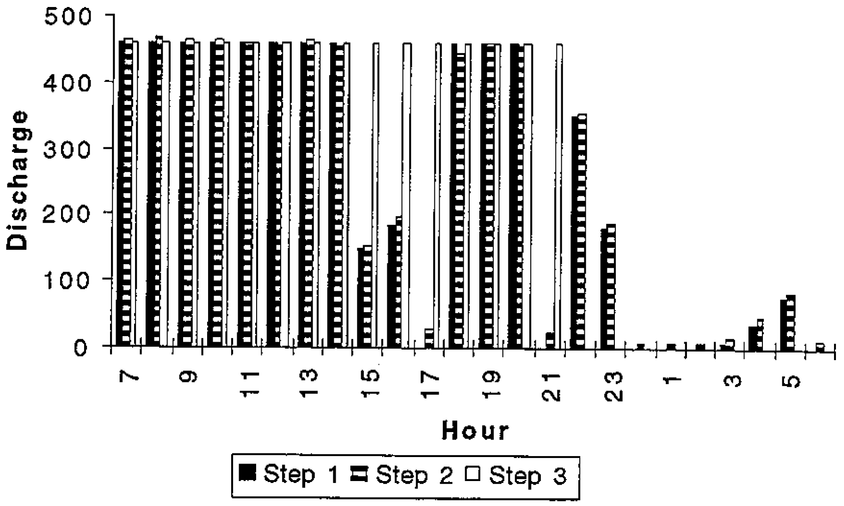


Fig. 6.12 Discharge schedule for a plant after the different steps for the medium load test day, with no extra load.

is much lower. The computation time and duality gap are 430 seconds and 0,22 per cent respectively. The error in calculation of the generation because of head dependence is 0,11 per cent (0,40 per cent after step 1). Four plants have their final reservoir contents outside the specified interval.

There are two main differences in the results for the second case with extra load compared with the above cases. The first difference is that the reserve constraints will be active for the hours with the lowest load, see Fig. 6.13. This comes from that it is expensive to use the water since we have recalculated the water value in order to get more thermal units on-line. Without the reserve constraint there would not be enough hydro committed to provide the required spinning reserve.

The second difference is that hydro units are sometimes started or stopped for one hour only. This comes from two reasons: that the flow in the river is low and that I have set the prices for purchase and sale in such a way that power almost never is purchased or sold. Because of the low flows almost all plants will operate on points lower than the continuous part. A change in hydro generation will cause a change in thermal generation since almost no exchange is performed. The spread in marginal costs in the thermal generation can lead to that the thermal system is not used as a "slack" in the same way as the exchange in the example with no extra load. Thermal generation

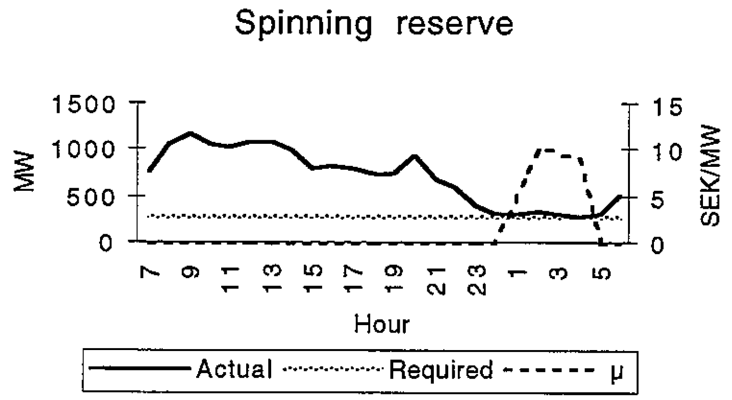


Fig. 6.13 Actual and required spinning reserve in the second case with extra load for the medium load test day.

is avoided in plants with higher cost than water storage value and used to maximum in plants with higher marginal cost than the water value. This means that the hydro system tries to fulfil the load not covered by the thermal system as exactly as possible while operation only is allowed at discrete points. To do this hydro units have to be started and stopped often. This will decrease the objective function since the start-up cost for the hydro units is lower than the difference in marginal cost between the thermal plants.

I have compared these results with a test with the exchange prices for the first 100 MWh equal to the water storage plus 10 SEK for purchase and minus 10 SEK for sale. This leads to three stops for one hour during periods of low load to avoid going outside the 100 MWh interval. In this low load hours there are few units committed and thereby harder to stay in the interval without these stops for one hour.

It can be questioned if the hydro model is used outside its area of validity in the case since the amounts of hydro and thermal generation are in the same range. Still hydro generation is only allowed at discrete point and at the upper continuous part while thermal generation is allowed from minimum to maximum generation level. For some hours the thermal generation is more than two times higher than the hydro generation, see Fig. 6.14 In this case a wider range of the hydro domain might have to be used. Otherwise, hydro

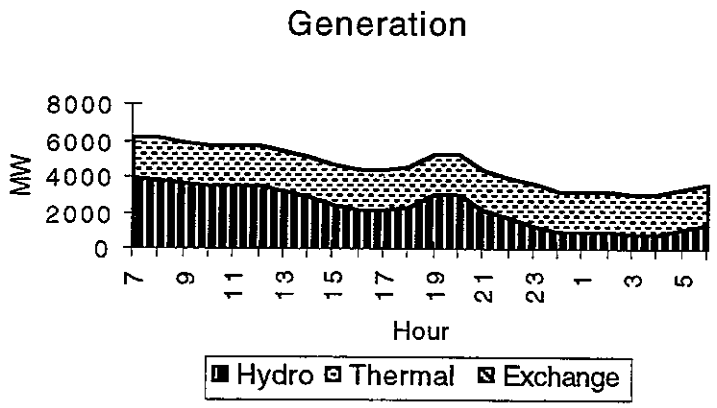


Fig. 6.14 The generation for the second case with extra load for the medium load test day.

might be started and stopped even though it should be better to deviate somewhat from the local best-efficiency points.

If the operation is allowed around the points with local best-efficiency the developed concept will still be valid. We can use a piecewise model for the description of the generation characteristic around the local best-efficiency point. The break points can be the local best-efficiency point and the upper and lower limit for generation around that point. The variable splitting can then be used in the same way as earlier to decompose the problems into subproblems. In heuristic search the operation around the local best-efficiency point can be treated in the same way as discharge at the continuous part.

Finally, the computation time and duality gap for this test case are 1094 seconds and 0,70 per cent. The average error in the calculation of the generation is 0,23 percent (0,33 percent after step 1). The case with the parameter settings giving the lowest objective function for this load situation has the duality gap 0,57 per cent. There are eight plants with final reservoir contents outside the specified interval.

The low load test day

The results for the low load test day show the same tendencies as the result for the medium load test day. For the test with no extra load and the first test with extra load there will be some starts and stops of hydro units for one hour

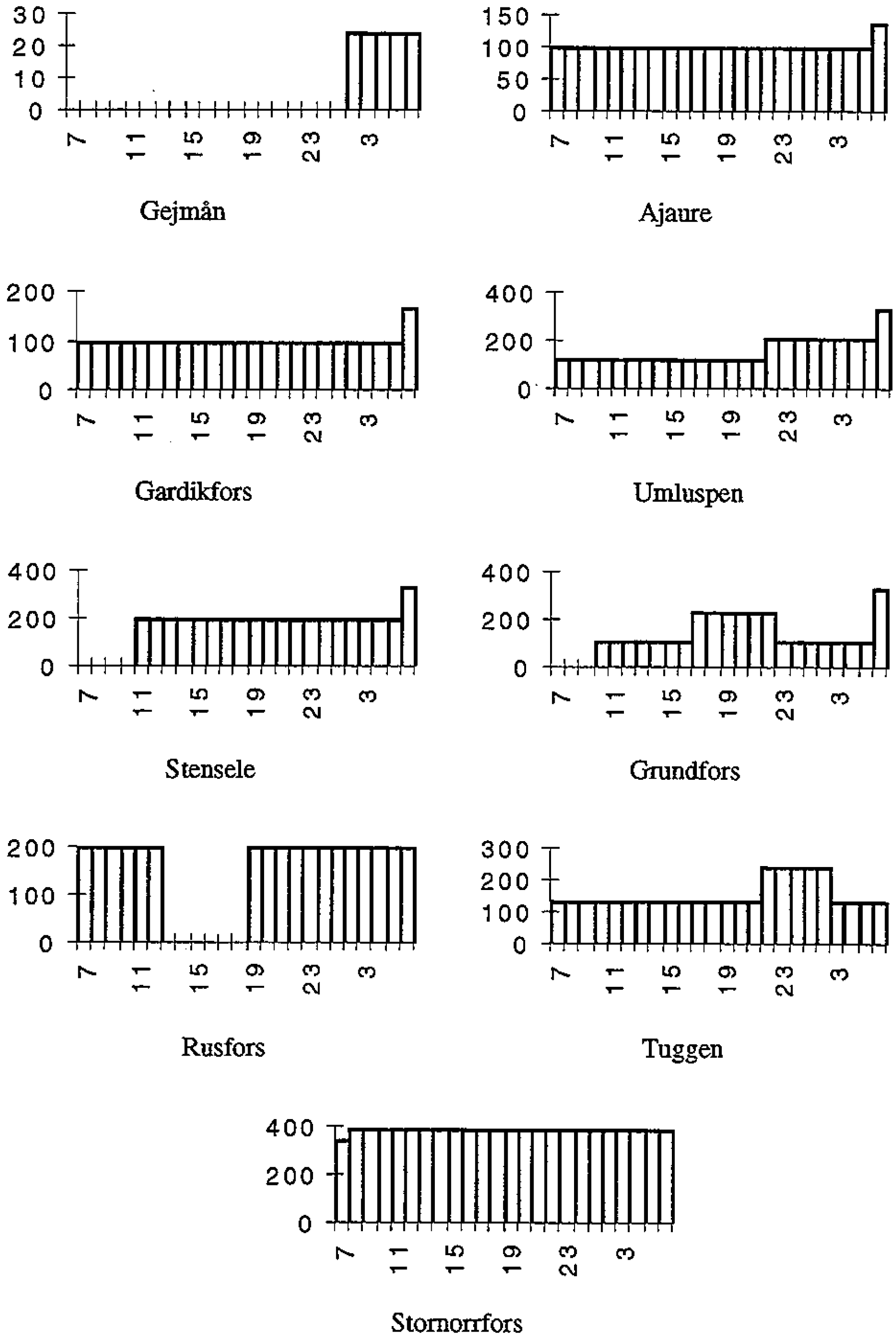


Fig. 6.15 Discharge schedule for the plants in the Ume River for the low load case without any extra load.

only and sometimes operation between local best-efficiency points. This will be changed by the third step. For the second case with extra load there will be some starts and stops for only one hour. For one plant and hour there will also be spillage which is much lower than the difference between the local best-efficiency points. Fig. 6.15 shows the schedule for the plants in the Ume river for the low load test day without any extra load. The reason for the discharge increase for the last hour is that the load increases at the end of the planning period. In the solutions there are final reservoir contents outside the specified interval for 3-5 plants.

The computation times are 473, 458 and 1328 seconds respectively. The corresponding duality gaps are 0,58, 0,097 and 0,64 per cent.

The difference between the low load test day and the other test days are that the correction for the head dependence does not work as well. For the case with no extra load the average error in the calculation of the generation increases from 0,24 per cent after step 1 to 0,30 per cent after step 3. For the second case with extra load the increase is from 0,26 per cent to 0,30 per cent. For the first case with extra load the error on the other hand decreases from 0,29 per cent to 0,14 per cent. The reason for that the head dependence compensation does not work so well for the two cases above is that the schedules from step 1 are changed quite much to step 3. These changes come from that there are several operation points in step 1 that are not allowed in step 3. Since most plants operate below the continuous part the schedules for some plants will be changed quite much after step 1 and 2 in order to fulfil the discharge domain constraints and at the same time all the other constraints.

The results for three test days are summarized by table 6.3.

It could be questioned if we should update the head dependence again after step 3. It is easy to do since we only have to rerun step 2 with given hydro commitment. However, step 2 does not change the schedule significantly, which was exemplified by Fig. 6.7 and Fig. 6.12. After step 3 we are also more restricted in changing the schedule since many plants operate at a fixed point and the rest in a given interval. This means that a new optimization with head dependence would not change the schedule so much. This is specially true for cases with low generation since almost all plants operate at fixed points.

Test day	Extra load	Changed water storage value	Duality gap (per cent)	Generation error after step 1 (per cent)	Generation error after step 3 (per cent)	Computation time (sec)
High load	No	No	0,08	0,30	0,17	372
High load	Yes	No	0,37	0,39	0,15	322
High load	Yes	Yes	0,08	0,31	0,11	1110
Medium load	No	No	0,57	0,33	0,20	441
Medium load	Yes	No	0,22	0,40	0,11	430
Medium load	Yes	Yes	0,70	0,33	0,23	1094
Low load	No	No	0,58	0,24	0,30	473
Low load	Yes	No	0,10	0,26	0,30	458
Low load	Yes	Yes	0,64	0,29	0,14	1328

Table 6.3: Summary of the results

6.3 Test 2: Unavailability costs in a river

In chapter 4 I discussed start-up costs for hydro units. Parts of these start-up costs could be assigned to unavailability costs. First, start-ups led to more frequent maintenance. Second, a start-up sometimes led to a malfunction in the control equipment.

In this section I will give an example of how a short term planning method can be used to estimate the costs of unavailable units. This could be used for planning of investments in different units. In order to simulate the operation of the river I have used the linear network programming model. Instead of planning the generation according to a known load, I have assumed that the generation is planned for a known spot price. This will be equivalent to the relaxation in section 5.3, which makes the solution very fast. The difference in profit between the case with and the case without an outage will be the cost for the specific outage. On a higher level we have used Monte-Carlo simulation to simulate the operation for different scenarios of price, reservoir levels and river inflow.

In Appendix D I have presented the Monte Carlo model I have used to simulate the costs of unavailable generating units.

If the results are used for planning investments the investment cost should be compared with the calculated unavailability cost multiplied by the change in probability for outage caused by the investment. If the calculated cost is used as part of the start-up cost the calculated cost of a malfunction during the start-up should be multiplied by the probability of such an outage.

I have calculated the unavailability cost with all the plants Lule river as test system.

The results of the simulations are shown in Fig. 6.16-6.18. Fig. 6.16 shows the average daily cost having the unit unavailable, Fig. 6.17 shows the average cost of an outage during operation and Fig. 6.18 shows the average cost of an outage during the start-up phase. Please, observe that there are different scales on the axis in the figures. Do also observe that units 30 and 33 are "must run"-units and will not have any start-ups and thereby no cost of an outage during the start-up phase.

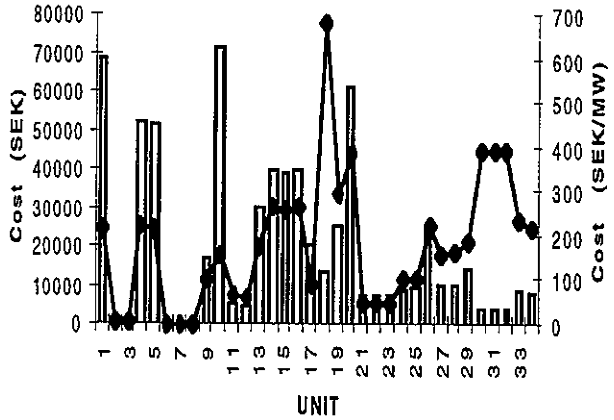


Fig. 6.16 The daily costs of a long outage. The bars represent the cost and the dots the cost per MW installed capacity.

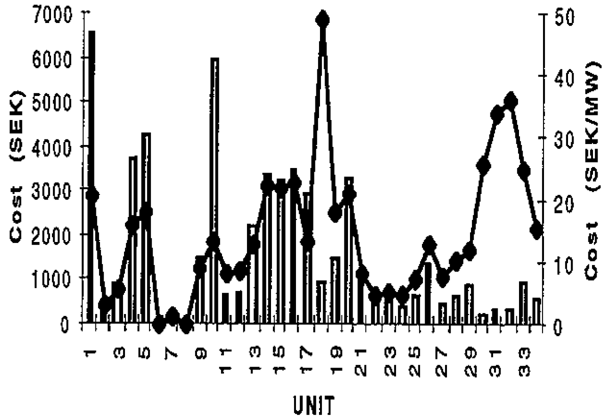


Fig. 6.17 The costs of an outage during the operation. Same representation as in Fig. 6.16.

From the results it is possible to discuss many interesting things. First, we can see a common pattern in both the cost and the cost per installed capacity in the three figures. The pattern shows in which units an outage will have the most economic consequences. Another observation is that the average cost of an outage during operation is for most units higher than the average cost of an outage during the start-up phase. Our explanation is that if a unit is on-

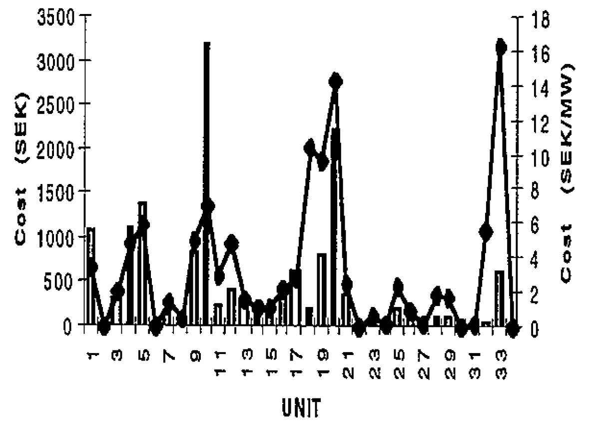


Fig. 6.18 The costs of an outage during the start-up phase. Same representation as in Fig. 6.16.

line for the whole planning period it will not be started. The units which have a large difference in cost for the two cases are much more often on-line for the whole planning period than started and stopped during the period. An outage in a unit which is on-line for the whole period will be more expensive than if the unit is only on-line for a shorter period, since the unit in the second case can be rescheduled and it is also a lower risk of spillage.

Another interesting observation is that units 6-8 will have almost zero cost for a long outage. These units belong to the same plant as units 9-10. Units 6-8 were installed more than 15 years before the other two. The capacities and degrees of efficiency are lower for units 6-8, which means that in most cases units 9-10 are used. If the price varies much during the day some of the units 6-8 are used. This has the result that units 6-8 are not scheduled so often, and if not all three of them are scheduled at the same time, one of them can without any extra cost replace the failed one. The downstream plant has a small reservoir and a lower discharge capacity. This means that if all units 6-10 are scheduled during the whole planning period, the downstream plant has to spill. Thus it is unusual that units 6-10 are scheduled during the whole planning period at the same time. The conclusion is that when an outage occurs in one of units 6-8, the failed unit can easily be replaced by another, if not all of units 6-8 are on-line. If all of units 6-8 are on-line, one of them has probably been started during the planning period and it will lead to a quite low cost according to our earlier reasoning.

Units 2-3, which belong to the same station also have small costs for a long outage. This is mostly due to the fact that this plant has a large reservoir which can store the water which will not be used during the outage. Also the fact that the two units are quite identical will lower the outage cost since one unit can replace the other if only one of them is on-line. Still, there will be some cost since it sometimes is optimal to use both units at the same time.

Finally, some comments concerning the applied method. We have assumed that it is very unlikely that two units will have an outage during the same period. Therefore it will be even more unlikely that such a thing will happen in the same station. This assumption is valid for today's maintenance level. Since several outages in the same plant can lead to somewhat higher cost than each outage treated one at a time, it is important to know if that the assumption is valid. Another issue is that the costs of a long term outage are based on daily generation simulation. If an outage, as for units 2-3, will lead to that the water is stored in the reservoir which will result in a low cost, daily generation simulation might underestimate the cost if the outage is very long. This because the reservoir will sooner or later be full and the plant will be forced to spill. However, such long outages are unusual.

7 Conclusions and Future Work

7.1 Conclusions

This thesis presents a method for daily planning of a hydro dominated hydro-thermal power system. The aim of the planning is to minimize the operation cost subject to the forecasted load, river inflows and trading prices. The system must also provide enough spinning reserve.

The hydro model is mixed-integer. Discharges are allowed at zero flow, the local best-efficiency points and on the continuous part between the local best-efficiency point with the highest flow and the point with maximum flow. This last continuous part is modelled as a linear function. The difference between the maximum capacity of the on-line hydro units and their generation is the spinning reserve.

In order to take into consideration the start up-cost I have performed a survey among the power producers in Sweden. The survey shows that the power producers estimate the start-up cost to be around \$3 per MW unit nominal output on average. The start-up costs come from:

- Wear and tear of the windings owing to temperature changes during the start-up.
- Loss of water during maintenance.
- Wear and tear of mechanical equipment during the start-up.
- Malfunctions in the control equipment during the start-up.
- Loss of water during the start-up.

For the thermal plants I have used a standard model with polynomial operation cost, start-up costs and ramp-rate constraints. The model also includes the possibilities of purchasing and selling power to forecasted prices.

The solution techniques are based on Lagrange relaxation with variable splitting, dynamic programming, network programming and heuristic search. The head dependence is updated in the optimization process.

The models and the solution technique are tested for three different load situations; heavy, medium and light load. Loads in the test are based on real loads for the hydro system. In order to test the thermal part of the method I have different test cases with different extra load.

The results show that the computation time is reasonable, around 6-8 minutes on a Power Macintosh (120 MHz), for the test days with a normal amount of thermal generation committed. For the test where a large part of the power is generated in the thermal system the computation time is somewhat higher than acceptable, between 18-22 minutes. The reason is that the time for thermal EDC increases when more thermal generation is committed.

The duality gap is lower than 0,6 per cent for all test cases. This indicates that the found solutions are near optimal. Cases with only a few units scheduled at the continuous part will have a higher duality gap since this will make the nature of the problem more combinatorial. Also when more inequality constraints are active the duality gap increases.

For all test except the second case with extra load for the medium and low load day no hydro units are started or stopped for only one hour. For high load test day the schedule from step 1 (piecewise linear hydro model) will include some operation points between the local best-efficiency points. This is corrected by step 3 (mix-integer model) where no such operation is allowed. For the medium and low load test days there will also be units started and stopped for one hour only after step 1. For the second case with extra load there will also be some starts and stops for only one hour after step 3. This depends on that plants only are allowed to operate on discrete points. The starts and stops may be necessary to fulfil other constraints at lowest cost. It can therefore be discussed whether the model should be extended to allow operation around the local best-efficiency points if the load is low. The developed concept will still hold if the characteristic around the local best-efficiency point is piecewise linear. The variable splitting can be used to divide the problem into subproblems. In the heuristic search operation around the local best-efficiency point can be treated in the same way as operation on the continuous part.

The correction of the head dependence (step 2) results in that the average error in the calculation of the generation is below 0,2 per cent for the high and medium load test days. For the cases with the lowest hydro generation the results are not so favourable. This depends on that the schedule from step 1 will be a bad estimate of the final schedule. The reason is that the schedule from step 1 will include lots of operation points which are not allowed in step 3. The schedules therefore have to be changed quite a bit to fulfil these constraints and at the same time minimize the cost while fulfilling other constraints.

Sometimes the method gives small amounts of spillage, much less than the difference between two local best-efficiency points. The reason is that heuristic search fails to convert this spillage into discharge since discharge only is allowed at discreet points. In practice this is no problem since this spillage can be converted into discharge by deviating only slightly from the local best-efficiency point. If the generation is higher for the hours before or after the hour with the small amount of spillage, the spillage can be converted into discharge by decreasing generation later and increasing generation earlier respectively. It also possible to allow discharge in a small domain around the local best-efficiency point. This would lead to some modification but the method will still be able to solve the problem according to the discussion above.

The piecewise linear model is also applied to calculate unavailability costs in a river. The results show in which plants an outage has the highest impact on the production economy. The calculated outage costs and the probability of outages should then be compared with the investment costs and how investments can affect the probability of outages.

7.2 Future work

Some ideas about future work:

- *Better performance of optimization algorithm.* One way to improve the performance of the optimization algorithm could be to use a more advanced method than the subgradient method for updating the Lagrange multipliers. One example is the bundle method. Another way to speed up the calculations could be to solve the subproblems in the calculation of the dual objective function in parallel. The parallel computation is suitable for this kind of calculation since the subproblems can be solved independently of each other. If the method should be applied to a system with a large part of thermal generation it is possible that the methods for optimization of the thermal system should be more developed. For example it might be useful to change the thermal integer variables in the search for a good feasible solution. With large thermal part it might also be necessary to extend the allowed operation domain for the hydro plants to not only include the local best-efficiency points.

- *Comparison of methods.* During the 90's the interest for more detailed hydro models has grown. It has emerged several different ways in modelling and solving the short term planning problem with detailed representation of the hydro plants. Some examples are different kinds of Lagrange relaxation in combination with dynamic programming, mix integer-linear programming, genetic algorithms, linear and non-linear programming in combination with some heuristics and combination of the mentioned methods. As far as I know there is no study comparing the different methods. Such a comparison should use a simulation model with model of the hydro system as detailed as possible.

- *Use of weekly planning for setting the commitment states for hydro and thermal units.* Use some more simple models for weekly planning and use the results for setting final states for units and reservoir contents as discussed in section 2.1. This could for example be achieved by extending the horizon of step 1.

Appendix A
A Model for Head Dependency

This appendix gives a background to the model of head dependence, which I presented in section 3.2. The modelling of the hydro power generation dependence of the head height is based on [67]. Let me first define a few things:

- *The nominal head* is the difference between the upstream reservoir level and the downstream reservoir level subtracted with the head losses between the end of the tailrace tunnel and the downstream reservoir.
- *The actual head* is the nominal head subtracted with the losses in the intake gate.
- *The reservoir factor* describes how much a change of head means in the change of volume. It is usually expressed in HE/m. If the reservoir is small this factor will be constant. If the reservoir is large the reservoir factor will normally be a function of the reservoir level. However, during a 24 hour period the changes of the reservoir level will be quite small. For small changes of the reservoir level the factor can be approximated to a constant.

The losses in the intake gate as well as the losses between the end of the tailrace tunnel and the downstream reservoir are increasing with increasing discharge. The losses could be different for the same discharge if we use different unit combinations. The individual efficiency for the unit will also affect the total efficiency for different unit combinations. As mentioned in section 2.2.1 we want to find the combination with the best-efficiency for a specified discharge. Let us assume that we have the output power as a function of the discharge for optimal unit combination at different reservoir levels and thereby different nominal heads, see Fig. A.1.

We can now get the power as a function of the discharge within two different levels. If the reservoir is small, these levels can be the upper and lower bounds of the reservoir level. If the reservoir is large the reservoir can be

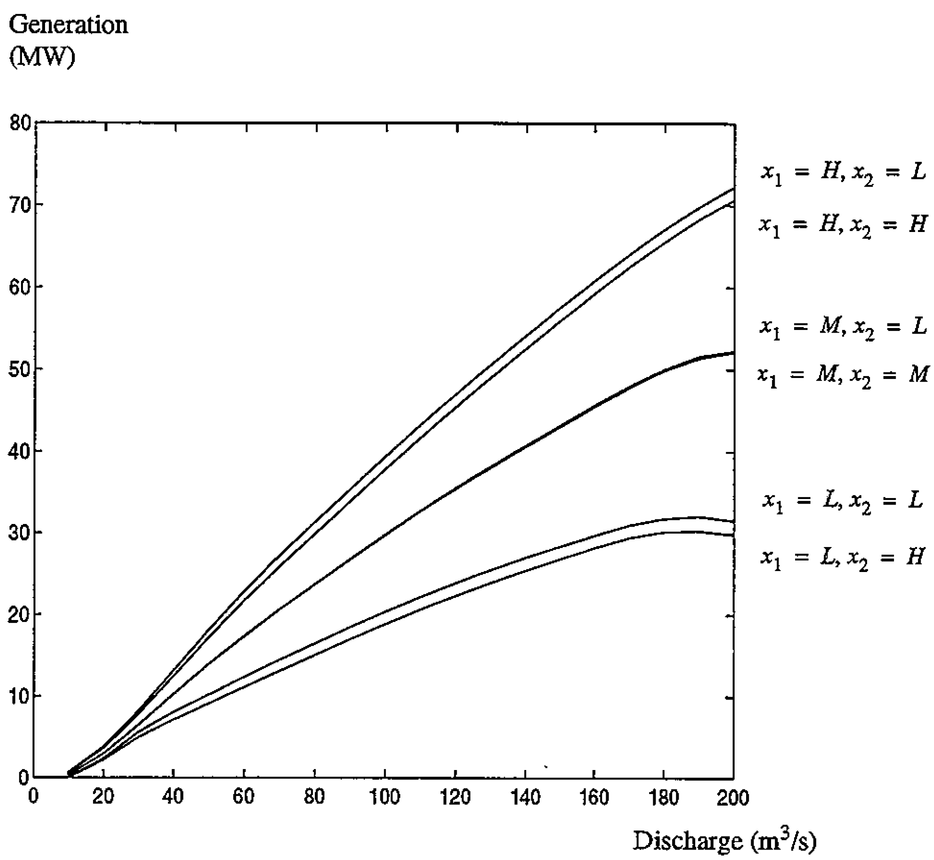


Fig. A.1 An example of the generation as function of the discharge at different reservoir levels. H = highest level, M = medium level and L = lowest level.

divided into horizontal sections. Depending on in which section the actual reservoir level is, the upper and lower levels of the actual section are used.

Level 1 is when the upstream reservoir is at its lower level in Fig. A.2 and the downstream reservoir is at its higher level in Fig. A.2. Level 2 is vice versa. Let us define:

- $h_1(u)$ nominal head, level 1 and discharge u.
- $h_2(u)$ nominal head, level 2 and discharge u.
- $p(u, h_1)$ power, level 1 and discharge u.

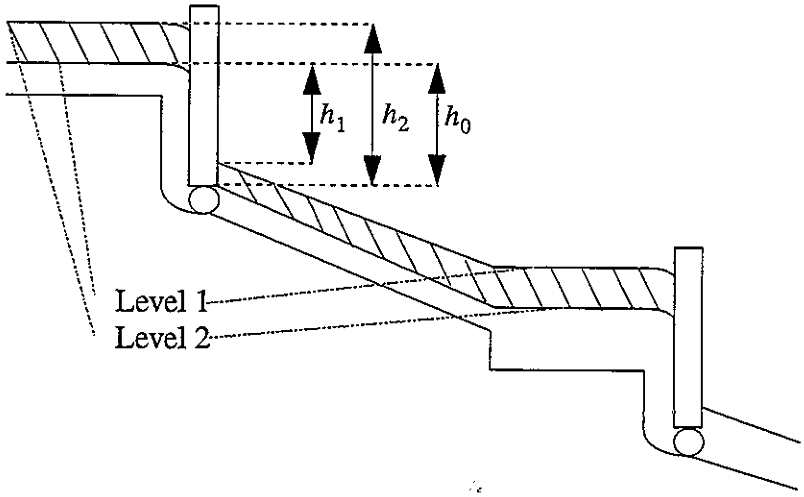


Fig. A.2 The minimum and maximum heads.

- $p(u, h_2)$ power, level 2 and discharge u.

It can be noticed that $h_1(u)$ and $h_2(u)$ depend on the discharge, since the losses between the tunnel and downstream reservoir increase when the discharge increases. For nominal heads between h_1 and h_2 the generation can be calculated by linear interpolation. The nominal head, h_{b_r}, can be written as a function of up- and downstream reservoir levels:

$$h_{b_r}(u) = h_0(u) + \alpha' x(j) - \beta'(u) x(d_j) \tag{A.1}$$

where

- $h_0(u)$ is the nominal head when both reservoirs are at their lower levels in Fig. A.2. Notice that h_0 is a decreasing function of the discharge.

- α' is a constant, which shows how much an increase of the reservoir contents for the upstream reservoir gives in increased head. This is 1 divided by the earlier mentioned reservoir factor.

- $\beta'(u)$ shows in the same way how much a change in the reservoir contents for the downstream reservoir affects the nominal head. This is 1 divided by the reservoir factor multiplied by a function, which is dependent on the discharge. This function shows how much lowering of the downstream reservoir level gives in increased head. This function is assumed to be known.

By linear interpolation we can construct an expression for produced power for heads between heads h_1 and h_2.

$$p(u, h_{br}) = \frac{p(u, h_2) - p(u, h_1)}{h_2 - h_1}(h_{br} - h_1) + p(u, h_1) \tag{A.2}$$

but from (A.1) we will get

$$h_1 = h_0(u) - \beta'(u)\bar{x}(d_j) \tag{A.3}$$

$$h_2 = h_0(u) + \alpha'\bar{x}(j) \tag{A.4}$$

(A.3) and (A.4) in (A.2) gives

$$\begin{aligned} p(u, h_{br}) &= \frac{p(u, h_2) - p(u, h_1)}{h_0(u) + \alpha'\bar{x}(j) - (h_0(u) - \beta'(u)\bar{x}(d_j))} \cdot \\ &[h_0(u) + \alpha' x(j) - \beta'(u)x(d_j) - (h_0(u) - \beta'(u)\bar{x}(d_j))] \\ &+ p(u, h_1) = p_0(u) + \alpha''(u)x(j) - \beta''(u) \cdot x(d_j) \end{aligned} \tag{A.5}$$

where

$$p_0(u) = \frac{p(u, h_2) - p(u, h_1)}{\alpha'\bar{x}(j) + \beta'(u)\bar{x}(d_j)}\beta'(u)\bar{x}(d_j) + p(u, h_1) \tag{A.6}$$

which is the power produced when the nominal head is h_0, which means that $p_0(u) = p(u, h_0)$.

$$\alpha''(u) = \frac{p(u, h_2) - p(u, h_1)}{\alpha'\bar{x}(j) + \beta'(u)\bar{x}(d_j)}\alpha' \tag{A.7}$$

$$\beta''(u) = \frac{p(u, h_2) - p(u, h_1)}{\alpha'\bar{x}(j) + \beta'(u)\bar{x}(d_j)}\beta'(u) \tag{A.8}$$

What is the benefit of this work? Now there are no functions depending on the head. Instead, we have the generation as a function of discharge, upstream and downstream reservoir contents. There are three functions depending only on the discharge:

- $p_0(u)$, which is the relationship between water discharged and generation.
- $\alpha''(u)$, which is the coefficient for the contents of the upstream reservoir. $\alpha''(u)$ multiplied by the contents of the upstream reservoir is the head correction for upstream reservoir.
- $\beta''(u)$, which is the coefficient for the contents of the downstream reservoir. $\alpha''(u)$ multiplied by the contents of the downstream reservoir is the head correction for downstream reservoir.

How do we express those functions explicitly depending on the discharge? In section 3.2.2 I described how to model p_0. α'' is going to be an increasing function of the discharge. This because a higher upstream level will give an increase of power. This increase will be larger with larger discharges. β'' will be positive, but it is not that easy to know how it actually depends on the discharge, owing to low quality of data. It is likely that β'' can have different properties for different plants. Fig. A.3 and Fig. A.4 show estimates of α'' and β'' respectively for the same plant as in Fig. A.1. I have assumed that α''

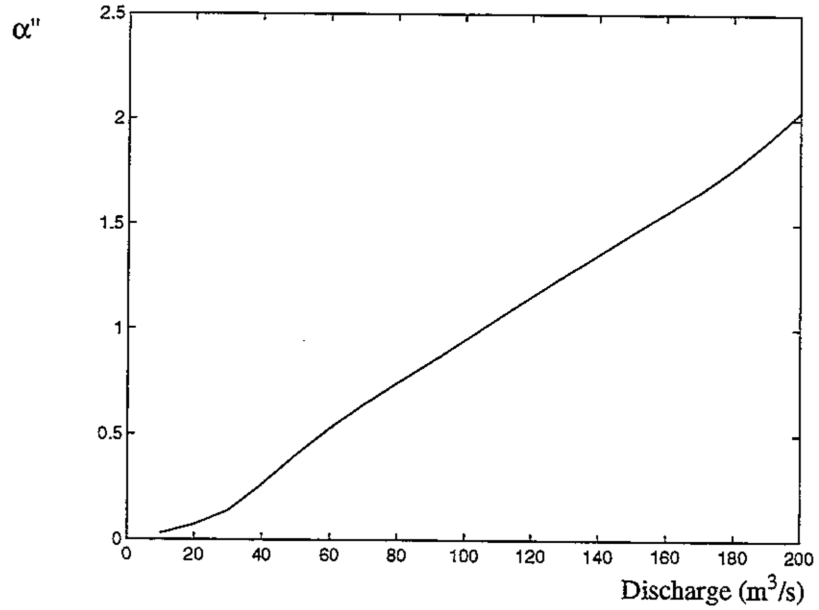


Fig. A.3 An example of an estimate of α''.

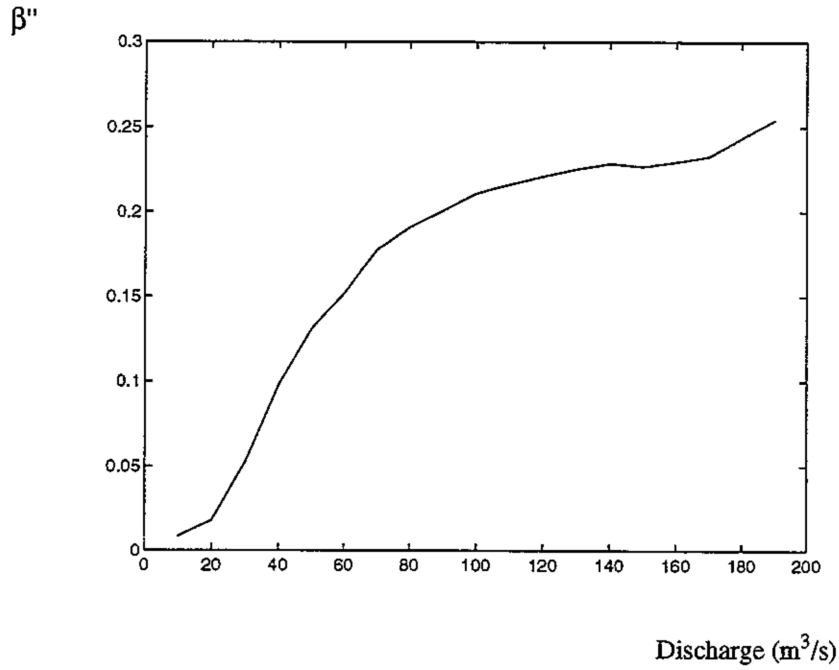


Fig. A.4 An example of an estimate of β".

can be approximated to a linear function. But I have also assumed that β" can be approximated to a linear function. This gives:

$$\alpha''(u) = \alpha u \tag{A.9}$$

$$\beta''(u) = \beta u \tag{A.10}$$

where α and β are calculated by least squares approximation of (A.7) and (A.8) respectively. This gives the generation as a function of discharge, upstream and downstream reservoir contents:

$$p(u, x(j), x(d_j)) = p_0(u) + \alpha u x(j) - \beta u x(d_j) \tag{A.11}$$

Appendix B
Variable Splitting

In this appendix I will describe a variant of Lagrange relaxation that can be advantageous to use in certain circumstances. This relaxation is called variable splitting [34]. Assume that we have an optimization problem with two different kinds of constraints and that the problem is hard to solve

$$
(O)\quad \begin{array}{l} \min c_O\omega \\ A_1\omega = W_1 \\ A_2\omega = W_2 \\ \omega \in \Omega \end{array} \tag{B.1}
$$

Assume also that relaxing $A_1\omega = W_1$ or $A_2\omega = W_2$ will give a much easier problem. However, if we relax one of the constraints we may lose important information. Instead we reformulate the problem by introducing separate sets of variables for the different sets of constraints

$$
(O')\quad \begin{array}{l} \min c_O\omega \\ A_1\omega = W_1 \\ A_2\omega' = W_2 \\ \omega = \omega' \\ \omega \in \Omega \\ \omega' \in \Omega \end{array} \tag{B.2}
$$

Both sets also have the same feasible domain and each variable in the ω-set has to be equal the corresponding variable in the ω'-set. Now we can relax the constraint coupling the two original sets of constraints

$$
(L') \qquad \begin{aligned} &\min c_O\omega + \pi(\omega - \omega') \\ &\quad A_1\omega = W_1 \\ &\quad A_2\omega' = W_2 \\ &\quad \omega \in \Omega \\ &\quad \omega' \in \Omega \end{aligned} \tag{B.3}
$$

This means that the problem falls into two parts

$$
(L'_1) \qquad \begin{aligned} &\min (c_O + \pi)\omega \\ &\quad A_1\omega = W_1 \\ &\quad \omega \in \Omega \end{aligned} \tag{B.4}
$$

$$
(L'_2) \qquad \begin{aligned} &\min -\pi\omega' \\ &\quad A_2\omega' = W_2 \\ &\quad \omega' \in \Omega \end{aligned} \tag{B.5}
$$

Both the problems are easy to solve since (L'_1) is the problem we would have got if we relaxed $A_1\omega = W_1$ in the original problem and (L'_2) is the corresponding problem for $A_2\omega' = W_2$. In this relaxation we keep more of the original structure of the problem. This will lead to a lower duality gap compared with relaxing one of the original constraints.

In chapter 5 I have used this method as part of the solution method for solving the problem with the mixed integer hydro power model. When the load balance equation is relaxed, two different kinds of constraints remain for the hydro system. These are the reservoir balance constraints and the integer constraints for the operation of the hydro plants. If we relax the reservoir balance constraints, which I have done in my licentiate thesis, we get a subproblem for each plant [64]. These subproblems can easily be solved by dynamic programming. However, the network structure of the reservoir balance is lost. If we instead relax the integer constraints, we get a subproblem with piecewise linear hydro power models. This subproblem can easily be solved by network programming. However, it might be hard to find good feasible solutions in the integer constraints from that relaxation. If we instead use the variable splitting with one set of variables for the hydro balance and another set for the integer constraints, we will get one subproblem for hydrological constraints and another subproblem for integer constraints.

Appendix C
Implementation of Heuristic Search Method

This appendix gives a detailed description of how the heuristic feasible solution method in section 5.5.3 can be implemented in practice. First I will go through the algorithm in the case of no run-of-the river plants and no soft constraints for final reservoir contents. I will come back to how we need to modify the algorithm for these cases. The algorithm is shown in Fig. C.1.

The basic idea of the algorithm is to check for one plant at a time, if it is possible to decrease the objective function or the infeasibility by changing one integer variable or discharge at the continuous part. This will change the discharge. The discharge change is balanced with a change in the continuous variables: discharge on the continuous part for another hour, final reservoir contents or spillage.

If we are in mode 1 in Fig. 5.17 the goal is to find a feasible solution and if we are in mode 2, the goal is to decrease the objective function. In the following I will assume that we are in mode 2. However, the only difference if we are in mode 1, is that we should calculate how the changes affect the infeasibility instead of how they affect the objective function. If two changes have the same effect on the infeasibility, the one having the best effect on the objective function should be ranked before the other.

We start in box 0 and go to box 1 where we set the counter for plants equal to zero. In the next box we go to the next plant and initialize the counter for hours. In the third box we check if there is spillage some hour. If this is the case the next step is to check if it is possible to decrease the spillage by increasing the final reservoir contents. This situation may occur if the reservoir contents of the plant or the downstream plant have been changed in an earlier iteration.

Each search for the best change of the discharge starts with a sorting of the possibilities of balancing the change with the continuous variables I mentioned above. This is done in boxes 4 and 5.

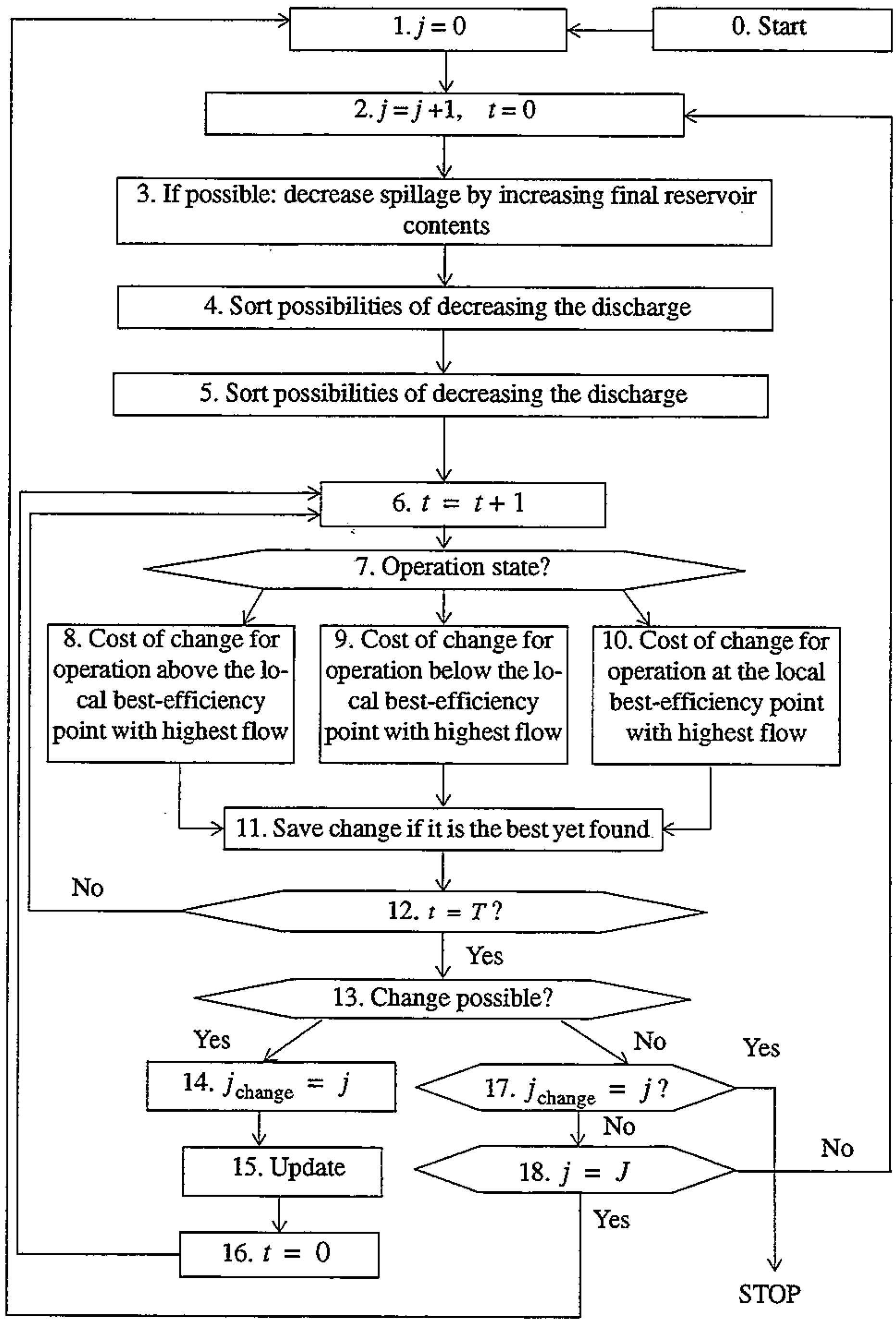


Fig. C.1 Flow chart of the heuristic algorithm.

Boxes 4 and 5 sort all the possibilities of balancing the change of an integer variable or discharge at the continuous part. Box 4 sorts the possibilities of decreasing the discharge, spillage and final reservoir without changing integer variables, in accordance with how this would affect the objective function (least increase per discharge first). If the discharge is decreased, the purchase in the power exchanges must increase. Call this list A.

In the same way box 5 sorts the possibilities of increasing the discharge and final reservoir without changing integer variables, in accordance with how this would affect the objective function (maximum decrease per discharge first). If the discharge is increased, the purchase in the power exchanges must decrease. Call this list B.

After that we start to go through the possibilities of changing the discharge for different hours. The items in lists A and B are used to balance the discharge change.

In box 6 where the counter for hours is updated. Then we go to box 7: If the plant is operating above the local best-efficiency point with highest flow (but not at the lower limit) during hour t, go to box 8. If the plant is operating at a lower state than the continuous part during hour t go to box 9, else go to box 10.

Box 8 checks the total change in the objective function per change of discharge for a small change of the discharge for hour t. The changes of discharge must be balanced with changes of discharge, spillage or final reservoir contents according to lists A and B, where list A is used for increase of discharge and list B is used for decrease of discharge. Here we will use the first item in the list if it is possible according to reservoir limits, otherwise the second and so on. It is not always possible to find a way to change the discharge in this manner. If it is not possible, we can set the cost to infinity. If we instead go to box 9 the procedure is almost the same as box 8, with the difference that we check the effect on the objective function of changing the discharge to points on change of status (off-line or on-line) for one unit. The effect on the objective must include change of start-up costs. Here it is possible that we have to use several items in list A or B. The third alternative is box 10 where we use the method in box 8 to calculate the cost of an increase of the discharge and use the method of box 9 to calculate the cost of a decrease.

If this change of the objective function is lower than zero and the best yet found, then save it (box 11). Now check if we have calculated the cost change for all hours. If not go back to box 6. Otherwise go to box 13, where we check if it was possible to decrease the objective function by changing the discharge for some hour. If it was possible we go to box 14, otherwise to box 17.

In box 14 we store number of the plant, for which it was possible to update the discharge. Then we go to box 15 where we update the discharge, reservoir contents and spillage in the way that gives the largest decrease in the objective function. Update lists A and B. After that, reset the counter for hours in box 16 and go to box 6.

In box 17 it is checked if we have gone through all plants without finding a possibility of decreasing the objective function. If this is the case we stop, otherwise we go to box 18. In this box it is checked if we are at the last plant. In this case go to the first plant (box 1). Otherwise go to the next plant (box 2).

In the case when we have run-of-the river plants where the discharge for one hour leads to generation for other hours, see section 3.2.2. This means that if the discharge change is balanced with a discharge change for another hour, these discharge changes can affect the cost function for the same hour. Then the discharge changes cannot be treated independently of each other as above. So, instead of boxes 4 and 5 in Fig. C.1 we have to find the best ways to balance the discharge change in boxes 8, 9 and 10 for each possible discharge separately. As a result the calculations will be somewhat more time consuming since we cannot use the same list of cost for balancing discharge change in boxes 8, 9 and 10.

A similar problem might occur if we use soft constraints for final reservoir contents. If we hit a breakpoint in the cost for the final reservoir contents the cost for changing these contents will change. Then we have to resort the list A or B. The best way to do this is to copy the current list and work on the copy in boxes 8, 9 or 10 so that we can go back to the original list while we are in the loop from boxes 6-12.

Appendix D
Monte Carlo Simulation of Hydro Unit Outage

In this appendix I will present the Monte Carlo simulation model I have used to calculate the unavailability costs in chapter 6.

The hydro plants in a river are hydrologically coupled to each other. This means that the unavailability costs of one unit are dependent on how the other plants in the river are operated. In [82], for example, a model for unavailability calculations of thermal plants is presented. Since thermal plants are not coupled in the same way as hydro plants it is possible to perform the calculations with analytical models. In case of unavailability calculations for the plants in a river an analytical approach will have too many states. Instead we have used Monte Carlo simulation to calculate the cost of unavailability in a river.

Fig. D.1 shows a flow chart of the Monte Carlo simulation model [41] which we have used to estimate the unavailability costs of the units. In the first box we have to format data in such a way that they will be suited for the model. In box 2 we sample a scenario for the external factors price, initial reservoir levels and river inflow, c. f. section D.1. The next step (box 3) solves the base case with all units available. This step uses the LP-model presented in section 3.2. In the following steps we make one unit at a time unavailable and calculate the cost for this unavailability by using the planning model. The unavailability cost will be the difference in benefit of the base case (box 3) and the benefit of the different outage cases (boxes 5-7).

It is possible that several units are unavailable at the same time. However, in our simulations we have not taken this case into consideration for several reasons. First, the probability for two units being unavailable at the same time is very low. Secondly, we compare the benefit of a schedule with some unit unavailable with a schedule with the unit available. In most cases it is not likely that the difference in benefit for those two schedules does change significantly if another unit is unavailable or not. Thirdly, taking all possible combinations of outages into consideration will lead to an exponential increase of the number of outage cases as function of the number of units. To

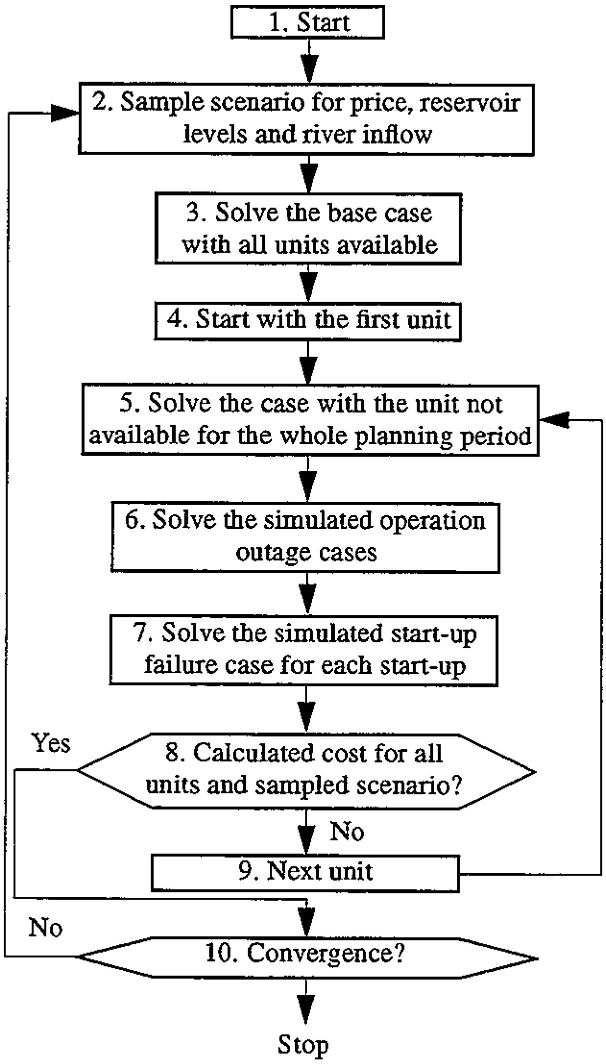


Fig. D.1 Monte Carlo simulation for unit unavailability cost calculation.

sum up, taking cases with several units unavailable at the same time into consideration will lead to many more simulations with only little influence on the result.

D.1 Model of external factors

The external factors: spot price, initial reservoir contents and river inflow vary during the year. The profile of the spot price also varies during the week. The river inflow in the Nordic countries has its maximum during the spring (April-June) when the snow is melting, and the load has its maximum during the winter (December-February). Since most of the water from the spring flood is stored in the reservoirs for the winter the reservoir contents of long term reservoirs will have their maximum during the summer and its minimum just before the spring flood. The smaller reservoirs, which are used for short term regulation, have shorter cycles from one maximum to the next. This cycle could for example be 24 hours. The initial reservoir contents are important in this study owing to three things. First, the spot prices are strongly correlated to the reservoir contents. If the total amount of energy stored in the reservoirs is lower than normal for a period, it is most likely that the spot price will be higher for that period. Secondly, if the reservoir contents are close to the maximum or minimum level during the planning period an outage can lead to spillage or a considerable rescheduling, which can be costly. The third fact is that reservoir contents affect the head of the hydro plants. The generation for a certain discharge will be lower if the head is lower. This means that a cost for an outage is lower when the head is lower if everything else is the same.

In order to sample scenarios for the random variables initial reservoir contents, river inflow and spot price we first sample a day of the year. Based on the type of day the random variables are sampled as follows:

<u>Model of initial reservoir contents.</u>

For the large reservoirs we have used statistics of the total contents of these reservoirs. Based on the minimum, mean and maximum values of the actual week we have sampled the reservoir contents (triangular distributed). All large reservoirs will get the same relationship between sampled contents and maximum contents. For small reservoirs we have assumed that these reservoirs are rather full in the beginning of the planning day. We have let the reservoir contents be between 80 and 100 per cent of their maximum, uniformly distributed. The contents of the small reservoirs are sampled independently of each other.

Model of river inflow.

There are statistics of the river inflow to the reservoirs available for 40 years. For the sampled week we sample one of the inflows in the statistics for that week.

Model of spot price.

Samples of spot prices are based on statistics from the Nordic spot market. We have divided the price scenarios into the groups: weekdays and weekends, so we have to sample which kind of day is the simulated day. If the sampled day is a weekday we will sample a price scenario valid for a weekday and the corresponding for weekends. The spot price of the Nordic market has varied between ca 20 SEK to 350 SEK ($3 to $50) per MWh. In order to take this into account we have sampled a factor for the price level. This factor is strongly negatively correlated to the deviation of the reservoir contents from the mean reservoir contents. The price factor will be sampled in the following way:

$$\varsigma = \vartheta' + \frac{\mathrm{cov}(\vartheta, \varsigma)}{\mathrm{var}(\varsigma)}\varsigma \tag{D.1}$$

where

- ϑ is the mean price of the day.
- ϑ' is the mean price sampled without correlation to the reservoir contents.
- ς is the sampled reservoir contents minus the mean contents, divided by the mean contents.
- $\mathrm{cov}(\vartheta, \varsigma)$ is the co-variance between ϑ and ς.
- $\mathrm{var}(\varsigma)$ is the variance of ς.

The sampled price profile is then multiplied by ϑ. In our example the correlation factor, $\mathrm{cov}(\vartheta, \varsigma)/\sqrt{\mathrm{var}(\vartheta) \cdot \mathrm{var}(\varsigma)}$, between ϑ and ς is about -0,86. I have calculated this value based on statistics from the Swedish-Norwegian power market and statistics from the KGS-pool where the power companies coordinated their planning before the deregulation. In the latter case I have assumed that the system marginal cost is a good estimate of what the spot price would have been if the market was deregulated.

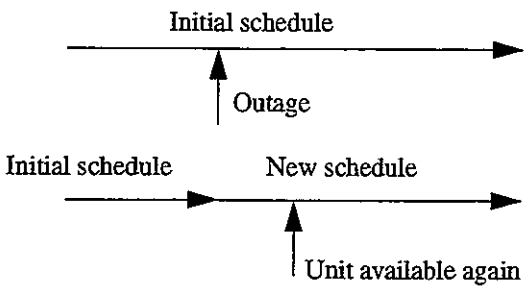


Fig. D.2 The estimation of the unavailability cost by comparison between the schedules without and with an outage.

D.2 Model of short outage

In order to estimate the cost of a short outage we will compare the cost of a schedule with the unit available during the whole period (box 3) with the cost of a schedule where the unit is unavailable for a random time (boxes 6-7). The second schedule will be the same as the first during the time before the outage since the planner does not know that the outage will occur. When an outage has occurred we assume that it is possible to give a rather good estimate of the duration of the outage. Based on this knowledge we can create a new schedule for the part of the planning period which is after the outage. Fig. D.2 shows a schematic picture of the estimation of the unavailability cost by comparison between different schedules.

The outage can occur during the operation (box 6) or during the start-up phase (box 7). This means that we have to treat these situations separately. If the outage occurs during the operation we have to sample the outage hour. If the outage occurs during the start-up phase we do not have to sample the outage hour since we know from the base case which hours the unit is started.

We have assumed that the outage time is Weibull distributed. The corresponding distribution function is

$$G(l) = 1 - e^{-Bl^{\Xi}} \qquad l \geq 0 \qquad \text{(D.2)}$$

A quite common model for the repair time is the exponential model. However, we were not able to fit the available data into the exponential distribution. Since the Weibull distribution has two parameters instead of one for the exponential distribution it was easier to fit the data to the Weibull distribution.

In the sampling of the outage time we have used the methods *stratified sampling* and *complementary random numbers* to reduce the variance [41]. *Stratified sampling* means that we have divided the distribution into parts. Then we have sampled outage times from each part. This will ensure that long outage times that have low probability, but will affect the cost significantly, will occur in the calculation of the outage cost for all units. *Complementary random numbers* means that if one sample gives a value with a probability of 0,2 for that specific value or lower, we will in the next sample use the value of the random variable that has a probability of 0,8 for that specific value or lower. We have used the complementary numbers inside the different parts.

D.3 Convergence

We will stop calculations (box 10) when the maximum relative uncertainty of the average cost is lower than 5 per cent. The relative uncertainty [88] for factor Y is defined as the estimated standard deviation of the expected value divided by the expected value:

$$\frac{sd[Y]/\sqrt{N}}{E[Y]} = \frac{sd[Y]}{E[Y]\sqrt{N}} \tag{D.3}$$

where

- $E[Y]$ is the estimated expected value of Y.
- $sd[Y]$ is the estimated standard deviation of Y.
- N is the number of samples.
- $sd[Y]/\sqrt{N}$ is the estimated standard deviation of $E[Y]$.

References

[1] N. H. Abbasy and M. K. Elfayoumy, "An Improved Two Stage Dynamic Programming/Artificial Neural Network Solution Model to the Unit Commitment of Thermal Unit", *Proceedings of IEEE/KTH Stockholm Power Tech, Power Systems*, Stockholm, Sweden, June 18-22, 1995, pp. 36-41

[2] I. Abrahamsson, "Energimodellen - Framtidens verktyg", *Fjärrvärmetidningen*, nr 8, 1996, pp 8

[3] M. Aganagic, B. Awobamise, G. Raina and A. I. McCartney, "Economic Dispatch with Generation Contingency Constraints", *IEEE Transactions on Power Systems*, Vol. 12, No. 3, 1997, pp. 1229-1234

[4] Y-S Baek and J-H Han, "Fast Economic Dispatch Approach Solution With Piecewice Quadratic Cost Function Using Lagrangian Multiplier", *Proceedings of IEEE/KTH Stockholm Power Tech, Power Systems*, Stockholm, Sweden, June 18-22, 1995, pp. 42-47

[5] W. R. Barcelo and P Rastqoufard, "Dynamic Economic Dispatch Using the Extended Security Constraints Algorithm", *IEEE Transactions on Power Systems*, Vol. 12, No. 2, 1997, pp. 961-967

[6] J. F. Bard, "Short-Term Scheduling of Thermal Generators Using Lagrangian Relaxation", *Operations Research*, Vol. 36, No. 5, September-October 1988, pp. 756-767

[7] J. Batut, T. Eve, F. Pellegrino and P. Vintache, "Optimal Hydroelectric Generation in Operation, *Proceedings of the 12th PSCC*, Dresden, Germany, August 19-23, 1996, pp. 390-396

[8] J. Batut, A. Renaud and P. Sandrin, "New Software For the Generation Scheduling In Future EDF National Control Center", *Proceedings of the 10th PSCC*, Graz, Austria, August 19-24, 1990, pp. 1163-1170

[9] H. Brännlund, *Network Programming Applied to Operation Planning of Hydrothermal Power Systems*, Department of Electric Power Systems Engineering, Kungliga Tekniska Högskolan, Stockholm, Sweden, 1986

[10] H. Brännlund, J.A. Bubenko, D. Sjelvgren and N. Andersson, "Optimal Short-Term Operation Planning of a Large Hydrothermal System Based on a Nonlinear Network Flow Concept" *IEEE Trans. on Power Apparatus. Systems.*, vol. PWRS-1 No. 4, 1986, pp. 1154-1161

[11] J-P Chancelier and A. Renaud, "Daily Generation Scheduling: Decomposition Methods to Solve the Hydraulic Problems", *Electric Power & Energy Systems*, Vol. 16 No. 3, 1994, pp 175-181

[12] H-C Chang and P-H Chen, "A Novel Economic Dispatch Approach-Implementation of Generic Algorithm", *Proceedings of IEEE/KTH Stockholm Power Tech, Power Systems*, Stockholm, Sweden, June 18-22, 1995, pp. 48-54

[13] S-C Chang, C-H Chen, I-K Fong and P. B. Luh, "Hydroelectric Generation Scheduling with an Effective Differential Dynamic Programming Algorithm", *IEEE Transactions on Power Systems*, Vol. 5, No. 3, 1990, pp. 737-743

[14] A. Chiva, F. J. Heredia and N. Nabona, "Network Model of Short-Term Optimal Hydrothermal Power Flow with Security Constraints", *Proceedings of IEEE/KTH Stockholm Power Tech, Power Systems*, Stockholm, Sweden, June 18-22, 1995, pp. 67-73

[15] M. Christoforidis, B. Awobarnise, S. Tong, M. Aganagic, R. J. Frowd and F. A. Rahimi, "Short-Term Hydro Generation and Interchange Contract Scheduling for Swiss Rail", *IEEE Transactions on Power Systems*, Vol. 11, No. 1, 1996, pp. 274-280

[16] A. I. Cohen and G. Ostrowski, "Scheduling Units With Multiple Operating Models in Unit Commitment", *IEEE Transactions on Power Systems*, Vol. 11, No. 1, 1996, pp. 497-503

[17] A. Conejo, "Unit Commitment", *Lecture Notes: Advanced Planning Methods in Generation, Transmission and Distribution of Electric Energy Systems*, Univeridad Pontifica de Comillas, 1993

[18] C. L. Corrêa de Sá Jr and C. Lyro Filho, "Tuning the Planning Chain of Hydroelectrical Systems", *Proceedings of Hydropower 97*, Trondheim, Norway, June 30-July 2, 1997, pp. 267-270

[19] F. Dahlfors and K. Wilhelmsson, "Managing of Power Networks and Business in a Deregulated Market", *Proceedings of IEEE/KTH Stockholm Power Tech, Information and Control Systems*, Stockholm, Sweden, June 18-22, 1995, pp. 99-104

[20] G. Demartini, T. R. De Simone, G. P. Granelli, M. Montagna and K. Robo, "Dual Programming Methods for Large-Scale Thermal Generation Scheduling", *Proceedings of the 1997 IEEE PICA Conference*, Columbus, USA, May 11-16, 1997, pp. 232-238

[21] A. Elström, "Norway: Europe's Pumped-Storage System - Necessary modifications of Power Plants", *Proceedings of Hydropower 97*, Trondheim, Norway, June 30-July 2, 1997, pp. 173-179

[22] S. Feltenmark, *On Optimization of Power Production*, Ph. D. thesis, TRISA-MAT 97-OS1, Department of Mathematics, Kungliga Tekniska Högskolan, Stockholm, Sweden, 1997

[23] S. Feltenmark and P-O Lindberg, *Network Methods for Head-Dependent Hydro Power Scheduling*, TRITA/MAT-96-OS2, Kungliga Tekniska Högskolan, Stockholm, Sweden, 1996

[24] L Ferreira, "Short-Term Scheduling of a Pumped Storage Plant", *IEE Proceedings-C*, Vol. 139, No. 6, November, 1992

[25] M. L. Fisher, "An Application Oriented Guide to Lagrangian Relaxation", *Interfaces*, Vol. 15, pp 10-21, 1985

[26] M. L. Fisher, "The Lagrangian Relaxation Method for Solving Integer Programming Problems", *Management Science*, Vol. 27, 1981

[27] O. B. Fosso and A. Johannesen, "Security Constraint Short-Term Hydro-Thermal Scheduling in a Flexible Market Environment", *Proceedings of IEEE/KTH Stockholm Power Tech, Power Systems*, Stockholm, Sweden, June 18-22, 1995, pp. 74-79

[28] P. E. C. Franco, M. F. Carvalho and S. Soares, "A Network Flow Model for Short-Term Hydro Dominated System Hydrothermal Systems", *IEEE Transactions on Power Systems*, Vol. 9, No. 2, 1994, pp. 1016-1022

[29] X. Guan, P. B. Luh, and B. Prasannan, "Power System Scheduling with Fuzzy Reserve Requirements", *IEEE Transactions on Power Systems*, Vol. 11, No. 2, 1996, pp. 864-869

[30] X. Guan, P. B. Luh, H. Yan A. Amalfi, "An Optimization-Based Method for Unit Commitment", *Electric Power & Energy Systems*, Vol. 14 No. 1, 1992, pp 175-181

[31] X. Guan, P. B. Luh, H. Yan and P. Rogan, "Optimization-Based Scheduling of Hydrothermal Power System With Pumped-Storage Units", *IEEE Transactions on Power Systems,* Vol. 9, No. 2, 1994, pp. 1023-1031

[32] X. Guan, P. B. Luh and L. Zhang, "Non-Linear Approximation Method in Lagrangian Relaxation- Based Algorithms for Hydro Thermal Scheduling", *IEEE Transactions on Power Systems,* Vol. 10, No. 2, 1995, pp. 772-778

[33] X. Guan, E. Ni, R. Li and P. B. Luh., "Optimization-Based Scheduling of Cascaded Hydro Power System with Discrete Constraints", *Proceedings of the 28th North American Power Symposium,* Boston, USA, November, 1996, pp. 17-23

[34] M. Guagnard and S. Kim, "Lagrangean Decomposition: A Model Yielding Stronger Lagrangean Bounds", *Mathematical Programming 39,* 1987, pp 215-228

[35] F. Gubina, G. Zlatarev, J. Hrovatin and V. Kusakatov, "Optimal Short Term Operation of Variable Head Hydro Plants in Hydro-Thermal System by Minimum Norm Approach", *Cigré 1996:38-205,* Paris, France

[36] H. Habibollazadeh, *Application of Mathematical Programming to Short-Term Operation Planning of Hydrothermal Power Systems*, Department of Electric Power Systems Engineering, Kungliga Tekniska Högskolan, Stockholm, Sweden, 1984

[37] H. Habibollazadeh, D. Frances and U. Sui, "A New Generation Scheduling Program at Ontario Hydro", *IEEE Transactions on Power Systems,* Vol. 5, No. 1, 1990, pp. 65-73

[38] S. Hachem, F. Welt, A. G. Hammadia and M. Breton, " Dynamic Models for Real Time Management of Hydro Plants", *Proceedings of Hydropower 97,* Trondheim, Norway, June 30-July 2, 1997, pp. 341-346

[39] P. G. Harhammer, "Advanced Operation Planning - Second Generation Tools (Energy Transaction Management System)", *Proceedings of the 4th Conference on Power System Management and Control,* IEE Conference Publication No. 421, London, UK, April 16-18, 1996, pp. 7-11

[40] F. J. Heredia and N. Nabona, "Optimum Short-Term Hydro Scheduling with Spinning reserve through Network Flows", *IEEE Transactions on Power Systems,* Vol. 10, No. 3, 1995, pp. 1642-1651

[41] F. S. Hiller and G. J. Liebermann, *Introductions to Operations Research*, McGraw-Hill international editions, 1990

[42] W. J. Hobbes, G. Hermon, S. Warner, and G. B, Sheblé, "An Enhanced Dynamic Programming Approach for Unit Commitment", *IEEE Transactions on Power Systems,* Vol. 3, No. 3, 1988, pp 1201-1205

[43] S-J Huang and C-L Huang "Application of Genetic-Based Neural Networks to Thermal Unit Commitment", *IEEE Transactions on Power Systems,* Vol. 12, No. 2, 1997, pp. 654-660

[44] M. Hüsselman, T. Müller and H. Wetzlinger, " Economic Short Term Scheduling for a Run-of-River-Plant Chain by Combined LP and Generic Optimization, *Proceedings of the 4th Conference on Power System Management and Control,* IEE Conference Publication No. 421, London, UK, April 16-18, 1996, pp. 42-46

[45] A. Jiang and S. Ertem, "Economic Dispatch With Non-Monotonically Increasing Incremental Cost Units and Transmission System Losses", *IEEE Transactions on Power Systems,* Vol. 10, No. 2, 1995, pp. 891-897

[46] U. Kakko, T. Karttinen and J. Ruusunen, "A New Model for Optimization at Imatran Voima", *Proceedings of the 1995 IEEE PICA Conference,* Salt Lake City, USA, May 7-12, 1995, pp. 33-39

[47] T. Kashiwagi, T. Wakabayashi, Y. Hayashi and S. Iwarmoto, "ELD Calculation Using Quadratic Programming Based on Binary Search", *Proceedings of IEEE/KTH Stockholm Power Tech, Power Systems,* Stockholm, Sweden, June 18-22, 1995, pp. 55-60

[48] J. L. Kennington and R. V. Helgason, *Algorithms for network programming,* 1980, John Wiley & Sons

[49] L. Lafond, An Optimization Model for Regional Planning at Hydro-Québec", *Proceedings of Hydropower 97,* Trondheim, Norway, June 30-July 2, 1997, pp. 259-266

[50] L. L. Lai, J. T. Ma and K. P. Wong, "Evolutionary Programming for Economic Dispatch of Units With Non-Smooth Input-Output Characteristic Function", *Proceedings of the 12th PSCC,* Dresden, Germany, August 19-23, 1996, pp. 492-498

[51] C-A Li, E. Hsu, A. J. Svoboda, C-L Tseng and R. B. Johnson, "Hydro Unit Commitment in Hydro-Thermal Optimization", *IEEE Transactions on Power Systems,* Vol. 12, No. 2, 1997, pp. 764-769

[52] C-A Li, R. B. Johnson and A. J. Svoboda, "A New Unit Commitment Method", *IEEE Transactions on Power Systems*, Vol. 12, No. 1, 1997, pp. 113-119

[53] D. G. Luenberger, *Linear and Nonlinear Programming*, Addison Wesley, 1989

[54] P. B. Luh, D. Zhang and R. N. Tomastik, "An Algorithm for Solving the Dual Problem of Hydro Thermal Scheduling", *IEEE paper PE-601-PWRS-0-01-1997, IEEE/PES Winter Meeting,* New York, USA, February 2-6, 1997

[55] T. Magnusson, *The unit commitment problem a dual approach*, TRITA-MAT-1991-44, Department of Mathematics, Kungliga Tekniska Högskolan, Stockholm, Sweden, 1991

[56] J. W. Marangon Lima and A. M. Leite de Silva, "Spinning Reserve Requirements for Multi-Area Systems", *Proceedings of the 12th PSCC*, Dresden, Germany, August 19-23, 1996, pp. 724-729

[57] V. M. F. Mendes and L. A. F. M. Ferreira "Short-Term Resource Scheduling in Large-Scale Hydrothermal Systems: Dual Optimization by a Modified ε_Subgradient Algorithm, *Proceedings of IEEE/KTH Stockholm Power Tech, Power Systems*, Stockholm, Sweden, June 18-22, 1995, pp. 86-92

[58] V. M. F. Mendes, L. A. F. M. Ferreira, R. Roldao and R. Pestana, "Optimal Short-Term Resource Scheduling by Lagrange Relaxation: Bundle-type Versus Subgradient Algorithms", *Proceedings of the 12th PSCC*, Dresden, Germany, August, 19-23, 1996, pp. 375-381

[59] A. Merlin, B. Lauzanne, J. F. Maurras, J. Auge and M. Ziglioli, "Optimization of short-term scheduling of EDC hydraulic valleys with coupling constraints: the OVIDE model", *Proceedings of 7:th PSCC*, July 12-17, 1981, Lausanne, Switzerland

[60] A. Merlin and P, Sandrin "A New Method for Unit Commitment at Electricite de France", *IEEE Transactions on Power Apparatus and Systems*, Vol. PAS-102, No. 5, 1983, pp 1218-1225

[61] V. Miranda, D. Srinivasen and L. M. Proenca, "Evolutionary Programming in Power Systems", *Proceedings of the 12th PSCC*, Dresden, Germany, August 19-23, 1996, pp. 25-42

[62] K. Neumann, "Dynamic Programming", *Lecture Notes: Optimization in Planning and Operation of Electric Power Systems*, SVOR/ASRO, 1992, Switzerland

[63] A. Nilsberth and L. Söder "Planning of Electricity Contracts on a Deregulated Market", *DA/DSM Europe 96*, Vienna, Austria

[64] O. Nilsson, *Hydro Power Models For Short Term Generation Planning*, Licentiate Thesis, TRITA-EES-9502, Department of Electric Power Engineering, Kungliga Tekniska Högskolan, Stockholm, Sweden, 1995

[65] O. Nilsson, *Startkostnader för vattenkraftaggregat*,π Internal report, A-EES-9511, Department of Electric Power Engineering, Kungliga Tekniska Högskolan, Stockholm, Sweden, 1995

[66] O. Nilsson, *Vattenkraftmodeller för kortsiktig produktionsplanering*, Internal report, A-EES-9403, Department of Electric Power Engineering, Kungliga Tekniska Högskolan, Stockholm, Sweden, 1994

[67] O. Nilsson, *Älvoptimering*, Master Thesis, B-EES-9201, Department of Electric Power Systems, Kungliga Tekniska Högskolan, Stockholm, Sweden, 1991

[68] O. Nilsson and D. Sjelvgren, "A Mixed-Integer Model for Daily Hydro Planning", *Proceedings of IEEE/KTH Stockholm Power Tech, Power Systems*, Stockholm, Sweden, June 18-22, 1995, pp. 99-104

[69] O. Nilsson and D. Sjelvgren, "A Planning Model With Mixed-Integer Representation of Both Hydro and Thermal Plants", *Proceedings of the 4th Conference on Power System Management and Control*, IEE Conference Publication No. 421, London, UK, April 16-18, 1996, pp. 280-285

[70] O. Nilsson and D. Sjelvgren, "Hydro Unit Start-Up Costs and Their Impact on Short Term Scheduling Strategies of Swedish Power Producers", *IEEE Transactions on Power Systems*, Vol. 12, No. 1, 1997, pp. 38-44

[71] O. Nilsson and D. Sjelvgren, "Improved Mixed Integer Programming in Short-Term Hydro Scheduling", *Proceedings of the 12th PSCC*, Dresden, Germany, August 19-23, 1996, pp. 382-389

[72] O. Nilsson and D. Sjelvgren, "Mixed-Integer Programming Applied to a Hydro-Thermal System", *Proceedings of PICA*, Salt Lake City, USA, May 7-12, 1995, pp. 158-163 and *IEEE Transactions on Power Systems*, Vol. 11, No. 1, 1996, pp. 281-286

[73] O. Nilsson and D. Sjelvgren, "Variable Splitting Applied to Modelling of Start-Up Costs in Short Term Hydro Scheduling", *IEEE Transactions on Power Systems*, Vol. 12, No. 2, 1997, pp. 770-775

[74] O. Nilsson and L. Söder, "Daily Generation Planning of a Hydro Dominated Hydrothermal System", *Proceedings of Hydropower 97*, Trondheim, Norway, June 30-July 2, 1997, pp. 293-300

[75] O. Nilsson and L. Söder, "Short Term Planning Methods Applied to Evaluation of Unavailability Costs in a River", *Proceedings of IPEC 97*, Singapore, May 22-24, 1997, pp. 36-41

[76] O. Nilsson, L. Söder and D. Sjelvgren, "Integer Modelling of Spinning Reserve Requirements in Short Term Scheduling of Hydro Systems", *IEEE paper PE-142-PWRS-0-06-1997, IEEE/PES Summer Meeting*, Berlin, Germany, July 21-25, 1997

[77] G. G. Oliveira and S. Soares, "A Second-Order Network Flow Algorithm for Hydrothermal Scheduling", *IEEE Transactions on Power Systems*, Vol. 10, No. 3, 1995, pp. 1635-1641

[78] S. O. Orero and M. R. Irving, *A Genetic Algorithm Modelling Framework and Solution Technique for Short Term Optimal Hydrothermal Scheduling*, Brunel Institute of Power Systems, Brunel University, Uxbridge, UK

[79] S. O. Orero and M. R. Irving, "Large Scale Unit Commitment Using a Hybrid Genetic Algorithm", *Electric Power & Energy Systems*, Vol. 19 No. 1, 1997, pp 45-55

[80] J. W. O'Sullivan and M. J. O'Malley, "Economic Dispatch of a Small Utility with a Frequency Based Reserve Policy", *IEEE Transactions on Power Systems*, Vol. 11, No. 3, 1996, pp. 1648-1653

[81] C. K. Pang and H. C. Chen, "Optimal Short-Term Thermal Unit Commitment", *IEEE Transactions on Power Apparatus and Systems*, Vol. PAS-95, No. 4, 1976, pp 1336-1346

[82] A. D. Patton, C. Singh and M. Sahinoglu, "Operation Considering in Generation Reliability Modeling - An Analytical Approach", *IEEE Transactions on Power Apparatus and Systems,* Vol. 5, 1991, pp 2656-2663

[83] F. Pellegrino, A.Renaud and T. Socroun, "Bundle and Augmented Lagrangian Methods for Short-Term Unit Commitment", *Proceedings of the 12th PSCC,* Dresden, Germany, August 19-23, 1996, pp. 730-739

[84] W. L. Peterson and S. R. Brammer, "A Capacity Based Lagrangian Relaxation Unit Commitment with Ramp Constraints", *IEEE Transactions on Power Systems,* Vol. 10, No. 2, 1995, pp. 1077-1084

[85] M. R. Piekutowski, T. Litwinowicz and R.J. Frowd, "Optimal Short-Term Scheduling for a Large-Scale Cascaded Hydro System", *IEEE Transactions on Power Systems,* Vol. 9, No. 2, 1994, pp 805-811

[86] B. T. Polyak, "Minimization of Nonsmooth Function", *Journal of Computational Mathematics and Mathematical Physics,* vol. 9 No. 3, 1969, pp. 14-29

[87] R. A. Ponrajah, J. Witherspoon and F. D. Galiana, "Systems to Optimise Conversion Efficiencies at Ontario Hydro's Hydroelectric Plants", *Proceedings of the 1997 IEEE PICA Conference,* Columbus, USA, May 11-16, 1997, pp. 245-251

[88] J. Román-Ubeda, *Reliability Assessment of Composite Power System Planning Containing Mixed Hydro Thermal Optimization,* Ph. D. thesis, Department of Electrical Engineering and Electronics, University of Manchester, UK, 1991

[89] S. Ruzic, N. Rajakovic and A. Vuckovic, "A Flexible Approach to Short-Term Hydro-Thermal Coordination, Part I: Problem Formulation and General Solution Procedure", *IEEE Transactions on Power Systems,* Vol. 11, No. 3, 1996, pp. 1564-1571

[90] S. Ruzic, N. Rajakovic and A. Vuckovic, "A Flexible Approach to Short-Term Hydro-Thermal Coordination, Part II: Dual Solution Procedure", *IEEE Transactions on Power Systems,* Vol. 11, No. 1, 1996, pp. 1572-1579

[91] M. Sandqvist, *Stochastic Methods in Seasonal Planning of Hydro-Thermal Power Systems,* Licentiate Thesis, TRITA-EES-9504, Department of Electric Power Engineering, Kungliga Tekniska Högskolan, Stockholm, Sweden, 1995

[92] G. B. Sheblé, T. T. Maifield, K. Britting, G. Fahd and S. Fukurozaki-Coppinger, "Unit Commitment by Genetic Algorithm with Penalty Methods and a Comparison of Lagrangian Search and Genetic Algorithm - Economic Dispatch Example", *Electric Power & Energy Systems*, Vol. 18 No. 6, 1996, pp 339-346

[93] D. Sjelvgren, *Systemplanering för elektriska energisystem*, Department of Electric Power Engineering, Kungliga Tekniska Högskolan, Stockholm, Sweden, 1994

[94] G. Skugge, *Including Natural Gas In Hydro-Thermal Energy Balance - for Evaluation with Multiple Objectives and Uncertainties*, Ph. D. Thesis, TRITA-EES-9204, Department of Electric Power Systems, Kungliga Tekniska Högskolan, Stockholm, Sweden, 1992

[95] W. L. Snyder, H. D. Powell and J. C. Rayburn, "Dynamic Programming Approach to Unit Commitment", *IEEE Transactions on Power Systems*, Vol. PWRS-2, No. 2, may 1987, pp 339-350

[96] S. Soares and C. T. Salmazo "Minimum Loss Predispatch Model for Hydroelectric Power Systems", *IEEE Transactions on Power Systems*, Vol. 12, No. 3, 1997, pp. 1220-1227

[97] Svenska Kraftnät, *Handelsplats för el*

[98] Svenska Kraftnät, *The Swedish Electricity Market Reform and its Implications for Svenska Kraftnät*

[99] Svenska Kraftverksföreningen, *Kraftåret 1996*

[100] A. J. Svoboda, C-L Tseng, C-A Li and R. B. Johnson, "Short-Term Resource Scheduling with Ramp Constraints", *IEEE Transactions on Power Systems*, Vol. 12, No. 1, 1997, pp. 77-83

[101] L. Söder, "A Mixed Rule Based-Linear Programming Method Applied to Daily Operation Planning of a Hydro Power System", *Proceedings of IEEE/KTH Stockholm Power Tech, Power Systems*, Stockholm, Sweden, June 18-22, 1995, pp.93-98

[102] L. Söder, *Kortsiktig planering av vattenkraftsystem*, Inst för Elkraftteknik, Kungliga Tekniska Högskolan, Stockholm, 1996

[103] L. Söder, *Långsiktig kraftsystemplanering*, Inst för Elkraftteknik, Kungliga Tekniska Högskolan, Stockholm, 1996

[104] J. Tang and P. Luh, "Hydrothermal Scheduling Via Extended Differential Dynamic Programming and Mixed Coordination", *IEEE Transactions on Power Systems,* Vol. 10, No. 4, 1995, pp. 2021-2028

[105] H. Tomeoka, H. Nakajima, Y. Kawachi, S. Nakamura and M. Watanabe, "New Daily Scheduling System for Multi-Chain Hydro Power System", *Engineering Intelligent Systems* Vol. 2 No. 1, March, 1994, pp 23-30

[106] N. Tufegdzig, R. J. Frowd and W. O. Standin, "A Coordinated Approach for Real-Time Short Term Hydro Scheduling", *IEEE Transactions on Power Systems,* Vol. 11, No. 4, 1996, pp. 1698-1704

[107] Vattenfall, *Effektivare körning av vattenkraftstation*

[108] Vattenfall and Skanska, *Hydro Power in Sweden,* Kraftverksföreningen, Stockholm, 1987

[109] L. Vinogg, "Maximizing the Profit of Hydro Generation When Taking Into Account the Extra Costs of Co-Generation With Thermal Power", *Proceedings of Hydropower 97,* Trondheim, Norway, June 30-July 2, 1997, pp. 323-326

[110] X. Wang and J. R. McDonald, *Modern Power System Planning,* McGraw-Hill Book Company, London, 1994

[111] T. G. Werner and J. F. Verstege, "Evolutionary Strategies Combined With Mixed-Integer Programming for Short-Term Operation Planning of Hydrothermal Systems", *Proceedings of IPEC 97,* Singapore, May 22-24, 1997, pp. 24-29

[112] J. Wernérus and L. Söder, "Area Price Based Multi-Area Economic Dispatch With Transmission Losses and Constraints", *Proceedings of IEEE/KTH Stockholm Power Tech, Power Systems,* Stockholm, Sweden, June 18-22, 1995, pp. 710-715

[113] A. J. Wood and B. F. Wollenberg, *Power Generation, Operation and Control,* Second edition, John Wiley and Sons, New York, USA, 1996

[114] H. Yan and P. B. Luh, "A Fuzzy Optimization-Based Method for Integrated Power Systems Scheduling and Inter-Utility Power Transaction with Uncertainties", *IEEE Transactions on Power Systems,* Vol. 12, No. 2, 1997, pp. 756-763

[115] H. Yan, P. B. Luh, X. Guan and P. M. Rogan, "Scheduling Of Hydrothermal Power Systems", *IEEE Transactions on Power Systems*, Vol. 8 No. 3, 1993, pp 1358-1365

[116] X. Yan and V. C. Quintana, "An Efficient Predictor Interior Point Algorithm for Security-Constraint Economic Dispatch", *IEEE Transactions on Power Systems*, Vol. 12, No. 2, 1997, pp. 803-810

[117] H-T Yang, P-C Yang and C-L Huang, "A Parallel Genetic Algorithm Approach to Solving the Unit Commitment Problem: Implementation in Transputer networks", *IEEE Transactions on Power Systems*, Vol. 12, No. 2, 1997, pp. 661-668

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