Two-way Concrete Slabs with Openings

Experiments, Finite Element Analyses and Design

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MASTER OF SCIENCE PROGRAMME

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Division of Structural Engineering

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PREFACE

This thesis completes five years of my studies. Three of the first years I spent at Gdansk University of Technology and the last two at Luleå University of Technology. Both of these periods gave me very valuable experience which will have its application in the future. The last months when I have worked with the thesis have definitely been the most demanding but also the most exciting during my studies. In spite of all tough work I had to put into the research and writing it has been a great satisfaction to use the knowledge I have obtained and to see my first serious publication arising.

This thesis would not see the daylight without people who were giving me helping hand. First of all I would like to thank Professor Thomas Olofsson and Professor Björn Täljsten who gave me the possibility to write this thesis and to graduate from Luleå University of Technology. They made an effort not only to solve my problems arising during the writing process but also to allow me experience the professional research work.

Professor Lennart Elfgren is also the person that I would like to thank for his help during my stay in Sweden. Since the very beginning of my studies at Luleå University of Technology he has been giving me his support and has become my mentor during this time. He has never grudged me his time to develop my knowledge especially in bridge engineering, our common interest.

A priceless contribution in my Master’s Thesis has my friend and co-supervisor Ola Enochsson who taught me how to write a scientific publication and pay particular attention to aesthetics. I also thank him for never-ending discussions on structures’ behaviour and for convincing me, through his passion to research work, to continue the education as a Ph.D. student.
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I have never depreciated the role of Gdansk University of Technology in giving me the necessary knowledge to complete the studies. I would like to thank especially Professor Krzysztof Wilde who taught me structural mechanics and strength of materials, the most basic and the most important courses in civil engineer’s education. His brilliant and smart way of teaching helped me to obtain high skills in these subjects, what was very useful in further studying and writing my thesis.

Luleå, June 2005

Piotr Rusinowski
Abstract

Concrete slabs with openings are usually designed with help of traditional rules of thumb proposed by building codes. Such methods, however, introduce limitations concerning size of openings and magnitude of applied loads. Furthermore, there is a lack of sufficient information and instructions are needed to design fibre strengthening of cut-outs in existing concrete slabs.

This thesis is a part of a project carried out at Luleå University of Technology. It deals with the problem of openings in two-way concrete slabs. The project comprises full-scale tests on simply supported square slabs with square openings. The slabs are loaded with uniformly distributed load up to failure. The distributed load is applied by means of an underlying airbag. The aim of the research is to obtain knowledge concerning the influence of opening size and strengthening type on the load carrying capacity and the stress distribution. Slabs cast with an opening are reinforced by steel bars and slabs with a sawn up opening are strengthened with Carbon Fibre Reinforced Polymers (CFRP).

The aim of this thesis is to carry out finite element analyses of tested RC slabs, with a non-linear concrete model satisfying complex support conditions.

Four types of slabs have been analysed with help of the program ABAQUS/Standard. Despite difficulties met in the design of proper boundary conditions, the obtained results are satisfactory in comparison with the experiments. In the thesis questions are outlined for further research on finite element modelling of strengthening with fibre reinforcement.
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1 Introduction

1.1 Background and identification of the problem

In the design of concrete slabs with openings the building codes propose instructions that are not supported by the underlying theories. Slabs with small holes are traditionally designed using empirical methods which are described in Chapter 2. For larger holes the strip method is often used but this method is not accurate and underestimates in some cases the load capacity of the structure. Due to lack of accurate calculation methods the size of an opening and the magnitude of allowable load are limited by codes. Considering the relatively low price of steel nowadays it is hard to find economical aspects in possible improvements in this area. However, knowing the stress distribution in such structures would constitute a background to invent new ways to reinforce and make the designing more flexible.

The problem becomes more complex when openings are planned to be made in existing slabs. The most common way to substitute additional steel reinforcement is to apply fibre strengthening before cutting a hole. The designing methods in this case are proposed by Täljsten (2004) and consist in recalculating the amount of steel into an equivalent amount of carbon fibre, see section 2.4. However, this method is based on strengthening of beams where bending only in one direction occurs. Behaviour of strengthened concrete slabs, where the state of stresses is much more complex, needs to be further examined. Improvements in the design methods should take into account such aspects as possible debonding or delamination. Furthermore, in contrast to steel, carbon fibres are expensive and efficient methods that could decrease costs are needed.
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These problems cannot be solved by analytical methods since such structures like concrete slabs with openings, especially strengthened with carbon fibres, are too complicated. Nowadays, access to powerful computers and advanced software gives possibility to create accurate models by means of the finite element method. There is always a risk of wrong settings in computer program and therefore numerical calculations should be supported by experiments. Advanced finite element modelling is in most cases time-consuming and for this reason it might be too expensive for design offices or when the number of needed openings is limited. Accurate methods obtained from academic researches can prepare a background for evaluation of simple design methods and ready solutions for designers.

1.2  Aim and scope

This thesis is a part a project at Luleå University of Technology concerning analysis of square concrete slabs with square openings in the middle. A particular case of simply supported slabs subjected to uniformly distributed load is being considered. The research comprises experiments and calculations of weakened slabs, additionally reinforced and strengthened with CFRP (Carbon Fibres Reinforced Polymers). The slabs are loaded with uniformly distributed surface load up to failure. The distributed load is increased with 0,04 kN/slab/s and is carried out with an underlying embedded airbag. The aim of the project is to learn more about structures’ behaviour what will help to find new design methods, both computer aided and ready solutions in form of tables and diagrams. First part described by Ericsson & Larsson (2003) and Enochsson et al (2003 & 2004) consisted of experiments and a brief discussion of existing methods.

The next step of the project introduces finite element analysis and its comparison with experiments results. This thesis services as an introduction to finite element modelling of experimentally tested two-way RC slabs with or without sawn up openings.

- Discussion of selected experimental results
- Finding accurate finite element model for reinforced concrete slabs
- Explanation of phenomena not commented in earlier work
- Discussion of concrete cracking modelling

The main aim of the thesis is to create a background for finite element modelling of concrete slabs with an opening strengthened with CFRP. It also
gives suggestions for future experimental work to avoid problems that usually occurs in numerical analyses.

1.3 Types of slabs and their application

Slabs, in definition and the way of designing them, are structures that transmit loads normal to their plane. Concrete slabs are widely in use as floors not only in industrial and residential buildings but also as decks in bridges. The big advantage is flexibility in methods of manufacturing. They can be made in-situ as well as prefabricated and brought to construction site in full scale. For larger spans pre-stressed concrete is very often applied to increase capacity without extending slab height.

The biggest demand on concrete prefabricated slabs and shells appeared after II World War. Countries that suffered most needed a quick method to rebuild demolished residence buildings. That time lots of blocks of flats were built with prefabricated concrete shells and slabs, see Figure 1.1. Neither aesthetic nor economic aspects were of the highest importance, the meaning was to provide citizens with flats as soon as possible.

![Figure 1.1 Building made of concrete shells and slabs.](image)

Nowadays, mostly due to health reasons, concrete shells are used seldom for residential application. Walls are most often made of hollow masonry bricks or cellular concrete bricks which are more healthy materials for human. Concrete slabs, however, are still common in use as floors in multi-storey buildings. Here decide short time of construction, comfort of construction and the possibilities of architectural planning. The last aspect is strongly related with
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the static system and the design of reinforcement. Concrete slabs are commonly divided into the following types:

- **Beamless slabs**

  Beamless slabs are supported only on columns and are applied mostly in large industrial and public buildings. For lower loads they can have constant thickness and are called flat slabs or flat plates, see Figure 1.2a. To increase punching resistance flat slabs with drops and column heads are applied, see Figure 1.2b. The advantage of beamless slabs is their relatively simple structure and short time of construction even with an in-situ method. The alternative way to construct is the so-called “lift slab” method and is shown in Figure 1.3. A principle of this method is that a batch of prefabricated slabs is lifted by hydraulic jacks and successively connected to steel columns.

![Figure 1.2 Beamless slabs; a) flat slab (flat plate) and b) flat slab with drops and column heads.](image)

![Figure 1.3 Lift slab system.](image)
• **One-way slabs**

One-way slabs have the main reinforcement only in one direction and are applied in beam-and-girder system. The system contains a thin multispan slab carried by ribs, which are supported on frames, see Figure 1.4. Similar construction can be observed in heavy timber and steel structures. The advantage of such a system is that realistic moments can be calculated directly. Although a slab is designed as bent only in one direction it must be restrained also in the other, above a girder. Otherwise the upper side of the slab would crack in this region.

![Figure 1.4 Beam-and-girder system.](image)

• **Two-way slabs**

Two-way slabs are supported along all edges by beams or walls and are bent in two directions, see Figure 1.5. This type is a development of the beam-and-girder system and is widely applied in skeleton constructions. Two-way-slabs system is more aesthetic than the other one and is easier to construct due to less framework.
Two-way concrete slabs with openings

Figure 1.5   Two-way slab supported with beams.

• **Waffle slabs**

Waffle slabs have the shape of a set of crossing joists with a thin plate on the upper side. The meaning of this system is to increase the effective depth keeping a relatively low self-weight of the structure. Such slabs can be designed to work either as beamless or two-way slabs (see Figure 1.6) and are applied when long spans, up to 10m, are needed.

Figure 1.6   Arrangement of waffle slabs as flat slabs / two way slabs.

Obviously, the list of possible types could be much longer and the choice of reinforcing method and static system depends individually on the architectural plans. These types mentioned above, however, are the most common and constitute a base for other solutions. The subject is widely described by Park & Gamble (2000) and Cope & Clark (1984).
2 Design of two-way concrete slabs with openings

2.1 Design of additional reinforcement according to Swedish code

There are slight differences in methods of designing rectangular concrete slabs with rectangular openings between national codes. Swedish code, BBK 04 (2004) allows applying openings of dimensions not longer than \( \frac{1}{3} \) of the shortest slab span, see Figure 2.1.

![Figure 2.1 Limits of opening dimensions according to BBK 04 (2004).](image)

To calculate the necessary reinforcement first moments and shear forces should be found for a corresponding homogeneous slab. The moments and the shear forces from area of the openings should be divided and added to bands around the opening together with the existing moments and shear forces. The bands are to be considered as beams of a width not larger than three slab heights or 1/10 of the span length. The length of the reinforcement bars in these beams should be the same as those which had been designed for a homogeneous slab.
Two-way concrete slabs with openings

The Swedish code (BBK 04) gives only the outlines of the procedure. Instructions about calculation of design moments and distribution of reinforcement have to be considered according to other Swedish handbooks as e.g. The Concrete Handbook, Design Part.

2.2 Design of additional reinforcement according to Polish code

The Polish code for concrete structures (PN-B-03264) suggests traditional method of designing additional reinforcement. In this case the reinforcement is designed for a homogeneous slab and then the amount of the reinforcement from the opening is distributed around the opening, see Figure 2.2. For the additional reinforcement the anchorage length $l_{h,\text{net}}$ should be calculated. This method, however, introduces stricter limits of the application than in BBK 04. Opening edge cannot be longer than $\frac{1}{4}$ of the effective length of the corresponding slab span and the magnitude of the uniformly distributed load (over the self-weight) is not allowed to be larger than 10 kN/m$^2$. Some examples of effective span lengths are shown in Figure 2.3. If the needs exceed the limits concerning the opening dimension or the magnitude of the load, the code demands a kind of trimmer members to be designed in form of hidden beams. The width of such beams cannot exceed four slab heights. The method of hidden beams for one-way slabs is described by Leonhardt (1974) and the determination of moments in two-way slabs is proposed by Stiglat & Wippel (1973).

![Figure 2.2 Limits of dimensions for a small opening according to Polish code (PN).](image)
Design of two-way concrete slabs with openings

Figure 2.3  Definition of effective span lengths.

The code describes also easy and clear instructions concerning reinforcement in two-way slabs designed according to a linear elastic analysis. Calculations only of maximum span and support moments are necessary. The maximum span reinforcement is to be applied only in the middle part limited by a rectangle as shown in Figure 2.4.

Figure 2.4  Region of maximum span reinforcement.

Outside this region the amount of the reinforcement can be reduced to 50 %. In corners, due to uplift, top reinforcement must be provided and in this case also
Two-way concrete slabs with openings

the half of the designed main reinforcement can be applied. Furthermore, in the corner of two simply supported edges, bottom reinforcement perpendicular to bisector is to be provided. Its area per 1 m of a section width should not be less than that designed in the middle. The principle of reinforcing according to the Polish code is shown in Figure 2.5.

If an equilibrium method is applied, the reinforcement bars in the entire slab should have equal spacing and the corner reinforcement should be determined by calculations.

Figure 2.5  Reinforcement of two-way slabs according to Polish code, PN. Area marked 1 designates bottom reinforcement and area marked 2 designates top reinforcement.

2.3  Method of hidden beams

The method of hidden beams described by Leonhardt (1974) concerns only one-way slabs which are not the scope of this thesis. This publication, however, contains precise instructions for calculation of moments as well as recommended arrangement of reinforcement. An outline of Leonhardt’s method is presented below.

Let us consider a simply supported one-way slab with a rectangular opening, subjected to a uniformly distributed load $q$, see Figure 2.6. The slab is to be
Design of two-way concrete slabs with openings

strengthened within the width \( b_m \) in the direction of the main reinforcement. Furthermore, additional reinforcement has to be placed along opening’s edge in the other direction.

**Figure 2.6** Simply supported one-way slab with a rectangular opening.

The width \( b_m \), which demands strengthening, is to be calculated according to the following equation:

\[
b_m \approx \left(0.8 - \frac{b}{L}\right) L
\]

(2.1)

and the moment acting in this region:

\[
m_{ym} = \left[0.125 + 0.19 \frac{a}{L} \left(\frac{2b}{L}\right)^2\right] qL^2
\]

(2.2)
Reinforcement placed in direction $y$ corresponds to a trimmer member in timber structures. Moment $m_{xy}$ acting there is to be determined according to the equation:

$$m_{xy} = 0.125qa(a + 2b_m) \quad (2.3)$$

Equation (2.3) is valid when $b/a \geq 0.5$. Otherwise a moment for a slab simply supported on three edges is to be considered.

Additional reinforcement calculated for moments from equations (2.2) and (2.3), should be gradually concentrated nearer the opening’s edge.

Stiglat & Wippel (1973) proposed an evaluation of this method for one-way slabs with fixed edges, see Figure 2.7.

---

**Figure 2.7 One-way slab with fixed edges.**

The additional reinforcement is to be applied on width $b_m$ for span and on width $b_{me}$ for support, which are obtained from the following equations:

$$b_m \approx 0.6 \left( \frac{0.8 - \frac{b}{L}}{L} \right) \quad (2.4)$$

$$b_{me} \approx 0.18L \quad (2.5)$$

The span moment $m_{xm}$ is to be calculated as:
Design of two-way concrete slabs with openings

\[ m_{sm} = \left( 0.042 + 0.19 \frac{a}{L} \right) qL^2 \] for \( \frac{b}{L} \geq 0.4 \) (2.6)

or

\[ m_{ym} = \left( 0.042 + 0.33 \frac{b^3}{b_m L^3} \right) qL^2 \] for \( \frac{b}{L} < 0.4 \) (2.7)

In the case, when \( \frac{b}{a} \geq 0.5 \) the support moment \( m_{yer} \) is to be calculated with the following equation:

\[ m_{yer} = -\left( 0.083 + 0.33 \frac{b^2}{b_m L^2} \right) \left( 1.5 - \frac{b}{L} \right) qL^2 \] (2.8)

For larger values of \( \frac{b}{a} \) the support moment \( m_{ym} \) can be considered as for a slab fixed along one edge and simply supported along lines \( x = \pm a/2 \).

If \( \frac{b}{a} < 0.5 \), both the moments \( m_{yer} \) and \( m_{ym} \) are to be taken as:

\[ m_{yer} = m_{ym} = -\frac{qb^2}{2} \] (2.9)

The moment \( m_{xr} \) acting along the opening’s edge in the secondary direction is to be calculated in the same way as for the simply supported slab, see equation (2.3).

Stiglat & Wippel (1973) present also a consideration of two-way square slabs with a square opening in the middle, subjected to a uniformly distributed load \( q \), see Figure 2.8. This situation is very similar to the case examined in the thesis. The difference is that, in the publication, slabs with fixed edges are considered, whereas the project concerns only simply supported slabs.

Moments are calculated with equation:

\[ m_i = k_i qL^2 \] (2.10)

Table 2.1 contains values of the factor \( k \) with dependence on the ratio \( a/L \). No particular suggestions to reinforcement arrangement are described.
Two-way concrete slabs with openings

![Figure 2.8](image)

**Figure 2.8** Two-way square slab with a square opening.

<table>
<thead>
<tr>
<th>$a/L$</th>
<th>0.0</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{em}$</td>
<td>-0.052</td>
<td>-0.048</td>
<td>-0.036</td>
<td>-0.019</td>
<td>-0.005</td>
</tr>
<tr>
<td>$m_{r,\text{max}}$</td>
<td>0.018</td>
<td>0.022</td>
<td>0.010</td>
<td>0.004</td>
<td>0.001</td>
</tr>
<tr>
<td>$m_{re}$</td>
<td>0.018</td>
<td>0.015</td>
<td>0.008</td>
<td>0.003</td>
<td>0.001</td>
</tr>
</tbody>
</table>

### 2.4 Design of CFRP strengthening

In the case when a slab already exists and e.g. a changed usage demands that sawing up a hole, a common way of strengthening is to apply carbon fibres. The principle of calculations is very simple and is based on methods described in section 2.1 and 2.2. To begin the calculations of the amount of the polymer, the required area of the additional steel reinforcement must be known.

Let us consider a band around the opening with a basic reinforcement with a sectional area $A_{S1}$ and an additional reinforcement with a sectional area $A_{S2}$, see Figure 2.9.
The moment capacity $M_{d1}$ can be determined from an equilibrium equation at a point lying on the neutral line (NL):

$$M_{d1} = 0.6F_c x + (F_{s1} + F_{s2})(d - x)$$  \hspace{1cm} (2.11)

The next step is to substitute the additional reinforcement $A_{s2}$ with carbon fibre polymer, see Figure 2.10. The thickness of the polymer is around 0.1 mm so a negligible error will be committed if we assume that its centre of gravity lies on the concrete surface ($d_f = h$).

The moment capacity $M_{d2}$ of a section strengthened with CFRP is obtained the same way as $M_{d1}$:

$$M_{d2} = 0.6F_c x + F_{s1}(d - x) + F_f (h - x)$$  \hspace{1cm} (2.12)
Two-way concrete slabs with openings

The aim of the recalculation is to keep the same capacity of the section strengthened with additional steel reinforcement and CFRP, i.e. the moments $M_{d1}$ and $M_{d2}$ must be equal:

$$M_{d1} = M_{d2}$$  \hspace{1cm} (2.13)

Inserting expressions (2.11) and (2.12) into the equation (2.13) we obtain a relation between the forces in the additional reinforcement and the CFRP:

$$F_{s2}(d - x) = F_f(h - x)$$  \hspace{1cm} (2.14)

Expressing forces $F_{s2}$ and $F_f$ as products of stresses and sectional areas gives:

$$A_{s2}\sigma_{s2}(d - x) = A_f\sigma_f(h - x)$$  \hspace{1cm} (2.15)

Applying Hooke’s law yields:

$$A_f = \frac{E_s\varepsilon_s(d - x)}{E_f\varepsilon_f(h - x)}A_{s2}$$  \hspace{1cm} (2.16)

Assumptions that Bernoulli’s hypotheses yields and that perfect bond between the concrete and the carbon fibre is obtained, means that the formula for $A_f$ becomes dependent only on the geometry and the differences between the elastic modulus of the steel bars and the carbon fibres:

$$A_f = \frac{E_s(d - x)^2}{E_f(h - x)^2}A_{s2}$$  \hspace{1cm} (2.17)

Since compression height $x$ is not commonly used in design procedure, equation (2.17) can be presented in simplified form:

$$A_f = \frac{E_s(1 - \omega)^2}{E_f\left(\frac{h}{d} - \omega\right)}A_{s2}$$  \hspace{1cm} (2.18)

where: $\omega = \frac{x}{d}$  \hspace{1cm} (2.19)
2.5 Conclusions

The problem of openings in two-way concrete slabs is not widely and precisely described either in codes or literature. Very often only general instructions or suggestions are shown without giving any design method. Difficulties consist not only in a complex stress state but also in a large number of possibilities concerning boundary conditions, slab and opening dimensions or opening position. New calculation and reinforcing methods are needed and can be found especially with help of finite element methods.

In most cases only strengthening along opening edges is advised. One practical method of reinforcement arrangement that is omitted consists in applying skew bars in opening corners, see Figure 2.11. Big advantages in application of this method can be obtained since bars are situated perpendicularly to the direction where the main crack is to appear. There remain still questions of the needed reinforcement area and the anchorage length.

![Figure 2.11 Opening strengthened both along edges and with skew reinforcement in corners.](image)

The last conclusion concerns a calculation method for fibre polymers. Apart from the problem of different directions of stresses in two-way slabs, which was mentioned in the introduction, there is also one inconsistency in the model. The section is assumed to work in a plastic range. At the same time Hooke’s law is applied to derive the final equation which is not exactly correct considering the reinforcing steel. This fact may cause error which should be rather marginal but needs further investigations.
Two-way concrete slabs with openings
3 Experimental work

3.1 Introduction

The experimental work of the project was being done in 2002 and 2003-2004 at Testlab at Luleå University of Technology. The first year the program was financed by The European Union regional fund and Sto BPE Systems AB. The second year was financed by The Swedish Development Fund of Construction Industry (SBUF) and STO Scandinavia AB.

The tests comprise examinations of two-way concrete slabs with different dimensions of openings and different methods of strengthening. Both cases when a hole is made before and after casting are considered. As distinct from most of the experimental work, the slabs are subjected to uniformly distributed loads which correspond better to design procedures. For each slab a number of deflection and strain results were collected and compared.

It is worth to mention that the openings’ dimensions in the tested slabs exceed these limited by the codes. This fact decides that the experiments can judge not only methods of strengthening openings but also rightness of dictated limitations.

The slabs made with an opening have additional reinforcement designed with a method similar to these proposed by the codes. It is a mixture of different instructions though. Opening’s edges are reinforced with an area equal to this designed for a homogeneous slab at the place of the opening. Additional reinforcement bars are of the same type and length as all other and form a kind
of narrow beams. The experiments examine also benefits of applying skew reinforcement in the opening’s corners.

Ericsson & Larsson (2003) discussed tested slabs from a pilot study while this thesis is based on tests where the supporting structure was strengthened. Errors made in the first part of the experimental work do not allow a full analysis and comparison. Therefore, in this thesis the analysis supported by the experimental work must have been limited to the case with only a skew reinforcement and a strengthening applied. Following reasons decide about the limitation:

- Another type of steel reinforcement was used in the pilot study.
- Too little stiffness of the supporting structure which caused its bending and twisting.
- Unknown displacement of the supporting structure during the tests in the pilot study.

This material, however, gives enough background to create an accurate finite element model.

### 3.2 Specimens

All tested slabs are square and have the dimensions $2.6 \times 2.6 \times 0.1$ m. The boundary conditions allow uplift in the corners and for this reason only bottom reinforcement was applied. As materials, concrete C 32/40 and steel Nps 50 have been used in design.

There are two types of openings which are further called as “small hole” (S) and “large hole” (L). The former has the dimensions $0.85 \times 0.85$ m (see Figure 3.1a) and the latter $1.2 \times 1.2$ m (see Figure 3.1b). In both cases the openings are situated in the middle of a slab.

The slabs cast with an opening were additionally reinforced by skewed steel bars. For this purpose the pieces of the steel bars that would have crossed the opening were used, i.e. they have the same length as the edge of the opening. In each corner there are two reinforcement bars, see Figure 3.2.
Figure 3.1  a) Slab with a small hole and b) slab with a large hole.

Figure 3.2  Additional skew reinforcement in corners of opening.

The slabs strengthened with CFRP were cast as homogeneous and the openings were cut afterwards. Needed amount of carbon fibres was calculated according to the instructions described in 2.4. In opposite to practise the carbon fibres were bonded after a hole was made, see Figure 3.3. In the reality the strengthening must be applied before the holing.
Two-way concrete slabs with openings

![Diagram of slab strengthening process]

Figure 3.3  Preparing a slab strengthened with CFRP; a) Casting a homogeneous slab, b) cutting a hole and c) bonding of laminates.

As references to strengthened and additionally reinforced slabs three specimens were tested:

- Homogeneous slab (no opening)
- Slab with a cut small hole without strengthening
- Slab with a cut large hole without strengthening

The aim of applying additional reinforcement or strengthening is to obtain the same load carrying capacity as the homogeneous slab.

The program of the tests is presented in Table 3.1. Drawings of all the specimens are enclosed in Appendix A.
### Table 3.1  Test program.

<table>
<thead>
<tr>
<th>Test no.</th>
<th>Symbol of specimen</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>H</td>
<td>Reference homogeneous slab without any opening</td>
</tr>
<tr>
<td>2</td>
<td>Swe</td>
<td>Reference slab with cut small hole, without strengthening (weakened)</td>
</tr>
<tr>
<td>9</td>
<td>Lwe</td>
<td>Reference slab with cut large hole, without strengthening (weakened)</td>
</tr>
<tr>
<td>3</td>
<td>S</td>
<td>Slab cast with small hole additionally reinforced in corners by steel bars</td>
</tr>
<tr>
<td>4</td>
<td>L</td>
<td>Slab cast with large hole additionally reinforced in corners by steel bars</td>
</tr>
<tr>
<td>12</td>
<td>Sst</td>
<td>Slab with cut small hole strengthened in corners with CFRP</td>
</tr>
<tr>
<td>8</td>
<td>Lst</td>
<td>Slab with cut large hole strengthened in corners with CFRP</td>
</tr>
</tbody>
</table>

### 3.3  Loading

The tested slabs are subjected to uniformly distributed loads which are provided by a system of four airbags, see Figure 3.4. The bags are joined together and one of them is connected to the compressor. During the experiment the air is being pumped to the bags with a speed of 0.04 kN/slab/s.

The system of bags is laid on a wooden floor supported by four beams HEA 200. Laterally, bags are embedded by beams UPE 360. If a slab with an opening is tested an additionally interior embedding structure is applied. The fixture is connected with a supporting frame through four load cells in each corner. The fixture and the support construction demands that the specimens are tested “upside-down”. Figure 3.5 shows the test setup.
Uniformly distributed load has not been commonly used in earlier experiments, mostly because of its difficulties in the applying. It gives, however, a better consideration regarding comparison of design methods. Apart from the method with airbags also a loading method with high pressure water bags has been used before, see Mosalam & Mosallam (2003). Airbags propagate better deformations of a structure and are safer in the case of a possible damage than water bags.
3.4 Boundary conditions

The specimens are supported by a steel frame that should provide line supports distanced 100 mm from each slab edge. The supporting frame consists of four beams HEA 200 which were strengthened before the experimental work in 2004, see Figure 3.6. To avoid concentrations of stresses the reactions are transmitted by steel plates from the support to the slab surface.

Due to an uneven concrete surface and imperfections of the supporting frame there were large difficulties to obtain a line support. To reduce the irregularities of the concrete a layer of plaster was used in the tests but it did not solve the problem. Gaps between the support and the interlaying plates always appeared. The gaps were shimmed with short and thin steel plates, as shown in Figure 3.7. It does not mean, however, that a line support was provided. On the contrary, severe contact problem remains and even small gaps that can hardly be noticed, may change the stiffness of a slab drastically. Important fact is that the tested slabs are relatively stocky and need little deflections to plasticize the outer fibres. It has been observed that the best contact was most often received in the corners. Therefore, we can assume with high probability that the slab is supported only in the corners at the beginning of the experiment. It gets better contact along the edges later on but it may happen out of elastic range. As a prove of this, a finite element model has been done simulating the behaviour of a slab with a small hole, supported only in the corners. The model is precisely described in Chapters 5 and 6.
Let us consider a displacement of a point lying in the middle of the line along the slab edge, which should be continuously supported, see Figure 3.8. This displacement represents a closing gap and Figure 3.9 shows its relation towards the load.

Figure 3.7  Gaps shimmed with thin steel plates. Roughness of the concrete surface is levelled by a layer of plaster.

Figure 3.8  Model of the slab with a small hole supported only in the corners. Pinned support is applied and transferred to the slab through a rigid plate in order to avoid stress concentration.
Figure 3.9  Displacement of the middle point of the supporting line. Dashed lines show the moment when the deformations become plastic.

The diagram shows that in the case of a slab weakened by a small opening, if a gap is bigger than 1.2 mm, contact will not be reached in the elastic range. This value may decrease considering possible microcracks in the concrete.

Complex supporting conditions will cause serious difficulties in finite element analysis and approximations must be done. Solutions of this problem are proposed and discussed in 5.4. It is vital that even if ideal line supports were provided in the experiments, a consideration of the specimens would be too complex for analytical methods. Treating slab as simply supported in calculations would overestimate the specimen since the supports in the experiments allow uplift in the corners. Ordinary pinned support blocks the movements in both directions and makes the structure much stiffer.

Other aspect concerning the boundary conditions, which has not been taken into account in the earlier publications to the project, is possible vertical movements of the supporting structure. The discussion of the problem appeared after a comparison of the experimental results with a finite element analysis. The displacements measured on the slabs during the tests showed far larger inaccuracies than the strains. Therefore, in the last two experiments (slabs Sst and Sst2), the displacement of the supporting frame was checked in
Two-way concrete slabs with openings

two points. Figure 3.10 shows the tested slab Sst2, which is not in the scope of the thesis and is presented only for the needs of this discussion.

Due to the negligence done in measurements when Sst was tested, the displacement of the corner support cannot be taken into account. A diagram with the other results and the proposed approximation line are presented in Figure 3.11.

![Diagram of measurement points for determination of the displacement of the support structure.]

*Figure 3.10* Measurement points for determination of the displacement of the support structure.
Considering displacement in the middle of the supporting line, the curves obtained from two experiments are exactly the same. In this case we can assume that this behaviour is a rule for all the tests. Unexpectedly, the corner displacement is larger and differs much from the middle one. This fact could be explained only by a movement of the entire testing structure, including the fixture. Also the jump around load of 10 kN/m², which can be observed in most of the other experiments as well, indicates that problems with the fixture and the supporting structure occur. That explains why the diagrams of the slab displacements obtained from the tests show linearity before and after the jump. This sudden increase of the displacements, therefore, is not caused by processes in the specimen. A method proposed in the thesis is to subtract the approximated line of support displacements from the slab deflection diagrams. At the same time a jump, which is easy to notice and reduce visually, is to be kept unchanged. The approximation is represented by the following linear relation between the load and the displacement:

\[ P = 25u = 25qA \]  

(3.1)

where:  
\[ P \] is the total load [kN]  
\[ q \] is the uniformly distributed load  
\[ A \] is the loading area  
\[ u \] is the displacement of the supporting structure [mm]
Two-way concrete slabs with openings

The approximation is based only on the results from the middle point of the supporting line. Having only one curve for the corner point it is hard to judge if its displacement gives regularity.

3.5 Instrumentation

During the experiments the loads, the displacements and the strains were measured with help of electronic equipment of the type Spider 8 which transmitted the data to the computer program. The loading was detected by four load cells connecting the fixture with the supporting frame in each corner, see Figure 3.12.

The deflections of the specimens were measured by means of Strain Gauge based Displacement Transducers (SGDT). The strains were measured by Strain Gauges (SG) on the compressed concrete surface (50 mm glued SG), in the reinforcement (10 mm welded SG) and on the CFRP (10 mm glued SG). Figure 3.13 presents some examples of the measure instruments and Figure 3.14 their configuration in the experimental program.

Figure 3.12 Load cell connecting the fixture with the supporting frame and the spring balancing the self-weight of the support.
Figure 3.13  Measurement instruments: a) SGDT, b) glued SG for concrete, c) glued SG for CFRP and d) welded SG for steel reinforcement.
Two-way concrete slabs with openings

3.6 Material properties

3.6.1 Concrete

The specimens were cast in four batches and for each of them three concrete cubes 150x150x150mm were manufactured to measure the 28-days compressive strength. Additionally, for the slab H and Swe, similar cubes were made for split tests to check the tensile strength. The concrete mixture was designed to receive a 28-days compressive strength of C32/40 according to BBK 04 (2004).
Experimental work

- **Compressive strength**

The values of the compressive strength $f_{cc}$ and their mean value $f_{ccm}$ are calculated according to the following equations:

$$f_{cc} = \frac{F_{cc}}{A} \quad (3.2)$$

$$f_{ccm} = \frac{\sum_{i=1}^{n} f_{cc,i}}{n} \quad (3.3)$$

where:
- $F_{cc}$ is the maximal compressive force
- $A$ is the area of compressed cube face ($L \times B$)
- $n$ number of cubes

The Swedish code for concrete structures BBK 04 gives two methods to estimate the required compressive strength $f_{KK}$. In our case, when three cubes were tested, the following conditions must be fulfilled:

$$f_{KK} = f_{ccm} - 6 \quad [\text{MPa}] \quad (3.4)$$

$$f_{cc} \geq f_{KK} \quad (3.5)$$

The characteristic value of the compressive strength $f_{cck}$ is calculated with the equation:

$$f_{cck} = \frac{f_{KK}}{1.14} \quad (3.6)$$

For finite element analysis also the approximate yield strength is required. This can be assumed, according to BBK 04, as 60% of the compressive strength:

$$f_{yc} = 0.6 f_{cc} \quad (3.7)$$

- **Tensile strength**

The value of the tensile strength is estimated from splitting tests. Instructions for this procedure are described in the previous version of the Swedish code for concrete structures, BBK 94 (1994).
Two-way concrete slabs with openings

The values of the cube splitting strength $f_{ct,sp}$ are obtained with the equations:

$$f_{ct,sp} = \frac{2F_{ct,sp}}{\pi A}$$

(3.8)

where: $F_{ct,sp}$ is the splitting force

$A$ is the area of the face perpendicular to the direction of the force

The cube tensile strength $f_{ct}$ is determined as 80% of the splitting strength:

$$f_{ct} = 0.8f_{ct,sp}$$

(3.9)

The mean value $f_{cm}$ of the cube splitting strength is calculated according to the equation:

$$f_{cm} = \frac{\sum_{i=1}^{n} f_{ct,i}}{n}$$

(3.10)

The requested tensile strength, in the case when three cubes are tested, must fulfill the following conditions:

$$f_T \leq f_{cm} - 0.6 \text{ [MPa]}$$

(3.11)

$$f_{ct} \geq \begin{cases} f_T - 0.5 \\ 0.8f_T \end{cases}$$

(3.12)

**Application to finite element model**

For finite element models the concrete properties are adequate to be determined directly from the mean values of the cube compressive and the tensile strengths, $f_{ccm}$ and $f_{ctm}$ respectively. Since the splitting test was done for the concrete used only to make the slab H and Swe, it must be used as a reference to the other slabs. It is assumed, therefore, that the tensile-to-compressive strength ratio is constant for all the specimens.

The estimation of the modulus of elasticity $E_{ct}$ is based on table 7.223a in BBK 04. Table 3.2 contains the concrete properties and the complete data from the cube testing is enclosed in Appendix B.
Table 3.2  Actual concrete properties evaluated from the 28-days strength tests.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>H</th>
<th>Swe</th>
<th>S, L</th>
<th>Sst, Lst</th>
<th>Lwe</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compressive strength [MPa]</td>
<td>$f_{ccm}$</td>
<td>46.5</td>
<td>45.4</td>
<td>49.5</td>
<td>50.6</td>
</tr>
<tr>
<td></td>
<td>$f_{ccy}$</td>
<td>27.9</td>
<td>27.2</td>
<td>29.7</td>
<td>30.4</td>
</tr>
<tr>
<td></td>
<td>$f_{KK}$</td>
<td>40.5</td>
<td>39.4</td>
<td>43.5</td>
<td>44.6</td>
</tr>
<tr>
<td></td>
<td>$f_{ck}$</td>
<td>35.5</td>
<td>34.6</td>
<td>38.2</td>
<td>39.1</td>
</tr>
<tr>
<td>Tensile strength [MPa]</td>
<td>$f_{cm}$</td>
<td>3.1</td>
<td>3.0</td>
<td>3.3</td>
<td>3.4</td>
</tr>
<tr>
<td></td>
<td>$f_{T}$</td>
<td>2.5</td>
<td>2.4</td>
<td>2.7</td>
<td>2.8</td>
</tr>
<tr>
<td>Modulus of elasticity [MPa]</td>
<td>$E$</td>
<td>34</td>
<td>34</td>
<td>35</td>
<td>35</td>
</tr>
<tr>
<td>Strength class</td>
<td>C35/45</td>
<td>C35/45</td>
<td>C40/50</td>
<td>C40/50</td>
<td></td>
</tr>
</tbody>
</table>

3.6.2  Reinforcing steel

The reinforcing steel of class Nps 50 with a nominal yield strength $f_{yk} = 510$ MPa was ordered to be applied in the experiments. To check its actual tensile strength three pieces of reinforcing bars were used in the tensile tests. The tensile stresses are calculated from the following equation:

$$\sigma_t = \frac{4F_t}{\pi d^2} \quad (3.13)$$

where: $F_t$ is the tensile force  
$d$ is the diameter of the reinforcing bar

The diagrams obtained in proofs and the approximation functions are presented in Figure 3.15. The approximation curve, which will be further used in the finite element models, follows the shape of the proof curves containing relatively few points. The function is defined according to Table 3.3.
Two-way concrete slabs with openings

Figure 3.15  Stress-strain relationship from the tensile tests of the steel bars.

Table 3.3  Definition of the approximation curve used in the FE-analyses.

<table>
<thead>
<tr>
<th>Stress [MPa]</th>
<th>Strain [%]</th>
<th>Plastic strain [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>510</td>
<td>0.25</td>
<td>0</td>
</tr>
<tr>
<td>575</td>
<td>0.40</td>
<td>0.15</td>
</tr>
<tr>
<td>595</td>
<td>0.70</td>
<td>0.45</td>
</tr>
<tr>
<td>600</td>
<td>1.00</td>
<td>0.75</td>
</tr>
<tr>
<td>615</td>
<td>2.00</td>
<td>1.75</td>
</tr>
<tr>
<td>620</td>
<td>3.50</td>
<td>3.25</td>
</tr>
</tbody>
</table>

The steel used in the experiments is cold worked and in this case the yield strength should be estimated as 0.2% of the proof test, see Figure 3.16. The yield and the ultimate strengths for all the proofs, and their mean values are listed in Table 3.4. Inclination of the line used to find the yield strength was chosen individually for each proof curve. For further consideration, however, the modulus of elasticity is assumed to be equal $E = 209$ GPa.
Experimental work

Figure 3.16  Estimation of the 0.2 % yield strength from the tensile tests.

Table 3.4  Evaluated tensile strength of the tested steel bars.

<table>
<thead>
<tr>
<th>Proof no.</th>
<th>$f_t$ [MPa]</th>
<th>$f_{0.2k}$ [MPa]</th>
<th>Nominal $f_y$ [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>633.7</td>
<td>582.8</td>
<td>510</td>
</tr>
<tr>
<td>2</td>
<td>615.7</td>
<td>579.2</td>
<td>510</td>
</tr>
<tr>
<td>3</td>
<td>631.1</td>
<td>580.9</td>
<td>510</td>
</tr>
<tr>
<td>Mean value:</td>
<td>626.8</td>
<td>581.0</td>
<td></td>
</tr>
</tbody>
</table>

3.6.3  Carbon fibres

One type of carbon fibre polymers, BPE® Composite 200S, was used in the experiments described in the thesis. The standard properties of this material are shown in Table 3.5. A very important characteristic of carbon fibres is their constant elastic behaviour until failure.
Two-way concrete slabs with openings

**Table 3.5** Properties of the carbon fibre used in the corners.

<table>
<thead>
<tr>
<th>Type</th>
<th>BPE® Composite 200S</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus of elasticity [GPa]</td>
<td>&gt;228</td>
</tr>
<tr>
<td>Tensile strength [MPa]</td>
<td>&gt;4500</td>
</tr>
<tr>
<td>Density [g/m²]</td>
<td>200</td>
</tr>
<tr>
<td>Thickness [mm]</td>
<td>0.11</td>
</tr>
<tr>
<td>Failure strain [%]</td>
<td>~1.8</td>
</tr>
</tbody>
</table>
4 Discussion of experiments results

In this chapter a part of the results collected during the experiments is presented and compared. The aim is to judge the used methods of reinforcing and strengthening considering the deflections and the strains. Strengthened and the additionally reinforced slabs with an opening should have similar deflections and higher or an equal load capacity as the homogeneous slab. Furthermore, the results obtained from the tests of the weakened slabs show how the holing influences the load capacity and where the strengthening is needed most.

In the diagrams presented below symbols of the slabs are used, which are explained in Table 3.1.

4.1 Deflections

The deflections presented in this section were measured at the points lying on line 1 near the opening, see Figure 4.1. These results represent the largest possible displacements of the slabs with an opening and corresponding displacements of the homogeneous slab i.e. at the same location.
Figure 4.1  Location of the SGDT dependent of the size of the opening.

The magnitude of the deflections is very important in the discussion of the load carrying capacity of the slabs. Very often the specimens show large ultimate resistance, which is reached when the displacements are in the range of 50 mm. That, what interests us most, considering the design methods, is the behaviour when the structure begin to plasticize.

Considering the diagrams in Figure 4.2 we can conclude, that the proposed method of placing the additional reinforcement in slabs with a small opening show less effectiveness for the steel reinforced slab (S) than for the carbon strengthened slab (Sst). Although the failure of slab S occurred at a load of 42 kN/m$^2$, the displacements were already relatively large around a load of 26 kN/m$^2$. This fact means that the serviceability limit state was reached early and the load capacity of the reinforced slab with an opening is lower than for the homogeneous slab. The graphs show, however, that the reinforcing bars placed in the openings’ corners delay the propagation of main cracks and this is the cause of the hardening seen in the plastic range.

Much better results give the proposed method when carbon fibre is applied instead of reinforcement bars. The strengthening delays not only the propagating but also the arising of cracks. Although the amount of the carbon fibre polymers corresponds directly to the additional reinforcement area, the composite distributes narrower cracks in a wider region than the steel reinforcement. Not less important fact is that the concrete used in the casting of the strengthened slabs was strongest according to the cube proof, see Table 3.2.
Discussion of experiments results

Figure 4.2 Load-displacement relationship in the position Di1 for the slabs with a small opening, see also Figure 4.1. Results shown for the homogeneous (H), weakened (Swe), steel reinforced (S) and CFRP strengthened slab (Sst).

Considering deflections at point Di1L similar conclusions can be drawn as for the slabs with a small opening, see Figure 4.3. The additional reinforcement does not give satisfactory results and the slab L plasticizes relatively early. In opposite to the slab S and Sst, the specimens with a large hole show less sharp plastic strength, though bigger hardening after reaching it. The curve obtained for the CFRP strengthened slab with a large hole only confirms the argument that applying carbon fibres is more efficient than corresponding amount of steel reinforcement, at least in the case shown here.

It can be noticed easily that the true deflections of the slab L and Lst are less than these shown in the diagrams. Around the load of 10 kN/m² the specimen displaces immediately together with the supporting structure which is widely described in section 3.4. After this jump the results seem to be linear with a similar inclination, which excludes cracks in the concrete as a cause of a phenomenon.
4.2 Steel strains

The strain gauges were welded on the steel bars distanced 175 mm and 150 mm from the small and the large opening’s edge, respectively. Discussed data was obtained from the gauges which placement is shown in Figure 4.4. Only in one case, for the strengthened slab Lst, the strains in location St3yL are presented.

The diagrams of the steel strain depend directly on the crack pattern observed during the experiment. Immediate increases of the strains indicate that a slab cracks near a gauge. Figure 4.5 and Figure 4.6 present strains measured around a small hole and at a corresponding point in the homogeneous slab. In all of the cases larger strains were reached at St2xS which means that the main cracks appeared always in the diagonals, see Figure 4.6. The curves obtained for the strains in location St1xS show an influence of the additional reinforcement and
the CFRP strengthening on the crack distribution, see Figure 4.5. The strains at this point in the slab S and Sst are much larger than in the weakened slab.

The reinforcement and the strengthening perpendicular to the main crack stops its propagating and the stresses are redistributed to other regions. For the diagonal cracks in the weakened slabs it is much easier to propagate and other cracks, if occur, remain small and rather insignificant.

![Figure 4.4](image)

*Figure 4.4* Location of the welded SG dependent on the size of the opening.
Two-way concrete slabs with openings

Figure 4.5 Load-strain relationships in the location St1x for slabs with a small opening, see Figure 4.4. Results shown for the homogeneous (H), weakened (Swe), steel reinforced (S) and CFRP strengthened slab (Sst).
Similar behaviour as for the slabs with a small hole can be observed in the results obtained for the slabs with a large hole. The largest strains were detected in the openings’ corners. The curves presented in Figure 4.7 prove that the stress distributions and, what is its consequence, the crack pattern should be treated individually for separate slabs and their regions. Comparing the diagrams for slab L in Figure 4.7 and Figure 4.8, the first immediate increase of strains occurs at point St1xL. This fact could indicate that, unexpectedly, near this point a first crack appeared. It cannot be judged as a rule for this kind of slab though due to lack of results for substitute points. A cause of such behaviour may be local microcracks or support problems which are described widely in section 3.4.

An asymmetry of the specimens is confirmed by comparing the strains at the corresponding points in the slabs Lwe and Lst, see Figure 4.7. Considering slab Lst the strains at the point St1xL remain low until the end of the test, whereas the substitute gauge at point St3yL detected a plastic increase of strains around a load of 36 kN/m². This means that the slab cracked more severely along the line 3 than along the line 1 which is a prove that the substitute gauge cannot constitute an ideal reference.
Two-way concrete slabs with openings

Figure 4.7 Load-strain relationships in the location St1x for slabs with a large opening, see Figure 4.4. Results shown for the homogeneous (H), weakened (Lwe), steel reinforced (L) and CFRP strengthened slab (Lst).
Discussion of experiments results

Figure 4.8  Load-strain relationships in the location St2x for slabs with a large opening, see Figure 4.4. Results shown for the homogeneous (H), weakened (Lwe), steel reinforced (L) and CFRP strengthened slab (Lst).

4.3 Concrete strains

To measure concrete strains, gauges were glued on the compressed surface at points corresponding to the gauges welded to the reinforcing bars. The placement and the direction of the gauges are shown in Figure 4.9. The strains at location Co2vS and Co2vL represent the minimum principal strains on the slabs’ diagonals. Positive values in the diagrams mean compression.
Two-way concrete slabs with openings

![Diagram](image.png)

**Figure 4.9** Location of the strain gauges glued to the concrete dependent on the size of the opening.

The diagrams of the strains indicate that the stresses are similarly distributed for the tested slabs with a small hole, see Figure 4.10 and Figure 4.11. The strains increase more or less linearly until the main cracks along the diagonals begin to propagate. This moment can be noticed clearer in the graphs of the strains Co1xS where an immediate unloading occurs, see Figure 4.10.

In both regions the strengthened slab Sst reached relatively large strains, what is related to a significant contribution of the distributed fibre strengthening. In case of the tested slabs, when a failure never occurs on the compressed surface, the strains detected by the gauges depend on the tensile strength of the structure. High strains, especially in the elastic range, show that the CFRP strips restrain a stretched slab surface from cracking heavily.
Discussion of experiments results

Figure 4.10  Load-strain relationships in the location Co1x for slabs with a small hole, see Figure 4.9. Results shown for the homogeneous (H), weakened (Swe), steel reinforced (S) and CFRP strengthened slab (Sst).
Two-way concrete slabs with openings

Figure 4.11  Load-strain relationships in the location Co2y for slabs with a small hole, see Figure 4.9. Results shown for the homogeneous (H), steel reinforced (S) and CFRP strengthened slab (Sst).

Figure 4.12 and Figure 4.13 present graphs of concrete strains measured near a large opening and a corresponding point in the homogeneous slab. Due to damage of a gauge no data for Co1xL was detected during the test of the slab L. The results from the substitute gauge, Co3yL, are presented instead what should be taken into account in the comparison. Using data from the reserve gauges, as already mentioned in section 4.2, carries always some error since it demands an assumption of ideal symmetries of specimens, which cannot be provided in experiments.

Likewise in the case of slabs with a small opening, the following results show high efficiency of using CFRP strips for the strengthening. In the diagram of the strains Co1xL for the slab Lst a change of the inclination in the elastic range occurs, see Figure 4.12. Such phenomenon may be caused by a stiffening of the entire specimen through getting better support contact which is described in section 3.4.
Figure 4.12  Load-strain relationships in the location Co1x for slabs with a large opening, see Figure 4.9. Results shown for the homogeneous (H), weakened (Lwe), steel reinforced (L) and CFRP strengthened slab (Lst).
Two-way concrete slabs with openings

Figure 4.13 Load-strain relationships in the location Co2v for slabs with a large opening, see Figure 4.9. Results shown for the homogeneous (H), weakened (Lwe), steel reinforced (L) and CFRP strengthened slab (Lst).

4.4 CFRP strains

The results from measurements in CFRP presented here come from the strain gauges glued in the middle of the strips in direction of the fibres. The results are shown in form of diagrams through the cross section at points near the holes’ corners. For a comparison, similar graphs are presented for the reinforced slab S and L. Since the amount of the CFRP was recalculated from the additional steel bars used in the slab S and L (see section 2.4), the diagrams should be of a similar shape. The comparison is done in the elastic range with a load equal to 5 kN/m² and 10 kN/m², respectively. The graphs contain some error in connection with the location of the gauges on the concrete surface which are not exactly aligned with the gauges on the CFRP and the additional steel bars, see Figure 4.14.
Discussion of experiments results

**Figure 4.14** Location of the strain gauges glued to the concrete and the CFRP strips dependent on the size of the opening. The gauges for CFRP strips are not exactly aligned with the gauges on the concrete surface, which cause some errors in consideration of stress distribution through cross section.

The diagrams presented in Figure 4.15 and Figure 4.16 show a relatively similar strain distribution through the height. In both the cases, for the slabs with a small and a large hole, the strains on the compressed concrete surface are similar when an additional reinforcement or a fibre strengthening is applied. The slabs with a reinforced opening have, however, the neutral line displaced in the direction of the compressed surface, which should never happen in the elastic range. This fact shows that the inconsequence in the location of the strain gauges (see Figure 4.14) has a bigger influence on the sections with a discrete than on a distributed strengthening.

An important point in the discussion is the fact that the strains obtained in the testing slabs with a large hole are much lower than these measured near a small hole. It indicates that the less the dimensions of opening are, the more necessary is it to apply a skew strengthening also in its corner.
Two-way concrete slabs with openings

Figure 4.15  Strains through cross section in the locations Ca2v and Co2v for slabs with small hole. Results shown in the elastic regime for the reinforced (S) and CFRP strengthened slab (Sst).

Figure 4.16  Strains through cross section in the locations Ca2v and Co2v for slabs with large hole. Results shown in the elastic regime for the reinforced (L) and CFRP strengthened slab (Lst).


4.5 Conclusions

The data collected during the experimental work shows very good results of applying CFRP strengthening in the openings’ corners. The strengthened slab with a small opening as well as with a large opening had the highest load carrying capacity. Such kind of strengthening very effectively restrains the opening’s corners from cracking and redistributes the stresses in regions along its edges. Comparing with other specimens, including the homogeneous slab, the CFRP strengthened slabs reached much higher elastic strength.

The additionally reinforced slabs showed only a high ultimate strength but most capacity was obtained in the plastic range with large deflections. The uniformly distributed load when the slab S and L cracked severely was lower than for the homogeneous slab at the same failure level. Such behaviour emphasizes the advantages of a distributed fibre strengthening in comparison to a discrete steel reinforcement.

Considering the results of the experiments one can assume that when a hole in an existing slab is to be made, proposed method of a strengthening with CFRP strips is sufficient and can be applied. In the case of the slabs cast with an opening, the steel reinforcement placed only in the opening’s corners does not give satisfactory enough results and its application in a real construction can not be recommended.
Two-way concrete slabs with openings
5 Finite element analysis

5.1 Introduction

The finite element method was introduced in the early 1960s by scientists like Argyris, Clough and Zienkiewicz. Since then the method has been developed to be one of the most powerful methods to solve engineering problems. Finite element method is based on matrix algebra and its efficiency depends directly on the performance of the computer. Nowadays, when powerful computers are available, new methods of non-linear analysis are being developed. The finite element method is today widely used especially in mechanical and civil engineering.

Both analytical and finite element solutions are based on the governing differential equations. The largest difficulty using analytical methods is to find a function that fulfils the differential equation and the boundary condition over the entire body. Therefore analytical methods are limited to solving simple problems. Finite element method is an approximation method that leads to division a structure into finite number of connected elements. The set of elements is called finite element mesh and loads and boundary conditions are in form of concentrated forces in elements’ nodes. The accuracy of finite element analysis depends on size of elements and the order of so-called shape function. The latter is in most cases either linear or quadratic.

The fundamental equation of equilibrium of the finite element method has the form of a system of linear equations:

\[ K \mathbf{a} = \mathbf{f} \]  

(5.1)
Two-way concrete slabs with openings

where \( \mathbf{K} \) is global stiffness matrix
\( \mathbf{a} \) is nodal displacement vector
\( \mathbf{f} \) is force vector

The dimension of the system of linear equations is a product of number of nodes and degrees of freedom. A global stiffness matrix contains information on properties of entire body and is built up by the stiffness matrices of each element, \( \mathbf{K}^e \). The basic FE formulation can be described by following procedure:

- Create the element stiffness matrices \( \mathbf{K}^e \)
- Establish global stiffness matrix \( \mathbf{K} \)
- Aggregate element stiffness matrices \( \mathbf{K}^e \) into the global stiffness matrix \( \mathbf{K} \)
- Establish the nodal displacement vector \( \mathbf{a} \) and force vector \( \mathbf{f} \) from the boundary conditions
- Solve the equation \( \mathbf{K}\mathbf{a} = \mathbf{f} \)

A detailed explanation of the finite element method for different types of elements can be found e.g. in handbooks of Zienkiewicz & Taylor (1989) and Ottosen & Petersson (1992).

Finite element method is always less accurate than analytical methods. It has much larger application though and in most engineering problems where analytical solutions cannot be found is the only method that can be used.

The finite element analysis performed in this thesis have used the ABAQUS 6.4-3 program. This FE-program has been chosen mainly due to its wide possibilities in concrete modelling. The analysis described in this thesis comprises of five slabs:

- H – homogeneous slab
- Swe – slab weakened by a small hole
- S – slab with small hole strengthened with extra reinforcement
Finite element analysis

- Lwe – slab weakened by a large hole
- L – slab with large hole strengthened with extra reinforcement

5.2 Model of the concrete part

The concrete slabs are modelled as three-dimensional solid parts, whereas in most cases, such structures are designed as two-dimensional shells. The reason why it has been decided to use solid elements is the possibilities to model complex supporting conditions and application of discrete reinforcement. The latter is needed especially to define extra reinforcement in the corners of the hole. In the analysis, the symmetry of slabs is used to reduce calculation time and only a quarter of a slab is considered. Figure 5.1 shows the analysed quarter of the slab with a small opening.

![Figure 5.1 A quarter of slab with a small opening.](image)

The figure also shows the boundary conditions and the applied pressure. The partitioning of the model is made to assign the support conditions, which are described in more detail in section 5.4, and to determine the loading region.

The required elastic properties are the Young modulus $E$ and Poisson’s ratio $\nu$. The former has been estimated with help of Swedish code for concrete structures BBK 04 based on cube tests, see section 3.6. Since no tests was made to estimate Poisson’s ratio the value 0.2, advised by BBK04 was selected in the calculations. The selected non-linear material properties of the concrete part are described in more detail in Chapter 6.
Two-way concrete slabs with openings

5.3 Model of the steel reinforcement

ABAQUS provides two possibilities to assign reinforcement in three-dimensional concrete slab. Reinforcement can be designed either as embedded surface with rebar layer or embedded truss elements. In the first method program recalculates given arrangement of discrete reinforcement into an equivalent uniformly distribution. This method, however, demands regular reinforcement. Considering arrangement of extra reinforcement it was decided to apply discrete reinforcement in form of truss elements embedded in concrete part 20mm from the bottom surface. For simplification, it was assumed that all reinforcement bars, disregarding their direction, lie on the same level. The example of reinforcement type used in analysis is shown in Figure 5.2. Tolerances demanded to define embedded region properties have been applied as default in ABAQUS, see Figure 5.2.

![Figure 5.2](image)

Figure 5.2 Reinforcement of slab S and properties of embedded region.

Elastic properties for reinforcement steel have been assumed to be $E = 209$ GPa and $\nu = 0.3$, common for most steel types. The plastic properties is defined as a piecewise linear stress-plastics strain curve with help of Plastic option in ABAQUS (see Figure 5.3). The stress-strain curve is approximated from test on steel rebars, see section 3.6.2.
Finite element analysis

5.4 Boundary conditions

Since only quarter of the slab is considered in the analysis, the symmetry boundary conditions are applied. For three-dimensional solid elements symmetry boundary conditions are obtained by applying horizontal reactions on entire planes lying on symmetry axes, see Figure 5.1.

One of the most complicated parts of the model is proper design of the supporting conditions. As already described in section 3.4, in most cases the best support is provided in slab corners. Along edges there are gaps filled by thin steel plates and plaster layer which can be considered as elastic support. Such support conditions provide relatively high load capacity with larger deformations than for ideal line supports. It is assumed in calculations that slab is supported stiffly in the corners and elastically in four intermediate points along the edge. The intermediate supports are modelled as non-linear springs. Their location is similar for all considered slabs. Exemplary arrangement for
Two-way concrete slabs with openings

the slab with the small hole is shown in Figure 5.4. Figure 5.5 shows the properties for the two types of spring support (B and C) used in the analysis.

![Location of supports](image)

**Figure 5.4** Location of supports.

![Springs properties](image)

**Figure 5.5** Springs properties. Non-linearity of the springs represents improvement of the supporting conditions.

Springs are specifically designed to describe initial behaviour of the slabs. Stiffness of spring supports determines stiffness of entire structure. The curves
in Figure 5.5 show soft behaviour at the beginning of the test due to poor contact between the specimen and the supporting frame. At the certain point, when better contact is reached, the springs become stiffer providing better support conditions.

To avoid stress concentrations, the reaction forces are transferred to a slab through plates defined as discrete rigid bodies. Plates transferring reactions from the springs are connected to a slab using the “tie” option which means that parts cannot be disconnected during loading. Using this solution, especially with the rigid body option, causes some errors and using a contact analysis would describe the real behaviour better. However, contact analysis, complicates the problem and makes the convergence of the numerical solution much more difficult to achieve. A contact interaction is only defined in the corner of the slab (reaction A) which allows a possible uplift during loading.

5.5 Design of mesh

5.5.1 Mesh of concrete part

The concrete part of the model is divided into so-called brick elements to obtain a proper stress distribution in the 3D analysis. There are several types of brick elements available in ABAQUS. For the analysis, C3D8R elements have been chosen and the abbreviation stands for:

- C: Continuum stress/displacement
- 3D: Three-dimensional element
- 8: 8-node brick (linear order)
- R: Reduced integration

The best accuracy give elements with quadratic order but they cannot be used in contact problems. Among linear-order elements there are two types that have been considered in the analysis, i.e. fully and reduced integrated elements. In problems when a structure is subjected to bending fully integrated linear elements give poor results due to shear locking. The phenomenon can be
Two-way concrete slabs with openings

illustrated using an element subjected to pure bending. The realistic behaviour of such element is shown in Figure 5.6.

![Figure 5.6 Realistic behaviour of element subjected to pure bending. (ABAQUS Manual).](image)

The vertical dotted lines have the same length in deformed element whereas horizontal dotted lines deform with constant curvature and change their length. This fact indicates that only the normal stress $\sigma_{11}$ is non-zero. Furthermore, angles between vertical and horizontal dotted lines remain 90°, which means that shear stress $\sigma_{12}$ is zero.

The approximation of realistic behaviour in FE-analysis depends on order of elements and number of integration points. Integration points are Gauss points needed to integrate the polynomials used in element shape function. Fully integrated linear brick elements (C3D8) have 8 integration points, 2 in each direction. When such element is subjected to pure bending upper and lower fibres change their lengths but they cannot curve, see Figure 5.7. Vertical dotted lines led through integration points follow deformations changing angle towards horizontal dotted lines. The shear stresses introduced in this type of element subjected to pure bending leads to serious inaccuracies even if a model is finely meshed. In this case the strain energy creates more shear deformation than bending deformation and this phenomenon is called shear locking.

![Figure 5.7 Fully integrated linear element subjected to pure bending. (ABAQUS Manual).](image)

To diminish the problem with shear locking in linear solid elements reduced integration is used. This method consists of using one fewer integration point in each direction, i.e. linear solid element has only one integration point. As
shown in Figure 5.8, angle between horizontal and vertical lines passing integration point remains 90° and no shear stress is introduced. With relatively fine mesh, a model built of reduced integration linear bricks can give good correspondence with the real structure. However, the number of elements through the height of bent part must large enough to capture the bending of the structure.

Figure 5.8 Reduced-integration linear element subjected to pure bending. (ABAQUS Manual).

Therefore, to determine the number of elements needed to model the slab a finite element analysis of a simply supported homogeneous concrete slab was carried out. To simplify and make comparison possible, the slab tested here differs from the one used in experimental work. Its size is reduced to 2.4×2.4×0.1m, load is applied on entire top surface and boundary conditions applied to slab’s edges disables uplift, see Figure 5.9. Furthermore, no reinforcement was used in the analysis.

Figure 5.9 Homogeneous slab used to determine number of elements.

The result was compared with an elastic analysis and maximum deflection under the load of 5 kN/m² with different designs of the mesh. Size of elements in x-y plane varied from 30×30 mm to 50×50 mm and number of elements through slab height varied from 5 to 6. Only three-dimensional linear solid elements was being examined both for reduced and full integration.
Two-way concrete slabs with openings

As a reference the development of Lévy’s series described by Timoshenko & Woinowsky-Krieger (1959) is used. According to this approximation of theory of plates maximum deflection is to be calculated with following equation:

\[ w_{\text{max}} = 0.00406 \frac{qa^4}{D} \]  

(5.2)

where:  
- \( q \) is magnitude of uniformly distributed load  
- \( a \) is size of square plate  
- \( D \) is flexural rigidity of plate

Flexural rigidity of plate \( D \) is defined as:

\[ D = \frac{Eh^3}{12(1-\nu^2)} \]  

(5.3)

where:  
- \( E \) is Young modulus  
- \( h \) is height of plate  
- \( \nu \) is Poisson’s ratio

Data needed to calculate maximum deflection is listed in Table 5.1

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( q ) [kN/m(^2)]</td>
<td>5.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( a ) [m]</td>
<td>2.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( h ) [m]</td>
<td>0.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( E ) [GPa]</td>
<td>34.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \nu ) [-]</td>
<td>0.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.1  Slab properties.

Using data from Table 5.1 in equations (5.3) and (5.2), the maximum deflection of slab amounts to:

\[ w_{\text{max}} = 0.228\text{mm} \]  

(5.4)

Table 5.2 contains values of deflection for different mesh designs and their errors towards value obtained from theory of plates. Errors \( \Delta \) are calculated according following equation:
Finite element analysis

\[ \Delta = \frac{W_{\text{max,FEM}} - W_{\text{max}}}{W_{\text{max}}} \]  \hspace{1cm} (5.5)

Table 5.2 Slab deflection for different mesh designs.

<table>
<thead>
<tr>
<th>Integration</th>
<th>x-y size of elements [mm]</th>
<th>Number of elements on height</th>
<th>Total number of nodes</th>
<th>Total number of elements</th>
<th>Deflection [mm]</th>
<th>Error [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reduced</td>
<td>50×50</td>
<td>5</td>
<td>14406</td>
<td>11520</td>
<td>0.231</td>
<td>-1.3</td>
</tr>
<tr>
<td>Reduced</td>
<td>50×50</td>
<td>6</td>
<td>16807</td>
<td>13824</td>
<td>0.227</td>
<td>0.4</td>
</tr>
<tr>
<td>Reduced</td>
<td>30×30</td>
<td>5</td>
<td>39366</td>
<td>32000</td>
<td>0.232</td>
<td>-1.8</td>
</tr>
<tr>
<td>Reduced</td>
<td>30×30</td>
<td>6</td>
<td>45927</td>
<td>38400</td>
<td>0.229</td>
<td>-0.4</td>
</tr>
<tr>
<td>Full</td>
<td>50×50</td>
<td>6</td>
<td>16807</td>
<td>13824</td>
<td>0.208</td>
<td>8.8</td>
</tr>
<tr>
<td>Full</td>
<td>30×30</td>
<td>6</td>
<td>45927</td>
<td>38400</td>
<td>0.216</td>
<td>5.3</td>
</tr>
</tbody>
</table>

Comparison with the analytic solution of different mesh arrangements indicates that the biggest influence on accuracy has number of elements through slab height. Using 6 elements gives good correspondence with the theory. Fully integrated elements give poor results even with very fine meshes due to shear locking. In the further FE analysis, meshes with 6 elements through height and element size 50×50 mm will be used. Result obtained for such a mesh have the same absolute value of error as the finer mesh (30×30 mm), though the difference in time, related directly to total number of elements, is significant.

The mesh of the slab with a small hole is shown in Figure 5.9. Some mesh refinement is done along the supports to provide better conditions for the contact and tie formulation.
5.5.2 Mesh of steel reinforcement bars

Discrete reinforcement bars can be defined only by three-dimensional truss elements either in linear (T3D2) or quadratic order (T3D3). The former with approximate size of 50 mm are used in calculations for all reinforcement types.

5.5.3 Mesh of supporting plates

Supporting plates that transfer the reactions to the concrete elements are designed as discrete rigid plates and, in opposite to analytical rigid bodies, have to be meshed. ABAQUS provides two types of elements for such parts, quadrilateral (R3D4) and triangular (R3D3). Quadrilateral elements have been chosen in the calculations and their size is identical to the corresponding elements on the concrete surface.
6 Non-linear model of concrete

6.1 Introduction

ABAQUS/Standard provides two models for non-linear concrete behaviour, smeared cracking and concrete damaged plasticity. The former is commonly used rather for simple structures under monotonic loading. Smeared cracking method is based on damaged elasticity formulation and consists in detecting and propagating a number of micro-cracks. The method has convergence problems when applied in the type of models considered in this thesis. The problems are caused by presence of non-linear springs which change the support conditions and the stiffness of the structure. Less sensitive on such changes is concrete damaged plasticity which is mostly used in structures under cyclic or dynamic loading. Two different models have been used for each slab. Description of models and results of analyses are presented below.

6.2 Principle of concrete damaged plasticity formulation

The most significant difference between damaged plasticity and smeared cracking is ability to define compression and tension degradation. The damage property lowers the elastic stiffness when the element plasticizes. Therefore it cannot recover to its initial strength, which is especially important for cyclic loading. The range of degradation is defined by the user. The principle of damaged plasticity can easily be explained by means of Figure 6.1. The curves represent behaviour of concrete element under cyclic loading. The element is subjected to tension exceeding its tensile strength. Cracks cause partial damage of the material which can be defined by the parameter $d_i$ and when element is unloaded the modulus of elasticity has changed to $(1-d_i)E_0$. 
If the element is compressed afterwards, its elastic behaviour is determined by the \( w_c \) parameter and elasticity modulus in compression is defined as 
\[
(1-d_t+w_c d_t)E_0
\]
Assuming that cracks do not influence stiffness in compression, parameter \( w_c \) is to be defined as 1. When the value of the parameter \( w_c \) equals zero (full degradation), the stiffness in compression is identical to the tension stiffness. In a similar way the damage in compression can be described. Crushed section loses its initial properties in compression, defined by parameter \( d_c \), and initial properties in tension, defined by parameter \( w_t \).

![Figure 6.1 Behaviour of concrete element under cyclic loading. (ABAQUS Manual).](image)

The damage behaviour is isotropic, i.e. the stiffness is controlled equally in all directions by a plastic potential proposed by Lubliner et al (1989).

Concrete damaged plasticity parameters are to be filled in three groups in material property window: Plasticity, Compressive Behaviour and Tensile Behaviour. Settings used in analyses of concrete slabs are described below.

### 6.3 Plasticity parameters

There are five parameters that need to be defined to solve Drucker-Prager plastic flow function and yield function proposed by Lubliner et al (1989).
To obtain exact values of these parameters, number of additional tests would have to be carried out for the material used in the experiments. Due to lack of sufficient information, the default parameters in ABAQUS or proposed in other publication have been used. The parameters needed to describe the plastic properties of concrete are:

- **Dilation angle \( \psi \)**

  Dilation angle is the ratio of volume change to shear strain. In Drucker-Prager formulation the value of the dilation angle is to be determined for element under biaxial compression with high confining pressure. According to Vermeer and de Borst (1984) typical dilation angle for concrete is 12° and this value has been used in the models of concrete slabs.

- **Eccentricity**

  This parameter is the rate at which Drucker-Prager function approaches the asymptote. With eccentricity tending to zero the plastic flow tends to a straight line. In further calculations the eccentricity 0.1, which is default for ABAQUS, is used. This value is chosen to obtain a soft curvature of the potential flow and provides almost the same dilation angle for wide range of confining pressure values.

- **\( \sigma_{c0}/\sigma_{b0} \)**

  This parameter is needed to solve yield function and represents the ratio of initial equibiaxial compressive strength to uniaxial compressive strength. The default value 1.16 is used in calculations.

- **Viscosity parameter**

  Viscoplastic regularization can be used when convergence problems caused by softening behaviour and elasticity degradation appears. Since the slabs models did not cause severe convergence difficulties the viscosity parameter is assumed to be zero.

- **\( K_c \) parameter**

  The value of \( K_c \) parameter is to be determined considering yield surface in deviatoric plane, see Figure 6.2. The parameter is the ratio of the second stress invariant on the tensile meridian (T.M.) to the second stress invariant on the
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compRESSive meridian. Figure shows the yield surfaces for $K$ values 1 and 2/3. The value of 2/3 have been used in the calculations.

![Diagram showing yield surfaces for different $K$ values.](image)

**Figure 6.2** Yield surface in the deviatoric plane (ABAQUS Manual).

### 6.4 Compressive behaviour

An accurate model of the compressive behaviour is not necessary in case of examined slabs. Considering the strength of the used concrete and the amount of the reinforcement, the specimens will never reach the yield stress in compression. Simplified model for compressive behaviour is applied in the analysis, which assumes that yield strength is 60 % of ultimate strength and plastic strain at failure equal 0.15 %. Values of yield and ultimate strength are taken from Table 3.2. The figure shows the input window for compressive behaviour for slab weakened by a small opening (Swe).

For more complex problems suboption for compression damage is available. Since no crushing of compressed concrete surface is likely to occur, the compression damage parameters are omitted in the analysis.
Figure 6.3  Settings of compressive behaviour for slab Swe.

6.5  Tensile behaviour

The tensile behaviour is described by defining the post-cracking tension softening curve. The concrete damaged plasticity model in ABAQUS/Standard allows determining post-failure behaviour in tension in three ways: by defining strain, crack opening (displacement) or fracture energy towards plastic tensile stress. These three alternative options, however, are related to one another and choice of them depends on the knowledge of material and structural behaviour. The safest way for complex structures is to define the fracture energy $G_F$. The definition of fracture energy was proposed by Hillerborg (1974) and is described as the energy required to open a unit area of crack. The energy can be illustrated as the area under stress-displacement diagram, see Figure 6.4. The fracture energy value varies from 40 N/m for low-strength to 120 N/m for high-strength concrete.
Two-way concrete slabs with openings

Figure 6.4 Fracture energy at stress-displacement diagram.

If the fracture energy is set as definition of tensile behaviour, ABAQUS recalculates it into stress-displacement relation in a simplified triangular form, see Figure 6.5. The minimum value of stress, however, is set by program to 1% of tensile strength.

Figure 6.5 Stress-displacement diagram for \( G_F \) as a material property.

The fracture energy and the stress-displacement relation are connected directly to each other and can be used alternatively to define the tensile behaviour. They can be recalculated into stress-strain relation when crack band is known. The width of crack band depends on spacing of reinforcement bars and size of aggregate. Plos et al (2004) claim that for structures with distributed cracks, the distance between cracks is the crack band width at the same time and can be assumed to be equal to spacing of reinforcement bars.

In the calculations of the slabs, the two models of tensions stiffening have been used, defining fracture energy (GF) and stress-strain relation (ST). For concrete
C 35/45 used in experiments a reasonable value of fracture energy is 100 N/m. An approximate stress-cracking strain relation has been obtained using the element in the slab Swe that first reached the tensile yield strength, see Figure 6.6. This diagram constitutes a base for the other model of tension softening, assuming stress-strain relation as material a property. The aim of this model is to keep similar fracture energy using the non-linear tension softening and examining how it influences the post-failure behaviour.

![Stress-strain relation for GF and ST models.](image)

Due to the complex supporting conditions a specimen may be locally unloaded when springs become stiffer. To avoid closing cracks tensile damage is defined in the model. Figure 6.7 shows values of tensile damage factor $d_t$ and corresponding plastic strains for models GF and ST. Furthermore, it is assumed that in case of transition stress state into compression only 80 % of elasticity is recovered ($w_c = 0.8$).
Two-way concrete slabs with openings

Figure 6.7  Tensile damage factor.
7 Results of finite element analysis

7.1 Introduction

For each of the four examined slabs two analyses have been carried out. Results signed with “GF” have been obtained for analysis where the fracture energy was used as a material property. Notation “ST” stands for analysis with the tension softening defined as a stress-cracking strain curve.

This chapter contains the results of both type of analyses of the slab weakened by a small hole (Swe) and their comparison with the experimental results. For other slab types only the diagrams of maximum deflection are presented. The rest of the results can be found in Appendix C.

The experiments showed large deformations with hardening behaviour when the specimens were severely cracked. Such behaviour has not been obtained in finite element analyses due to numerical problems when large regions of a structure are fully plasticized. To make the comparison with the numerical analysis clearer, some curves obtained from the experiments have been clipped.

7.2 Deflections

Collected results concern maximum deflection of a specimen. For slab weakened by a small hole (Swe) the largest displacements appear in the middle of the opening’s edge. Figure 7.1 shows the comparison between experiments and finite element analyses.
Considering ultimate strength we can observe that the calculations underestimate the maximum loads in the experiments. However, the ultimate limit state could not be reached with the finite element analyses; possible cause of this phenomenon are the supporting conditions in the experiments which continuously change the static system of the structure. This problem could be solved using a more accurate definition of the non-linear springs at the supports. However, such formulation also causes serious convergence problems.

To provide better comparison of curves, the diagram from experiment has been clipped at displacement value of 10mm, see Figure 7.2. Curves for two different finite element analyses are of similar shape. The model with the defined tension softening (ST) shows softer behaviour though and has slightly higher load capacity. It can be noticed for both models that yielding starts already around a uniform load of 13 kN/m² but the properties of the spring supports stiffen and strengthen the structure.

The models show relatively good correspondence with experimental result until the yield plateau is reached. Therefore regarding the deflection, approximation of supporting conditions as non-linear supports is accurate enough to model tested specimens.

Figure 7.1  Maximum displacement of slab weakened by small hole.
Since the same supporting conditions have been applied for all slab calculations, the results obtained can be used as references in comparison of the load capacity. To carry out such comparison the maximum deflections of slabs are presented, both for experiments (see Figure 7.3) and finite element analyses (see Figure 7.4). The result from the “ST” analysis with defined stress-strain curve for tension softening, which show a better post-failure behaviour, are used in Figure 7.4. Only for the slab L we “ST” analysis has not been obtained due to numerical problems and the results from the analysis “GF” are presented instead.

The specimens tested in the experiments have higher load capacity and larger deformations. Especially slab L, the large hole strengthened with extra reinforcement, shows significant hardening behaviour after yielding. We can suppose that the hardening depends on the loading area, i.e. magnitude of total load. This fact can be related to supporting situation. The moment when specimen reaches final static system depends on total, not uniformly distributed load. If we consider the onset of yielding instead, when deflections are still reasonably low, all slabs with an opening have lower capacity than the homogeneous slab. The results from the FE analyses shown in Figure 7.4 support the judgement that applying extra reinforcement only in the opening’s
Two-way concrete slabs with openings corners is not sufficient. The small contribution of such reinforcement can be observed especially when comparing slabs S and Swe.

Figure 7.3  Maximum displacement of slabs in experiments.
Figure 7.4 Maximum displacement of slabs in FE analysis.

7.3 Steel strains

The steel strains from two finite element models (GF and ST) corresponding to the points in the experimental work, see section 4.2, are compared in Figure 7.5 for the slab Swe.
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Figure 7.5  Load-strain relationship in the reinforcing steel in the locations 1x and 3y for slab weakened by a small hole (Swe).

The Figure shows that the strain levels in the tests correspond relatively well with the FE analysis when the slab is un-cracked. After cracking the divergence between the curves is mainly caused by different crack patterns obtained in tests and calculations, see Figure 7.6. In the experiment one main crack propagates out from every corner. Some of them, like the ones shown in Figure 7.6, split into two cracks and propagated in relatively narrow band. For the finite element models two identical propagating cracks have been obtained, which are much more spread. This is probably caused by the formulation of the spring supports which is only an approximation. In reality the support conditions are much more complex. It is also believed that the spring support cause convergence problem and could be the root for not being able to proceed the calculations. The load–displacement curves in Figure 7.1 indicate snap back behaviour.

The difference in crack pattern is even more significant considering strains in the locations 2x and 2y since the reinforcement yields only in the cracks. For the tested slab Swe the main cracks passes close to the gauge and very high strains are recorded. For finite element models where main cracks appear in different places, strains in the location 2x are not particularly high since the cracks do not propagate in these positions. For this reason two more strain
Results of finite element analysis

curves are presented in Figure 7.7 at the locations of the propagating cracks. The notation “Crack” stands for the location where a crack appear in the finite element model.

![Image of cracks](image1.png)

**Figure 7.6** Comparison of crack pattern for slab weakened by a small hole (Swe).

![Graph of load vs. strain](image2.png)

**Figure 7.7** Steel strains St2x(2y) and “Crack” for slab Swe. Curves for “Crack” show steel strains in the cracks.
Two-way concrete slabs with openings

7.4 Concrete strains

Data for the concrete strains have been collected for points Co1x and Co2x on compressed surface. Similarly as for steel strains, due to possible asymmetry of the specimen, two concrete strain gauges have been used in the experiment.

The strain curves, Co1x(3y), presented in Figure 7.8 show very good correspondence between tests and finite element analyses. Since main cracks occur near diagonals, strains in the region along opening’s edges decrease what can be observed both in test and calculations. All graphs show also similar stiffening behaviour due to variable supporting conditions. The difference in crack pattern, described in 7.3, seems to have less influence on strains Co1x on compressed concrete surface than on corresponding strains in reinforcing steel.

![Figure 7.8 Concrete strains Co1x(3y) for slab Swe.](image)

Figure 7.9 shows the concrete strains, Co2x(2y), in the section where main crack occurs in test and in the finite element analyses, CoxCr. The results show good correspondence until the high yield strains are reached. The strains from calculations remain relatively low even in the section where the crack occurs. The difference in strains due to crack location can clearly be noticed comparing the curves of Co2x and CoxCr in the finite element analyses.
Results of finite element analysis

Figure 7.9  Concrete strains Co2x(2y) and “Crack” for slab Swe. Curves for “Crack” show concrete strains in the cracks.
Two-way concrete slabs with openings
8 Discussion

8.1 General conclusions

The thesis contains consideration of slabs tested in experimental program carried out in 2003 and 2004. Discussion of results leads to general conclusion that using extra reinforcement only in openings’ corners is not sufficient method of strengthening. Similar application of CFRP strips in strengthening of cut-offs gives better results though. This can be explained by the fact that fibre reinforcement is distributed directly on concrete surface in tension which delays cracking. Furthermore, very important in this case are material properties of carbon fibres. This type of material, in contrast to steel, keeps its elastic properties until failure. This may explain a large hardening in testing slabs strengthened with CFRP strips. When a crack occurs in the structure internal forces are redistributed and concentrated in a part of reinforcement or strengthening passing the crack. Considering steel reinforcement local yielding occurs and the structure deflects relatively easily. Since carbon fibres do not yield strengthened structures reach larger loads and have better stability after cracking.

In finite element analyses most problems have been met in modelling supporting conditions which appeared to be very complex in the experiments. Their definition as a set of discrete spring supports gives good correspondence with test data until failure. Convergence difficulties, possibly caused by snap back behaviour, cause limitations in further modelling of the post-failure behaviour. The damaged plasticity model is less sensitive to the variable supporting conditions than alternative smeared cracking model and is the only one used in FEM calculations.
Two-way concrete slabs with openings

The finite element analyses support the judgement that the extra reinforcement gives poor contribution to load capacity and thus cannot be the only strengthening applied to the structure.

8.2 Further research

This thesis is only an introduction to finite element analysis of two-way slabs with an opening planned in the project. The most effort was done to satisfy complex conditions met in experiments. Having an accurate model of concrete slab further research on reinforced concrete slabs can be done without experiments. Thus, the moment distribution and the strengthening needs around openings could be analysed for different types of slabs.

To create accurate finite element model of fibre strengthening some more tests should be done especially to determine the bonding strength. Experiments on fibre strengthened structures with simple supporting could be carried out as a reference.

The last proposition on future research concerns the needs of advanced concrete properties in using the type of model in FE calculations. In most cases only uniaxial strength is checked, whereas many parameters requested in ABAQUS are related to biaxial compression. Possible future research in this area could show dependence of these parameters on concrete strength and used aggregate.
REFERENCES


Two-way concrete slabs with openings


Appendix A  Drawings of tested slabs

A.1  Homogeneous slab. Scale 1:25

- strain gauges welded to the reinforcing bars
- strain gauges glued to the compressed concrete surface
Concrete: C 32/40
Steel: Nps 50, Ø5
A.2  Slab weakened by a small hole (Swe). Scale 1:25

- strain gauges welded to the reinforcing bars
- strain gauges glued to the compressed concrete surface

Concrete: C 32/40
Steel: Nps 50, Ø5
A.3  Slab with a small hole strengthened with steel reinforcement (S).
Scale 1:25

- strain gauges welded to the reinforcing bars
- strain gauges glued to the compressed concrete surface

Concrete: C 32/40
Steel: Nps 50, Ø5
Two-way concrete slabs with openings

A.4 Slab with a small hole strengthened with CFRP (Sst). Scale 1:25

- strain gauges welded to the reinforcing bars
- strain gauges glued to the compressed concrete surface
- strain gauges glued to the CFRP strips

Concrete: C 32/40
Steel: Nps 50, Ø5
A.5 Slab weakened by a large hole (Lwe). Scale 1:25

- strain gauges welded to the reinforcing bars
- strain gauges glued to the compressed concrete surface

Concrete: C 32/40
Steel: Nps 50, Ø5
A.6 Slab with a large hole strengthened with steel reinforcement (L).
Scale 1:25

- strain gauges welded to the reinforcing bars
- strain gauges glued to the compressed concrete surface

Concrete: C 32/40
Steel: Nps 50, Ø5
A.7 Slab with a large hole strengthened with CFRP (Lst). Scale 1:25

- strain gauges welded to the reinforcing bars
- strain gauges glued to the compressed concrete surface
- strain gauges glued to the CFRP strips

Concrete: C 32/40
Steel: Nps 50, Ø5
Two-way concrete slabs with openings
Appendix B  Comparison between FE-analyses and experiments

In this appendix the results of the FE analyses are presented and compared with the experiments. The results for slab weakened by a small hole (Swe) are shown and described in Chapter 7. In most cases no significant difference was observed between curves obtained from “ST” analysis (tension softening described by stress-strain) and “GF” analysis (tension softening determined by the value of fracture energy). Therefore only one of the two analyses, which showed better propagation, was taken into account.

Crack pattern for presented slabs obtained from the experiments differs from the models. The tested specimens have the main cracks mostly along the diagonals while these from FE analyses have the main cracks spread. The different crack pattern causes discrepancies in strains in the points located on the diagonals (St2x, Co2x). In these cases data from cracks in the modelled slabs is presented and signed “Crack”, see e.g. Figure B.5.

The finite element analyses stop after the structure plasticizes while the strains obtained from the experiments reach large values in fully plastic region. To make comparison in early stage clearer some graphs have been clipped.

B.1 Homogeneous slab (H)

Figure B.1  Comparison of crack pattern for homogeneous slab (H) between the experiment and the finite element model. For finite element model only a quarter of the slab is shown.
Two-way concrete slabs with openings

Figure B.2 Maximum deflection of homogeneous slab (H).

Figure B.3 Load-strain relationships in the reinforcing steel in the location Mx for homogeneous slab (H).
Figure B.4  Load-strain relationships in the reinforcing steel in the locations 1xS and 3yS for homogeneous slab (H).

Figure B.5  Load-strain relationships in the reinforcing steel in the locations 2xS, 2yS and “Crack” for homogeneous slab (H).
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Figure B.6 Load-strain relationships in the reinforcing steel in the locations 1xL and 3yL for homogeneous slab (H).

Figure B.7 Load-strain relationships in the reinforcing steel in the locations 2xL, 2yL and “Crack” for homogeneous slab (H).
Figure B.8  Load-strain relationships on the compressed concrete surface in the locations 1xS and 3yS for homogeneous slab (H).

Figure B.9  Load-strain relationships on the compressed concrete surface in the locations 2xS and 2yS for homogeneous slab (H).
Two-way concrete slabs with openings

Figure B.10 Load-strain relationships on the compressed concrete surface in the locations 1xL and 3yL for homogeneous slab (H).

Figure B.11 Load-strain relationships on the compressed concrete surface in the locations 2xL and 2yL for homogeneous slab (H).
B.2 Slab with a small hole strengthened with additional steel reinforcement (S).

Figure B.12 Comparison of crack pattern between the experiment and the finite element model for slab with a small hole strengthened with additional steel reinforcement (S).

Figure B.13 Load-displacement relationship in the location Di1 for slab S.
Two-way concrete slabs with openings

Figure B.14  Load-strain relationships in the reinforcing steel in the locations 1x and 3y for slab S.

Figure B.15  Load-strain relationships in the reinforcing steel in the locations 2x, 2y and “Crack” for slab S.
Figure B.16  Load-strain relationships in the reinforcing steel in the locations 2v and 4w for slab S.

Figure B.17  Load-strain relationships on the compressed concrete surface in the locations 1x and 3y for slab S.
Figure B.18  Load-strain relationships on the compressed concrete surface in the locations 2x, 2y and “Crack” for slab S.

Figure B.19  Load-strain relationships on the compressed concrete surface in the locations 2v and 4w for slab S.
B.3 Slab weakened by a large hole (Lwe)

*Figure B.20* Comparison of crack pattern between the experiment and the finite element model for slab weakened by a large hole (Lwe).

*Figure B.21* Load-displacement relationship in the location Di1 for slab weakened by a large hole (Lwe).
Two-way concrete slabs with openings

Figure B.22  Load-strain relationships in the reinforcing steel in the locations 1x and 3y for slab Lwe.

Figure B.23  Load-strain relationships in the reinforcing steel in the locations 2x, 2y and “Crack” for slab Lwe.
Figure B.24  Load-strain relationships on the compressed concrete surface in the location 1x and for slab Lwe. Data from the reserve gauge in the location 3y missing due to damage.

Figure B.25  Load-strain relationships on the compressed concrete surface in the locations 2x, 2y and “Crack” for slab Lwe.
B.4 Slab with a large hole strengthened with additional steel reinforcement (L)

Figure B.26 Comparison of crack pattern between the experiment and the finite element model for slab with a large hole strengthened with additional steel reinforcement (L).

Figure B.27 Comparison of crack pattern between the experiment and the finite element model for slab L.
Figure B.28  Load-strain relationships in the reinforcing steel in the location 1x for slab L. Data from the reserve gauge in the location 3y missing due to damage.

Figure B.29  Load-strain relationships in the reinforcing steel in the locations 2x, 2y and “Crack” for slab L.
Two-way concrete slabs with openings

Figure B.30  Load-strain relationships in the reinforcing steel in the locations 2v and 4w for slab L.

Figure B.31  Load-strain relationships on the compressed concrete surface in the location 3y and for slab L. Data from the gauge in the location 1x missing due to damage.
Figure B.32  Load-strain relationships on the compressed concrete surface in the locations 2x, 2y and “Crack” and for slab L.

Figure B.33  Load-strain relationships on the compressed concrete surface in the locations 2v and 4w for slab L.
Two-way concrete slabs with openings
**Appendix C  Cube tests of concrete**

During casting of each batch of slabs, three cubes of dimensions 150×150×150 mm were produced to control the compressive strength of concrete. Table C.1 contains the data obtained for 28-days-old concrete.

For the batch which contained slabs H (homogeneous) and Swe (weakened by a small hole) another three cubes were cast for splitting test to obtain the tensile strength of concrete. The tests were done also 28 days after casting and the data is presented in Table C.2.

*Table C.1  28-day compression tests on concrete cubes*

<table>
<thead>
<tr>
<th>Slab</th>
<th>Dimensions [mm]</th>
<th>Mass [g]</th>
<th>Axial force [kN]</th>
<th>Compressive strength [MPa]</th>
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<tr>
<td></td>
<td>L</td>
<td>B</td>
<td>H</td>
<td></td>
</tr>
<tr>
<td>H &amp; Swe</td>
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<td>150,7</td>
<td>150,4</td>
<td>8076</td>
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<td></td>
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<td>149,3</td>
<td>150,5</td>
<td>8058</td>
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<td></td>
<td>150,3</td>
<td>149,8</td>
<td>150,3</td>
<td>7976</td>
</tr>
</tbody>
</table>

Average: **46,5**

| S & L  | 151  | 151,2 | 150,2 | 8056           | 1012                    | 44,3                     |
|        | 150,2 | 151  | 150,3 | 8107           | 1012                    | 44,6                     |
|        | 151,3 | 149,7 | 150  | 8034           | 1071                    | 47,2                     |

Average: **45,4**

| Sst & Lst | 151,1 | 150,4 | 150,2 | 8222           | 1075                    | 47,3                     |
|           | 151,2 | 151,1 | 150,3 | 8254           | 1138                    | 49,8                     |
|           | 150,9 | 148  | 151,2 | 8227           | 1149                    | 51,4                     |

Average: **49,5**

| Lwe    | 151,5 | 149,3 | 150,1 | 7827           | 1131                    | 50                       |
|        | 151   | 150,3 | 150,6 | 7920           | 1163                    | 51,2                     |
|        | 150,8 | 149,2 | 150,7 | 7889           | 1142                    | 50,7                     |

Average: **50,6**
Two-way concrete slabs with openings

Table C.2  28-day splitting test on concrete cubes

<table>
<thead>
<tr>
<th>Slab</th>
<th>Dimensions [mm]</th>
<th>Mass [g]</th>
<th>Axial force [kN]</th>
<th>Splitting strength [MPa]</th>
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