Analyses of a Rotor Dynamic Testrigs

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Preface

Rotating machines and rotor dynamics is a specific area of dynamics and has been changed very little and in a low pace during the last decades. Early engineers did not know what would happen in case they could exceed to operate a system over the first natural frequency. Finally engineers like De Laval succeed to operate a rotor way above the critical speed. Nowadays, engineers try to work with more complicated problems or to develop techniques that could determine the dynamics of a real machine more accurately.

A similar study regarding rotating machines took place at the period Marc-July 2009, at Luleå University of Technology in Sweden. The main purpose of this study was to combine theoretical and experimental results in different systems. Furthermore, the use of Rotor Kit RK4 was part of this study to evaluate the dynamics of a small scale rotor.

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Furthermore, the writer dedicates the present report with all his heart to his family(Dimitrios, Efthymia, Georgios and Barbara) for their support and encouragement in times of difficulties and doubt.....
Abstract

This thesis report describes how to develop a mathematical model of a rotordynamical system, either the reader is a beginner or has a background in dynamical systems. The theory is described in order to conceive the main idea how to develop rotor dynamical models. The problems are based on FEM and described in detail.

In addition, the educational Rotor Kit RK4 will be introduced and analysed. This rotor equipment is especially developed for measuring and detecting different phenomena that occur under operation of a rotating system. Many setups and options will be shown in order understand how Rotor Kit RK4 can be used. That will help one to develop models from a simple to a more complicate system. Thus, according to each setup the mathematical model will be adapted once with a single mass, once with two masses, perturbation at different spans and so forth. For each system different weaknesses of the theoretical model is observed. The main purpose is to identify the reason of having a model unable to calculate the right results. That will be carried out by changing different values of the model like different number of elements, and by including other facts which were assumed that not exist but finally affect the final computation.

The mathematical models which were developed for this thesis are three. First model is a simple system with rigid points of support. The code calculates only the rotor regarding its properties and objects attached on it. The second model is a system that includes bearing properties. The system now can be slightly move up and down at the support which shows that the system became more sensitive. The third model includes the extent part of the rotor out of the bearings. The real rotor kit’s shaft does not stop at the bearings, but it has a small part of its length out of them at both ends. It is understandable that, improving a model by adding other facts like bolts, nuts and generally the environment in which the real system works, computations can be achieved to be closer to the real results.

The used equipment has unlimited choices in developing experimental tasks so long as someone has a vast fantasy. Moreover, the equipment provides the ability to easily adapt any real system, to almost the same in a smaller scale. Many experimental tasks are shown which were designed for students in order to get a better understanding about Rotor Kit and Rotor dynamics.
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Introduction

During the last 50 years engineers have developed several new techniques and solved many problems that industry is facing in dynamics of rotating machines. However the demands and reliability on machines that can operate in hard conditions are expected to increase. In this study one of the most significant goals was to analyse and evaluate the rotor kit RK4. The rotor is especially designed for educational purpose and it will be used in dynamic courses at LTU. Therefore, it is important to identify potential errors or assumptions that significantly affect the results by comparing experimental with theoretical results. Thus, a theoretical model was developed for the rotor kit equipment to extract information not only about the assumptions done in order to simplify the study of a system, but also to observe how the same environment in which the system operates affects the results.

In addition, several studies have been done with rotor kit which indicate the ability of unlimited configurations and experimental tasks that can be achieved. Some remarkable references are: Active vibration control of rotor by Kari Tammi with the use of special accessory to develop magnetic field round the rotor [1]. Response of a warped flexible rotor with a fluid bearing by Jim Meagher, Ci Wu and Chris Lencioni [2]. Investigation of vibration of a rotor system supported by absolutely rigid bearings with a shaft containing a notch by Petr Ferfecki, Jan Ondrouch and Tomas Lukas [3]. The main purpose was to study a rotor system with a shaft weakened by a notch. Determination of oil whip phenomena by Agnes Muszynska [4]. Experiment design for simulating different faults in a rotor kit by Enayet Halim [5]. A magnetorheological fluid damper for rotor applications by P. Forte, M. Paterno, and E. Rustighi [6].

The report describes the theory, equipment and experimental results of different setups. In the first section the theory used for developing the theoretical models is described. The theory shows how to develop matrices, build a FEM model and calculate the natural frequencies by using General Eigenvalue Problem. Section two describes the rotor kit RK4 and how it can be used for educational purpose. Third section presents experimental results and comparison with theoretical results.

The purpose of this thesis, as mentioned above, is to identify why calculations are not the same or sometimes not even close at all to the real values. Thus, the main goal is, to track down what more should be taken into consideration in the computation (like bearings, support and so forth). The thesis describes also several experimental setups that will give undergraduate students the opportunity to compare theory with the reality. Finally, the results are discussed together with suggestions how the experimental setups could be improved.
Chapter 1

Rotor dynamic modeling and analysis

1.1 Element matrices

This section will describe the theoretical method and techniques that were used to develop a mathematical model. In addition, explanations are given to better understand how the matrices are built.

1.1.1 Fundamental beam theory

In rotor dynamics, problems can be approached with different methods, like the use of classical formulas of physics or by FEM method. Even though FEM is more difficult to define, it is an effective method to simulate dynamical problems. In this section the main purpose is to describe the fundamental theory on how to build an element.

First, a prismatic beam, Figure 1.1, subjected to a constant force is described. Then the shape functions of an element can be found for static conditions. According to the theory [7] a beam has to satisfy the equation of the elastic line,

\[ \frac{\partial^2}{\partial x^2} \left( EI \frac{\partial^2 u}{\partial x^2} \right) = 0 \]  

thus, it can be assumed that,

\[ u(l,t) = c_1(t)l^3 + c_2(t)l^2 + c_3(t)l + c_4(t) \]  

Where, \( c_1, c_2, c_3 \) and \( c_4 \) are the constants that can be found from the boundary conditions, and \( x_1, \vartheta_1, y_1, \varphi_1, x_2, \vartheta_2, y_2 \) and \( \varphi_2 \) for the displacements and rotations respectively. The behavior of a prismatic beam can be
described with a shape function graph. From the theory, book [8], of FEM and prismatic beams, the four shape functions for the element in Figure 1.1, will be,

\[
\begin{align*}
  h_1(l) &= 1 - 3s^2 + 2s^3, \quad h_2(l) = L(s - 2s^2 + s^3) \\
  h_3(l) &= 3s^2 + 2s^3, \quad h_4(l) = L(-s^2 + s^3)
\end{align*}
\] (1.3)

where for \( s = l/L \) and \( L \) the length of the entire element. The same shape functions 1.3 stand for the y direction. From the general solution 1.2 now can be found that:

\[
\begin{align*}
  u(l,t) &= h_1(l)x_1 + h_2(l)\vartheta_1 + h_3(l)x_2 + h_4(l)\vartheta_2 \\
  u(l,t) &= h_1(l)y_1 + h_2(l)\varphi_1 + h_3(l)y_2 + h_4(l)\varphi_2
\end{align*}
\] (1.4)

The kinetic energy, \( T \), can be introduced as,

\[
T(t) = \frac{1}{2} \int_0^L \rho A \left( \frac{du(l,t)}{dt} \right)^2 dl
\] (1.5)

which should be equal to,

\[
T(t) = \frac{1}{2} \{ \dot{u}^T \} [M^e] \{ \dot{u} \}
\] (1.6)

Since it is assumed that the equation 1.5 and 1.6 are equal, the \( M^e \) mass matrix can be determined. A similar approach is used for the \( K^e \) stiffness matrix. For the stiffness matrix the kinetic energy will be,

\[
T(t) = \frac{1}{2} \int_0^L EI \left( \frac{\partial^2 u(l,t)}{\partial l^2} \right)^2 dl
\] (1.7)

which should be equal to,

\[
T(t) = \frac{1}{2} \{ u^T \} [K^e] \{ u \}
\] (1.8)
After the integration the elements of matrix $K^e$ can be identified. Every part that can be described with matrices, from now on can be called a finite element. The inverse of the stiffness matrix $A^{(e)} = K^{(e)}\,^{-1}$ is the flexibility matrix [9].

### 1.1.2 Finite element method

Finite element is today a frequently used method which help engineers to create models. The accuracy of the results is related on how many elements are used for a problem. However, that does not imply that one has to use as many elements as possible. For instance, if it is of interest to find the 1$^{st}$ and 2$^{nd}$ natural frequency of a beam problem it is enough to use only two-four elements.

Nevertheless, a higher number of elements are necessary, if we are interested in beams of more complex shapes or in finding mode shapes of the beam. In addition, the element type is selected due to the complexity of the problem. In this study were chosen to use 8$\times$8 element matrices.

\[
\delta^{(e)} = \{\chi_1 \ y_1 \ \varphi_1 \ \Theta_1 \ \chi_2 \ y_2 \ \varphi_2 \ \Theta_2\} \tag{1.9}
\]

The above vector represents the displacements and rotations at the left and right node of an element. The following matrix is the stiffness matrix that was used at the present study.

\[
K^{(e)} = \frac{EI}{L^3} \begin{bmatrix}
12 & 0 & 0 & 6L & -12 & 0 & 0 & 6L \\
0 & 12 & -6L & 0 & 0 & -12 & -6L & 0 \\
0 & -6L & 4L^2 & 0 & 0 & 6L & 2L^2 & 0 \\
6L & 0 & 0 & 4L^2 & -6L & 0 & 0 & 2L^2 \\
-12 & 0 & 0 & -6L & 12 & 0 & 0 & -6L \\
0 & -12 & 6L & 0 & 0 & 12 & 6L & 0 \\
0 & -6L & 2L^2 & 0 & 0 & 6L & 4L^2 & 0 \\
6L & 0 & 0 & 2L^2 & -6L & 0 & 0 & 4L^2
\end{bmatrix} \tag{1.10}
\]

This is an $8 \times 8$ matrix where, $E$ is the Young’s modulus, $I = \pi r^4/4$ is area moment of inertia, $r$ the radius of the element and $L$ the element length. Assuming $L_{Total}$ the total length of the uniform rotor and $n$ the number of the elements, a single element length will be equal to $L = L_{Total}/n$.

The following matrix determines the mass of each element. The mass matrix takes into account only the mass of the rotor, therefore, it will later be shown how the mass of the disc can be added to the right node.
\[ M^{(e)} = \frac{m}{420} \begin{bmatrix}
156 & 0 & 0 & 22L & 54 & 0 & 0 & -13L \\
0 & 156 & -22L & 0 & 0 & 54 & 13L & 0 \\
0 & -22L & 4L^2 & 0 & 0 & -13L & -3L^2 & 0 \\
22L & 0 & 0 & 4L^2 & 13L & 0 & 0 & -3L^2 \\
54 & 0 & 0 & 13L & 156 & 0 & 0 & -22L \\
0 & 54 & -13L & 0 & 0 & 156 & 22L & 0 \\
0 & 13L & -3L^2 & 0 & 0 & 22L & 4L^2 & 0 \\
-13L & 0 & 0 & -3L^2 & -22L & 0 & 0 & 4L^2 
\end{bmatrix} \tag{1.11} \]

The bearings of the rotor kit were considered to be very stiff, thus, both cases were included in the final computations, with stiff bearings and flexible bearings. The gyroscopic effect results in torques on the system at the node on which a polar inertia is attached. One method is to directly add the moment of inertia at the right node, something that will be explained further at the assembly section. Thus, the damping matrix can be of 4 \times 4 size which in fact represents only a single node. This matrix will directly be placed in the final matrix at the right node.

\[ C_d = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & J_p\omega & 0 \\
0 & 0 & -J_p\omega & 0 
\end{bmatrix} \tag{1.12} \]

Note that this is the method which was used for this thesis, however, one can use matrices of 8 \times 8 size, which is suggested in some books, for instance [9]. Usually those matrices include both nodes which overlap each other, and therefore they are always multiplied by 1/2. Matrix 1.12 is for a disc on one node, where \( \omega = \frac{n2\pi}{60} \) is the driving frequency with \( n \) the rotation speed in rpm. The \( J_p = \frac{1}{2}m_dR^2 \) is the polar mass moment of inertia, where \( m_d \) the mass of the disc and \( R \) the radius.

The last matrix is the \( M_d \) matrix that contains information about the disc. The disc mass matrix is developed to describe only where the disc is positioned,

\[ M_d = \begin{bmatrix}
m_d & 0 & 0 & 0 \\
0 & m_d & 0 & 0 \\
0 & 0 & J_d & 0 \\
0 & 0 & 0 & J_d 
\end{bmatrix} \tag{1.13} \]

where, \( m_d \) is the disc mass, \( J_d = 2\left[\frac{mR^2}{8} + \frac{m}{6} \left(\frac{t}{2}\right)^2\right] \) the mass moment of inertia and \( t \) the width of the disc.

### 1.1.3 Bearing matrices

If a rotor boundary conditions are flexible, then the bearing conditions can be added to the node as a damping and stiffness matrix. The node stiffness matrix due to concentrated bearing properties is,
\[ K_{\text{bearing}}^{(n)} = \begin{bmatrix} K_{xx} & K_{xy} & 0 & 0 \\ K_{yx} & K_{yy} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \]  

(1.14)

and the node damping matrix due to the damping of the bearing is

\[ C_{\text{bearing}}^{(n)} = \begin{bmatrix} C_{xx} & C_{xy} & 0 & 0 \\ C_{yx} & C_{yy} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \]  

(1.15)

These are the matrices which can increase the sensitivity of a system, since they take into account the journal bearing motion. Including bearing conditions to a system, will give lower frequency. The final matrices now will be,

\[
\begin{align*}
M_f &= M_r + M_e \\
C_f &= C_r \\
K_f &= M_r + K_r
\end{align*}
\]  

(1.16)

where, \( M_r \)\&\( K_r \) are the rotor’s element mass and stiffness matrix respectively (equation 1.11 and 1.10), and \( M_r, C_r \& K_r \) are the matrices for disc mass, bearing damping and bearing stiffness matrix respectively. The \( M_r, C_r \& K_r \) are the matrices in which are placed the \( M_d \) (1.13), \( C_d \) (1.12), \( C_{\text{bearing}} \) (1.15) and \( K_{\text{bearing}} \) matrix (1.14) which are of \( 4 \times 4 \) size. In the following sections the assembly process is described to develop a system.

### 1.2 Assembly process

**Introduction**

From the theory is known that a model can be assembled in different ways. The present study is based on Finite Element Method and in the following sections the process is described in detail.

#### 1.2.1 System matrix assembly

According to Finite Element Method, a beam or a rotor is considered to be a system that can be easily described with one or more elements. It is obvious that more elements results in more realistic solutions. Nevertheless, it is not always suitable to use extremely high number of elements due to time consuming computations.
The system matrix assembly can be developed with elements that overlap each other at the common nodes, as indicated in Figure 1.2. To begin with, a beam can be divided in smaller elements that are connected at their ends(nodes). Each element consists of two nodes($n_1$ and $n_2$) and the element that connects those two. In this case each element is a matrix. From the figure is obvious that four elements are coupled in the assembled matrix(Element 1,2,3 and 4). The symbols $\delta_1, \delta_2, \delta_3$ and $\delta_4$, are the vectors for each element and in the present case they represent the displacement vectors. At the connection point the second node $n_{12}$ of the first element must to be linked with the first node $n_{21}$ of the second element. The same procedure will be repeated for all nodes.

![Figure 1.2: Overlap matrices - Beam matrix.](image)

The final result is a matrix with a size related to the number of assembled elements. The connected parts can be presented as matrices that overlap at the common nodes in order to make a system that interacts physically, (A,B and C are the overlap parts). Regarding the previous sections and the introduced matrices, each node consist of four quantities ($\chi_i, y_i, \varphi_i, \Theta_i$). Thus, the matrix in Figure 1.2 will be of $20 \times 20$ size.

### 1.2.2 Assembly of the rigid disc

There are different ways to include an additional inertia matrix in rotor dynamics. One simple way is to consider mass and inertia of the disc as nodal properties and just include them to one ($4 \times 4$ matrices) or two nodes ($8 \times 8$ matrices). One node gives a point property and two nodes are used when the thickness is included [9]. Inertia properties can also be included with consistent matrices as in [9].

Figure 1.3 shows how a $4 \times 4$ disc matrix is placed in the global matrix. This kind of matrix is diagonal and the imaginary dash lines in the figure split the main matrix in submatrices of $4 \times 4$. Since the node on which the disc is attached is known, a disc matrix $4 \times 4$ can be placed on the diagonal of the matrix size[$N$], at the position which represents the node where the disc
is attached. The $4 \times 4$ matrix used in this report is similar to 1.13. In this case the disc is at the second node, and as a result the disc matrix is placed at the second diagonal dashed square on the global matrix (matrix area: $m_{55} \div m_{88}$). In the figure, the left matrix represents $M^e_1$ where $(1),(2),(3)$ and(4) are overlap matrices similar to mass matrix (1.11), and the right the $M_r$ with $M_d(1.13)$ placed at the second diagonal dashed square. According to equation (1.16) these two matrices can be added together, in order to get the final matrix $M_f$.

1.2.3 Boundary conditions

Boundary conditions are important for the final results. From now on, boundary conditions will be noted as BC. If the BC can be determined with good accuracy, then a mathematical model will be able to provide more realistic results.

To be able to determine BC it is important to get some information about the system. To become more specific, information about the bearings and the general environment in which a rotor operates. For instance, what kind of material do the bearings have and which properties, oil or any other kind of lubrication between the contact surfaces and so forth. On the other hand, if the support points could be considered stiff, then they can be discarded from the global matrix, since the final displacement $\Delta \chi = 0$.

To give a better understanding about the BC, a rotor with $n$ elements is shown in Figure 1.4. The previously defined mass matrix is taken as an example. The rotor is supported with bearings that enable rotation about $x$ and $y$ axis. Here it has to be mentioned, that after a laboratory control, the bearings of Rotor Kit allow the rotor operating in slight rotations about $x$ and $y$ axis, as indicated in Figure 1.5.a. In the case of Figure 1.4 it is obvious (if the support is considered to be very stiff) that the rotor is not
able to move in x and y direction at the 1st and nth node. Thus, the columns and rows of the matrix that represent those nodes in x and y direction have to be discarded or replaced with zero(0). In the figure those columns and rows are shown with BC lines.

There are some cases where a deeper study is necessary and the behavior of the bearings might be of a big interest. In this case the nodes that represent the BC of the system should not be discarded, but in a contrary a damper-spring system shall be added at the main system that can describe the motion of the support. Thus, it is evident that if the same system is supported with a damper-spring system, the values for the natural frequencies would be lower in relation with a rigidly supported system.

Figure 1.5: Cross correlation of a bearing with a damper-spring system

Figure 1.5.a shows an element whose left node is located in a slipping-bearing. These couples of damper-spring system in x and y direction, rep-
resent the system’s BC. Further more, Figure 1.5.b shows how the damper-
spring system of a BC can be described. It is known from Rotor Dynamics,
that any friction inside a system develops damping forces \[9\]. Detail A shows
what occurs at the BC while a system is operating. While the system is op-
erating, bowing of the rotor occurs due to grown vibrations. Thus bearing
surface is under pressure \(p\) and that grows friction forces \(F_f\). A simple way
to determine the spring stiffness is by using Hooke’s law, as:

\[ Kx = F \leftrightarrow \sigma A = F \]  

(1.17)

thus, the spring stiffness can be easily found by applying a known force \(F\)
on the bearing and measuring the displacement \(\Delta x\).

In Detail B is shown the bearing’s behavior under pressure of an \(F\) force.
Moreover, similar phenomena occur at the attached points. Attached point
is called any point along the span on which a rigid disc or other object is
mounted.

1.3 The general eigenvalue problem

The general eigenvalue problem is a useful method for problems that in-
clude damping. This method is easy to use compared with other methods
that can be used on complex systems. Furthermore, the general eigenvalue
method provides more information about a system’s behavior. With the
development of programs, this method is today frequently more suitable for
very complex problems. Once the eigenvalues are solved one can directly
get information about natural frequency, damped natural frequency and the
damping ratio.

1.3.1 General eigenvalues

From the classic dynamics is known that a system can be defined from the
Second Law of Newton. According to the 2nd law of Newton, a system can
be described as,

\[ M\ddot{x} + C\dot{x} + Kx = 0 \]  

(1.18)

where, \(M\) is the mass matrix, \(C\) the damping matrix which can be replaced
with \(D = C + \omega J_p = C_f\), where \(J_p\) is the damping matrix due to gyroscopic
effect, \(K\) the stiffness matrix, and \(\ddot{x}, \dot{x} & x\) the vectors for the acceleration,
velocity and displacement respectively. If they state vectors are defined as
\(\dot{y} = (\ddot{x} \ \dot{x})'\) and \(y = (\dot{x} \ x)'\), then the equation 1.18 according to [10] can be
written as,

\[
\begin{bmatrix}
-K & 0 \\
0 & M
\end{bmatrix} \dot{y} + 
\begin{bmatrix}
0 & K \\
K & C
\end{bmatrix} y = 0
\]  

(1.19)
or

\[
\begin{bmatrix}
-I & 0 \\
0 & M
\end{bmatrix}
\dot{y} + \begin{bmatrix}
0 & I \\
K & C
\end{bmatrix} y = 0
\]

where, \( I \) is the unit diagonal matrix and \( 0 \) zero matrix. Assuming that the first matrix is \(-S\) and the second \(R\), the equation 1.19 becomes,

\[-Sy + Ry = 0 \quad (1.20)\]

Assuming a solution as \( y = CYe^{\lambda t} \), then the problem becomes an eigenvalue problem, and the equation 1.20 can be turned into,

\[
[R - \lambda S]Y = 0,

[A - \lambda I]Y = 0, \quad (1.21)
\]

or

\[
\begin{bmatrix}
R^{-1}S - \frac{1}{\lambda}I
\end{bmatrix} Y = 0
\]

The eigenvalues \( \lambda \) can now be determined from the equations 1.21, since it is known that \( A = S^{-1}R \). With the use of a mathematical program, the eigenvalues can be extracted directly from \( A \). In rotordynamics the most interesting is to plot the imaginary part of the eigenvalues to get the Campbell diagram, see Figure 1.6. Since, the eigenvalues are known the \( Y \) can be easily determined.

\[
y_h(t) = \sum_{i=1}^{n} C_i Y_i e^{\lambda_i t} \quad (1.22)
\]

The \( C_i \) can be solved for known initial conditions with a mathematical program. If \( E \) is the matrix with the eigenvectors,

\[
E = [Y_1, Y_2, ..., Y_n] \quad (1.23)
\]

then the constants can be determined from the following equation:

\[
C = E^{-1}y_h(0) \quad (1.24)
\]

Having the solved constants from 1.24, in equation 1.23 the motion of a unforced system can be analysed.
1.3.2 Campbell diagram & speeds

Since the main topic deals with rotor dynamics, it is important to mention about the Campbell diagram. As shown in Figure 1.6, once the eigenvalues are solved, then the imaginary part can be plotted in order to see how the gyroscopic effect acts on the rotor. Moreover, Campbell diagram shows the resonances of a system, either these are operating forward or backward.

To begin with, Campbell diagram includes, two straight lines $\omega$ which are the driving frequencies for forward and backward direction, and several lines $\omega_{n+1}, \ldots, \omega_{n+6}$ whose intersection with the driving frequency (dash lines) shows the resonance. The curves $\omega_{n+1}, \omega_{n+2}, \omega_{n+3}$ are positive which indicates that the whirl has the same direction as the angular velocity of the rotor, and is called **Forward precession** [9]. The curves $\omega_{n+4}, \omega_{n+5}$ & $\omega_{n+6}$ are negative, which indicates that the whirl has opposite direction related to the rotor’s, and is called **Backward precession** [9]. At the right of the same figure are the mode shapes ($W_i$) of the shaft for backward and forward frequency.

In Figure 1.6, is the Campbell diagram of a Jeffcott rotor. The disc is placed at the midspan of the shaft and as it can be seen from the diagram there is no gyroscopic effect at the first resonance speed. The eigenfrequencies $\omega_{n+3}$ and $\omega_{n+4}$ are just straight lines with the same value for forward and backward precession.

1.3.3 Particular solution

The general eigenvalue problem gives solution for systems without external force. This force can be a small unbalanced mass that develops centrifugal forces. Those centrifugal forces increase the systems amplitude, especially when a rotor operates close to resonances. Note that, if one is aiming to
exceed the critical speed (which in fact is the most crucial), the rotor should accelerate fast through this critical area. According to [10] for particular solution the 2nd law of Newton becomes,

\[ M \ddot{x} + D \dot{x} + K x = f_s \sin(\omega t) + f_c \cos(\omega t) \quad (1.25) \]

where, the two vectors \( f_s \) and \( f_c \) are forces. According to the theory [10], a harmonic input on a linear system can only result in a harmonic output of equal frequency, and therefore it can be assumed that,

\[ x = a \sin(\omega t) + b \cos(\omega t) = as + bc \quad (1.26) \]

where, \( a \) and \( b \) are real vectors. Thus, the equation of motion becomes,

\[ [K - \omega^2 M][as + bc] + \omega D[ac - bs] = f_s s + f_c c \quad (1.27) \]

By separating the equation 1.27 in \( s \) and \( c \) term, it becomes two equations,

\[
\begin{align*}
[K - \omega^2 M]a - \omega Db &= f_s \\
[K - \omega^2 M]b - \omega Da &= f_c 
\end{align*}
\quad (1.28)
\]

To simplify the equations 1.28, it is assumed that \( \tilde{K} = [K - \omega^2 M] \), so the equation becomes,

\[
\begin{align*}
\tilde{K} a - \omega Db &= f_s \\
\tilde{K} b - \omega Da &= f_c 
\end{align*}
\quad (1.29)
\]

Solving the equations 1.29 by the terms \( a \) and \( b \), the problem is solved.

\[
\begin{align*}
\hat{b} &= \left[ \tilde{K} + \omega^2 D \tilde{K}^{-1} D \right]^{-1} \tilde{K} f_c - \omega D \tilde{K} f_s \\
\hat{a} &= \tilde{K}^{-1} f_s + \omega \tilde{K}^{-1} Db
\end{align*}
\quad (1.30)
\]

Note that the \( \omega \) is the driving frequency of the system. In addition, this a method that assumes that the system includes damping that is not equal to zeros, \( D \neq 0 \). The amplitude can be found from the resultant of \( a \) and \( b \), and if \( R \) is the amplitude then,

\[ R = \sqrt{a^2 + b^2} \quad (1.31) \]

If the \( R \) is plotted, then the resonance frequencies can be found for actual forcing.
Chapter 2

Rotor Kit-Bently Nevada

2.1 Introduction

Bently Nevada Rotor Kit RK-4, is a rotating machine that can be adapted in different configurations and is suitable for educational purposes. The Rotor Kit comes along with different accessories that can provide different options and a wide number of experimental tasks.

- Rotor speed
- Shaft bow
- Rotor stiffness
- Amount and angle of unbalance
- Shaft rub or hitting condition
- Rotor-bearing relationships

The user is able to perform many different experiments and compare the final results with a theoretical model. The advantage of this experimental equipment, is that one can verify that theoretical calculations give realistic values, or values that are close to the real ones. Moreover, this is one way to verify that assumptions in a theoretical model are valid.

2.2 Rotor Kit

The rotor kit is shown in Figure 2.1. In the mounting blocks probes are used to measure the displacement of the rotor at different positions of the shaft. All the outputs will be displayed by a simple program in LabView or an oscillator. It is important that the rotor is set up on a rigid base to avoid disturbances that can influence the measurements.

Figure 2.1 shows one of the numerous setups that can be achieved with this equipment. Two disc are available that can be placed in any position on the rotor. Further more, constant forces and unbalanced masses can easily
added to develop a more complicate system. Two interesting options that can be applied are, *Oil Whirl/Whip option* and *Perturbator option*, Figure 2.2.

![Figure 2.1: Rotor Kit](image)

2.2.1 Oil whirl & whip

The oil whirl/whip option, Figure 2.2.a, can be applied by using the special equipment Oil Whirl/Whip kit. The Oil kit develops a special environment for the rotor by operating in an oil reservoir. Due to the transparent material it is possible to observe the oilfilm during the experiments. For this option a preload frame can be added which can remove the effect of gravity and position the journal at a desire eccentricity. With this equipment both oil whirl and oil whip can be studied. Whirl can be developed or prevented by applying preload in any direction. The preload can be regulated by changing the oil flow or pressure(for further information look [11]).
2.2.2 Perturbator

The perturbator option, Figure 2.2.b, includes a perturbator disc which accomplishes asynchronous rotation of an unbalance in both forward and backward direction. A second motor rotates the perturbator disc in different speeds independently of the rotor speed. In addition, a weight can be attached on the perturbator disc, in order to develop a constant force that is pulling the rotor towards one direction.

In appendix D can be found some experimental suggestions for all of three options above.

2.3 Probes & calibration

![Figure 2.3: Proximity probes-Standard Mount](image)

There are eight inductive probes included in the rotor kit which are of high accuracy and work with DC voltage. The eight probes that constitute the system are: 2 for measuring the speed, 2 for getting the pulses and 4 for measuring the displacement of the rotor at different positions along the shaft. The probes which are measuring the displacement can be aligned in x and y direction, or even in an angle of 45°. A probe is consisted from the parts shown in Figure 2.3.

The part No.1 is the probe tip that collects all the information directly from the shaft surface (More information about the probes in Appendix B). The probe reacts to a displacement with a voltage. Therefore, this voltage has to be translated in (mm). Thus, according to Table 2.1 which describes the correlation between voltage and displacement, each output can be transformed directly in (mm).

Table 2.1 contains the values that describe the correlation between the voltage and displacement. In fact, more detailed measurements should be done at the laboratory for shorter magnitudes, in order to get the smallest possible error. Interesting to note is that the probes behaved linearly for amplitudes between 0,25±1,78mm. Thus, the approximation of the displacement could be done by considering a linear behavior of the probes. A polyfit
Table 2.1: Calibration Table. Each probe was calibrated against the shaft and the bearing. The ordering 1,2,3 & 4 is based on the input canals of the monitor for getting the measurements.

<table>
<thead>
<tr>
<th>Distance (mm)</th>
<th>Probe-Shaft</th>
<th>Probe-Bearing</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>No.1</td>
<td>No.2</td>
</tr>
<tr>
<td>0.25</td>
<td>2.90</td>
<td>2.25</td>
</tr>
<tr>
<td>0.51</td>
<td>4.97</td>
<td>4.14</td>
</tr>
<tr>
<td>0.76</td>
<td>7.09</td>
<td>6.23</td>
</tr>
<tr>
<td>1.02</td>
<td>9.26</td>
<td>8.39</td>
</tr>
<tr>
<td>1.27</td>
<td>11.30</td>
<td>10.41</td>
</tr>
<tr>
<td>1.52</td>
<td>13.14</td>
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<td>1.78</td>
<td>14.30</td>
<td>13.70</td>
</tr>
<tr>
<td>2.00</td>
<td>14.39</td>
<td>14.35</td>
</tr>
<tr>
<td>2.25</td>
<td>14.45</td>
<td>14.43</td>
</tr>
<tr>
<td>2.50</td>
<td>14.48</td>
<td>14.47</td>
</tr>
</tbody>
</table>

Polyfit line can be used to describe the displacements from the measured voltage of each probe. Polyfit line can be easily calculate by using Matlab. Matlab finds the coefficients of a polynomial \( p(x) = p_1x^n + p_2x^{n-1} + \ldots + p_{n-1}x + p_n \) of n degree that fits the data from \( p(x(i)) \) to \( y(i) \). This method approaches the least error or the line that fits best through the measuring points. Furthermore, it has to be mentioned about mils or mil, which is a length unit equal to 0.001 inch (0.0254 mm) or milli-inch. Mils are used primarily in the U.S to express small distances and tolerances. One mil is equal to 25.4 microns.

In addition, the probes were calibrated against different materials to observe how it affects the accuracy and compared with the rotor’s (material). Note, that Rotor Kit RK4 comes along with certification (Final Test Results) of each probe. This certification shows the correlation between the voltage and displacement, and the potential error given by the same probe at different gaps for a polyfit line. In Table 2.1 lists the values that are interpolated in order to get a characteristic curve from which the program will calculate the right signals. By generating a polyfit line through the points 0.25 ÷ 1.78 mm for each probe, then the error will be according to Figure 2.4.

Notification: Figure 2.4 shows the error for a polyfit line. The reason for this study is to make a comparison between the experimental and factorial error. In fact the program that treats the raw data, is relying on a calibration table, Table 2.1, that develops interpolation between those points, from 0.25 ÷ 2.50 mm. Thus, the error is even less than the one shown in Figure 2.4.

In this figure the error is shown which affects the accuracy of the measurements. From Figure 2.4 can be observed the calibration curve from Bently Nevada does not completely match to the Lab error.

To begin with, Rotor Kit RK4 has been manufactured in U.S and one potential reason is that the calibration was done with different voltage (110 ÷ 120 Volts at 60 Hertz). Thus, it is unknown whether or not, the very same calibration equipment should give the same error for the probes in a higher voltage, similar to European (220 ÷ 230 volts at 50 Hertz). Moreover, the
Figure 2.4: Final Test Results. *Comparison of the error taken directly at the lab with Bently Nevada results.*
calibration process was accomplished with different equipment than the one used by Bently Nevada. The calibrator used for this purpose is described in Appendix C. At the lab was marked that the probes gave a characteristic line that was parallel to Bently Nevada’s, but slightly higher along the voltage axis. One additional reason can be the environmental temperature and the wiring from the output to voltmeter. Finally, one reason can be that the calibration was done against another material.
Chapter 3

Experimental & theoretical results

The present chapter compares experimental and theoretical results. The purpose is to scale down the problem before it will be adapted in real dimensions. The experiments which will be explained in this section, were accomplished with Rotor Kit RK4 Bently Nevada. Furthermore, discussion will take place for each experiment to evaluate theoretical models which were developed in order to describe the same experimental systems.

3.1 Rotor effect of a disc mass & perturbation

This task shows how a system responds, while a perturbator attached on the shaft rolling back or forth. In fact, perturbator is an independent part of the system, whose main purpose is the determination of the system’s dynamic stiffness [12] related only to the rotational velocity. In this case a system has constant speed and the perturbator is running gradually. Furthermore, this is one method to identify potential lower modes of the system that were muted while using synchronous perturbation for unknown reasons.

Last, according to [12] the given information of a nonsynchronous perturbation, in comparison always with a synchronous perturbation, are more fruitful since, as it was mentioned above, a synchronous perturbation is affected by the very same system as a result of information that can not be observable.

3.1.1 Jeffcott rotor

This subsection describes the experimental result of a Jeffcott Rotor and compares with the theoretical model of the same system. Note that, Jeffcott rotor is called every system with the disc mass placed at the midspan of the rotating shaft. To begin with, the demonstration was executed with a disc
mass at the midspan of the shaft and the perturbator disc very close to its inboard side. Note that, the inboard side is the side with the motor and outboard where the shaft ends.

![Experimental results](image)

Figure 3.1: Experimental results. Amplitudes for forward & backward perturbation. Orbits for both cases as they were conceived during the experiment.

Parameters of experiment: shaft length $L=0.48\text{m}$, shaft diameter $d=0.01\text{m}$, density $\rho = 7850\text{kg/m}^3$, Young’s modulus $E = 206 \times 10^9 \text{Pa}$, unbalance mass $m_u = 0.0004\text{kg}$, unbalance distance from the shaft center $u=0.03\text{m}$, disc mass $m_d = 0.80630\text{kg}$, bearing block stiffness $k_b = 5.75 \times 10^5 \text{N/m}$.

According to the theory, Jeftott Rotor should have equal first critical
speed for forward and backward rotation. As it can be seen from Figure 3.1, indeed the first critical speed matches for both cases with a very little divergence, forward 1930rpm and backward 1921rpm. Thus, for both cases the first critical speed is approximately \( \omega \approx 203 \text{rad/s} \). The first natural frequency is very obvious, due to the fact that suddenly a multiperiodic solution becomes a large orbit of single frequency as shown in the figure. In this phase the orbits have the highest amplitude, and therefore, rotating machines should pass rapidly through this area because the consequences can be severe.

These experiments as can be seen from the graphs, were executed for speeds 200rpm-3000rpm. However, interesting was the behavior of the shaft when it was rapidly accelerated above 5000rpm for a very short period. The shaft behaves as almost a perfect balanced system in velocities above the first natural frequency, and the orbit gets smaller and smaller until the next critical speed. Furthermore, even if the shaft changes amplitude, it is whirling round the same axial line.

![Graph showing critical speeds](image)

**Figure 3.2:** Matlab results. a. Simple model of Rotor Kit, b. including bearing system & c. including the extent part after the bearings

Regarding the theory and the mathematical model developed for this purpose, in Figure 3.2 are indicated the results of three different mathe-
Table 3.1: Jeffcott Rotor results

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental results</td>
<td>203(rad/s)</td>
</tr>
<tr>
<td>Simple model(Matlab)</td>
<td>215(rad/s)</td>
</tr>
<tr>
<td>Simple model &amp; bearings</td>
<td>210.6(rad/s)</td>
</tr>
<tr>
<td>Entire shaft &amp; bearings</td>
<td>210.5(rad/s)</td>
</tr>
</tbody>
</table>

Mathematical codes. To begin with, the campbell diagram in figure a, describes a simple model of Rotor Kit constituted by a shaft and a disc (bearing stiffness equal to infinity). The final result of this model is 215rad/s. The second campbell diagram in figure b, shows a model that includes bearing system with the first natural frequency being at 210.6rad/s. The last diagram in figure c, gives slightly lower value, 210.5rad/s, for the first natural frequency and the mathematical model includes the extent parts after both bearings.

Thus, relying on Table 3.1, it can be identified that a simple code model can give good results, but a more detailed can provide more precise information. The approach is very good, even though a small divergence of approximately 7rad/s still exists. Unfortunately, it was impossible to reach the second frequency, since the system’s parameters where such that it was above the speed limit of the experimental facility(10000rpm).

Orbits & shapes

An interesting observation during the experiments is the difference in the orbits shape during different speeds.

Figure 3.3: Orbits in double perturbation speed. Forward perturbation to the left & backward perturbation to the right

In Figure 3.3 the orbits for both cases can be seen while the perturbator is running almost with double speed (constant:1000rpm-perturbation:1990rpm). It is obvious why that happens. A system needs some time to create an orbit circle, similarly the perturbator disc needs the same time. Nevertheless,
in this case the perturbator is running with double speed, and thus, in the same time is able to achieve two orbit circles.

To give a better understanding, in Figure 3.4 it can be seen how two independent forces are moving. In figure a, is the case of forward perturbation. Both forces are moving at the same direction and they will be met at the aligned point at a side. At that point grows the highest excitation \( F_{\text{max}} = F_s + F_p \), where \( F_s \) and \( F_p \) are the system and perturbation force respectively. Since the perturbation speed is double, while \( F_s \) will be at the b side of the aligned point, the \( F_p \) will be exactly at the opposite side(side a), thus at this instant on the system acts the minimum force, \( F_{\text{min}} = F_s - F_p \). It can be identified that the system will have two orbits with different amplitudes as it is shown in Figure 3.3 forward.

If it is analyzed the orbit of Figure 3.3 backward, then from Figure 3.4.b can be seen that with backward perturbation, \( F_s \) and \( F_p \) will be aligned six times for every single rotation of the system, three \( F_{\text{max}} \) and three \( F_{\text{min}} \). In Figure 3.4.b the positive aligned points that abstain 120° to each other are shown. Now, it is obvious why three peaks grow in backward perturbation with double perturbation speed. For more information about orbits in different speeds check Appendix E.1. and E.2.

### 3.1.2 Disc mass at 2/3 of the span

The second experimental task is to displace the mass disc at the 2/3 of the span. The perturbation disc is as before very close to the mass disc side towards the inboard bearing. All the experimental parameters for this experiment are the same with the experiment of Jeffcott rotor.
Figure 3.5: Experimental results. Amplitudes for forward & backward perturbation at 2/3 of the span. Orbits for both cases as they were conceived during the experiment.
From the theory and since the mass is displaced, it is expected to have two different first critical speeds for back and forth perturbation. From Figure 3.5 it is obvious that both cases give different critical speeds. Furthermore, the experimental result shows higher speed for forward perturbation, as it was expected. Thus, the first critical speed for forward perturbation is $\omega \approx 223 \text{rad/s}$ and backward $\omega \approx 222 \text{rad/s}$. In addition, it can be seen from the figure how the orbits in both cases become. At this point the orbits get the highest amplitude, therefore, as it was mentioned in the previous section all rotating machines should pass as fast as possible through the first critical area.

These experiments were executed for speeds 200rpm-3000rpm. However, it can be seen that the orbits are getting smaller above 5000rpm. That happens due to the fact that the rotor gets stiffer while running in speeds much higher the first natural frequency. Furthermore, the shaft always moving in ellipsoid orbits (whirling) round the same imaginary axis even if the shaft changes amplitude.

![Graph](image)

Figure 3.6: Matlab results. a. Simple model of Rotor Kit, b. including bearing system & c. including the extent part after the bearings

Adapting the matlab code for mass disc at 2/3 of the span, the outcome will be Figure 3.6. As before, in figure a stands a simple model of the
very same system without any bearing system, with forward frequency at 238rad/s and backward at 237rad/s. The second model includes bearing system and obviously lower results of 231.5rad/s for forward and 231.0rad/s for backward frequency. The last code shows a model that describes the entire system, including the extent parts after both bearings, with forward frequency at 231.4rad/s and backward at 230.8rad/s. Thus, the results will be as shown in Table 3.2.

Having now all the results, Table 3.2, it is understandable that including more components that affect the entire system, the theoretical results can be closer to the experimental ones. By including the entire shaft and the bearing system, it was achieved to reduce the difference from 15rad/s to 7rad/s. As it can be seen from Figure 3.6 the second critical speed is quite high, therefore, it was impossible to reach it.

**Orbits & shapes**

Similarly with the previous section, here it can be seen how the orbit becomes when the perturbation speed reach double value in comparison with the system’s speed.

In Figure 3.7 is shown orbits similar of Figure 3.3, and it is known hence-
forth why the orbit change in these modes. Something interesting to be mentioned, is that for speed multiple to the constant(three, four, five,... times), these orbits will get proportionally one more sub-orbit. For instance, the orbits in the figure will get, one more sub-orbit for perturbation speed 3000rpm, or two more sub-orbits for perturbation speed 4000rpm and so forth.

3.2 Journal bearing

Journal bearing is an option for observing different phenomena that occurs while a shaft operates in an oil environment. Oil grows forces on a journal surface in erratic orbits. In the following subsections is shown those phenomena through different tasks by using Rotor Kit RK4 equipment.

3.2.1 Oil wedge force

In Figure 3.8.a is shown the journal in a slow roll speed with only the gravity acting on it. In addition, the preload frame applies a very small load on the shaft. The rotor rolls counter clockwise and the oil wedge displaces the shaft from point a to point b.

In Figure 3.8.b the journal is placed at the center of bearing clearance, point a. In this case the rotor is in a complete immobility. If now a force $F_1$ and $F_2$ applied in x and y direction respectively, the journal will be displaced along the same directions. As indicated in the same figure, the center of journal is moving for point a to point b under the force $F_1$, and back at a after the removal of the force. Similarly in x direction, from point a to point c.

Thus, it is obvious that the journal is moving almost parallel to any force acting on it, due to the fact that no fluid wedge force exist.

If now speed up the shaft till 1500 rpm and try to apply a vertical force F as in Figure 3.8.c the new position of the journal will be different than the
expected one. Instead, the journal is moving in a new direction which is the resultant of force $F$ and fluid wedge forces, point $e$. Thus, fluid forces can be easily verified, since the applied force and the new direction is known.

3.3 Whirl & whip

Whirl and whip are two phenomena which are of big importance in rotor dynamics. The experimental task of Figure 3.9, shows the results of Journal-Bearing demonstration on Rotor Kit RK4 Bently Nevada. The task was fulfilled in 6psi oil pressure and angular velocities from 30rpm to 3000rpm. During this task main purpose was to verify all which are known from the theory.

In the figure can be seen the journal and midspan orbits. At the first graph is shown the shaft operating at 700rpm. The journal remains almost at the center of the bearing, with the midspan moving softly in a small orbit. The oil force is enough to keep the journal align with the bearing.

Increasing the speed gradually at 1340rpm, the shaft starts whirling. The midspan main orbit has become wider with quasiperiodic orbits. At that point it was very easy to get back the initial stability, with only a small tension in one of the preload springs. Increasing now the angular speed above
1340rpm (which in fact is where the area of critical speed starts) and much higher of it (f.e 3000rpm) to make sure that the rotor is whipping, even the double tension was not able to stabilize the shaft in smooth operation. From the picture can be seen how the orbit became at 3000rpm. In conclusion, it is more difficult to make the instability to go away while the rotor is whipping.

Something that should be done comprehensible, is the difference between whirl and whip. Whirl can take place sometimes even at low speeds, and its frequency is normally the half running speed (the oil film speed less than 50% of the journal surface speed, [12]). If this frequency coincides the natural frequency, then the oil whip occurs. Something remarkable is that whip locks the shaft at the same orbit due to the high frequency, which is obvious from the experiments done with Rotor Kit. In addition, the fluid stiffness is increasing due to lower circumferential velocity [11].

3.3.1 Oil pressure control

Oil pressure is very important to find out when the oil whirl/whip will take place. The oil pressure determines the size of concentrated radial forces which direct the journal.

In this experiment main purpose was to make an instability to go away by increasing the oil pressure. With rolling speed at 4280rpm and 4.5psi, as it can be seen from Figure 3.10 the journal and midspan starts whipping. The rolling speed is enough higher of the first natural frequency to unsure that the shaft is whipping. In the same figure now can be observed that the instability which whipping the shaft, went away by increasing the oil pressure from 4.5psi to 13.5psi.

Higher oil pressure increases the fluid film radial stiffness. Thus, the journal can be stabilized and directed to the center of the bearing. In fact, the oil pressure is high enough to overcome the dynamic forces of the journal and obtain higher circumferential velocity. Circumferential velocity usually $0.42 \div 0.48 \times$ running speed, [13], makes a journal almost to whirl.

3.3.2 Journal bearing whirl-whip phenomena

An interesting part of detecting the behavioral progress of a running journal in an oil bearing, is to observe how the midspan and journal orbits gradually change and how the journal is being aligned to the bearing due to growing dynamical forces. In this section will be shown the whip effect of an experiment done with Rotor Kit RK4. This and all the previous tasks were executed according to arrangement Appendix D.4.

As it can be seen from Figure 3.11, the journal operated in 5.5psi for velocities from 200rpm to 2200rpm. From both graphs is obvious that the journal is not perfectly aligned to the bearing (it is always difficult to direct
Figure 3.10: Behavior of the shaft by changing oil pressure. *Shaft whipping* in 4.5psi, left. *Shaft align with the bearing* in 13.5psi, right.

Figure 3.11: Amplitude graph for x and y direction. *The task was demonstrated* in 5.5psi from 200rpm to 2200rpm.
a rolling journal at the right position.), thus, the journal is running in an eccentric position inside the bearing.

To begin with, each graph can be divided in three areas. The first area a, as it can be seen from the graph, is the area where the journal is being aligned gradually as the angular velocity is increasing. The journal increases the oil speed as well as the oil stiffness [11], as a result of changing position in a new equilibrium area inside the bearing.

The second area b, is the area of oil whirl. The journal starts whirling and rapidly is placed in higher orbit similar to the bearing’s clearance. In this area any instability can be easily disappeared with the application of only a small force, since the dynamic forces of the journal are still weak.

The last area c, is where the journal starts whipping. In fact, whip will lock whirl’s frequency and maintain it while operating into higher and higher speeds [13]. According to [11] the shaft is in a very high position from the bearing center, therefore, a whipping shaft demands more force to stabilize its orbit.

As can be seen from the figure, the axial line is the imaginary bearing center, and the dash lines the bearing clearance. It is obvious that the journal will never exceed to have contact with the bearing surface due to oil wedge. It should be perfect lubrication, in order to avoid contact between the journal and the bearing.

Similarly to Figure 3.11, in Figure 3.12 one can see how the midspan of the shaft is moving in relation with the journal. According the figure, it is seen the different positions and orbits the shaft gets during the experiment. In 200rpm the shaft is almost aligned with the journal obviously in an eccentric position lower from the imaginary center. In 900rpm, with the midspan almost at the same position, the journal is directed towards the imaginary center in a smaller orbit. It is remarkable this behavior due to the higher oil circumferential velocity. In fact, if it can be achieved to have high circumferential velocity, vibration problems can be avoided according to [14]. That can be carried out by preturbating the oil flow inside the bearing.

As the circumferential velocity is increasing the journal orbit is getting smaller and smaller. That is obvious from the figure with the journal running in 1460rpm. At the next phase the journal starts whirling in 1590rpm. According to [13], any instability that exceeds orbit amplitude more than 50% of the clearance, is considered to be excessive.

The journal now is operating at its first natural frequency, 1620rpm. The journal is whirling in an orbit whose amplitude is growing towards the clearance limits, and the midspan running in erratic orbits. Last, running the shaft in a higher velocity, 2200rpm, the journal is henceforth whipping with orbit amplitude as high as possible.
Figure 3.12: Orbits of the journal and midspan. *Behavior of the shaft for different speeds running counter clockwise.*
Conclusion

The present thesis reports all the experimental tasks done in order to compare and verify the analytical solution of a system. All the experimental tasks were executed by using Rotor Kit RK4. The interesting part is, why theoretical results do not match with the experimental.

First of all, in this thesis perturbation was of main importance and that is why it was probed deep into it. From the experimental results it is seen that all three theoretical model give higher first critical speeds. It is believed that small errors as it can be seen from Table 3.1 and 3.2, can occur. For Jeffcott Rotor the error is: 5,58% for a simple model without bearings, 3,61% for a model that includes bearing stiffness and 3,56% if it will be taken into account the extent parts out of the bearings. Similarly for Disc mass at 2/3 of the span the error is: for a simple model 6,30% forward and 6,33% backward, including the bearing stiffness 3,67% forward and 3,90% backward, and including the extent rotor length out the bearings 3,63% forward and 3,81% backward. It is natural to have some difference between the theoretical and experimental results, since the theoretical method does not take in a consideration surrounding factors that can affect the final results. Besides, including only the bearing properties it can be realized that the error is very little. It is still unknown whether or not the friction at the attach points of the rotor (f.i the place where the disc is attached on the span) could change the physical properties of the rotor at that point, for instance, different stiffness of the rotor and damping due to the friction between the rotor and the disc. Another potential factor for lower experimental first natural frequency, the transportation of vibrations from the motor via the coupling.

Many of the reasons which have already been mentioned are: mistakes during the calibration. For instance, the calibration and the values on which the program relies on, done under different conditions like, calibration environment and temperature. Further more, one more reason is that the chosen bearing stiffness was taken according to [5], however, even if the theoretical model relied on this stiffness it is still not sure whether or not this value is true. Thus, the bearing stiffness might be higher than the real one, and therefore the theoretical model gives higher first critical speed.

In addition, the rotor’s body was considered to be rigid. In fact, all members of a system get some kind of distortion and obviously if someone does not include them in the final calculation, the result will be a stiffer system.

Moreover, it has to be mentioned that Rotor Kit RK4 was adjusted on a wooden bench whose potential weakness transferred vibrations on the main system which met the first natural frequency earlier. The wooden bench add some kind of elasticity on the system, something that was not taken into account at the theoretical models.
Last but not least, it is understandable now that by making a model that describes exactly the real environment and situation in which a system operates, the approach can be closer. This is something difficult to make in reality, since the description of a real system becomes very complicate and makes it difficult for engineers to compute such a model. Thus, from above we conclude that the assumptions gave us higher frequency.

In conclusion, the development of different experimental tasks brought the undergraduate students closer to the topic by raising questions and points that could be included at the present work or a nice suggestion for future research.
Appendix A

Rotor Kit specifications

ELECTRICAL

<table>
<thead>
<tr>
<th>Power Input</th>
<th>95 to 125 Vac, 45 to 65 Hz, 3 Amp, single phase, or 190 to 250 Vac, 45 to 65 Hz, 3 Amp, single phase</th>
</tr>
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</table>

Proximitor

<table>
<thead>
<tr>
<th>Power Output</th>
<th>-18.0 Vdc ± 0.8 Vdc at 140 mA max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Speed</td>
<td>10,000 rpm, typical</td>
</tr>
<tr>
<td>Ramp Rate Control</td>
<td>115,000 rpm/min, typical</td>
</tr>
</tbody>
</table>

MECHANICAL

<table>
<thead>
<tr>
<th>RPM Range</th>
<th>To 10,000 rpm</th>
</tr>
</thead>
</table>

DIMENSIONS

<table>
<thead>
<tr>
<th>Length</th>
<th>780 mm (30.8 in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width</td>
<td>340 mm (13.4 in)</td>
</tr>
<tr>
<td>Height</td>
<td>165 mm (6.5 in)</td>
</tr>
</tbody>
</table>

WEIGHTS

<table>
<thead>
<tr>
<th>Rotor Kit</th>
<th>14.5 kg (32 lbs)</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>Additional Weights</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Procimitor Assembly</td>
<td>0.9 kg (2 lbs)</td>
</tr>
<tr>
<td>Motor Speed Control</td>
<td>3.5 kg (7.8 lbs)</td>
</tr>
<tr>
<td>Oil Whirl Option</td>
<td>7.3 kg (16 lbs)</td>
</tr>
<tr>
<td>Perturbator Option</td>
<td>10.0 kg (22 lbs)</td>
</tr>
</tbody>
</table>

ENVIRONMENTAL CONDITIONS

<table>
<thead>
<tr>
<th>Operational Temperatures</th>
<th>25°C ± 10°C (77°F ± 18°F)</th>
</tr>
</thead>
</table>
Appendix B

Proximity probe

Figure B.1: Proximity probe-Standard Mount

1. Probe tip, 5.26mm (0.207 in) maximum diameter
2. Hexagonal nut
3. Case thread
4. Wrench flats
5. 750 cable, 2.8mm (0.11 in) maximum outside diameter, 7.6mm (0.30 in) maximum outside diameter of armor
6. 3.23mm (0.127 in)
7. Uthreaded length "A"
8. Miniature male coaxial connector, 7.23mm (0.285 in) maximum outside diameter "D"
9. Case length "B"
10. 2.92mm (0.115 in) maximum
11. Total length "C", -30%,-0%

These are many specifications for this type of probe. The environment and other factors that affect the results, are of main importance, and for this reason someone should consult the manual that provides deeper description of Transducer Systems. For those who want to have some further information can visit the following link, http://www.gepower.com/prod_serv/products/oc/en/downloads/147386.pdf.
Appendix C

High magnification extensometer calibrator

Instron Extensometer Calibrator.

Catalogue Numbers:
2602-001
(G-55-1)
2602-004
(G-55-1M)

This instrument is a calibrator of high ration of output to displacement. Accurate and rugged units, this calibrator is adaptable to most types of extensometers, and has a range of 1-inch (25.4mm-metric) units.

Figure C.1: High Magnification Calibration Scales (Inch System).

The High Magnification Extensometer Calibrator, Figure C.1, consists of a large micrometer head mounted on a stand. It has two thumbscrew
couplings for mounting various size and shape adapter spindles supplied with the unit to accommodate most extensometers. The dial on the metric micrometer head indicates directly to 0.002mm and by a four line vernier to 0.0005mm. Its accuracy is ± 0.00038mm at any setting. Figure C.1 shows a vernier based in inch measuring system (The vernier that was used to calibrate the probes was in metric system).

![Figure C.1](image)

Figure C.1: Vernier.

Figure C.2: Calibrator.

In Figure C.2 can be seen the parts that constitute the Calibrator. This instrument was used to ensure the probe accuracy. The probes were adjusted on the mobile part of the calibrator and the shaft was stabilized on the opposite side.
Appendix D

Experimental suggestions

The present section introduces some experimental suggestions from Bently Nevada. Of course there are numerous combinations that someone can accomplish on this equipment in order to achieve a more advanced model. The experimental models should be carried out with all the necessary instructions for a proper environment. **Something very crucial:** be sure that the foundation on which Rotor Kit RK4 is accommodated, is rigid.

D.1 Disc & unbalance mass

![Disc options diagram]

Figure D.1: Disc options: Rotor with, a) two discs, b) one disc at midspan (Jeffcott rotor), c) three discs(two regular and one perturbator disc) & d) one disc at any position.
In Figure D.1 are indicated many disc configurations. As it can be seen in Figure D.1.a, one interesting arrangement is with two discs on the shaft. The discs can be placed in different positions along the shaft. The distances $\alpha$ and $\beta$ can be equal or not. Bently Nevada recommends $\alpha, \beta = 25\%$ in relation with the effective shaft length. In Figure D.1.b the disc is placed at the midspan, something interesting in case someone aims to work and detect phenomena of Jeffcott rotor.

In Figure D.1.c there are three disc which according to Bently Nevada, a good suggestion is to use distance $\alpha, \beta = 25\%$ and $\gamma = 50\%$ in relation with the effective shaft length, or $12\%$ and $50\%$ respectively. The second configuration is more preferable in case the bearings are not very stiff. In the present and all forth coming options an unbalance mass can be placed in order to develop higher excitations in x and y direction. The unbalance mass can be easily placed/threaded on the discs, since each disc brings peripheral holes with thread as it is shown in Figure D.1.c.

The last configuration Figure D.1.d, a disc is placed at any position for further treatment. The distance can be chosen from the executor of the task.

### D.2 Preload condition

Figure D.2: a) Preload frame, b) Preload rod

To develop preload condition can be either applied with a preload frame, Figure D.2.a, or a nylon rod manually, Figure D.2.b. The second is obvious from the figure that is used manually to force the shaft during demonstrating a task. The preload frame is usually used with the oil whirl/whip option. As can be seen from the figure the preload frame has four springs that can apply controlled forces in different directions. The application of a preload
force is interesting task in showing how the shaft position and the orbit shape changes.

**D.3 Rub condition**

This task can be carried out by using a screw made out of copper which can be also called rub screw. The main purpose is to develop rub(friction) by adjusting the rub screw at the Probe Mount until it has surface contact with the shaft.

**D.4 Oil whirl/whip**

![Figure D.3: Journal-Bearing option](image)

Oil whirl/whip option is task that can be achieved by a special shaft which is accommodated in a plastic bearing. The bearing is supplied continuously with oil. Running a task by using the setting up of Figure D.4, the executor can observe different phenomena. Oil whirl which is a self excited vibration because of the oil forces that are growing within bearing. Oil whip which takes place while the oil whirl approaches the first critical speed. The whip will be maintained at the first natural frequency while the shaft’s rotational speed increases. In addition, this option gives the chance to accomplish observations like, oil distribution, oil wedge and so forth. Further information about oil whirl/whip look [11].

**D.5 Perturbator**

Perturbator as can be seen from Figure D.1.c, is a task that can execute nonsynchronous rotation in both directions. In the figure the Perturbator disc(middle disc) is rotating independently by another source. This option can be arranged in different configurations and combined similarly with all the other option mentioned above. This is a task that observes when the system becomes unstable(backward and forward frequencies) with the main system running at a stable velocity.
Appendix E

Perturbation orbits

The following orbits were captured during the experimental task with perturbator disc running in asynchronous rotational speed. The experiments were accomplished for two configuration, one at the midspan (Jeffcott Rotor) and one at the 2/3 of the span. The experiments were executed for rotational speeds: constant speed 1000rpm and perturbator speed 200rpm÷3000rpm.
E.1 Jeffcott Rotor forward

Constant speed: 1000rpm
Perturbator speed: 1000rpm

Constant speed: 1000rpm
Perturbator speed: 1300rpm

Constant speed: 1000rpm
Perturbator speed: 1490rpm

Constant speed: 1000rpm
Perturbator speed: 1660rpm

Constant speed: 1000rpm
Perturbator speed: 1790rpm

Constant speed: 1000rpm
Perturbator speed: 1930rpm

Constant speed: 1000rpm
Perturbator speed: 1990rpm

Constant speed: 1000rpm
Perturbator speed: 2990rpm
E.2 Jeffcott Rotor backward

- Constant speed: 1000 rpm
- Perturbator speed: 1009 rpm

- Constant speed: 1000 rpm
- Perturbator speed: 1326 rpm

- Constant speed: 1000 rpm
- Perturbator speed: 1742 rpm

- Constant speed: 1000 rpm
- Perturbator speed: 1921 rpm

- Constant speed: 1000 rpm
- Perturbator speed: 1990 rpm

- Constant speed: 1000 rpm
- Perturbator speed: 2318 rpm

- Constant speed: 1000 rpm
- Perturbator speed: 2485 rpm

- Constant speed: 1000 rpm
- Perturbator speed: 2980 rpm
E.3 2/3 of the span forward

\begin{align*}
\text{Constant speed:} & \quad 1000\text{rpm} \\
\text{Perturbator speed:} & \quad 1329\text{rpm} \\
\text{Constant speed:} & \quad 1000\text{rpm} \\
\text{Perturbator speed:} & \quad 1656\text{rpm} \\
\text{Constant speed:} & \quad 1000\text{rpm} \\
\text{Perturbator speed:} & \quad 2127\text{rpm} \\
\text{Constant speed:} & \quad 1000\text{rpm} \\
\text{Perturbator speed:} & \quad 2975\text{rpm}
\end{align*}
E.4 2/3 of the span backward
Bibliography


