ORBITAL ENVIRONMENT CONSIDERATIONS DURING THE CLOSE APPROACH PHASE OF MISSIONS TO SMALL BODIES

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ABSTRACT

Missions to small bodies have become increasingly attractive in recent years, firstly, due to their scientific value, but also because of their potential risk to Earth and prospective economic return. A variety of missions have been proposed, ranging from manned exploration to commercial mining missions. There have already been missions to asteroids (e.g. Hayabusa) which brought samples and scientific data, while successor spacecraft are on their way to new targets. For such and future missions, it is essential to perform in-situ observations by landers in order to enhance scientific return. Simple, reliable and low-cost lander modules would satisfy the desired observational capability by exploiting the natural dynamics of these bodies. Therefore, CubeSat systems are good candidates to fulfill the aforementioned exploration demands. This research considers a mission that is targeted to binary asteroid system, which constitute 15% of NEA population. The mission architecture includes a mothership carrying one or several CubeSats. CubeSat deployment is performed by a spring mechanism which is limited for maximum velocity. Natural landing trajectories are investigated after deployment for an unpowered CubeSat within the dynamics of binary system by using the frame of Circular Restricted Three Body Problem (CR3BP). Landing is envisaged in local vertical direction in order to avoid damage to the CubeSat. Dynamical model is propagated backwards from the surface in a novel bisection algorithm to obtain lowest energy trajectories. CR3BP only considers point mass gravity in the model, therefore a perturbation analysis is carried to find out when solar radiation pressure would dominate the evolution of trajectories. The research provides new insights into the regions and sizes of binary systems that could potentially be explored by a simple, underactuated lander with very little control. Suggestions are also made for a CubeSat that could possibly be employed as a lander for small body exploration.

Keywords:

Small body exploration, Binary asteroids, Ballistic landing, Natural trajectories, CubeSat, Circular Restricted Three Body Problem
ACKNOWLEDGEMENTS

Although the cover page contains only my name, this work would not be possible without the help and support of many people around me. My experience during this work was unforgettable and I would like to thank all of them.

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this is going to end one day, and we will be back together again. Your love, patience, support made everything possible during past two years. With all my love, I would like to dedicate this work to you. Jag älskar dig.
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<th>Abbreviation</th>
<th>Full Form</th>
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<tr>
<td>AIDA</td>
<td>Asteroid Impact &amp; Deflection Assessment</td>
</tr>
<tr>
<td>AIM</td>
<td>Asteroid Impact Mission</td>
</tr>
<tr>
<td>BYORP</td>
<td>Binary Yarkovsky-O'Keefe-Radzievski-Paddack Effect</td>
</tr>
<tr>
<td>Bi-CR3BP</td>
<td>Bi-Circular Restricted Three Body Problem</td>
</tr>
<tr>
<td>CR3BP</td>
<td>Circular Restricted Three Body Problem</td>
</tr>
<tr>
<td>DART</td>
<td>Double Asteroid Redirection Test Mission</td>
</tr>
<tr>
<td>DLR</td>
<td>German Aerospace Agency</td>
</tr>
<tr>
<td>ESA</td>
<td>European Space Agency</td>
</tr>
<tr>
<td>GEO</td>
<td>Geostationary Earth Orbit</td>
</tr>
<tr>
<td>IAA</td>
<td>International Academy of Astronautics</td>
</tr>
<tr>
<td>JAXA</td>
<td>Japanese Aerospace Exploration Agency</td>
</tr>
<tr>
<td>JHU/APL</td>
<td>Johns Hopkins University / Applied Physics Laboratory</td>
</tr>
<tr>
<td>MASCOT</td>
<td>Mobile Asteroid Surface Scout</td>
</tr>
<tr>
<td>MIT</td>
<td>Massachusetts Institute of Technology</td>
</tr>
<tr>
<td>NASA</td>
<td>National Aeronautics and Space Administration</td>
</tr>
<tr>
<td>NEA</td>
<td>Near Earth Asteroid</td>
</tr>
<tr>
<td>NEAR-Shoemaker</td>
<td>Near-Earth Asteroid Rendezvous – Shoemaker Mission</td>
</tr>
<tr>
<td>OCA</td>
<td>Observatoire de la Côte d'Azur</td>
</tr>
<tr>
<td>OSIRIS-REx</td>
<td>Origins-Spectral Interpretation-Resource Identification-Security-Regolith Explorer</td>
</tr>
<tr>
<td>PPOD</td>
<td>Poly Picosatellite Orbital Deployer</td>
</tr>
<tr>
<td>SRP</td>
<td>Solar Radiation Pressure</td>
</tr>
<tr>
<td>USSR</td>
<td>Union of Soviet Socialist Republics</td>
</tr>
<tr>
<td>YORP</td>
<td>Yarkovsky-O'Keefe-Radzievski-Paddack Effect</td>
</tr>
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</table>
1 INTRODUCTION

Small bodies are attracting significant interest in last couple of decades. One reason is to understand the evolution of solar system and find out the mysteries of life that is engendered on Earth. They are also among the easiest objects that could be reached from the Earth (Yarnoz, Sanchez, & McInnes, 2013). Additionally, spacecraft to be sent to those targets are also a significant technological challenge to be tackled.

The motivations of small body research range from pure science to planetary defence. Even though all these struggle to understand the nature of those would be simplified as science, the purpose of small body exploration extends to even commercial ways. Thus, it is essential to understand the insights of these motivations in order to identify specific needs and requirements for space projects.

In next sections, motivations of small body explorations are explained in detail. Moreover, space missions to date and up to near future are given to show how these motivations are addressed.

1.1 Motivations of Small Body Exploration

1.1.1 Science and Technology

The most primitive and humble motivation to small body exploration is perhaps scientific curiosity. The huge distance between Mars and Jupiter and Titius-Bode Law, which implies a relation for the ratio of orbital radii of the other planets, made scientist to think the existence of another planet in between those in early 1800s. This idea led them to the discovery of Main Belt Asteroids (Peebles, 2000). Number of theories are pronounced since then, about their source and nature. They are abundant all over the solar system and outside.

They are among the most primitive bodies in the solar system, only remnants of the first days. They collided with other bodies, merged and disrupted over the history of solar system. Each one of them have unique properties, though some
of them have similarities. Today, it is hypothesized that life might have engendered via asteroid and comet collisions.

Today’s observation capabilities are significantly advanced compared to past. However, it is still insufficient to draw very accurate conclusions regarding their nature. Thus space missions that are targeted those bodies have an importance to expand our knowledge.

However, space missions bring technological challenges. Small bodies are significantly different than planets, e.g., their gravitational field are very low. In addition to that, there are other non-gravitational perturbation sources which must be overcome that are not present or negligible in planetary exploration. Examples of which would be solar radiation pressure or comet outgassing. Moreover, multiple visits to those have an importance for small body exploration. This could be achieved only with optimised trajectories, which are another challenge to be tackled by engineers.

A recent NASA roadmap document states that there will be a manned asteroid exploration mission in near future (NASA, 2015). That will be a precursor mission before the ultimate destination, Mars. In order to do that, a very small asteroid is planned to be redirected to an orbit around Moon, where astronauts will visit it. Whilst it is a difficult task to bring an asteroid already, a manned exploration would definitely push our limits.

Human imagination brought the idea of colonising the solar system, even other star systems. For such purposes it is essential to exploit resources all around us. The idea of producing propellant for spaceships, from materials contained in asteroids, is nothing new. Although it seems like a far-fetched idea, it is still another motivation for small body exploration. A similar idea today is tried to be employed in a much more pragmatic way, for commercial purposes, which will be discussed in next section.

1.1.2 Asteroid Mining

The Earthly sources are not infinite; however the abundance of asteroids would provide nearly unlimited resources for humanity. Asteroid mining is a seriously
considered idea nowadays, and there are two companies, which are known to be investing on it, that are called Deep Space Industries and Planetary Resources Inc. Their near-term goal is mainly concentrated on surveying economically viable near-Earth asteroids. General path of both companies is to survey asteroid with small spacecraft first, and then analyse the feasibility of asteroid from an economic perspective (Deep Space Industries, 2015; Planetary Resources, Inc, 2015). Even though ground-based observations provide initial idea about the internal composition of an asteroid, rendezvous missions or in-situ observations are essential to find out the actual composition.

Asteroid mining offered inspirations to new research projects and academic studies related to mining and space mission design, in addition to existing body of research.

1.1.3 Planetary Defence

Planetary defence implies, loosely speaking, to protect the Earth from impacts of small bodies. The idea arose first in late 18th and early 19th century in English literature (Peebles, 2000); however, interpreting this possibility as a serious matter begins in the second half of 20th century. It was the result of the fact that dinosaurs became extinct by an impact 65 million years ago and similar faith might be coming to humanity, as well (Peebles, 2000).

The first engineering challenge on planetary defence is dated to 1967. The students of the course “Advanced Space Systems Engineering” at Massachusetts Institute of Technology (MIT) were given a task to design a spacecraft to prevent the impact of asteroid 1566 Icarus to protect the Earth. They were given a very tight time frame, as well as resources. It was the first such project that defined the requirements of such a mission and importance of the danger to some extent (Peebles, 2000).

However, it was another incident that made the danger clearer. In 1993, a comet orbiting Jupiter is discovered by Eugene and Carolyn Shoemaker and David Levy. The calculation of the orbit showed that it will impact Jupiter in July 1994. The impact occurred as expected and fragments were visible by Hubble space
telescope and effects are measured by Galileo, Ulysses and Voyager spacecraft. It was widely covered by media and made the idea of planetary defence apparent in public (Peebles, 2000).

Since the relevant methods are developed, the orbits of observed small bodies are determined accurately. With the development of computers, more accurate estimates of orbits are possible, as well as impact probabilities.

Asteroid deflection methodologies are widely researched. Several different deflection methods are already presented in the literature (Sanchez, Colombo, Vasile, & Radice, 2009). Stardust research network is dedicated to asteroid and space debris manipulation, “to save our future” (Stardust Network, 2013). Also, Planetary Defence Conference is organised since 2009 at which researchers have a medium to discuss their results and findings (International Academy of Astronautics (IAA), 2015).

The joint NASA-ESA Asteroid Impact & Deflection Assessment (AIDA) Mission is designed to crash on the smaller companion of a binary asteroid Didymos in order to test deflection methods (ESA, 2015).

1.2 Space Mission to Small Bodies

Space missions targeted to small bodies can be divided into two, i.e. flyby and rendezvous missions. Flyby missions generally provide much more rough information than rendezvous missions, for which spacecraft usually spends more time on body than a flyby mission. Examples of missions to comets and asteroids are given in next sections.

1.2.1 Comet Exploration

The first comet exploration mission is targeted to comet Halley in 1986 with huge collaboration of NASA, ESA, USSR and Japan (Scheeres, 2012). Within this huge collaboration, Giotto was the European flyby mission to Halley, which was initially a collaborative mission between ESA and NASA. It was also the first interplanetary mission of Europe (ESA, 2013). With the help of this mission, rotation state, composition and shape of the comet was roughly determined.
However, three dimensional shape and accurate mass information could not be gathered (Scheeres, 2012). Giotto mission then visited comet Grigg-Skjellerup, as well (ESA, 2013).

![Image of Comet Halley taken by Giotto Spacecraft (ESA, 2013)](image)

**Figure 1-1 Image of Comet Halley taken by Giotto Spacecraft (ESA, 2013)**

The NASA’s asteroid mission DeepSpace-1 was extended twice after its asteroid visit in order to flyby to comet Borelly in 2001 and valuable information about instruments had been obtained. Images of bifurcated shape of comet Borelly was also sent back to the Earth (Scheeres, 2012).

Stardust mission was a NASA mission that is targeted to comet Wild-2 and it was the first sample return mission from a comet’s coma. Rendezvous was performed in 2004 and sample was returned to the Earth in 2006 (Scheeres, 2012).

Deep Impact was also a NASA mission which was launched in 2005 and targeted to comet Tempel-1. It carried an impactor on it by which it was aimed to create a crater on comet and to observe the strength of the comet. Dust level after the impact turned out to be too high, which limited observational capability of the spacecraft. However this crater was observed by Stardust spacecraft in 2011. After primary mission goals were fulfilled, the mission was extended further and another flyby with comet Hartley-2 was performed in 2010 (Scheeres, 2012).
Rosetta mission is perhaps the most popular comet exploration mission at the time of writing of this thesis. It was launched in 2004 and targeted to comet Churyumov-Gerasimenko. Rendezvous happened in the early days of 2014, and spacecraft is operational since then. It provides valuable information about shape, composition, surface features. Its attempt to deploy the lander Philae in November 2014 was partially successful. The operational life of Rosetta is planned to end by August 2015 but possible extension is considered (ESA, 2015).

<table>
<thead>
<tr>
<th>Spacecraft</th>
<th>Year</th>
<th>Target</th>
<th>Flyby/Rendezvous</th>
</tr>
</thead>
<tbody>
<tr>
<td>ESA/Soviet/Japan Collaboration</td>
<td>1986</td>
<td>Halley, Grigg-Skjellerup (Giotto only)</td>
<td>Rendezvous (Halley), Flyby (G-S)</td>
</tr>
<tr>
<td>DeepSpace-1</td>
<td>2001</td>
<td>Borely</td>
<td>Flyby</td>
</tr>
<tr>
<td>Stardust</td>
<td>2004</td>
<td>Wild-2</td>
<td>Rendezvous</td>
</tr>
<tr>
<td>Deep Impact</td>
<td>2005</td>
<td>Tempel-1, Hartley-2</td>
<td>Rendezvous (Tempel-1), Flyby (Hartley-2)</td>
</tr>
<tr>
<td>Rosetta</td>
<td>2014</td>
<td>Churyumov-Gerasimenko</td>
<td>Rendezvous</td>
</tr>
</tbody>
</table>
1.2.2 Asteroid Exploration

The very first asteroid mission was Galileo mission in 1991 which was originally designed to orbit Jupiter. Its journey to Jupiter was extended to flyby two asteroid. Those were the asteroids Gaspra and Ida. Asteroid Ida was found out to be a binary asteroid with its companion Dactyl, which was the first binary asteroid system observed. The measurements were not very precise, however it provided the first information regarding asteroids from close encounter (Scheeres, 2012).

DeepSpace-1 mission visited asteroid Braille in a flyby mission in 1999. While testing new technologies for instruments, it provided images of the asteroid (Scheeres, 2012).

NEAR – Shoemaker mission, named after Eugene Shoemaker, was targeted to one of the largest near-Earth asteroid (NEA), Eros. It is an uncharacteristic asteroid among the other NEAs, it is 15 km in diameter and has a nearly homogenous composition (Scheeres, 2012). The mission was aimed to understand its composition, mineralogy, morphology, internal mass distribution and magnetic field of the asteroid, as well as interaction with solar wind and surface regolith properties (NASA, 2015). It was the first ever spacecraft which was attempted to land on an asteroid. It provided valuable images from the close distance to surface; however contact was lost with the spacecraft two weeks after this operation was performed (NASA, 2015).
The Japanese mission Hayabusa visited asteroid Itokawa in 2005 and returned samples of the asteroid back to the Earth in 2010. Its stay was short due to several failures but samples were returned. Its small lander Minerva (1 kg) failed to land on the asteroid (Scheeres, 2012). Asteroid Itokawa was later proved to be contact binary, which means two asteroid companion is orbiting around their common centre of mass while touching each other (Demura, Kobayashi, & Nemoto, 2006).

The Rosetta mission performed two flybys with asteroids Steins (2008) and Lutetia (2010) on its way to its target comet. It performed several measurements during these flybys (ESA, 2015).
The most recent mission to an asteroid, Dawn, is targeted to asteroid Vesta and recently named as dwarf planet Ceres. The aim is to investigate these two small bodies, which evolved differently through the history of solar system (NASA, 2015). The mission continues successfully at the time of writing.

Another recent asteroid mission is Hayabusa-2, which was launched in December 2014. It is again a sample return mission to near-Earth asteroid 1999 JU3 (JAXA, 2008). The rendezvous is planned to be in 2018. It carries Minerva lander on it, as well as German lander spacecraft MASCOT (JAXA, 2015).

Two future missions are planned to be sent to asteroids. OSIRIS-Rex, a NASA mission is going to be launched in 2016, will rendezvous with near-Earth asteroid 1999RQ36, also called Bennu, in 2018 and will obtain samples in 2019. The return of the capsule is expected to be in 2021 (NASA, 2015). The other mission, called AIDA, is a joint NASA-ESA mission to asteroid Didymos. It contains two spacecraft. NASA spacecraft will be an impactor spacecraft and crash on the companion of Didymos whereas ESA spacecraft will observe the effects of this impact. The mission will provide insights into asteroid deflection. It will be launched in 2020 and rendezvous is expected by 2022 (ESA, 2015).
A summary of asteroid missions to date is given in Table 1-2.

Table 1-2 Asteroid exploration missions to date or to be planned

<table>
<thead>
<tr>
<th>Spacecraft</th>
<th>Year</th>
<th>Target</th>
<th>Flyby/Rendezvous</th>
</tr>
</thead>
<tbody>
<tr>
<td>Galileo</td>
<td>1991</td>
<td>Gaspra, Ida &amp; Dactyl</td>
<td>Flyby</td>
</tr>
<tr>
<td>DeepSpace-1</td>
<td>1999</td>
<td>Braille</td>
<td>Flyby</td>
</tr>
<tr>
<td>NEAR - Shoemaker</td>
<td>2001</td>
<td>Eros</td>
<td>Rendezvous</td>
</tr>
<tr>
<td>Hayabusa</td>
<td>2005</td>
<td>Itokawa</td>
<td>Rendezvous</td>
</tr>
<tr>
<td>Rosetta</td>
<td>2008, 2010</td>
<td>Steins, Lutetia</td>
<td>Flyby</td>
</tr>
<tr>
<td>Dawn</td>
<td>2011, 2014</td>
<td>Vesta, Ceres (dwarf planet)</td>
<td>Rendezvous</td>
</tr>
<tr>
<td>Hayabusa-2</td>
<td>2018</td>
<td>1999 JU3</td>
<td>Rendezvous</td>
</tr>
<tr>
<td>OSIRIS-REx</td>
<td>2018</td>
<td>Bennu</td>
<td>Rendezvous</td>
</tr>
<tr>
<td>AIDA</td>
<td>2022</td>
<td>Didymos</td>
<td>Rendezvous</td>
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1.3 The Scope of This Research

The motivations of small body exploration and missions to date is already provided in preceding sections. For such small body exploration missions, it is important to be able perform in-situ observations by landers. Small and simple lander modules would provide desired observations at low cost and low complexity. CubeSats would be put forward to fulfil this task. CubeSats offer standardised way of developing small spacecraft for very low cost. Especially a three unit (3U) CubeSat would provide an intermediate step between very small (Minerva) and medium (MASCOT) scale landers.

Landers that are used to date for small body exploration were deployed in very close distances to small bodies, because the risk to bounce off the surface and escape was high due to very little gravity of those. That sort of landing is required for soft landing; however, it is also dangerous for instruments on-board of mother spacecraft. Thus, another way of landing is necessary. An underactuated landing from a distance would be performed by the help of gravitational pull of the body or bodies in the system. The Circular Restricted Three Body Problem (CR3BP) dynamical model would allow to find unaided landing trajectories for a lander, under the point mass gravitational attraction of two massive bodies compared to lander.
Thus, a binary asteroid system is of interest in this thesis research. The secondary body in binary system, which is smaller and orbiting the primary body, is targeted for landing. Proposed landing trajectories are generated within CR3BP dynamical model under the gravitational pull of two bodies in binary asteroid system. The research provides new insights into sizes and regions of binary asteroid systems to be landed by a simple underactuated lander modules. Along with landing trajectory search, a preliminary perturbation analysis has been performed in order to understand when would solar radiation pressure starts dominating spacecraft over gravitational perturbation. Additionally, a preliminary mission design ideas are presented for a CubeSat to be landed on a binary asteroid.
2 THE THEORY OF BINARY ASTEROID SYSTEMS

2.1 How Likely Is It to Find Binaries Among Small Bodies?

It is generally accepted that about ~15% percent of the near-Earth asteroids (NEA), which have the diameter larger than 200 m are in binary character (Margot, Nolan, Benner, & et al., 2002). This result is reached basically using mainly Earth-based observations, which include radar observation mostly based in Arecibo and Goldstone observatories, lightcurve analysis which is performed by investigating the light intensity of a particular celestial object, colour and spectroscopy analysis. The abundance of NEA binaries are tried to be explained by different formation theories.

Polishook & Brosch however speculates that there may be more than 15% of binaries among NEA, based on their researches among Aten family of asteroids (Polishook & Brosch, 2006). They show that it may be around 63%, according to their work, which sampled 8 members of Aten family asteroids, and found that 5 of them were binaries. However, their sample study is quite small compared to thousands of bodies in near-Earth, therefore their conclusions should be taken carefully.

Recent studies suggest that binary lifetime may be less than expected before. Tidal effects and planetary flybys were considered as the primary causes of binary formation by earlier studies. However, radiation-related forces (Yarkovsky-O'Keefe-Radzievski-Paddack) seem more effective than tidal forces due to smaller size of NEAs (<10 km) and shorter timescales. Radiative forces influence on a small body one order of magnitude shorter in timescale than tidal effects. Pravec et al. claims that binary systems concentrate among NEAs smaller than 2 km in diameter and the fraction goes down considerably among larger NEAs (Pravec & Harris, 2007). Especially after the introduction of binary YORP (BYORP) concept, the researchers hypothesized that the abundance can be explained by this effect. BYORP has not been observed yet, however if the mechanism exists, it may be argued that every NEA becomes binary at least once.
Many asteroids are said to be rubble piles because their internal structures resembles that of a pile of rubble or sand, i.e., only kept together by a minimum amount of cohesion. A rubble pile asteroid can be described as “moderately porous, strengthless body with constituents bound only by their own gravity” (Walsh & Richardson, 2006). Another definition describes this structure as gravitationally bound rock collection in different sizes (Jacobson & Scheeres, 2011). During the first close observations achieved by the Hayabusa mission on small asteroid Itokawa, where this different sized collection of particles became apparent (Jacobson & Scheeres, 2011). They consists of few bigger bodies as a core and small rocks, dust particles covering the body.

A binary system is argued to be formed by material dissipation from the surface of primary due to rotational break-up. The primary is considered in an ellipsoidal shape initially. Although this is not entirely true due to irregular shape of the body, is still a good enough approximation. Primary asteroid spins in the axis of maximum moment of inertia which corresponds to the shortest distance from the centre of mass. Primary’s spin speed increases (spin period decreases) due to different perturbations, such as tidal forces, planetary encounters and YORP effect, which will be discussed later in this review. Once the spin reaches its critical limit, or a little less than that according to some researches (Jacobson & Scheeres, 2011), surface of asteroid begin to dissipate materials and these materials start orbiting around the primary body and create satellite.
2.3 Different Perturbation Sources for Possible Formation Mechanisms

2.3.1 Different Models Used to Understand the Formation

The only close range observation of a small binary asteroid was performed by Hayabusa mission on the contact binary asteroid Itokawa. The radar, lightcurve and other type of Earth-based observation methods still have limitations and are open to misunderstanding. Therefore some researchers have developed theoretical models to understand the driving mechanism of binary formation. There are three methods observed in the literature, which are N-body approach, continuum approach and rigid body approach.

N-body approach is a computational model which is generated using small bodies to create a rubble pile structure as a gravitational aggregate without cohesion but non-fluid. The stress behaviour of such bodies can be represented with angle of friction between bodies (Walsh, Richardson, & Michel, 2008). For angle of friction of ~40°, this model has considerable consistency with continuum model explained below (Holsapple, 2010).

Continuum model is applied by using the well-understood soil and rock mechanics in geological research. In this approach continuum deformation on the structure is investigated in which a granular structure smaller than entire body is needed. Several sets of equations are solved for the balance of mass, momentum and energy (Holsapple, 2010). In his paper, Holsapple explains the limitations of N-body and rigid approaches in which he states that neither N-body approach nor rigid body approach accounts for inter-particle bonding (2010).

The rigid body approach uses kinematics of individual bodies resting on each other by examining the stability and equilibrium conditions (Scheeres, 2009). It does not consider deformations, which may happen inside the structure.
2.3.2 Perturbation Sources for the Formation

Early work on small binary formation considered the tidal disruption during a planetary encounters as primary source among the other possible mechanism, such as formation by capturing two independent asteroids or catastrophic impact cratering due to low probability of encounters and energy required for impact, respectively (Margot, Nolan, Benner, & et al., 2002). Tidal disruptions due to planetary encounters seemed reasonable to researchers because the progenitor (later primary) is required to have no tensile strength to shed mass due to spin up. According to the theory, the binaries grow during subsequent planetary encounters (Walsh & Richardson, 2006). This theory is also consistent with the work of Bottke & Melosh, whom tried to explain the doublet crater on three different planets, Earth, Venus and Mars (1996). The required timescale for formation by this mechanism is $\sim 10^6$ yr (Margot et al., 2002; Cuk & Burns, 2005).

However, more recent studies suggest another mechanism of formation; so called Yarkovsky-O'Keefe-Radzievski-Paddack (YORP) effect. The term and concept is introduced to the community of small solar system bodies by Rubincam (2000). It is a variation of the well-known Yarkovsky effect, which states that when infrared radiation caused by Sun escapes the body it applies a momentum on and accelerates it. YORP generally depends on the primary’s shape, size and orbit around the Sun (Walsh, Richardson, & Michel, 2012). Due to small amount of the effect, its outcomes are generally observed on km-sized NEAs rather than big bodies in main belt.

The mechanism is simple as explained before: the reemission of solar radiation from the progenitor slowly accelerates it, and spins it up until the critical point is reached. The timescale for this is given as $\sim 5 \times 10^5$ yr (Cuk & Burns, 2005). When the progenitor exceeds critical spin limit, it starts dissipating mass, so-called “landslide” starting from the edges of its equator (Walsh et al, 2012; Scheeres, 2009). Once the binary system is formed, YORP effect continue to shape the primary and the system.
Most recently another mechanism was hypothesized in the field that is the so-called: binary YORP (BYORP) (Cuk & Burns, 2005). The effect is same in principle, as explained above, however it is not considered for the centre of mass of primary but the centre of mass of the mutual orbit. BYORP effect is taken into account when the secondary reaches sufficient mass (30% of primary) and distance from the primary (Cuk & Nesvorny, 2010). BYORP is more complex effect in nature compared to YORP. It depends on the relative sizes, mutual orbit and the spin states of the members of the system (Walsh, Richardson, & Michel, 2012). In long term, BYORP effect may cause orbital instabilities, which may result in ternary (triple asteroid) systems, contact binaries and so on (Jacobson & Scheeres, 2011). Its effect may enlarge or contract the orbit which in turn influence the evolution of the binary system (Scheeres, 2009). The observational evidence of BYORP is still insufficient to draw a conclusion.

YORP is the most accepted formation mechanism today by many researchers. The abundance of small binary systems and the short formation timescale makes the YORP most probable formation mechanism. However, tidal disruption is still accounted for ~1-2% of the near-Earth binary formation, and it is always considered for the further evolution after formation (Pravec & Harris, 2007).

2.4 Properties of Binary Asteroid Systems

The generally agreed formation mechanism for binary asteroids, namely the rotational break-up, suggests that the primary asteroid shall change its shape when it starts shedding mass from its equator. The radar imaging of the near-Earth binary asteroid 1999 KW4 shows that the primary has an equatorial bulge with an oblate shape where secondary has more elongated shape (Ostro, Margot, Benner, & et al., 2006). This result is consistent with the simulations performed by (Walsh & Richardson, 2006). Primary becomes oblate if it is already spherical. If it is already prolate, it evolves “favourable shape” before mass shedding. (Walsh, Richardson, & Michel, 2008). This “favourable shape” can be described as low equatorial elongations (Walsh, Richardson, & Michel, 2012).
It can be claimed that secondary is substantially formed by primary, according to the results of observations and theoretical models. Walsh et al. found that 70-90% percent of the material of secondary is originated from the primary (2008). Additionally, mass shedding from the equator uncovers 15-35% of the surface particles which are not originally on the surface (Walsh, Richardson, & Michel, 2012). These particles are uncovered in poles firstly. In their computational model, Walsh et al. investigated effect of different rubble pile asteroid types, namely the ones formed by equal size spheres (nominal case), unequal size spheres (intermediate case) and by large cores with smaller spheres. The satellite formation started in nominal case when 2.5% of the systems mass is ejected. Same result came up at around 7% for intermediate case and third case (Walsh, Richardson, & Michel, 2012).

The density predictions range from 1.33 g/cm$^3$ to 3 g/cm$^3$ according to different researchers which corresponds to porosity of 35% to 60% (Ostro et al., 2006, Walsh & Richardson, 2006). Pravec & Harris states that this density range covers about 2/3 of the binary asteroid systems (2007). Porosity of the secondary is expected as the same or lower than primary due to smaller size.

The secondaries are not larger than half the diameter of the primaries and, in most cases, smaller than that. For small binary asteroid systems (< 10 km), especially the systems with primaries smaller than 1 km diameter, the system usually has a diameter ratio of secondary to primary of 0.5 or lower (Pravec & Harris, 2007). The most observe NEA binaries have the mass ratio of 0.2 or lower (Jacobson & Scheeres, 2011). According to summary given by Walsh et al., the secondaries have around 50% of the total ejected mass (2012). This result can be supported by the computational simulations of Walsh et al., where they found out that secondary evolution stops around the half size of primary (2008). This result suggests that the mass transfer should stop at some point in the binary evolution or there should be another mechanism to explain this phenomenon.
2.5 Orbital Properties of Binary Asteroid Systems

For asteroids larger than 200 m in diameter, it is unusual to observe spin period of less ~2.2 h (Jacobson & Scheeres, 2011; Walsh & Richardson, 2006). Additionally, the majority of the very small binaries (< 1 km diameter of primary) have the rotation period less than 3 h, only with two exceptions, Hermes (13.89 h) and 2000 UG11 (4.44 h) (Walsh & Richardson, 2006). The observations show that the primary of the binary asteroid 1999 KW4 has the spin period of ~2.8 hours, almost same as 2000 DP7 (Ostro et al., 2006; Margot et al., 2002). This spin is around the maximum moment of inertia of primary where corresponds to the shortest distance from the centre of mass.

It is widely accepted that in the most of the small binary systems observed in NEA (<10 km) the secondaries are said to be synchronous, meaning that their spin periods are equal with the orbital period (Jacobson & Scheeres, 2011). The semi-major axis of the orbit is in average between 1.5 to 5 primary radius with nearly spherical primary and elongated secondary (Jacobson & Scheeres, 2011; Walsh & Richardson, 2006, 2008). Eccentricity of the secondary orbit around primary is usually closer to 0, which points out a circular orbit. However, Taylor et al. defined high-eccentricity binary systems also, which are asynchronous and low secondary-to-primary mass ratio systems (Taylor, et al., 2008). The orbital period of binary is ranging between 12 h to 21 h, with exception of 2000DP7 which has the period of 42 h (Walsh & Richardson, 2006).

2.6 Orbital Stability of Binary Asteroid Systems

The introduction of the BYORP concept gives the researchers an important tool to develop new models. Lack of observational data about the orbital evolution of the small binaries, especially of secondary in the system which has a very small diameter (sometimes < 100 m) increases the importance of theoretical models. After the first effort on BYORP effect by Cuk & Burns (2005), the further development in the model and research of possible outcomes of the effect on long-term term evolution is performed by Cuk & Nesvorny (2010) and Jacobson.
& Scheeres (2011). Observational evidences of the results are linked to the Hayabusa mission data (Scheeres, et al., 2007) and a solid ground is tried to be established.

As previously discussed, the secondary grows up to 0.3-0.4 magnitude of primary radius and this suggests a stopping point for the mass transfer process. In fact, the mass transfer process does not stop but the BYORP effect becomes more evident on the system which makes the mass transfer less important. It is thought that small binary systems may undergo subsequent YORP-related cycles, this may result in that small binaries may be shorter-lived than thought before.

In their work, Cuk & Nesvorny modelled the system by “turning-off” the mass transfer process, to see how the system will evolve under the BYORP effect (2010). They found out that the semi-major axis of the orbit tends to increase, so does the eccentricity and goes to a chaotic state. After a while it comes back to a stable state and goes back to the chaotic state until a more stable state is found. They explain this as the orientation change of the secondary. However, these re-established stable states may be completely stable if dissipation is accounted for secondary. Especially when secondary is in a close separation, it is likely to have dissipation due to primary’s tidal forces. With sufficiently close separations, the secondary and primary may merge due to tidal forces or unstable orbit, otherwise some material has to be dissipated at least to have subcritical rotation of one body. If secondary is in a relatively large distance away and primary is still rotating fast, then there is a possibility to create another secondary and a triple system. This situation may result in a more stable binary system at the end, either by ejected member of the system or by strike of one of the members to primary. It is though that this is a cycle in binary life and continue again and again (Cuk & Nesvorny, 2010; Jacobson & Scheeres, 2011).

There may be an observational example to support which is given by (Scheeres, et al., 2007). The asteroid Itokawa has no craters on it which implies that it is a young asteroid. Contact binary population is thought of 9% of the NEA population
and its characteristics can be explained with the theory above (Walsh, Richardson, & Michel, 2012). It is a contact binary, meaning that contains two bigger bodies, once orbiting about each other. The body and head sizes are 490 m and 230 m, respectively (Demura, Kobayashi, & Nemoto, 2006). It shows a consistent proportion between primary and secondary which is slightly less than 0.5. The surface material is not significantly heterogenous, thus Itokawa was most probably formed from a one progenitor body (Mazrouei, Daly, Barnouin, & et al., 2014). It is claimed that the force due to YORP and BYORP effect may have affected this system as explained above and the result may be a low-speed impact of two members. Third body may have been ejected in this process.

The most recent observations of the asteroid 1996 FG3 may give some more information about the BYORP effect. It is found that the BYORP effect can be tracked in a binary system by observing change in mean anomaly of the secondary orbit around the primary quadratically [°/yr²] (Scheirich, Pravec, Jacobson, & et al., 2015). The drift values is calculated for some of the well-known asteroids by using the known information. The value for the asteroid 1996 FG3 is 0.89 °/yr². The detected value 0.04 +/- 20 °/yr² which is fairly away from the estimation (Scheirich, Pravec, Jacobson, & et al., 2015). Although there are more observational evidences needed, Scheirich et al. concludes that BYORP may not be the driving mechanism for long-term evolution of binary systems (2015). According to them, this early conclusion brings the tidal effects into bigger picture again (2015).

### 2.7 Conclusion

A review on the theory of formation and evolution of small binary asteroid formation is given. In this review, due to the abundance of binary asteroid systems, only near-Earth asteroid binaries are considered. Small asteroid binaries are formed from rotational break-up of a progenitor body (later primary) due to increase in rotational acceleration. Main source of rotational acceleration is thought to be YORP effect where irregular shape of the body allows YORP to accelerate it. The secondary body is formed from the primary body, according to
observations and theoretical models, and density, porosity and material composition is nearly identical to primary body. The shape of the secondary is elongated, where primary becomes more oblate over the course of binary evolution. Nearly spherical shapes are observed within the binary systems.

The mutual orbit of the binary system is nearly circular with semi-major axis of 1.5-5 primary radius and synchronous, meaning that the secondary is spinning with the same period as the mutual orbit. Once binary system is formed and secondary grew around 0.3-0.4 primary radius, the same radiative forces influence on secondary, as well. This expands and shrinks the mutual orbit. Together with this orbital cycle and tidal dissipation, new secondaries may form in the system, or some of them may merge with their primaries. Some of the well-known binary systems may be explained by using this theory. This cycle allows us to suggest that small binary cycle may be shorter than expected, couple of hundreds or thousands of year according to the recent theories, and to explain the abundance in small binary systems in NEAs and in solar system.
3 PERTURBATION ANALYSIS IN SMALL BODY ENVIRONMENT

3.1 Introduction

Orbiting bodies or particles around a major body, such as the Sun, the Earth, etc. experience various torques caused by the environment. As an example, in an ideal case the Earth is considered as a perfect sphere; however this is not the case in reality. In fact, the Earth is an ellipsoid and its gravitational attraction varies throughout the orbit. Hence, the motion of a satellite orbiting the Earth is perturbed by this non-spherical shape. Another example would be atmospheric drag and solar radiation pressure (SRP). Although atmosphere gets thinner when altitude increases, it still affects satellites in lower altitudes. Atmospheric drag is generally taken into account up to 1000 km of altitude (Fortescue, Swinerd, & Stark, 2011). SRP is usually lower in magnitude compared to other torques in Earth orbit. This is because of the massive effect of the major body and the air drag. However, SRP becomes effective when the area to mass ratio is high, this is the case for solar sailed spacecraft, and often for GEO satellites with very large solar panel area (Fortescue, Swinerd, & Stark, 2011). There are some other additional perturbations that may also be considered, such as third body perturbations caused by other major bodies, like the Sun, Jupiter, Moon, etc. (Sidi, 2005).

In the case of small bodies, particularly for asteroids and comets, perturbations like non-spherical shape and SRP become essential. The gravitational attraction of the small bodies are considerably low compared to major bodies. Additionally, if irregular shapes of minor bodies are considered, non-spherical shape perturbation becomes the major perturbation source for spacecraft in the vicinity of the body (Scheeres, 2012). SRP also perturbs the motion of orbiting spacecraft or particles whose magnitude depends on the area-to-mass ratio (Fortescue, Swinerd, & Stark, 2011). Moreover, spacecraft to small bodies essentially orbits the Sun while investigating their target. Hence, third body perturbations shall also be considered in order to model the motion realistically. Likewise, Jupiter may
also be considered as perturbation source, as some missions pass close by these bodies like Rosetta (Scheeres, 2012).

In this chapter, two different perturbation sources, namely non-spherical shape perturbation and SRP, are investigated. The results obtained here will be used to discuss implications of adding more perturbations.

3.2 Non-spherical Shape Perturbation

One of the main distinctive feature of s bodies are their non-spherical, generally irregular shapes. The reason behind this is their non-homogenous mass distribution, and this results in strong perturbation on spacecraft's motion. One of the possible ways to describe the gravitational field of the minor body is the spherical harmonics model. Although it will be assumed here homogenous mass distribution, it still gives good approximation to the problem.

Gravitational model can be described by spherical harmonics given below (Hausmann, et al., 2012):

\[
U(r, \phi, \lambda) = \frac{GM}{r} \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \left( R^n P_{nm}(\sin \phi) \left( C_{nm} \cos(m\lambda) + S_{nm} \sin(m\lambda) \right) \right)
\]

where \( R \) is the reference radius of body, \( C_{nm} \) and \( S_{nm} \) are spherical harmonic (Stoke’s) coefficient and \( P_{nm} \) is Legendre polynomial with degree \( n \) and order \( m \). For the simplified case of triaxial ellipsoid with homogenous mass distribution and semi-axes a, b, c the spherical coefficients can be found as follows (Balmino, 1994):

![Figure 3-1 The triaxial ellipsoid considered (Yarnoz, Sanchez Cuartielles, & McInnes, 2013)]
Legendre polynomials can be written as (Montenbruck, Gill, & Lutze, 2002):

\[ P_{nm} = \left(1-u^2\right)^\frac{m}{2} \frac{d^m P_n(u)}{du^m} \]  

There are several recursive relationships could be considered between these polynomials. For the case \( n > m+1 \), the recursive relations are given as (Montenbruck, Gill, & Lutze, 2002):

\[ P_{nm}(u) = \frac{1}{n-m} \left((2n-1)u P_{n-m}(u) - (n+m-1)P_{n,m}(u)\right) \]  

Also for \( n=m \):

\[ P_{nn} = (2m-1)\left(1-u^2\right)^\frac{1}{2} P_{m-1,m-1} \]  

And

\[ P_{m+1,m}(u) = (2m+1)u P_{m,m}(u) \]  

Here in this case, 2\textsuperscript{nd} order polynomials are considered, hence:

\[ P_{20} = \frac{1}{2} \left(3u P_{10}(u) - P_{00}(u)\right) = \frac{1}{2} (3u^2 - 1) \]

\[ P_{22} = 3(1-u^2)^\frac{3}{2} P_{11} = 3(1-u^2) \]

where \( P_{00}(u)=1 \).
For the $S$ coefficients, the values can be found with a little algebra (Herring, 2013): $S_{21} = 0$ if the $Z$-axis along with the maximum moment of inertia, which corresponds to the shortest axis for a triaxial ellipsoid. $S_{22}$ coefficient is related to the moment of inertia $I_{12}$, which is zero due to the shape of the body.

Hence, the ultimate gravitational potential sum can be written as:

$$U(r, \theta, \gamma) = \frac{GM R^2}{r} \left[ \frac{1}{2} \left( 3 \cos^2 \theta - 1 \right) C_{20} + 3 \left( 1 - \cos^2 \theta \right) C_{22} \cos(2 \gamma) \right]$$

(3-8)

where $\theta$ is co-latitude and $\gamma$ gamma is longitude.

### 3.3 Solar Radiation Pressure (SRP) Perturbation

SRP perturbation is here computed using particle lightness number $\beta$, which is the ratio of SRP to gravitational attraction of Sun and given below (Yarnoz, Sanchez Cuartielles, & McInnes, 2014):

$$\beta = \frac{LQS}{4\pi c \mu m}$$

(3-9)

$L$ is solar luminosity and the value is $3.846 \times 10^{26}$ W, $Q$ is reflectivity ($Q=1$ completely absorbing, $Q=2$ completely reflecting), $S$ is area of dust particle or a small satellite, $c$ is speed of light, $\mu$ is gravitational parameter of the Sun and $m$ is the mass of particle or spacecraft.

### 3.4 Analysis

For non-spherical shape perturbations, asteroid reference masses between $10^7$ and $10^{15}$ are considered. Based on the ellipsoidal shape given in Figure 1, the reference radii ($R$) are calculated. A spacecraft is considered which has an orbit around an asteroid with 2R, 4R, 10R apart. The maximum non-spherical shape perturbation is found to be at the tip of ellipsoid in equator where maximum elongation is observed. Therefore, these values are calculated and considered as the worst case scenario.

In SRP for dust particle case, no $\beta$ values are calculated specifically but generated between $10^{-6}$ and $10^{-2}$. That corresponds a millimetre to a micrometre.
size particle (Bewick, Sanchez, & McInnes, 2012). Then, the perturbing acceleration is calculated based on given formulation above.

For spacecraft case, the parameters determine the $\beta$ value are area normal to solar radiation and the mass of spacecraft. Areas are varied from very small ($10^{-4}$ m$^2$) to very large ($10^{-5}$ m$^2$). The base value of the masses was chosen of 1U cubesat mass and varied up to 1000 kg. The $\beta$ value is then calculated and SRP acting on spacecraft is calculated based on the given procedure above. Perturbations acting on a spacecraft in different orbits are shown in figures below.
As expected, when spacecraft is orbiting farther from the target body, the magnitude of the non-spherical shape perturbation decreases and SRP becomes dominant perturbation. At 4R distance and more, all spacecraft, which are taken as reference for $\beta$, are dominated by SRP perturbation.

In this chapter, two different perturbation sources on a spacecraft around a minor body (in this case, asteroid) are investigated. The models are derived accordingly, then their effect in terms of perturbing acceleration is computed for increasing distances (2R, 4R, 10R) from the centre of mass of a hypothetical asteroid. The SRP perturbing acceleration is also computed for wide range of $\beta$, which could represent test cases from dust particles to spacecraft in orbit. It is observed that a dust particle will be under the effect of SRP in vicinity of asteroid only when its size is very large. For instance, at 2R distance, a dust particle with 0.1 mm size is still under effect of non-spherical perturbation. Same applies to a spacecraft. Spacecraft’s orbit appear to be dominated by the SRP perturbation only beyond an orbit at 4R distance from the centre of mass. From sufficiently large distances from the body, e.g. 10R, the SRP dominates all the reference spacecraft, which are represented with their $\beta$ number.
The analysis here provides an initial look to the perturbations around an asteroid with different orbits. Given that small body gravitational attraction much lower than planetary bodies, other perturbations than non-spherical shape also become important. At this point, deployment distance is critical to draw a conclusion regarding dominating perturbation. And the distance is determined by size of target body. However, it is for sure that adding more perturbation to trajectory generation will increase the fidelity of the dynamical model.
4 DYNAMICAL MODEL: CIRCULAR RESTRICTED THREE BODY PROBLEM

4.1 Introduction

The well-known two body problem has been widely used successfully for decades. In this problem, a particle is assumed to be bound gravitationally to a larger body. General gravitational problem states that two bodies would rotate around common centre of mass; however it can be approximated as a massless body rotating around the larger body when there is a significant mass difference. This approach is very useful and provides straightforward solutions to the vast majority of astrodynamical problems. It even works effectively for Earth-Moon case where the mass of Earth is only about $10^2$ times larger than mass of the Moon. Note that this factor is very small. If one compares with the fact that the mass of the Earth is $10^{19}$ times larger than the mass of the International Space Station, which is the largest Earth-orbiting object (NASA, 2015)

The two-body approximation has certain limitations. Although the problem could be approximated as one body orbiting the other, the third body gravitational field become still effective. Those are attempted to be represented as perturbations and added to the motion. Nevertheless, the more complex but more accurate solutions to astrodynamical problems could be achieved by considering three body in the system.

The three-body problem is not as straightforward as the two body problem, although analytical solutions, under certain assumptions, were already showed by Lagrange (Schaub & Junkins, 2009). The so-called Circular Restricted Three Body Problem (CR3BP) is one example. In CR3BP, there are two massive bodies which are rotating in their common centre of mass in a Newtonian/Keplerian fashion. The third body, which has negligible mass and gravitational effect compared to the other two, moves under the gravitational influence of the other two body in the system. For instance, third body would be exemplified as a spacecraft for an unpowered flight, which is moving under the gravitational influence of the Earth and the Moon for the transfer between these two. CR3BP
is derived in rotational frame of reference, which is more convenient given the fact that two massive bodies are rotating in their common centre of mass. This method of derivation results in several outcomes.

One of the most famous of these outcomes would be the Lagrange Points, named after French mathematician Joseph Lagrange. They are five stationary points in three-body model when viewed from rotational frame. A body is assumed to stay there under the three-body motion. The Lagrange Points exist in any three-body system which could be simplified as CR3BP. These points are increasingly attractive for new space missions, especially for scientific ones. Moreover, the results of CR3BP are also attractive for the design of low-thrust, low-energy, non-Keplerian trajectories for different targets in solar system.

In this section, the equation of motion of the CR3BP is developed. Alongside the equation of motion, zero-velocity surfaces, which a representation of energy in CR3BP, and Lagrange points are explained. Finally, a general bisection method of transfer orbit generation is explained, which will later be used for problem of landing trajectories.

### 4.2 Equations of Motion

Before start developing the equations of motion of the CR3BP, three body model should be visualised and inertial components should be expressed in rotating frame. Figure 1 shows the illustration of CR3BP. Here, the centre of the rotating frame is the barycentre of two massive body, m1 and m2. The motion of m1 and m2 is circular about their common centre of mass with angular velocity $\omega = \omega \hat{e}_3$. The angular velocity magnitude can be expressed as:

$$\omega = \frac{2\pi}{T}$$  \hspace{1cm} (4-1)

where $T$ is period of the motion $T = 2\pi \sqrt{\frac{\mu}{r_{12}^3}}$ Here $\mu$ is gravitational constant of the system and can be expressed as:
Figure 4-1 CR3BP Illustration (Schaub & Junkins, 2009)

\[ G(m_1 + m_2) = \mu_1 + \mu_2 \] (4-2)

Therefore magnitude of angular velocity is

\[ \omega^2 = \frac{G(m_1 + m_2)}{r_{12}^3} = \frac{\mu_1 + \mu_2}{r_{12}^3} \] (4-3)

and the position vector can be expressed in rotational frame:

\[ \mathbf{r} = r_x \hat{e}_r + r_y \hat{e}_\theta + r_z \hat{e}_3 \] (4-4)

The derivative of \( \mathbf{r} \) in order to find the acceleration, while keeping the rotational motion in mind, results in

\[ \dot{\mathbf{r}} = \left( \dot{r}_x - 2\dot{r}_y \omega - r_x \omega^2 \right) \hat{e}_r + \left( \dot{r}_y + 2\dot{r}_x \omega - r_y \omega^2 \right) \hat{e}_\theta + \dot{r}_z \hat{e}_3 \] (4-5)

Note that angular velocity is constant, hence its derivative is zero. The gravitational force acting on mass \( m \) from \( m_1 \) and \( m_2 \) is therefore,
Then equations of motion will become

\[ \ddot{r}_x - 2\dot{r}_y \omega - r_x \omega^2 + \frac{\mu_1 (r_x + r_i)}{\rho_1^3} + \frac{\mu_2 (r_x - r_2)}{\rho_2^3} = 0 \]

\[ \ddot{r}_y + 2\dot{r}_x \omega - r_y \omega^2 + \left( \frac{\mu_1}{\rho_1^3} + \frac{\mu_2}{\rho_2^3} \right) r_y = 0 \]  

\[ \ddot{r}_z + \frac{\mu_1}{\rho_1^3} r_z = 0 \]

where \( \rho_i \) is relative distance with respect to \( m_1 \) and \( m_2 \)

\[ \rho_{i=12} = \sqrt{(x - r_i)^2 + y^2 + z^2} \]  

It is possible to write equations of motion in a non-dimensional form. To do that, several parameters are used. All the distance are normalised by the distance between two massive bodies in the system, \( r_{12} \). Non-dimensional distances could be represented as follows:

\[ x = \frac{r_x}{r_{12}}; \quad y = \frac{r_y}{r_{12}}; \quad z = \frac{r_z}{r_{12}} \]

\[ x_1 = \frac{r_1}{r_{12}}; \quad x_2 = \frac{r_2}{r_{12}} \]
This would make the distance between $m_1$ and $m_2$ equals to 1. The mass parameter is also defined as

$$\mu = \frac{m_2}{m_1 + m_2}$$  \hspace{1cm} (4-10)

The barycentre of the system is found by

$$m_1x_1 + m_2x_2 = 0$$  \hspace{1cm} (4-11)

By using mass parameter, same expression is obtained

$$\left(1 - \mu\right)x_1 + \mu x_2 = 0$$  \hspace{1cm} (4-12)

Noting that the non-dimensional distance between $m_1$ and $m_2$ equals to 1, as well as the expression above, the non-dimensional coordinates of $m_1$ and $m_2$, can be found in terms of $\mu$:

$$x_1 = -\mu$$  
$$x_2 = 1 - \mu$$  \hspace{1cm} (4-13)

Apart from distances, all times are non-dimensionalised by $T/2\pi$, the time required to sweep one radian angle in circular orbit, and as a consequence of this, all velocities are non-dimensionalised by the tangential velocity of the circular motion. Finally, non-dimensional equations of motion can be written as follows:

$$\ddot{x} - 2\dot{y} = \frac{\partial U}{\partial x} = x - \frac{(1 - \mu)(x + \mu)}{\rho_1^3} - \frac{\mu(x - 1 + \mu)}{\rho_2^3}$$

$$\ddot{y} - 2\dot{x} = \frac{\partial U}{\partial y} = y - \frac{(1 - \mu)y}{\rho_1^3} - \frac{\mu y}{\rho_2^3} = y\left(1 - \frac{1 - \mu}{\rho_1^3} - \frac{\mu}{\rho_2^3}\right)$$  \hspace{1cm} (4-14)

$$\ddot{z} = \frac{\partial U}{\partial z} = -\frac{(1 - \mu)z}{\rho_1^3} - \frac{\mu z}{\rho_2^3} = z\left(-\frac{1 - \mu}{\rho_1^3} - \frac{\mu}{\rho_2^3}\right)$$
where $U(x, y, z)$ is called potential function and defined as
\[
U(x, y, z) = \frac{1}{2} (x^2 + y^2) + \frac{1 - \mu}{\rho_1} + \frac{\mu}{\rho_2}
\]  
(4-15)

4.3 Zero Velocity Surfaces

For an unpowered spaceflight, trajectories of a mass $m$ under the gravitational effect of other two bodies are defined by its initial conditions. These initial conditions also determine so called Jacobi Constant, denoted by $C$, measure of energy of $m$ in rotational frame. The expression given below defines the relationship between Jacobi Constant, kinetic energy and potential function
\[
C = \frac{1}{2} V^2 - U
\]  
(4-16)

where $V$ is velocity.

This expression describes a negative energy measure of the system, where the increase in $C$ means an increase in energy.

One implications of Jacobi Constant is so called zero velocity surfaces, which tells the possible regions of motions for a given energy level in an unpowered flight. In an extreme case, e.g., $V = 0$, there exists regions where spacecraft motion is not possible, bounce back trajectories, or apogee-like locations occur (Schaub & Junkins, 2009). Gradually increasing energy makes more regions possible for motion. The increase in energy can be exemplified by apogee kick motor burn, upper-stage injection to transfer orbit, or continuous transfer burn by electric engines.

Gradual energy increase and zero velocity surface change can be seen in Figure 4-2. In the figure, green regions shows the possible regions for motions, and white regions shows the so-called “forbidden regions.” Keeping energy level below the
level in which the zero velocity surfaces are closed would ensure that the body cannot escape the given region.

Figure 4-2 Five regimes of motion defined by Jacobi Constant (white regions are forbidden)
As illustrated in the figure, “the gates” to make regions possible for motion opens up beginning from L1 point, then continues following L2 and L3, respectively. L4 and L5 points require the highest energy, and they open up simultaneously.

Note that zero velocity surfaces would only be defined in rotational frame, as it is function of time.

4.4 Lagrange Points

Lagrange Points are stationary points in rotational frame in CR3BP. Once a spacecraft is placed, it presumably stays there. There are five of them in any three body system that can be simplified as CR3BP, named as L1, L2, L3, L4 and L5. The last two terms is named as triangular points, and their locations are found using the condition below in equations of motion:

\[ \begin{align*}
\dot{x} &= \dot{y} = \dot{z} = 0 \\
\ddot{x} &= \ddot{y} = \ddot{z} = 0
\end{align*} \tag{4-17} \]

The collinear points, L1, L2 and L3, can be located by using the equality \( y = 0 \) and \( z = 0 \).

The collinear points are known to be unstable, whereas triangular points are known to be stable. It is not a scope of this work, a detailed analysis can be found in (Schaub & Junkins, 2009).

Earth-Moon Lagrange points are shown in Figure 4-3.
The most common interests are concentrated on Sun-Earth, Earth-Moon, Earth-Mars, Sun-Jupiter Lagrange points. Sun-Earth L1 is popular for Sun observation missions, whereas L2 point is popular for astronomy missions. Earth-Moon L1 and L2 points are considered for futuristic space station design concepts, as well as low energy transfers between these two. Earth-Mars Lagrange points are also considered for low thrust transfers. Moreover, Trojan asteroids are present at Sun-Jupiter L4 and L5 points.

In this research, L2 point is of importance. For transfer trajectories coming outside region of zero velocity curves, L2 point offers the lowest energy gate to the inner realm of the three-body system, as explained in Section 4.3.
4.5 Bisection Method of Transfer Orbit Generation in CR3BP

Here the general form of transfer orbit generation by using a simple bisection algorithm is explained. It is initially given in the work of Hechler et al. (2002), then in Ren & Shan (2014). The algorithm is used for generating landing trajectories in CR3BP by using the L2 gate. The details, dedicated adaptation of the algorithm to the problem is explained in the next chapter.

An example can be given for a spacecraft in a parking orbit around secondary body in a three body system, which is shown in Figure 4-4. Three body system now has its zero velocity surfaces for the energy level defined by the motion. If some manoeuvre velocity gradually is added to spacecraft orbital velocity, zero velocity surfaces changes, as explained in section 4.3. If the critical energy is not achieved after manoeuvre, spacecraft bounces back in zero velocity surfaces and no transfer occurs to outer realm of zero velocity surfaces. On the other hand, if the critical energy is exceeded, then spacecraft would escape to outer realm.

To do that two surfaces are defined, $\Sigma^-$ and $\Sigma^+$ where Lagrange point of interest is located in between, which determines inner and outer borders of transfer and non-transfer trajectories. The border locations are determined as a fraction of Lagrange point location (Ren & Shan, 2014):

\[
\begin{align*}
\Sigma^- &= (1-p) \times R_{L_i} \\
\Sigma^+ &= (1+p) \times R_{L_i}
\end{align*}
\]

(4-18)

$p$ is the fraction and $R_{L_i}$ is the chosen Lagrange point distance. Note that the fraction should be selected carefully. It should be large enough to contain Lagrange point orbits, and small enough to ensure that secondary body does not cross the border and stays inside of inner border. Another important point is time-of-flight, which should be long enough to understand transfer or non-transfer trajectories. An illustration is given in Figure 4-4.
The algorithm requires two velocities, lower and upper boundaries, which must ensure that lower boundary does not transfer whereas upper boundary does. This is determined by an initial scan (Ren & Shan, 2014).

The algorithm flow chart is given in Figure 4-5 below. If it is to be explained by words, the algorithm steps can be listed as follows:

**Step 1.** Average value of upper and lower boundary velocities is computed.

**Step 2.** Average velocity computed in Step 1 is used as initial condition to solve the equations of motion of CR3BP, alongside position vector of spacecraft on orbit around secondary. Time of flight (simulation duration), tolerances and special events to flag or to terminate solution are also added.

**Step 3.** Solution obtained from CR3BP is investigated whether trajectory is a transfer or non-transfer trajectory. Flagged events are used to decide.
Step 4. If trajectory is a transfer trajectory, then upper limit velocity is equated with average velocity and interval is shrunk to the lower half. If it is not a transfer trajectory, then lower limit velocity is equated with average velocity and interval is shrunk to the upper half.

Step 5. If tolerance are not reached, the algorithm turns to Step 1. Otherwise, the algorithm goes to Step 6.

Step 6. If tolerance is reached, then solution is obtained.
Figure 4-5 Flowchart of bisection method of transfer trajectory generation
5 PROBLEM STATEMENT AND ANALYSIS

5.1 Introduction

In this chapter, problem statement is given and relevant analysis that had been performed is presented. Problem statement includes the mission architecture envisaged, relevant requirements determined for landing problem, the algorithm developed and the cases that are considered for this problem. In analysis part, trajectories are analysed, in terms of energy levels, landing velocities and deployment velocities.

5.2 Mission Architecture

This research considers a mission architecture that includes a mother ship carrying one or several CubeSats. The mission is targeted to a binary asteroid system, where there are primary and secondary bodies, secondary is orbiting primary. The mother ship is in an operational orbit in such a way that the possibility of collision, or contamination of instruments, due to mass shedding of asteroid is ruled out. That is under the assumption of asteroids in binary system are spherical. This is the equivalent of zero-velocity surfaces closed at the L2 point, while the mother ship would orbit in the exterior region. CubeSat deployment will be performed whilst mother ship is orbiting, the deployment would provide sufficient velocity to open up the zero velocity surfaces at the L2 point, and hence CubeSat can then ballistically land onto the secondary in binary system. An illustration of proposed mission architecture is given in Figure 5-1.
A set of requirements are defined for proposed mission architecture in order to prevent any damage to CubeSat and avoid drifting after bouncing off the surface whilst considering simplicity. These requirements are:

- **CubeSat deployment shall be performed by means of a spring mechanism.**
- **Spring mechanism shall provide sufficient velocity to CubeSat to open the zero-velocity surfaces near the L2 point.**
- **CubeSat deployment velocity shall be limited to maximum of 2 m/s.**
- **The landing velocity shall be such that the final touchdown velocity is in local vertical direction.**
- **The magnitude of landing velocity shall be sufficiently small to prevent damage to CubeSat.**

Spring mechanism is set for simplicity, nevertheless it shall be capable of providing required velocity to open-up L2 gate as well as maximum of 2 m/s. Maximum deployment velocity is set by considering standardised CubeSat
deployer, named as Poly Picosatellite Orbital Deployer (PPOD), which is the common deployment mechanism for piggy-back CubeSats (CalPoly, 2014).

Figure 5-2 Poly Picosatellite Orbital Deployer (PPOD) (CalPoly, 2014)

The rationale behind the landing velocity magnitude and direction is to prevent damage to CubeSat and mission itself. Large magnitude of velocities on landing would cause structural damage, which in turn would affect performance of electronics in it. On the other hand, if direction of the landing velocity is other than local vertical, i.e. if there are non-zero horizontal velocities, CubeSat drifts after bouncing from the surface, which could result in extra damage on landing. In the worst case, CubeSat may escape from binary system and be lost in space forever.

5.3 Generation of Landing Trajectories

5.3.1 Methodology

The ultimate goal of the research to provide insights about the regions and sizes of binary asteroid systems that could potentially be explored by a simple, under-actuated system. After defining the mission architecture, the next step will be landing trajectory generation. General form of bisection method of transfer orbit generation is given in Chapter 4. For ballistic landing problem, the algorithm given is adapted to the purposes of this research. The mission architecture states that deployment is performed exterior region of zero-velocity surfaces, however it is
not specified where the deployment should exactly occur. Similarly, the regions that could be reached are unknown due to unknown deployment position, velocity and time.

The approach here is to assume whole body of secondary is reachable. Similar to meshing, landing locations (points) are predefined with latitudes and longitudes over the surface of the secondary. The equator is defined as $0^\circ$ latitude, whereas the longitude that crosses x-axis at the closest point to L2 point is defined as $0^\circ$ longitude. Then trajectories are propagated backwards from the surface through the L2 gate within the CR3BP, by taking the advantage of the fact that the problem is symmetric, i.e. backwards integration is the symmetrical image of forward integration. Backwards integration offers more deployment options to be investigated for given duration of landing compared to forward integration, in which few particular points must be chosen and landing duration must be known to reach the surface of secondary.

![Figure 5-3 Representation of binary asteroid system](image)

Several termination and escape conditions are defined in order to generate desired trajectories. Once trajectories are generated, Jacobi Constants for energy levels and landing velocities are found.

The resulting trajectories are then used to find the times when mothership is in exterior region of zero-velocity surfaces, in order to find deployment positions, velocities and times. Details of these steps are given in the next section.
5.3.2 The Adapted Bisection Algorithm

The bisection is algorithm in Chapter 4 is a general form the algorithm, which needs to be adapted for the ballistic landing problem here. The most distinctive adaptation is the backwards integration, i.e. CubeSat launches off the surface of secondary, which corresponds to landing in forward problem. Another application of similar approach in forward problem can also be used for termination orbits in end-of-life disposal orbits of Lagrange point spacecraft (Olikara, Gomez, & Masdemot, 2013).

General bisection algorithm considers two borders around Lagrange points, inner and outer borders, in order to generate Lagrange point orbits. Here, only one border is considered, which is defined on the L2 point. That is because the general bisection algorithm is used to generate periodic Lagrange point orbits (Ren & Shan, 2014), only the transit trajectory through L2 gate is generated in this research.

Solution is terminated if any trajectory ends up crashing on one of the bodies in the system. In this case algorithm assumes transfer trajectory is not found and algorithm interval is shifted to upper half of it, assuming that tolerance is not reached.

Transfer trajectory to the exterior region of zero velocity surfaces (once again, landing trajectories in forward problem) are ensured with two conditions below:

1. Trajectory must transit through L2 distance at least once.
2. Trajectory must spend at least 8 hours of time at a distance larger than the L2 distance.

If these two conditions are not satisfied, the algorithm assumes transfer trajectory is not found and the average of the upper and lower boundary is set as lower boundary velocity, i.e. shifting the interval to the upper half of the previous one.

The bisection algorithm requires initialisation of an interval as explained in Section 4.5. The initialisation must be such that lower boundary velocity must be small enough not to allow transfer trajectory, whereas upper boundary must be
large enough to allow transfer trajectory. The details of initialisation is explained in next section.

5.3.2.1 Initialisation

Initial interval of bisection algorithm is determined by using L2 energy. The assumption of local vertical landing provides the directions of initial velocities over the surface of secondary. An example is shown in

![Figure 5-4 Directions for initial velocities](image)

Given that L2 point is stationary where the velocity at that point, \( v_{L2} = 0 \), then Eq. (4-16) becomes,

\[
C_L = -\frac{1}{2}(x^2 + y^2) - \frac{(1-\mu)}{\rho_1} - \frac{\mu}{\rho_2}
\]  

(5-1)

The Eq. (5-1) gives general form of the equation for Jacobi Constant, and hence the energy, of Lagrange points. The velocities on the surface of the secondary at the L2 point energy can then be found. In order to do that, geometry on the secondary surface should be defined.
From Figure 5-5, positions can be derived by following equations:

\[
\begin{align*}
    x &= 1 - \mu + r_s \cos(\lambda) \cos(\alpha) \\
    y &= r_s \cos(\lambda) \sin(\alpha) \\
    z &= r_s \sin(\lambda)
\end{align*}
\]  

(5-2)

where, \( \mu \) is mass parameter, \( \lambda \) is latitude, \( \alpha \) is longitude, and \( r_s \) is normalised secondary radius.

The set of equations above defines the components of positions on the surface of secondary in barycentric frame. Hence, position in barycentric frame can be found by using

\[
r = \sqrt{x^2 + y^2 + z^2}
\]

(5-3)

As described in Section 4, positions in primary and secondary frame can be given as:
\[ \rho_1 = \sqrt{(x + \mu)^2 + y^2 + z^2} \]
\[ \rho_2 = \sqrt{(x - 1 + \mu)^2 + y^2 + z^2} \] (5-4)

The values found in Eqs. (5-3) and (5-4) together with the L2 energy is substituted in general formula for Jacobi Constant. The rearranging Eq. (4-16) gives the velocity:

\[ V = \sqrt{2\left(C_{L2} + \frac{1}{2}(x^2 + y^2) + \frac{(1-\mu)}{\rho_1} + \frac{\mu}{\rho_2}\right)} \] (5-5)

Using same value as the L2 point energy keeps the L2 gate closed (it is just a point though), and makes sure that trajectories do not transit outside. In order to make sure that upper boundary enables transfer trajectory, a sufficiently large arbitrary coefficient would be multiplied by \( v \). In this work, it is selected as 12. Then, initial boundary of bisection algorithm becomes as follows:

\[ v_{\text{initial}} = \begin{bmatrix} v & 12v \end{bmatrix} \] (5-6)

Figure 5-6 shows the velocities \( v \) over the secondary surface.
Figure 5-6 L2 energy velocities over the surface of secondary

As seen in Figure 5-6, the highest initial velocities are observed in equatorial regions, and the lowest are observed at poles or near pole locations.

Figures below show initial trajectories given in upper and lower boundary of initial interval. The algorithm then finds the minimum velocity required in order to land on the secondary by opening up the L2 gate.

Figure 5-7 Lower boundary non-transfer trajectory
5.3.3 Energy of Landing Trajectories

Once trajectories are generated, resulting velocity found is the landing velocity. Within the dynamics of CR3BP problem, Jacobi Constant defines the energy level of a trajectory. Jacobi Constant on landing must be larger than Jacobi Constant of L2 point, it is the result of the fact that transfer trajectories transit through L2 gate and it needs to be opened.

As explained in Section 4.3, having these gates closed in zero velocity surfaces would make sure that spacecraft cannot escape from the region enclosed by zero velocity surfaces. Hence, if some of energy on landing is damped by some means (actively or passively), CubeSat is made sure that it will stay in region enclosed. In order to find out how much energy to be damped on landing, in terms of percentage, can be found by following formula:

\[
\text{% of energy to be damped} = \left| \frac{C_{\text{landing}} - C_{L2}}{C_{\text{landing}}} \right| \times 100
\]  

(5-7)
The values found here, along with landing velocities and asteroid surface properties, can be used for CubeSat design and/or operational decisions before landing.

### 5.3.4 Deployment Velocity

As mentioned in Section 5.2, deployment velocity of CubeSat must be such that the L2 gate of zero velocity surfaces would allow to be opened up. This is forward problem description, which is to be achieved. However, solution to the problem here is reversed. Thus, in backwards description it corresponds to closing of L2 gate. Note that any energy change on mother ship would result in a change on zero velocity surfaces. Therefore, problem can be seen as a manoeuvre performed by mother ship with a given $\Delta v$. Then this $\Delta v$ would make the energy such that energy of the motion exceeds the L2 energy. The Eq (4-16) for L2 gate can be written in a new form by considering $\Delta v$ to be given (Olikara, Gomez, & Masdemot, 2013):

$$C_{L2} = \frac{1}{2} \left( V + \Delta v_{\text{min}} \right)^2 - \left[ \frac{1}{2} \left( x^2 + y^2 \right) + \frac{1-\mu}{\rho_1} + \frac{\mu}{\rho_2} \right]$$

Rearranging the equation to obtain $\Delta v_{\text{min}}$ gives

$$\Delta v_{\text{min}} = \sqrt{2 \left( C_{L2} + \left[ \frac{1}{2} \left( x^2 + y^2 \right) + \frac{1-\mu}{\rho_1} + \frac{\mu}{\rho_2} \right] \right) - V}$$

$V$ is the mother ship velocity at the time of manoeuvre when the L2 gate is closed. Note that $\Delta v_{\text{min}}$ is the magnitude of the manoeuvre velocity. If the direction is of interest, then it should be investigated specifically for that particular orbital position that is defined in trajectory solution.

One important point should be noted that $\Delta v$ computed is the minimum manoeuvre velocity to reach the L2 energy in a particular orbital position. In order to allow the motion to inner region of zero velocity surfaces, given $\Delta v$ must be larger. Infinitesimally small fraction of velocity larger than $\Delta v_{\text{min}}$ allows motion to inner region.
Δν > Δν_{min} \quad (5-10)

5.4 Simulation Cases

5.4.1 Hypothetical Binary Asteroid

This case is one of the two cases used in simulations. Secondary radius and orbit semi-major axis is proportional to the radius of the primary, as commonly mentioned in Chapter 2. Density of the asteroid is assumed homogenous and density value is used as given in (Yáñez, Sanchez Cuartielles, & McInnes, 2014). Binary system is assumed to have circular orbit about its common centre of mass and orbital period is computed by using Keplerian orbit period formula. Whilst creating this hypothetical asteroid, the proportions are arranged such that the created asteroid is consistent with observed binary asteroids, as explained in Chapter 2.

The hypothetical radius is selected as 1000 m, which is in fact a large size of binary among the ones observed in NEAs so far. Tardivel & Scheeres used the binary 1996FG3 in their work, which has a diameter of 1900 m (2013). If the research here ensures that such a large asteroid is feasible for landing, then it can be concluded that it is feasible to land most of the NEA binaries.

Properties of hypothetical binary asteroid is given Table 5-1.

<table>
<thead>
<tr>
<th>Properties of hypothetical binary asteroid</th>
<th>Primary</th>
<th>Secondary</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius [m]</td>
<td>1000</td>
<td>0.35 x R_{primary}</td>
</tr>
<tr>
<td>Density [g/cm(^3)]</td>
<td>2.6 (Yáñez et al., 2014)</td>
<td></td>
</tr>
<tr>
<td>Mass [kg]</td>
<td>1.1 x 10^{13}</td>
<td>4.7 x 10^{11}</td>
</tr>
<tr>
<td>Mass Parameter ((\mu))</td>
<td>0.0411</td>
<td></td>
</tr>
<tr>
<td>Orbit semi-major axis [m]</td>
<td>3.25 x R_{primary}</td>
<td></td>
</tr>
<tr>
<td>Orbital period [h]</td>
<td>11.74648 h</td>
<td></td>
</tr>
<tr>
<td>Sphere of Influence [m]</td>
<td>18952.93</td>
<td></td>
</tr>
</tbody>
</table>
5.4.2 Binary asteroid 1996GT (65803) Didymos

Asteroid Didymos is the target of Asteroid Impact & Deflection Assessment (AIDA) Mission. AIDA is a collaborative mission of ESA, German Aerospace Agency (DLR), Observatoire de la Côte d’Azur (OCA), NASA and John Hopkins University Applied Physics Laboratory (JHU/APL) (ESA, 2015). It is a combination of NASA and ESA missions which has primary goal of demonstrating capabilities of asteroid deflection. NASA spacecraft Double Asteroid Redirection Test (DART) is going to impact on the secondary of Didymos, whereas ESA Asteroid Impact Mission (AIM) is going to rendezvous with Didymos and perform the observation part in order to understand the outcomes of the impact (ESA, 2015).

Following the announcement of the AIDA mission, ESA called for a SysNova challenge for CubeSat mission proposals of which AIM will carry two, in addition to MASCOT lander to be deployed on the secondary body (ESA, 2015). Primary goal of CubeSats are one of the secondary goals of AIM, testing inter-spacecraft links in deep space missions and demonstrating capabilities of CubeSats in interplanetary medium (ESA, 2015).

Considering the relevance of mission to the research here, Didymos case is taken into consideration, as well. Didymos is much smaller asteroid compared to the hypothetical binary asteroid and its density is also much lower. Several different sources are investigated for properties of the binary system. At the end, the online database Johnston’s Archive is used (2015). Orbit semi-major axis is computed from observed period by using Keplerian orbit period formula.
Table 5-2 Properties of binary asteroid 1996GT (65803) Didymos

<table>
<thead>
<tr>
<th></th>
<th>Primary</th>
<th>Secondary</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius [m]</td>
<td>375 ± 50</td>
<td>85 ± 15</td>
</tr>
<tr>
<td>Density [g/cm$^3$]</td>
<td>1.7 ± 0.4</td>
<td></td>
</tr>
<tr>
<td>Mass [kg]</td>
<td>$3.75 \times 10^{11}$</td>
<td>$4.37 \times 10^9$</td>
</tr>
<tr>
<td>Mass Parameter ($\mu$)</td>
<td>0.0115</td>
<td></td>
</tr>
<tr>
<td>Orbit semi-major axis [m]</td>
<td>1056.2</td>
<td></td>
</tr>
<tr>
<td>Orbital period [h]</td>
<td>11.8992</td>
<td></td>
</tr>
<tr>
<td>Sphere of Influence [m]</td>
<td>4868.81</td>
<td></td>
</tr>
</tbody>
</table>

5.5 Analysis of Landing Trajectories

5.5.1 Equatorial Landing Trajectories

Trajectories are analysed in terms of energies by looking at Jacobi Constants, but comparative analysis of lower or higher regions are analysed based on the percentage of energy difference relative to L2 energy. Then velocities on landing are shown, as well as deployment windows of each longitude over the course of operational time are analysed. Hypothetical case is examined first, whereas Didymos case results are given later. An example equatorial trajectory is shown in Figure 5-9.

Figure 5-9 An example of equatorial landing trajectory
One important information to be noted before summarizing the results is that secondary orbit in a binary system is generally tidally locked. That means secondary always faces the same region of primary, like the Earth and the Moon case. That would provide much better understanding when talking about regions feasible for landing.

5.5.1.1 Hypothetical Case

The first point which can be mentioned is that the results are not symmetric over whole longitudes. It means that the results in the first 180° are not repeating itself for remaining half. The reason is the rotation of binary system around their common centre of mass. If landing is to be performed in the opposite direction of the rotation, binary system appears to be “capturing” lander and pulls it gravitationally. That eventually results in much lower energy levels for trajectories. On the other hand, if lander is going for landing in rotation direction, lander must “catch” secondary, thus it requires more energy. Energy levels of trajectories are given in Figure 5-10.

![Hypothetical Case - Energy Levels of Equatorial Landing Trajectories](image)

**Figure 5-10 Hypothetical case: Energy levels of landing trajectories**

The main interest is L2 energy landing trajectories as it provides the lowest energy gate. It can be seen that the regions that “faces” L2 point have the lowest energy trajectories. Those correspond to 0°-95° and around 320° to 360° of longitudes. Effectively 1/3 of longitudes in total are available for low energy
landings. After the first quarter of longitudes, the required energy for landing opens up L3 gate, as well. The difference in energies between Lagrange points are generally very tiny. As it is mentioned before, L4 and L5 gates open up simultaneously for given energies. The required energy to land “back” region of secondary (relative to L2 point) is much higher. Here, zero velocity surfaces are opened up completely and allows the motion in all directions.

![Diagram of Lowest energy trajectories](image)

**Figure 5-11 Hypothetical case: Regions of lowest energy trajectories**

An important point to be mentioned is so called “no-landing” trajectories. The sharp increase in energy levels after about 170° of longitude is resulted from crash on primary body. Those trajectories need to pass through primary before reaching secondary, which will result in crash on primary in reality. For around 25° of region in the back, there are no landing opportunities exists. An example of a “no-landing” trajectory is given in Figure 5-12.
Figure 5-12 A "no-landing" trajectory (crashing to primary)

Note that values are scaled in contour map, i.e. if a trajectory has Jacobi Constant larger than 0, it is equated to 0. Otherwise, since some trajectories have very high energies, they shadow the low energy trajectories in contour map.

Even though Figure 5-10 and Figure 5-11 gives an idea of regions where low energy landings possible, they do not say about more details. A percentage of energy difference compared to L2 point energy can be shown in order to show more detail about regions which require lesser or higher energy trajectories than the others.

This analysis also has an importance for a mission analysis of landing. It can be decided by this how much energy would be required to damp in order to decrease energy of trajectory to L2 point energy level. By doing that, it can be ensured that lander would not be able to escape from binary system. An illustration of the result of energy damping is shown in Figure 5-13 and the results of how much energy to be damped by region is shown in Figure 5-14.
Figure 5-13 Closing of L2 gate by energy damping on landing (white regions are forbidden)

Figure 5-14 Hypothetical case: Energy to be damped to L2 point energy

Consistent with energy levels of trajectories given in Figure 5-10, lowest energy trajectories have minimum difference with the L2 energy. Nevertheless around 70° part of L2 facing region require only about 5% more energy than L2 point energy. Also, it is worth noting that maximum difference is about 20% up to the “no-landing trajectories” which cross primary in order to reach secondary.
Figure 5-15 Hypothetical case: Regions within 5% of L2 point energy

Similar to the previous analysis, if energy is more than 100% of L2 energy it is equated to 100%. Because, it is also not possible to damp more than 100% of energy in a passive sense.

Another important analysis to be made is for velocities on landing. Those are the resulting velocities after tolerances reached in bisection algorithm.

Two-body escape velocity from the system in hypothetical case is found as 0.74 m/sec, which is also important to assess reliability of landing in case of bounce from surface of asteroid. Results are given in Figure 5-16.
Figure 5-16 Hypothetical case: Velocities on landing

As one can expect from previous analysis on energy levels, lowest landing velocities are observed in lowest energy trajectories. However, the minimum velocity on landing is found as 0.29 m/sec and it is observed in about 45°-50° region in the L2 facing side. Up to crashing trajectories, no landing more than about 0.5 m/sec is observed. Only some trajectories landing to back side of secondary experiences landing velocities more than escape velocity.

Velocities larger than 1 m/sec is equated to 1 m/sec, as previously done for energies. It allows to see clearer values in contour map.

Figure 5-17 shows the lowest landing velocity region on a pie chart figure for the equatorial region.
Figure 5-17 Hypothetical case: Regions of lowest velocities on landing

The last analysis for hypothetical case is for landing duration and deployment velocities. It provides the insights about time required for each longitude at equator. The time required can be interpreted essentially as the time needed for landing operations once mothership is ready to deploy lander. Thus, it would affect decisions for landing site and time selection. For such landing, two criteria appears to be important. First of all, landing site must be suitable to aforementioned requirements for safe landing. Second, landing duration must be as short as possible, preferably in working hours over the course of one working shift of personnel available. Short landing is also important for another reason, which is related to the requirements as well. Lander is under effect of various perturbation sources throughout landing period, none of them are considered for analysis other than the point mass gravitational attraction of two bodies. If landing takes longer, lander’s motion is more likely and strongly to be perturbed. Thus, the considerations for landing is a trade-off between duration time and reliability.
The analysis is performed based on the method given in section 5.3.3 and results are shown in Figure 5-18. The figure can be interpreted as deployment windows, similar to that of launch windows. It gives the information about landing duration on vertical axis, manoeuvre velocity in contour that is necessary to open up zero velocity surfaces from L2 point for corresponding longitude at equator. However, it does not provide any information about deployment direction, which should be computed for the specific longitude, landing duration and therefore deployment velocity from a given set of solutions for trajectories.

The results show that options are limited for landings which take less than 6 hours. A region with high energy trajectories and a small portion of low energy regions have such landing options. Though deployment velocity is within the given limit (less than 2 m/sec), energy level is higher than proposed L2 landing.

![Hypothetical Case - Deployment Velocities for Equatorial Landing Trajectories [m/s]](image)

**Figure 5-18 Hypothetical case: Deployment options**

For 6 hours and more there are plenty of landing opportunities. One of the most desirable landing duration options would be 6-12 hours. For that window, there are options in lower energy regions with deployment velocity less than 0.5 m/sec. Deployment velocities in general seems within the limit.
5.5.1.2 Didymos Case

Similar to the hypothetical case, results are not symmetric in Didymos case. The reason for it is given in Section 5.5.1.1. The energy levels of trajectories is shown in Figure 5-19.

![Didymos Case - Energy Levels of Equatorial Landing Trajectories](image)

**Figure 5-19 Didymos case: Energy levels of landing trajectories**

Lowest energy landings appears to be possible for around 140° out of 360° longitudes in L2 facing region. The difference between energy levels of all gates are very small, especially between L3 and L4/L5 transition, and it takes only 10 more longitudes for zero velocity surfaces to be all open at around 120°. There is no transition between those two on the other end, a sharp decrease in energy occurs at around 320°.
“No-landing” trajectories, which are mentioned in Section 5.5.1.1, are observed in regions similar to the hypothetical case, which are around 170° to 200°.

The upper limit of the contour map is determined as Jacobi Constant equals to 0, similar to the previous case, in order to represent lower energies much better in the colour scale.

A more detailed comparative analysis between different regions relative to L2 point energy can be seen in Figure 5-21.
Landing on Didymos system appears to be much softer than that of the hypothetical system. Lowest energy trajectories are within the limit of 5% of L2 energy. There are no trajectories that have energy more than 10% of L2 energy up to very high energy trajectories in back side. That would be because of lower density and dimensions of the system, which results in L2 points to be closer to the binary system compared to higher density and larger dimension hypothetical model.
Figure 5-22 Didymos case: Regions within 1% of L2 energy

Note that values exceeding 100% is equated to 100% for the reasons given in Section 5.5.1.1.

The next analysis is for landing velocities. As previously mentioned, these are velocities that are obtained when tolerance reaches in bisection algorithm. Two-body escape velocity is found to be 0.37 m/sec (Ernst & et al., 2014). Figure 5-23 shows the landing velocities computed for Didymos case.

Landing velocities less than 0.1 m/s is observed in majority of the equatorial region. It is on the order of 0.05-0.06 m/sec for energy levels for the longitudes very close to L2. It can be claimed that a very soft landing can be performed with such velocities. Moreover, escape velocity is exceeded only for the regions where high energy trajectories exist.
Deployment options is the last analysis for Didymos case for equatorial case. The options for landing durations of less than 6 hours appear to be limited, that are
available only in the higher energy trajectories. Though landing velocities are very low, energy to be damped for those is very high, some of which is not even possible for an underactuated lander. Such landing may not be feasible in the first option. From 6 hours onwards, and up to 2 days there are options with low deployment velocities.

![Figure 5-25 Didymos Case: Deployment Options](image)

**Figure 5-25 Didymos Case: Deployment Options**

It is easily seen that deployment velocities are very low, sometimes as low as 0.01 m/sec. In colour scale some of the values are not represented very clearly. Deployment velocities for those are as low as $10^{-4}$ m/sec, it is two order of magnitude lower than the colour map values that are shown. Feasibility of such small velocities on a mother ship may be argued, however, those may be considered as among the possible options. One point relevant to such discussion is worth noting. For the problem statement here, there is no requirement for minimum deployment velocity is determined. In SysNova challenge, ESA requirement for deployment velocities of CubeSat payloads is given as 0.5 m/sec to 2 m/sec initially. These values also seem under assessment, though (Carnelli, Galvez, & Walker, 2015). If this requirement is to be followed, some of the options might need to be extracted from the possible options. Moreover, the maximum deployment velocity of 2 m/sec is reached for some regions.
5.5.2 Landing Trajectories in 3D

In this section, a broad analysis about landing trajectories is performed. The trajectories are analysed over whole secondary body in both cases, by providing the energy levels, velocities on landing and deployment velocities. However, deployment velocities are not given over the whole body but only for a certain latitude, which is selected to be as 60°. It is an arbitrary latitude, there is no particular reason other than being higher than equatorial latitude.

As claimed in Section 5.5.1, solution is not symmetrical in longitudinal direction for equatorial longitudes. That is also true for whole body, with the reasons given in the same section. However, in latitudinal direction solution is symmetric. The reason behind is that binary systems are created as two spheres with homogenous density. The initial conditions are the same in northern and southern hemisphere, except that they have different signs that represents different directions. As explained in Chapter 4, the resulting trajectory is determined by its initial conditions. Therefore, trajectories are the same in northern and southern hemisphere in terms energies, resulting landing velocities, etc., provided that the same algorithm is applied.

The results for both cases are given in next two sections.

5.5.2.1 Hypothetical Case

Similar to equatorial landing trajectories, lowest energy trajectories are observed in L2-facing or near L2-facing sides. However, when latitude increases, lesser longitudes appear to be reachable by low energy trajectories, which eventually extend up to 80° of latitude in north-south direction where eventually L3 and L4/L5 transition is rapid.
“No-landing” trajectories still exist at around 170° to 200°. They are decreasing in number when latitudes are increasing and extends up to about 20° latitudes. Nevertheless very high energy trajectories are not observed only for crashing ones. Some trajectories in the longitudes interval between 250° to 300° have very high energies, and they extend up to near polar latitudes. Their magnitude of energy is much higher around near equatorial latitudes, however. The reason for those trajectories is that they rotate around primary body before reaching the
secondary due to the higher gravitational attraction of primary. An example trajectory is given in Figure 5-28.

As previously done for equatorial trajectories, Jacobi Constant values greater than 0 is equated to 0.

The 5% regions extend up to 60°-65° latitudes in L2-facing region up to around 90° in horizontal direction in near equator. Figure 5-29 shows the results. As done before, the maximum value for it is 100% and all values that are greater than that are equated to 100%. For very high energy trajectories mentioned previously and shown in Figure 5-28, the percentage of energy is greater than or equal to 100% they extend up to near polar latitudes.

Figure 5-28 A very high energy trajectory
The minimum landing velocity observed as 0.29 m/sec. They are observed in lowest energy region, as expected. The majority of trajectories end up with velocity less than the escape velocity, which is previously found as 0.74 m/sec. Only higher energy trajectories end up on surface with velocities higher than the escape velocity.
Figure 5-31 Hypothetical case: Velocities on landing for all trajectories

Figure 5-32 Hypothetical case: Velocity on landing on spherical surface (L2-facing region (left))

For deployment windows (or options), 60° latitude is selected as a reference, as previously mentioned. The result is given in Figure 5-33.
The options appear to be much more limited than equatorial trajectories. It was observed previously that both cases have options for deployment at which landing takes less than 6 hours, even though those are very high energy trajectories. There still exists less than 6 hours landings, but they are limited to high energy regions now. The options exist in low energy side is beginning from exactly 6 hours at the tip of secondary.

There are number of opportunities for 6-12 hours of landing and more, both in low and high energy regions. Majority of the options are available for landing duration 12 hours or more. Note that there are no “no-landing” trajectories exist at this latitude, thus options are spread over the whole longitude.

All deployment velocities are less than 1.8 m/sec, which is below the upper limit for it. Following to the previous discussion, very small deployment velocities, and thus deployment options, exist which couldn’t be represented very well in colour map.

5.5.2.2 Didymos Case

The smaller size and lower density of Didymos result in closer L2 point to the secondary. Thus, lower energy trajectories extend much wider longitudes in equatorial region. Also, they extend up to polar latitudes in L2-facing region. Note that this valid only for low energy trajectories. If one considers the reachability of
whole longitude, then, it should be considered after about 20° latitude. Because crashing trajectories no longer exist after that latitude, and whole longitude becomes reachable.

![Diagram](image-url)

**Figure 5-34 Didymos Case: Energy levels of all landing trajectories**

**Figure 5-35 Didymos Case: Energy levels on spherical surface (L2-facing region (left))**

When one looks at the energies that are need to be provided in order to open the L2 gate, i.e. they are also the energies to be damped in order to close the zero velocity surfaces, it is observed that providing about 5% more of L2 gate energy
would be enough for worst case. In L2-facing regions, it decreases down to 1%. The results are given in Figure 5-36.

Figure 5-36 Didymos Case: Energy to be damped for all trajectories

Figure 5-37 Didymos case: Energy to be damped on spherical surface (L2-facing region (left))

Landing velocities as low as 6 cm/sec is observed in very low energy trajectory regions, generally around L2-facing region. Additionally, up to 10 cm/sec is observed up to “no-landing” trajectories in equatorial regions, which increases with increasing latitude. However, majority of the surface of Didymos’ secondary
are reachable with landing velocities less than two-body escape velocity of 37 cm/sec. Only exception of this results are observed in high energy trajectory regions. Landing velocity over the secondary surface is shown in Figure 5-38.

![Figure 5-38 Didymos Case: Velocity on landing for all trajectories](image)

**Figure 5-38 Didymos Case: Velocity on landing for all trajectories**

![Figure 5-39 Didymos case: Velocity on landing on spherical surface (L2-facing region (left))](image)

**Figure 5-39 Didymos case: Velocity on landing on spherical surface (L2-facing region (left))**

The last analysis for Didymos case is deployment options for higher latitudes. As it is done for the hypothetical case, 60° degree of latitude is selected as the reference. The results are shown in Figure 5-40.

81
Figure 5-40 Didymos Case: Deployment options for 60 degree latitude

The advantage of L2 facing regions for lower deployment velocities with reasonable 6-12 hours landing duration in equatorial regions appears to be vanished with limited trajectory options with more than 12 hours of landing. In higher energy trajectories, though, the options with less-6-hours of landing is still feasible.

The deployment velocities are well within the limits for this case, as well. The maximum of 1.4 m/sec is observed, whereas the lowest is on the order of $10^{-3}$ - $10^{-4}$ m/sec. That would lead a discussion of such precision. Rosetta deployed Philae lander with high precision deployment mechanism which is capable of providing deployment velocities between 0.05 m/sec to 0.52 m/sec, with an emergency system that is capable of 0.17 m/sec (Ulamec & Biele, 2009). The author believes that future spacecraft would be developed with the capability to provide much lower deployment velocities with higher precision, if higher latitudes are to be selected as the landing site. This might not be specifically happening for CubeSats but other small surface explorer on small bodies.
6 RESULTS AND DISCUSSION

This thesis is an effort of 9 months on astrodynamics of landing for binary asteroid exploration. It contains the theory of binary systems, a perturbation analysis around small bodies, methodology, the problem statement and results. The author hopes that the results would be useful input to future robotic missions to small bodies, particularly for binary asteroid exploration. In the remaining chapters, main research findings will be presented, along with suggestions for future missions and further research.

6.1 Summary of the Main Findings

In Chapter 3, non-spherical shape perturbations and solar radiation pressure (SRP) was compared in terms of their effect in relation to distance from the small body. Provided that small body gravitational field is extremely weak, other perturbations become more dominant. Small body model is taken as an ellipsoid, which is better approximation than a sphere, and gravitational field is computed using spherical harmonics model. For SRP, particle lightness number $\beta$ is generated between $10^{-6}$ to $10^{-2}$ which corresponds to mm to $\mu$m size particles. Non-spherical shape perturbation is largest at the tip of the ellipsoid and dominating a spacecraft up to the distance 4 times the radius (4R). After 4R distance, the spacecraft are dominated by solar radiation pressure. This analysis needs to be considered in conjunction with deployment position, because if the distance of deployment position to the small body is outside of non-spherical perturbation threshold, then SRP is needed to be added to the trajectory generation.

Chapter 4 draws the outline of Chapter 5, in which the dynamical model and algorithm used is given. First few sections of Chapter 5 explains the problem considered, i.e. mission architecture that is envisaged, adapted algorithm and methodology of analysis.

The mission architecture considers a mother ship and one or more several CubeSats. Mother ship has an operational orbit outside of zero velocity surfaces with L2 gate closed. Deployment of CubeSat would provide sufficient velocity to
open up L2 gate and make landing possible to smaller companion of binary system. Local vertical landing is considered which provides maximum energy damping.

Two cases are considered for analysis. Initially a hypothetical binary asteroid case is created by using the existing literature on binary asteroids. A second case is taken in to consideration after AIDA mission is announced by ESA (2015). The binary asteroid 1996GT (65803) Didymos is a smaller and less dense binary system compared to the hypothetical case.

The analysis of trajectories is carried out by considering energies of trajectories in general that are represented by Jacobi Constants, energy levels with respect to L2 gate which is important for energy damping, landing velocities and deployment options that is based on the velocity required to open-up L2 gate.

Initial analysis is performed on equatorial landing trajectories. In general, existing orbit in binary system result in asymmetry in energy levels. Trajectories to land in the direction opposite to the orbital rotation require less energy than for trajectories in the direction of orbital rotation. It is observed for both cases that around 1/3 of longitudes in equator are feasible with energy levels by only opening up L2 gate. These longitudes are facing L2 point. For other regions, the required energy is higher, such that L3 and L4/L5 regions become feasible. There are longitudes where no landing observed, that is because those trajectories are crashing to primary body in the system before reaching the secondary.

The size and density of the binary have an effect of landing trajectories, as well. The smaller and less dense Didymos is feasible for landing with lesser energy with respect to L2 gate. Only up to 1% more of L2 energy is enough for L2-facing regions in Didymos case, whereas the hypothetical case requires around up to 5% for same regions. On the primary facing or back regions, more than 100% of L2 energy is observed for both cases.

Landing velocities are the velocities once the bisection algorithm reaches its tolerance in backwards integration. As observed in energy levels, landing velocities are higher in hypothetical case than Didymos case. However, the
calculated two-body escape velocities are not exceeded for majority of the regions. The exception of this situation is higher energy landing trajectories where more than 100 % of L2 energy needs to be provided in general. Lowest landing velocities are observed in L2 facing regions where lowest energy trajectories are present.

The last analysis for equatorial trajectories are performed for deployment options. Deployment options are computed for any point outside of L2 point. The velocity required to open up L2 gate is taken as deployment velocity, and duration necessary to reach to that point is basically the landing duration to land on that longitude. In both cases, landing durations less than 6 hours are observed to be limited, and only observed in high energy trajectories. From 6 hours onwards, there are number of options in low energy side. For deployment velocities, the values are well within the limit for Didymos case, however there are values that are exceeding the given limit in the hypothetical case, especially for higher energy trajectories, on the back side of secondary. It is worth noting that very small deployment velocities are computed for Didymos case, for those engineering feasibility needs to be assessed.

The remaining part of Chapter 5, the analysis to land whole surface of secondary is presented. Similar to the equatorial case, the results of energy levels, landing velocities and deployment options are presented. However, deployment options are presented only for one latitude, which is 60° degrees, due to the ease of computation.

As it can be expected from equatorial results, L2 facing regions have more latitudes and longitudes to be reached by opening up only L2 gate. At the longitudes that are directly opposite to L2 gate, low energy trajectories are feasible up to polar latitudes. Those are decreasing with increasing longitudes. Similar to equatorial results, up to 1-5 % more of L2 energy is enough to land for Didymos case by low energy trajectories, that is 5-10% for the hypothetical case. Trajectories that crash on primary still exist up to low latitudes. If one considers low energy trajectories, it can be concluded that equatorial latitudes offer most
number of longitudes feasible for landing. Otherwise, all longitudes are reachable at higher latitudes without any crashing on primary.

Landing velocity results are also similar to one observed for equatorial trajectories. For both cases, CubeSat experiences very low landing velocities, they are lower than two body escape velocity. The velocities that are higher than the escape velocities are observed in high energy trajectories, which are mainly in back side of secondary body.

As mentioned before, deployment options are considered only for 60 degree latitude. There were no reason for this particular latitude other than being higher latitude. Deployment options are much more limited at this altitude, even 6-12 hours deployment appear to be limited. However, deployment velocities are well within the limits.

6.1.1 Conclusions

It can be concluded that for a reliable landing on a binary asteroid secondary, a reasonable choice for landing site appear to be on L2 facing side of the asteroid in equatorial region. This region is same in any orbit configuration, as binary systems are tidally locked. The analyses show that L2 facing, equatorial regions require less energy for trajectory, as well as lander experiences lower velocities on landing. Duration of landing would be between 6-12 hours. Rosetta’s lander Philae landed on the comet in 7 hours, thus such choice of landing duration is still reasonable. Given that the CubeSat lander is considered as underactuated, such considerations are much more important for landing site. It is obvious that landing site is not just depended on trajectories but also surface properties. It also should be ensured that local vertical landing requirement is fulfilled.

6.2 Suggestions for Future CubeSat Missions as Lander Modules

The CubeSat concept is revolutionising the use of small systems for space exploration purposes, with the miniaturisation of electronics used. Although there is no CubeSat yet that reached any solar system body, their reliability is
increasing with number of missions launched on Earth orbit. Using such systems, along with other technological advancements would enhance scientific return.

In particular for small body exploration, for example, opportunities for multiple visits to small bodies have an importance, as such bodies are unique in properties. The author believes that, by the aid of optimised low-thrust trajectories, several small bodies can be visited in one launch with mothership concept. Several CubeSats on-board would be benefited as a standardised lander, decreasing costs whilst increasing the scientific return. Same types of instruments may be employed in all these CubeSats.

Several types of scientific data can be gathered by using CubeSat with today’s technology. Subsurface mapping can be performed by using miniaturised spectrometers, radar and seismometers. Similarly gravitational measurements can be done by accelerometers. Surface imaging can be obtained by one or several cameras.

It should also be explained how a landing would be performed. It is already explained in Chapter 5 that landing is envisaged to be performed in local vertical direction. This offers the maximum possible energy damping in landing. Again in the same chapter, the analysis given on how damping of some of the energy on landing would close the zero velocity surfaces back and restricts the motion to interior region of zero velocity surfaces. The CubeSat here is considered to have no actuator to perform damping actively.

An ESA study investigates shock alleviation techniques for landers in two different landing techniques, which are namely penetrator (hard) landers and semi-hard landers (Doengi, Burnage, Cottard, & Roumeas, 1998). Penetrator landers are mainly employed for high impact velocities, such as Moon and Mars landings, whereas semi-hard landers are used for low or medium level of landing velocities, such as small body missions. Rosetta’s lander Philae is used with penetrator, although the landing is soft, however it appears to be doubtful even at the time of the project development (Doengi, Burnage, Cottard, & Roumeas, 1998).
Among the ones taken into consideration for ESA study, the shock absorption devices based on irreversible deformation is the interest here. Crushable aluminium honeycomb structures, foams and deformable tubes offer the highest specific energy absorption, and their behaviour over deformation period is predictable (Doengi, Burnage, Cottard, & Roumeas, 1998). Al honeycomb structures have very low density, and can be deformed up to 80% of its initial length (Doengi, Burnage, Cottard, & Roumeas, 1998). They also have a heritage on Eagle lander of Apollo program and still into consideration for new lunar lander modules (Pham, et al., 2013). The specific energy absorption of Al foams are lower than honeycomb structures. Deformable tubes appear to be creating issues for scaling down the results for different cases (Doengi, Burnage, Cottard, & Roumeas, 1998).

![Figure 6-1 Al honeycomb structure and foam (Doengi, Burnage, Cottard, & Roumeas, 1998)](image)

For their performance and low mass, it is believed that a CubeSat-like system would be used as a lander with given Al shock absorption material, shown in Figure 6-1, easily. The velocities on landing are very tiny, unlike planetary
landing, therefore a small thickness of such materials can be employed on landing surface of CubeSat without occupying too much space. There must be further research on this, surely, however, this approach seems promising for increasing landing reliability.

6.3 Further Research

Several issues need to be highlighted about this research before commenting on further research.

- First of all, uncertainties in trajectory generation are not taken into consideration.
- Secondly, this research considers no perturbation other than point mass gravitational attraction of two bodies. As it is shown in Chapter 3, SRP would be very effective near a small body based on the deployment distance.
- Thirdly, some binary systems are populated in the main belt, in between Mars and Jupiter. The gravitational effects of those bodies (and the Sun) are not presented here.
- Shape, density, surface and gravitational models are used roughly here, without taking any inhomogeneity, roughness, and irregular shape. Those issues would be addressed too.
- Scientific research opportunities need to be identified in order to utilise this approach for wide range of applications.

In the light of issues spotted above, there needs to be further research to be done in order to obtain more about the insights of landing trajectories for binary asteroid exploration. The first step for that would be to perform an uncertainty analysis in trajectory generation. It can be observed how trajectory evolves with given initial perturbations. A second step to add SRP to trajectory generation, in order to see how trajectories are affected by the presence of another perturbation. On top of that non-spherical shape perturbations can also be added by the help of advanced dynamical models. Moreover, gravitational effect of other bodies in trajectory generation also requires different and advanced level of dynamical
models. One of these can be Bi-Circular Restricted Three Body Problem (Bi-CR3BP) which is essentially Restricted Four Body Problem.

The application of all these models to real cases with accurate surface, shape, density, gravity is important in order to see the robustness of such approach. Since all small bodies have unique properties, some very well-known small bodies can be used as reference. Those are observed by ground-based methods for years and their properties are known with higher precision compared to others.

Whilst improving the dynamical model, novel mission opportunities should be investigated in order to utilise CubeSats in interplanetary environment, especially for small body exploration in relation to the research here.
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