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Passive control of a swept-wing boundary layer using ring-type plasma actuators

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Application of the ring-type plasma actuators for passive control of laminar-turbulent transition in a swept-wing boundary layer is investigated thorough direct numerical simulations. These actuators induce a wall-normal jet in the boundary layer and can act as virtual roughness elements. The flow configuration resembles experiments of Kim et al. (2016). The actuators are modelled by the volume forces computed from the experimentally measured induced velocity field at the quiescent air condition. The natural surface roughness and unsteady perturbations are also included in the simulations. The interaction of generated vortices by the actuators with these perturbations is investigated in details. It is found that for a successful transition control the power of the actuator should be increased to generate a jet velocity one order of magnitude higher than that in the considered experiments.

Key words: boundary layer control, boundary layer receptivity, ring-type plasma actuators, crossflow vortices

1. Introduction

Turbulent friction drag on swept wings of modern aircraft accounts for a large proportion of the total drag (Schrauf 2005). Thus, it is desirable to delay transition from laminar to turbulent flow for both environmental and economical reasons. In swept-wing flows, the so called crossflow instability typically dominates the transition process. This instability leads to growth of both streamwise orientated steady and travelling crossflow vortices depending on the external disturbance environment (see reviews by Bippes 1999; Saric et al. 2003). Non-stationary disturbances may dominate the route to transition in environments with rather high levels of freestream turbulence, e.g. wind tunnels. At free-flight cruise conditions, characterised by rather low levels of freestream turbulence, steady crossflow vortices excited by natural surface roughness prevail. Irrespective of the disturbance environment, crossflow vortices generate localised high-shear layers which trigger strong secondary instabilities prior to transition (Wassermann & Kloker 2002, 2003).
Several approaches to control transition in boundary-layer flows have been proposed in the literature. The review by Saric et al. (2011) summarises the main passive laminar-flow control techniques for swept-wing boundary layers. An encouraging method to control crossflow instabilities is the application of spanwise-periodic distributed micron-sized roughness (DMSR) elements placed near the leading edge of the swept wing. Such elements spaced narrower than the spanwise wavelength of the naturally most amplified mode, excite steady subcritical mode, commonly called the ‘control’ mode, and induce a useful mean-flow distortion. As a consequence, the growth of the naturally most unstable mode is attenuated and transition to turbulence is delayed. The application of DMSR elements for transition delay have been successfully shown in wind tunnel experiments (Saric et al. 1998; Kachanov et al. 2015), free-flight experiment (Carpenter et al. 2009) and numerically confirmed through direct numerical simulations (DNS) (Hosseini et al. 2013; Rizzetta et al. 2010) and through nonlinear parabolised stability equations (PSE) (Malik et al. 1999; Li et al. 2009).

In recent years, there has been increasing interest in the application of dielectric-barrier-discharge (DBD) plasma actuators for (mainly) active flow control. They have the benefit of being fully electric with low power consumption, fast response time and without moving parts. Plasma actuators generate a body force that locally accelerates its surrounding fluid. Thus, they can be used as an alternative tool to mimic blowing/suction or surface roughness elements for passive flow control. Furthermore, the amplitude of the generated body force can be tuned through the power supply to adapt to required conditions for flow control.

In an experimental investigation, Schuele et al. (2013) successfully employed an array of plasma actuators on a supersonic circular cone to excite subcritical crossflow modes and suppress the growth of naturally most unstable mode. In a more recent numerical study, Dörr & Kloker (2015b, 2016) investigated the applicability of plasma actuators for transition delay in a crossflow dominated boundary layer through DNS. In the former work, two plasma actuators per fundamental spanwise wavelength were employed to stabilise the boundary layer by base-flow manipulation and reduction of the mean crossflow, thus, weakening the primary crossflow instability. In the latter work, the plasma actuators were placed at selected spanwise positions to attenuate the nonlinear steady crossflow vortices and thus the associated secondary instability. In both studies, the effect of plasma actuators were modelled by a localised body force in the numerical simulations. Dörr & Kloker (2016) used one actuator per fundamental spanwise wavelength and applied the body force either in the direction of the crossflow or against it. Both setups were found to be effective in transition delay with the latter being less sensitive to the spanwise location of the actuator. The application of two plasma actuators per spanwise wavelength were also studied and found to be less effective for control of nonlinear disturbance state than applying one actuator per fundamental wavelength. This is because a significant
part of the actuator force can not directly attenuate the crossflow modes but merely reduces the crossflow in the mean, similarly to the work of Dörr & Kloker (2015b). Direct attenuation of the nonlinear crossflow mode is more effective approach for transition control compared to the base flow manipulation and reducing the crossflow mean, however, the latter technique hinders the growth of the unstable crossflow modes and is more robust (Dörr & Kloker 2016).

In a recent experimental investigation, Kim et al. (2016) studied the application of ring-type DBD plasma actuators, acting as virtual roughness elements, for transition delay on a swept-wing boundary layer. The ring-type plasma actuators generate a wall-normal jet while the aformentioned works were based on actuators inducing a wall-parallel jet. Kim et al. (2016) used rows of actuators with different spanwise spacing and chordwise locations. They observed that the actuators successfully excite the modes corresponding to their spanwise wavelength. However, they were not effective for transition delay and in some actuator configurations transition was promoted by 1.5% chord-length.

This study aims to numerically investigate the experiment of Kim et al. (2016) in order to gain a better understanding of the transition process and the role of plasma actuators. First, we study the effect of plasma actuators on the evolution of the primary steady crossflow modes. Then, unsteady disturbances are introduced in the simulations and the evolution of the primary and secondary modes is studied with and without active plasma actuators.

2. Flow configuration & numerical setup

The wing geometry used in the present investigation is a swept ONERA-D airfoil with a chord length of \( c = 0.35m \) normal to the leading edge. The geometry is invariant in the spanwise direction. The flow configuration follows experiments by Kim et al. (2016), performed within the European project BUTERFLI in ONERA TRIN1 wind tunnel, where a sweep angle \( \phi_\infty = 60^\circ \) and an angle of attack \( \alpha = -8^\circ \) has been used. The unit Reynolds number is \( Re_L = Q_\infty L/\nu = 4.4 \times 10^6 \), with \( L = 1m \) being the reference length scale, \( Q_\infty = 70m/s \) the total incoming freestream velocity and \( \nu \) the kinematic viscosity. \( Q_\infty \) is used to normalise velocities in this work. The ring-type plasma actuators with spanwise spacing of \( L_{pa} = 3.5mm \) and diameter \( d_{pa} = 1mm \) are placed in the leading-edge region of the swept wing at the chordwise location of \( x/c = 0.019 \) in the experiments. This location is too close to the stagnation point, \( x/c = 0.013 \), and does not allow for introduction of stationary and non-stationary disturbances upstream of the actuators in the numerical setup. Therefore, center of the actuators are shifted to the chordwise location of \( x/c = 0.05 \) which is slightly downstream of the neutral point of stationary crossflow modes. Boundary layer is more receptive to the plasma actuators at this location, thus, increasing the effect of the actuators on the evolution of the disturbances.
Figure 1: Swept ONERA-D airfoil with sweep angle $\phi_{\infty}$ and the total incoming velocity of $Q_{\infty}$. $\mathbf{(x, y, z)}$ and $(\xi, \eta, z)$ denote the cartesian and the curvilinear coordinate systems respectively. $c$ denotes the chord length normal to the leading edge. The wing is under an angle of attack $\alpha = -8^\circ$.

Figure 1 shows the airfoil shape and the coordinate systems used in this study. The Cartesian coordinates $\mathbf{(x, y, z)}$ denote the chordwise, normal-to-the-chord and spanwise directions and the corresponding velocity components are denoted as $(u, v, w)$. The body-fitted curvilinear coordinates $(\xi, \eta, z)$ define the tangential, wall-normal and spanwise directions with the corresponding velocity components denoted as $(u_\xi, v_\eta, w)$.

2.1. Direct numerical simulations

We consider incompressible Navier-Stokes equations subject to constant fluid properties together with the continuity equation,

\[
\frac{\partial \mathbf{u}}{\partial t} = -\mathbf{u} \cdot \nabla \mathbf{u} - \nabla p + \frac{1}{Re} \nabla^2 \mathbf{u} + \mathbf{f}, \quad \nabla \cdot \mathbf{u} = 0, \quad (1)
\]

where $\mathbf{u} = \{u, v, w\}^T$ is the vector of velocity components in the $x$-, $y$- and $z$-directions, $p$ represents the pressure and $\mathbf{f}$ is the body force term. Equations (1) are integrated in time using \texttt{Nek5000} code developed by Fischer et al. (2008). \texttt{Nek5000} is based on spectral element method (SEM) proposed by Patera (1984) which combines geometric flexibility of finite element method with spectral accuracy. The physical domain in SEM is decomposed into spectral elements where the local approximation of flow field is obtained as a sum of Lagrange interpolants defined by an orthogonal basis of Legendre polynomials up to degree $N$. Polynomial order $N$ is the same in all spatial directions. Following the $P_N - P_{N-2}$ spatial discretisation (Maday & Patera 1989), $N+1$ Gauss-Lobatto-Legendre (GLL) nodes are used to build velocity Lagrange polynomial interpolants and $N - 1$ Gauss-Legendre (GL) nodes for pressure Lagrange polynomial interpolants (two orders less than the velocity field) in every spectral element. Here we have used $N = 11$ for most of the simulations. The equations are advanced in time using a third-order conditionally stable backward differentiation and extrapolation scheme (BDFk/EXTk), employing an implicit treatment of the diffusion term and explicit treatment of the advection term. \texttt{Nek5000} is highly parallelised and scalable on thousands of threads (Tufo & Fischer 2001). Current results are obtained using up to 4096 processors.
Several sets of simulations have been carried out in this work; (I) laminar base flow is computed through a two-dimensional simulation in which the spanwise velocity component is computed using the temperature equation, (II) steady crossflow vortices are excited by means of natural surface roughness, (III) natural transition is triggered by introducing non-stationary perturbations inside the boundary layer, and (IV) change of transition location due to the action of distributed ring-type plasma actuators is studied.

2.2. Computational domain & boundary conditions

Prior to performing DNS computations, a complementary RANS solution is obtained around the same geometry with identical flow configuration, depicted in figure 2(a). The computational domain in the RANS computation includes the side walls of the experimental test section. Two different numerical domains are used in the DNS computations, the boundaries of which are shown in figure 2(a). The larger domain with two outflow boundaries is used for the laminar base flow computation. Although we are only interested in the flow field on the upper wing side, base flow domain extends to the lower wing side to account for the asymmetry of the configuration. The lower wing part and the leading edge region are discarded in the simulations of the perturbed flow which are performed in the smaller domain shown in figure 2(c). This is possible since the disturbances are introduced locally within the boundary layer and freestream disturbances, such as sound waves or freestream turbulence, are absent. The inflow and freestream boundary conditions of the perturbed flow are of Dirichlet type and taken from the laminar base flow solution. The outflow boundary conditions are the natural boundary conditions ($1/Re(\nabla u) - \mathbf{p} \mathbf{n} = 0$) derived from the weak form of the Navier-Stokes equations. The boundary conditions for the base flow computation are described in the following.

Dirichlet boundary conditions are set at the inflow plane using the RANS solution

$$\{u, v, w\}^T = \{u_{\text{trans}}, v_{\text{trans}}, w_{\text{trans}}\}^T \quad \text{on } \partial \Omega_{\text{inflow}}. \quad (2)$$

The outflow boundary conditions are a modified version of the natural boundary conditions

$$\frac{1}{Re} \frac{\partial u}{\partial x} - p = -p_a, \quad \frac{\partial v}{\partial x} = 0, \quad \frac{\partial w}{\partial x} = 0 \quad \text{on } \partial \Omega_{\text{outflow}}. \quad (3)$$

Here, $p_a$ stands for the ambient pressure and $p_a = 0$ results to the standard natural boundary conditions which is used in the perturbed flow simulations. It is worth mentioning that the natural conditions are not enforced explicitly but satisfied intrinsically by the numerical solver used in this work. In boundary-layer flows, the standard natural boundary conditions evaluates the pressure value at the junction point of the outflow and the wall to be zero, $p_{\text{wall}} = 0$, since the streamwise derivative of the velocity vanishes at the wall ($\partial_x u = 0$ at the wall). Evaluation of the pressure value to zero is not problematic since a correct pressure gradient is achievable with only one fixed pressure value in the domain. However, in flow configurations with multiple outflows, using the
standard outflow boundary conditions results in fixing the pressure value to zero at multiple locations. Therefore, the pressure gradient would depend on the relative location of the outflow boundaries which makes the results domain dependent. This problem is avoided by prescribing \textit{a priori} known pressure at the outflow boundaries which in turn sets the correct pressure gradient in a domain with multiple outflows. In this work, pressure field obtained from RANS solution is used to set the pressure at the outflow boundaries for base flow computation, $p_a = p_{\text{rans}} - Re^{-1}\partial_x u_{\text{rans}}$.

The freestream boundary conditions are of Dirichlet type in the streamwise and spanwise directions along with the modified natural condition normal to the surface

$$u = u_{\text{rans}}, \quad \frac{1}{Re} \frac{\partial v}{\partial y} - p = p_a, \quad w = w_{\text{rans}} \quad \text{on } \partial\Omega_{\text{free-stream}}. \quad (4)$$

The ambient pressure at the free stream is set to $p_a = p_{\text{rans}} - Re^{-1}\partial_y v_{\text{rans}}$ to account for the non-zero pressure gradient around the wing.
In all the simulations except the base flow computation, sponge regions are inserted in the vicinity of the outflow boundaries to avoid numerical instabilities. Within the sponge region, the flow is forced towards the laminar base flow. This is achieved by adding the forcing term

\[ F(x, t) = A_f \lambda_f(x)[U_f(x) - u(x, t)] \]  

on the right-hand side of the momentum equation. The field \( u = (u, v, w)^T \) stands for the instantaneous velocity field and \( U_f \) denotes the target solution (the laminar base flow). The function \( \lambda_f \) takes the form of a smooth step function which varies between zero and one in the sponge region and vanishes elsewhere. The coefficient \( A_f \) determines the maximal strength of the forcing and is \( A_f = 100 \) in this study. The length of the sponge region in the streamwise direction is 5% chord-length.

Distribution of the spectral elements in the computational domains used in the base flow and receptivity simulations are shown in figure 2(b,c) respectively. The mesh has been generated using the gridgen–c code by Sakov (2011) which provides quasi-orthogonal grids. The base flow domain extends from \( x/c = 0.018 \) on the lower side of the wing to \( x/c = 0.5 \) on the upper wing side. The domain for receptivity computations extends from \( x/c = 0.01 - 0.35 \) on the upper wing side. The 2D mesh for the base flow and the receptivity computations consists of 6300 and 6210 spectral elements respectively.

In the experimental study of Kim et al. (2016), an spanwise array of ring-type plasma actuators is inserted near the leading edge of the swept wing as a passive control mechanism to delay transition. The spacing between the actuators is \( L_{pa} = 3.5 \text{mm} \) which corresponds to approximately 2/3 of the wavelength of the naturally most unstable stationary crossflow mode (Saric et al. 1998; Hosseini et al. 2013). In order to excite the most unstable stationary mode and the control mode (induced by wall-normal jet of the plasma actuators) simultaneously, a spanwise length of \( L_z = 3 \times 3.5 = 10.5 \text{mm} \) is considered which dictates a fundamental spanwise wavenumber \( \beta_0 = 2\pi/L_z = 598 \text{rad/m} \). We mimic the actuator row by prescribing spanwise periodic boundary conditions. The two-dimensional mesh presented in figure 2(c) is extended in the spanwise \( z \)-direction. 36 spectral elements are uniformly distributed in the spanwise direction to resolve a span of length \( L_z \). The total number of three-dimensional spectral elements amounts to 223560 and using a polynomial order of \( N = 11 \) for the velocities results to \( \approx 386 \times 10^6 \) grid points.

### 2.3. Stationary perturbations

In the experimental studies, stationary crossflow vortices including the naturally most unstable one are excited due to the presence of the natural surface roughness in the leading edge region of the wing. In this work, the natural surface roughness which is localised along the chord and periodic in the spanwise direction is modelled by

\[ h(x, z) = \varepsilon h_x(x)h_z(z), \]  

where \( \varepsilon \) is the roughness parameter, \( h_x(x) \) is the chordwise variation of the roughness, and \( h_z(z) \) is the spanwise variation of the roughness.
where $\varepsilon_h$ is the maximum amplitude of the roughness bump and $h_x(x)$ and $h_z(z)$ are shape functions in the streamwise and spanwise directions. The shape function $h_x(x)$ is described by

$$h_x(x) = S\left(\frac{x - h_s}{h_r}\right) - S\left(\frac{x - h_e}{h_f} + 1\right),$$

where $S$ is a smooth step function defined in Schrader et al. (2009). The roughness bump in the streamwise direction starts at $x = h_s$, rises smoothly along the distance $h_r$ and ends at $x = h_e$ with a falling distance of $h_f$. The center of the roughness is located at $x_r = (h_s + h_e)/2$. This shape of roughness contains a broad spectrum of streamwise wavenumbers including the unstable ones. In this study, the natural surface roughness is centred at $x/c = 0.015$ with a total width of $1\text{mm}$. The rise and falling distances are equal and set to $h_r = h_f = 0.2\text{mm}$. The spanwise periodic shape function $h_z(z)$ is defined as

$$h_z(z) = \sum_{n=1}^{5} \sin(n\beta_0 z + \phi_n^{\text{rand}}),$$

where $\phi_n^{\text{rand}}$ are random phases and $\beta_0 = 2\pi/L_z$ with $L_z = 10.5\text{mm}$.

The natural surface roughness is not meshed but modelled by inhomogeneous boundary conditions at the wall. The no-slip conditions along the roughness $h(x, z)$ are projected from the bump surface to the wall via a Taylor series expansion,

$$\{u, v, w\}_w^T = \{-h(x, z) \frac{\partial U}{\partial y}, 0, -h(x, z) \frac{\partial W}{\partial y}\}_w^T, \quad h_s \leq x \leq h_e,$$

where $U$ and $W$ are the laminar base flow velocities. Since the roughness height $\varepsilon_h$ is assumed to be small, the Taylor series is truncated at the first order. The roughness height $\varepsilon_h$ is chosen such that the r.m.s. value of the $h(x, z)$ at the roughness centre is matching the reported r.m.s. roughness height of $1\mu\text{m}$ in the experiment.

2.4. Non-stationary perturbations

The growth of unsteady instabilities, both primary and secondary, requires the presence of unsteady disturbances such as acoustic noise or freestream turbulence. In this study, non-stationary perturbations are artificially induced inside the boundary layer by employing a weak randomly pulsed volume force. The forcing acts only in the wall-normal direction and reproduces the same effect that tripping strips have in wind-tunnel experiments. The forcing is located downstream of the natural surface roughness at $x/c = 0.024$ and its shape is attenuated by a Gaussian in the streamwise and wall-normal directions. The specific form of the forcing is given in Hosseini et al. (2016). The spectral content of the forcing is defined by two parameters, the temporal and the spanwise cut-off scales. The temporal cut-off scale corresponds to an angular frequency of $\approx 7000\text{Hz}$ and the spanwise cut-off scale corresponds to $30\beta_0$. The
amplitude of the forcing is chosen such that the transition is obtained at \( \sim 20\% \) chord, similar to the transition location observed in the experiments.

### 2.5. Plasma actuator body-force field

The ring-type DBD plasma actuator produces a wall-normal jet in the experiments which acts as a virtual roughness element. The action of the actuators in the numerical simulations is incorporated by a body force which corresponds to the velocity field produced by an actuator in quiescent air. Such a velocity field is provided by Kim & Choi (2016) who placed an actuator sheet on a flat plate and measured the induced velocity field by the actuators in quiescent air. The actuators were active under 5.8 kV power supply and at 40 kHz frequency. The measured velocity field is averaged in time and space which results to a steady body force in the numerical simulations. For a ring-type plasma actuator the induced velocity field is symmetric in the radial direction around the center of the actuator. Figure 4(a,b) shows the experimental radial and wall-normal velocity fields induced by the ring-type plasma actuator. In this section, velocity and body force are presented in dimensional values.

Nek5000 is used to calculate the body force from experimentally measured velocity field induced by a ring-type plasma actuator in quiescent air. The steady body force can be obtained by evaluating

\[
-f = -\mathbf{u} \cdot \nabla \mathbf{u} - \nabla p + \frac{1}{Re} \nabla^2 \mathbf{u},
\]

where \( \mathbf{u} \) and \( p \) are the experimental velocity and pressure fields. Instead of evaluating the individual terms in the the right hand side of equation (10), the whole sum can be evaluated by computing the first order time derivative using the numerical solver,

\[
\frac{\mathbf{u}^{(1)} - \mathbf{u}^{(0)}}{\Delta t} = -\mathbf{u} \cdot \nabla \mathbf{u} - \nabla p + \frac{1}{Re} \nabla^2 \mathbf{u}.
\]
Here $\mathbf{u}^{(0)}$ and $\mathbf{u}^{(1)}$ are the velocity vectors at the zeroth and first time step respectively. This procedure can be interpreted physically as balancing unsteadiness in the flow (in the absence of body force) with a force field, thus, the flow with such force field becomes steady. Using this method has two main advantages. First, the original velocity field is made divergence free with Nek5000 to any desired numerical tolerance. Secondly, the pressure gradient term can be kept in the calculation of the force field which is commonly neglected in other methods of computing the body force based on velocity information (for pressure gradients effect on the body force see Dörr & Kloker 2015a).
The experimental velocity field is interpolated on a three-dimensional Cartesian domain with the center of plasma actuator positioned at the origin. Since Navier-Stokes equations are solved in the Cartesian coordinates by Nek5000, the experimental radial velocity component (symmetric around the center of plasma actuator) is projected in the streamwise and spanwise directions. Homogeneous Dirichlet boundary conditions are applied on all the far-field boundaries of the domain in the streamwise and wall-normal directions. Periodic boundary conditions are applied in the spanwise direction. The time integration scheme is set to first order with a time step $\Delta t = 10^{-4}$. The solver tolerance for both velocity and pressure solvers is chosen to be $10^{-8}$ and the polynomial order is set to $N = 8$.

Figure 4(a,b) shows the experimental radial and wall-normal velocity components in the wall-normal spanwise plane at $x = 0$. The divergence free velocity fields $u^{(0)}$ are shown in figure 4(c,d) and the computed body force based on the evaluation of equation (11) is demonstrated in figure 4(e,f).

2.5.1. Validation

The computed force field is validated by performing a simulation in quiescent air and comparing the induced velocity field by the body force with the original velocity field. For this purpose the boundary condition at the most downstream streamwise plane is changed to outflow condition (instead of homogeneous Dirichlet condition used in computation of the force term). The boundary condition at the inflow plane and at the far-field in the wall-normal direction is kept as homogeneous Dirichlet conditions. Figure 5 shows the steady state solution resulting from the plasma actuator body force in quiescent air. Comparison of figure 5 and the divergence free original velocity field in figure 4(c,d) demonstrates that the computed plasma actuator force field in the numerical simulations generates the corresponding velocity field produced by the plasma actuators in the experiments.
3. Results

3.1. Laminar base flow

Since base flow is homogeneous in the spanwise direction ($\partial_z p = 0$), the spanwise momentum equation is decoupled from the other two momentum equations and acts as a passive scalar transport. Therefore, laminar base flow is computed, at a significantly lower numerical cost, via a two-dimensional simulation in which the spanwise velocity component is computed as a passive scalar

$$\rho c_p \frac{\partial W}{\partial t} = -(u \cdot \nabla)W + \kappa \nabla^2 W,$$

where the thermal conductivity is set to $\kappa = Re^{-1}$ and $\rho c_p = 1$.

The pressure coefficients, $C_p$, computed from the DNS and the RANS solutions are compared in figure 6(a). There is a close match on both the upper and the lower wing sides. This validates the modified outflow boundary conditions presented in section §2.2 and demonstrates domain independent solutions.

Further validation of the base flow is performed by solving the quasi-three-dimensional, fully non-similar boundary-layer equations (BLE), where the flow conditions required at the boundary-layer edge are retrieved from the pressure coefficient of the DNS solution. The equations are solved by employing second-order finite differences in both the $\xi$- and the $\eta$-directions starting from the stagnation point and neglecting curvature effects. The boundary layer profiles of the tangential velocity obtained from the DNS and BLE are compared in figure 6(b). The velocity profiles of the DNS are normalised by the maximum value in the boundary layer. The present results confirm the validity of the numerical setup for DNS computations.
In figure 7 the stability characteristics of the boundary layer under consideration is presented in terms of linear $N$-factor values for different frequencies and spanwise wavenumbers. The stability analysis is performed in framework of the nonlocal/PSE theory.

### 3.2. Effect of control on stationary disturbances

In this section we investigate the control capability of ring-type plasma actuator by studying its effect on primary stationary crossflow disturbances. To this end, two setups are considered. In the first setup, stationary crossflow vortices are excited by means of natural surface roughness only and in the second case, plasma actuator body force is activated in the presence of natural surface roughness. In the following these two cases are referred to as the 'natural' and the 'controlled' case respectively.

In order to compare the natural and the controlled case quantitatively, amplitude of individual crossflow modes is obtained by means of Fourier transformation in the spanwise direction, i.e.

$$
\hat{u}_\xi(\xi, \eta, z) = \sum_{n=0}^{N} \hat{u}_\xi(\xi, \eta, n\beta_0) e^{in\beta_0 z}.
$$

Here, $\beta_0 = 2\pi/L_z$ is the fundamental spanwise wavenumber dictated by the spanwise length of the domain. Amplitudes of individual modes for both the natural and the controlled case are plotted in figure 8 as

$$
A_u(x) = \max_{\eta} \frac{|\hat{u}_\xi|}{Q_{\infty}},
$$

where, $\hat{u}_\xi$ is the Fourier amplitude of the tangential velocity and $Q_{\infty}$ denotes the total freestream velocity. Linear stability analysis of the underlying base flow predicts that the most unstable stationary crossflow mode is the $2\beta_0$ mode. However, for both cases, amplitude of the fundamental $\beta_0$ mode dominates the amplitude of its superharmonics. Although all the excited modes by natural
surface roughness have identical amplitudes at the roughness location, the fundamental mode is the most receptive to the surface roughness. This is due to the fact that the neutral point of stationary modes is located around $x/c \approx 0.04$ and the surface roughness is positioned upstream of this location at $x/c = 0.015$. Therefore, the modes excited by the natural roughness first decay in amplitude, with different decay rates, until they reach their corresponding neutral point and then grow and evolve moving downstream. The growth rate (slope of the amplitude curve) of the $2\beta_0$ mode is however larger than the fundamental mode, conforming the linear stability prediction.

Figure 8 also shows the amplitude evolution of different modes obtained by linear and nonlinear PSE computations on the underlying base flow. The initial amplitude of modes for nonlinear PSE computations are extracted from DNS solutions at $x/c = 0.03$ and $x/c = 0.075$ for the natural and controlled case respectively. Despite slight discrepancies between nonlinear PSE and DNS, the overall agreement is found to be very good. For both natural and control cases, the $\beta_0$ mode shows linear growth up to $x/c \approx 0.25$ and the $2\beta_0$ mode behaves linearly up to the chord length $x/c \approx 0.20$. No nonlinear amplitude saturation is observed through the computational domain.

Figure 9(a) compares the amplitude evolution of the first three modes for both cases. Excitation of the $3\beta_0$ mode by plasma actuators in the controlled case is apparent. Subsequently, the $\beta_0$ mode is attenuated but the amplitude of $2\beta_0$ mode is increased compared to the natural case. In order to obtain the overall effect of control on the stationary disturbances, we extract the total amplitude of stationary disturbances by computing wall-normal maximum of the spanwise r.m.s. amplitude of the disturbances. The disturbance field itself is obtained by subtracting the spanwise mean velocity from the total velocity.
Figure 9: (a) Amplitude evolution of the stationary modes for both (-- --) natural and (---) controlled case in the absence of unsteady perturbations. (b) Amplitude (spanwise r.m.s) of the total disturbances.

field. Figure 9(b) shows the evolution of total disturbances amplitude evaluated for both the natural and the controlled case. It is apparent that excitation of the control mode slightly decreases the total amplitude of the stationary disturbances. However, there is no visible local effect on the total disturbance amplitude around the location of the plasma actuator.

3.3. Unsteady disturbances and secondary instabilities

In the previous section we showed the effect of control on evolution of primary stationary crossflow disturbances in the absence of unsteady perturbations. Because of the high accuracy of the DNS code, there were no background noise in the simulations, thus, flow did not transition to turbulence. In this section a more realistic setup is considered. Unsteady perturbations are introduced in the boundary layer by means of random volume force described in section §2.4. The amplitude of the random force is chosen such that transition is obtained close to the experimental transition location. Similar to the previous section, we study a natural and a controlled case. In the former setup, steady crossflow disturbances are excited by the surface roughness model while unsteady disturbances are introduced by employing random volume force. In the controlled case, plasma actuator body force is added to the natural setup in order to excite the steady control mode.

Figure 10 shows isosurfaces of instantaneous spanwise velocity for both the natural and the controlled case. Similar to the experimental observations, it is apparent that employing the control by plasma actuators have promoted transition. A quantitative measure for transition location is obtained by evaluating wall-friction coefficient. A breakdown to turbulent flow is accompanied by a strong increase in this coefficient. Figure 11 shows time-averaged friction coefficient $C_f = 2\mu(\partial u_s/\partial \eta)$ for both the natural and the controlled case where
Figure 10: Isosurfaces of instantaneous spanwise velocity field (a) natural and (b) controlled case. For better visualisation, domain is duplicated in the spanwise direction.

Figure 11: Time averaged friction coefficient for the natural and the controlled case.

\[ u_s = u_x \cos(\phi_\infty) + w \sin(\phi_\infty) \]

is the velocity in the direction of incoming freestream. The time averaging is required due to unsteadiness of the transition location for both cases (±3% chord-length). In the natural case, transition starts at \( x/c \approx 0.175 \) with appearance of turbulent spots and flow becomes fully turbulent at \( x/c \approx 0.22 \). In the controlled case, transition location moves upstream to \( x/c \approx 0.11 \), with a fully turbulent flow at \( x/c \approx 0.16 \).

In order to characterise the disturbance environment and the transition mechanism, the total velocity field is decomposed into time-periodic Fourier modes. This will allow us to capture the evolution of both steady and unsteady perturbations. Additionally, the onset of secondary instabilities can be identified by an explosive growth of high-frequency modes similar to the observations in previous studies (Malik et al. 1999; Wassermann & Kloker 2002, 2003; White & Saric 2005; Hosseini et al. 2013). To this end, amplitude of individual crossflow
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Figure 12: Amplitude evolution of steady and unsteady disturbances for (a) natural case and (b) controlled case. The gray lines represent unsteady disturbances plotted at a constant frequency step of 730Hz.

Amplitudes of individual modes for both the natural and the controlled case are plotted in figure 12 as

\[ A_u(x) = \max_{\eta,z} \frac{|\tilde{u}_\xi|}{Q_{\infty}}, \]

where, \( \tilde{u}_\xi \) is the time Fourier amplitude of the tangential velocity and \( Q_{\infty} \) denotes the total freestream velocity. Amplitude of steady disturbances is obtained by subtracting the mean flow, i.e. the (0,0)-mode, from the zero-frequency mode. Owing to the complexity of the set up, the amplitude evolution of disturbances is rather complicated. Both steady and unsteady disturbances grow throughout the domain. The amplitude of steady and low-frequency disturbances are of similar order and in the controlled case, low-frequency disturbances exhibit even higher amplitudes than the stationary disturbances. Complimentary linear PSE computations suggests that the most unstable crossflow disturbance has angular frequency of \( \omega \approx 800\text{Hz} \) and spanwise wavelength of \( \beta \approx 7\text{mm} \). Therefore, the low-frequency disturbances can be associated with unsteady primary crossflow disturbances.

In both cases, high-frequency disturbances are excited and just upstream of the transition location they undergo sudden amplification and reach the amplitude of primary modes within few percent chord length. This is a characteristic behaviour of secondary instabilities; once the primary crossflow waves reach high amplitudes, secondary instabilities are destabilised and grow to large amplitudes over a very short streamwise distance and lead to breakdown and turbulence (White & Saric 2005). Here, the amplitudes of primary disturbances,
both the steady and unsteady ones, reach \( \sim 10\% \) of the total freestream velocity prior to transition. Due to complex disturbance environment in our setup, it is not easy to isolate the secondary instability mechanism. Under more controlled conditions, it has been shown that transition is induced either by stationary or travelling saturated crossflow disturbances (Högberg & Henningson 1998; Malik et al. 1999; Wassermann & Kloker 2002, 2003). Moreover, Lerche (1996) and Lerche & Bippes (1996) observed that a superposition of both stationary and travelling primary modes can destabilise the secondary instability, while individually they are not strong enough to produce sufficient mean-flow deformation and trigger the secondary instability. In this case, transition is induced by co-existing steady and unsteady crossflow disturbances which exhibit similar amplitudes.

Based on disturbance energy production, secondary instabilities are often categorised into two types (Malik et al. 1996, 1999); \( z \)- and \( y \)-type modes that are produced by the spanwise and wall-normal gradients of the mean streamwise velocity, respectively. Figure 13 shows streamwise velocity component of the modified mean flow along with the distribution of the energy production term associated with a \( z \)-type mode, \(-\tilde{u}_\zeta \tilde{w} \partial \tilde{U}_\zeta / \partial z\), for the high-frequency mode with frequency 14600Hz. The sign of the production term indicates whether the local transfer of kinetic energy associated with it acts as stabilising (negative) or destabilising (positive). The wall-normal plane \((\eta, z)\) is chosen at \( x/c = 0.09\), an upstream position of the transition location for both natural and controlled cases. Although, the crossflow vortices are not saturated in either case, the mean-flow modification by the control mode has resulted in formation of stronger crossflow vortices. The maximum of the production is located on the updraught side of the primary vortices with slightly larger values in the controlled case. Therefore,
the secondary instabilities are destabilised earlier than the uncontrolled case which is followed by promotion of transition to turbulence.

3.4. Improved control & transition delay

In this section we aim to improve the control and delay transition using ring-type plasma actuators acting as virtual roughness element. There are several design parameters in this control technique which could be investigated for improvement, i.e. the streamwise location or the spanwise spacing of plasma actuators and the strength of actuators in terms of magnitude of the induced velocity (which corresponds to the height of a roughness element). Actuators should be located at streamwise locations where the boundary layer is more receptive to the excitation of the control mode. Here we have positioned the actuators slightly downstream of the neutral point of the stationary crossflow vortices. The spanwise spacing of the actuators is chosen smaller than the spanwise wavelength of the naturally most unstable mode so that the control mode is subcritical with respect to the most amplified mode and decays far downstream. Following the suggestion by Saric et al. (1998), we have spaced the plasma actuators with a spanwise wavelength equal to $2/3$ of the wavelength of naturally most unstable mode. Carpenter et al. (2009) placed roughness elements at $1/2$ of the wavelength of the most unstable crossflow mode in their free-flight experiments which was found to be effective in delaying transition. Choosing higher harmonics of the most unstable mode is also possible but the control mode may become too damped and therefore not effective.

In the successful applications of this control technique, Wassermann & Kloker (2002) and Hosseini et al. (2013) have shown the amplitude of control mode is around one order of magnitude larger than the most unstable stationary mode in the upstream regions of their flow cases (e.g. see figure 3 of the latter study). Furthermore, Wassermann & Kloker (2002) show that increasing the amplitude of control mode results in further suppression of the most unstable stationary modes and hence a more effective control (e.g. see their figure 24). Moreover, suppression of unsteady disturbances by nonlinear steady crossflow modes have been shown by Bonfigli & Kloker (1999). In our flow case, the amplitude of control mode, shown in figure 8(b), is one order of magnitude smaller than the amplitude of the most unstable mode and it is too weak compared to the cases in which the control have successfully delayed transition. Therefore, among the design parameters mentioned, we choose to investigate the plasma actuator strength for improvement of the control. To this end, plasma actuator body force is increased by a factor of 100 which corresponds to increasing the magnitude of the induced jet velocity by one order of magnitude from $0.2m/s$ to $5.0m/s$.

Figure 14(a) shows the amplitude evolution of individual stationary crossflow modes in the absence of unsteady perturbations for the new controlled case with higher plasma actuator body force. Amplitude of the $3\beta_0$ mode after excitation by the plasma actuator experiences a sharp increase and becomes
Figure 14: (a) Amplitude evolution $A_u$ of the stationary modes in the absence of unsteady perturbations for the improved controlled case with higher plasma actuator body force. Circle symbols show the amplitude evolution predicted by nonlinear PSE computation. (b) Amplitude of the total disturbances for the natural and the controlled case.

Figure 15: (a) Time averaged friction coefficient for the natural and the improved controlled case. (b) Amplitude evolution of steady and unsteady disturbances for the improved controlled case with higher plasma actuator body force. The gray lines represent unsteady disturbances plotted at a constant frequency step of 730Hz.

the dominant mode. Farther downstream, at about $x/c \approx 0.21$ it becomes stable and decays. The qualitative behaviour of control mode in this case is similar to the ones reported by Wassermann & Kloker (2002) and Hosseini et al. (2013). Evolution of total disturbance amplitude for both the natural and controlled case is shown in figure 14(b). Unlike the previous controlled case, shown in figure 9(b), excitation of the control mode leads to total disturbance amplitudes which are 3 times higher than those of the natural case. However,
Figure 16: Spatial distribution of the production term $-\tilde{u}_\xi \tilde{w} \partial U_\xi / \partial z$ for the high-frequency mode with frequency 14600Hz along with contours of modified mean flow (tangential velocity component). (a) natural case and (b) controlled case. The wall-normal ($\eta, z$) plane is located at $x/c = 0.17$.

far downstream at about $x/c \approx 0.21$ it decays and for $x/c > 0.23$ the total amplitude level is slightly below that of the uncontrolled case.

In order to evaluate the control capability of the plasma actuators with stronger body force in a realistic more complex disturbance environment, unsteady perturbations are introduced in the domain. Figure 15(a) shows the time-averaged friction coefficient for both the natural and the controlled case. It is clear that the new control has delayed transition to turbulent flow by about 3% chord-length. Fourier transform of flow field snapshots in time, characterises the disturbance environment into steady and unsteady parts. Amplitude of the individual modes, as defined in equation (16), for the controlled case are plotted in figure 15(b). The steady disturbances are clearly dominant in this controlled case. The initial amplitude of unsteady perturbations, both primary and secondary, are similar to the uncontrolled case up to the location of plasma actuators at $x/c = 0.05$. Thereafter, they experience a sharp decay up to $x/c \approx 0.1$ where they start to grow again. However, the unsteady disturbances are attenuated compared to the natural case. Furthermore, the sudden growth of high-frequency disturbances is shifted downstream which corresponds to the beginning of the transition location in the friction coefficient curve.

In order to understand further the mechanism of secondary instabilities, transfer of energy between the base flow and high-frequency modes is investigated. To this end, spatial distribution of energy production term associated with $z$-type modes, $-\tilde{u}_\xi \tilde{w} \partial U_\xi / \partial z$, for the high-frequency mode with frequency of 14600Hz along with contours of modified mean flow is shown in figure 16. The wall-normal plane is chosen at $x/c = 0.17$, prior to the transition location in both controlled and uncontrolled cases. The maximum production is located on the upwelling zone of the stationary crossflow vortices where the shear is strong. The destabilising energy production is clearly weaker in the controlled case as
compared with the natural case. Although figure 14(a) shows dominance of the control mode in the absence of unsteady disturbances, the mean flow structure in both natural and controlled cases is dominated by the most unstable \(2\beta_0\) mode in the presence of unsteady perturbations. In order to investigate this behaviour, disturbances are assumed to be spanwise- and time-periodic and may be decomposed into Fourier components of the form

\[
\bar{u}(\xi, \eta, z, t) = \sum_{m=0}^{M} \sum_{n=-N}^{N} \tilde{u}(\xi, \eta, n\beta_0, m\omega_0) e^{i(n\beta_0 z + m\omega_0 t)}.
\]

Amplitudes of individual modes for both the natural and the controlled case are plotted in figure 17 as

\[
A_u(x) = \max_{\eta} \frac{\mid \tilde{u} \mid}{Q_\infty}.
\]

In the following individual Fourier modes with a spanwise wavenumber \(n\beta_0\) and a frequency \(m\omega_0\) are represented by \((m, n)\). The fundamental frequency resolved here is 730Hz. In the natural case, steady and unsteady disturbances exhibit similar amplitude levels with the \((0,1)\)-mode being dominant up to \(x/c \approx 0.13\) where the \((0,2)\)-mode becomes dominant. In the controlled case, excitation of the control mode clearly attenuates the unsteady modes as well as the \((0,1)\)-mode. The dominant mode throughout the domain is the \((0,2)\)-mode in contrast to dominance of the control \((0,3)\)-mode in purely steady disturbance environment. This demonstrates the significant effect of unsteady disturbances in the receptivity process.

4. Conclusions

Passive control of a swept-wing boundary layer using ring-type plasma actuators is investigated thorough direct numerical simulations. The flow configuration
conforms to experiments by Kim et al. (2016) in which a spanwise array of ring-type plasma actuators were mounted close to the leading edge. Such actuators generate a wall-normal jet and act as virtual roughness elements to excite subcritical stationary control modes. The action of actuators are incorporated in the numerical simulations by including the corresponding body force of the induced velocity field by actuators. Steady crossflow disturbances are excited using a simplified model of natural surface roughness on the wing surface.

First, the effect of control on the evolution of stationary crossflow disturbances in the absence of unsteady perturbations is investigated, i.e. transition is not studied. Excitation of the control mode attenuates the amplitude of most unstable mode as well as the total amplitude of stationary disturbances. The amplitude of dominant mode in both the natural and the controlled case is rather low and not saturated within the computational domain. In the next step, a more realistic disturbance environment is considered by adding unsteady disturbances in the leading edge region using random volume force in the wall-normal direction. The amplitude of unsteady perturbations is adjusted such that the transition location is close to the experimental one. Employing control in this setup leads to promotion of transition which is in qualitative agreement with the experimental observations. The breakdown from laminar to turbulent flow is caused by explosive growth of secondary instability modes. In both the natural and the controlled case, stationary and travelling crossflow modes exhibit similar amplitudes and a combination of both modes triggers the secondary instabilities.

An improvement in the control is demonstrated by employing stronger plasma actuators with one order of magnitude larger jet velocity. In the absence of unsteady perturbations, the amplitude of control mode for such plasma actuator initially dominates the amplitude of the stationary modes and decays far downstream. This behaviour is in qualitative agreement with previous studies (e.g. Wassermann & Kloker 2002; Hosseini et al. 2013). It is shown that employing stronger plasma actuators in a disturbance environment with both steady and unsteady perturbations successfully delays transition. Amplitude of unsteady disturbances, both primary and secondary, is attenuated when control is applied. However, the modified mean flow is still dominant by the naturally most unstable crossflow mode. This is confirmed by spatio-temporal Fourier decomposition of disturbances. The results of the simulations show a complex interaction between stationary and unsteady vortices indicating the importance of unsteady crossflow vortices in transition for the case under consideration.

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