Numerical Simulation of Shape Rolling

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Preface

The work presented in this thesis has been carried out at the research institute MIKRAB in Borlänge. At the institute, I have been working with Björn Sjögren and Göran Engberg and I have them to thank for being such valued colleagues and great comrades to me. I give my thanks to all colleagues at Dalarna University for the friendly ambience at work I have enjoyed.

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Abstract

In the first part of this thesis, the FE program MSC.Marc is applied for coupled thermo-mechanical simulations of wire-rod rolling. In order to predict material behaviour of an AISI 302 stainless steel at high strain rates generated during wire-rod rolling, a material model based on dislocation density is applied. Then, the evolution of temperature, strain rate and flow stress is predicted in the first four rolling passes of a wire block.

In the second part of the thesis, an alternative approach to simulation of shape rolling is evaluated. The approach is applied in order to save the computational time in cases where many shape-rolling passes are to be simulated. The approach is a combination of the slab method and a 2D FEM with a generalized plane-strain formulation. A number of various isothermal shape-rolling passes are simulated applying the simplified approach. The simulations are carried out using an in-house 2D FE code implemented in Matlab. The results are compared to fully 3D FE analyses. The comparison shows that the simplified approach can predict roll forces and roll torques with a fair accuracy, but the predicted area reductions are a bit underestimated. The reasons for the deviations between the simplified approach and the 3D FEM are discussed.

Keywords: shape rolling, wire-rod rolling, finite-element method, generalized plane-strain, dislocation density, material model
Appended papers

Paper A: S. Riljak,
*Experimental and Numerical Analysis of Hot Rolling in Wire Block,*
Conference Proceedings of the 7th Esaform Conference on Material Forming,

Paper B: S. Riljak,
*Prediction of Roll Force and Roll Torque in Shape Rolling,*
Conference Proceedings of the 8th International Conference on Technology of
Plasticity, Verona, October 9-13, 2005.

Paper C: S. Riljak,
*A Finite Element Code for Simplified Simulations of Shape Rolling,*
submitted for publication.
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1 Introduction

1.1 Shape rolling

Shape rolling is one of the primary metal-forming processes. The process starts from blooms or billets, delivered from a continuous caster or a bloomery mill. After reheating, the stock passes through a succession of rolling stands. In each pass, the stock's cross-section is reshaped and reduced. After the final pass, the stock comes off the rolling mill in the form of a long bar. This gives the name “long products”, often used to refer a shape-rolling product. Significant for the shape rolling is the great variety of the profiles produced. Basic classification of the long products is shown in Figure 1.

![Figure 1. Classification of long products.](image_url)

In this work, we focus on the process of bar and wire-rod rolling. The whole process can be divided into the following steps:

1. re-heating of billets
2. roughing rolling
3. intermediate rolling
4. finishing rolling
5. cooling and/or heat treatment.

The reheating of the billets is often done in a so-called walking-beam furnace. The temperature of the furnace and the pace of the billet's “walking” through the furnace are set to provide equally distributed temperature at the withdrawal. The temperature is typically in a range of 1100-1200°C, depending on the steel grade. The re-heated billets are transferred to the roughing mill. This is traditionally a two-high or a three-high reversing rolling stand. The purpose of the roughing rolling is to break down the as-cast microstructure and quickly reduce the cross-section of the billet. The stock is then transferred to the intermediate mill where it passes through the rolling stands in a continuous manner, further reducing the cross-section to a profile appropriate for the finishing rolling. In the last finishing passes, the reductions may be milder in order to guarantee accurate profile within dimensional tolerances. In the wire-rod mill, the finishing rolling is performed in a wire block. Typical for the wire block is the high rolling speed. The wire can exit such a block at a speed as high as 100 ms⁻¹, depending on the steel grade. After the rolling is completed, the bar or wire could have experienced as many as forty or more rolling passes.
Modern bar and wire-rod rolling mills are built entirely in the continuous arrangement, so there are no reversing passes involved. This increases productivity of the mills and enables more accurate control of the rolling process. The most innovative rolling mills are integrated with metallurgical processing. The billets are then delivered directly to the mill from a continuous caster positioned in front of the rolling line. An example of such a rolling mill is ABS Luna - Endless Casting and Rolling Plant, Udine, Italy.

1.2 Roll-pass design

Roll-pass design is an essential part of shape-rolling technology. The primary goal of the roll-pass design is to ensure production of a correct profile, free of defects, with good surface quality. Traditionally, the roll-pass design was taken more as art than science. In 1969 a major work, systemizing the principles of roll-pass design, was published by Wusatowsky [1]. Since then, the complexity of shape-rolling processes has increased. Modern wire-rod and bar rolling mills are built as continuous or half continuous production lines. In addition, many new steel grades have been developed, requiring new rolling strategies, such as controlled rolling and controlled cooling, known as thermo-mechanical processing. Today, such processing is common practice, improving product quality by optimizing the microstructure obtained by deformation and temperature driven processes, such as recrystallization and grain growth. Obviously, the roll-pass design becomes even more complicated when optimising the final microstructure and roll-pass designers need to apply sophisticated material and process models in their work. Fortunately, in the last decade, computer aided simulations have become feasible for such an approach, gradually reducing the costly trial and error approach for optimising the shape-rolling process.

Figure 2. Model of a roll assembly in a two-high stand for reversing roughing rolling.
1.3 Objectives of this work

This work concerns the process of bar and wire-rod rolling and the possibilities to model this process using numerical methods such as the finite-element method. The rolling process is a multi-step hot-forming process, so the challenge is to model succession of the forming steps tracing the whole deformation history of the workpiece. This requires fast and accurate modelling tools.

Thus, the objective is to evaluate an alternative numerical approach to simulation of the shape-rolling process. The approach shall not contain the empirical essence of the traditional mathematical models and it shall be far more computationally efficient than the full 3D FE models.

To achieve the goal, an approach based on the so-called generalized plane-strain formulation has been chosen for the evaluation. The formulation is used in the 2D FE method, which is further combined with the so-called slab method. The combination of these methods is implemented in Matlab, which is an effective programming environment for the development of mathematical algorithms. The accuracy and the efficiency of the whole approach are validated. For that purpose, 3D FE simulations of rolling are performed using the program MSC.Marc. Details and results are presented in this thesis and in the appended papers.
2 Mathematical models of shape rolling

Based on everyday practice and extensive experimental work, the rolling engineers recognized a long time ago, that the main factors influencing the free spread of a material in a roll groove are:

1. shape of the groove, also called caliber
2. reduction \( \Delta h \rightarrow \text{spread} \)
3. roll radius \( R \rightarrow \text{spread} \)
4. friction \( \mu \rightarrow \text{spread} \)
5. temperature of the material being rolled \( T \rightarrow \text{spread} \)
6. rolling velocity \( v \rightarrow \text{spread} \)
7. material being rolled and its chemical composition that influences the growth of oxide scale, which in turn influences the friction.

To predict these observations by means of mathematical formulations is a task all but trivial. Nowadays, there are basically two approaches to the mathematical modelling of shape rolling.

The first, traditional, empirical approach in which relatively simple mathematical formulas have been derived in order to predict the global process parameters such as free spread, roll separating force and roll torque. Typically, the formulas contain sets of coefficients that approximate the effect of various rolling conditions. The approach can provide mean values of stress, strain and strain rate, experienced by the material in a rolling pass.

The second, up-to-date approach is based on numerical methods, mainly the finite-element method. Using the FEM, not only the global forces and displacements are successfully predicted, but also the local distribution of strains, strain rates, stresses and temperatures in the deformation zone of a workpiece can be followed. Obviously, the numerical approach is superior to the empirical one, but sometimes, its practical application may be hindered by its complexity and required computational power.

2.1 Empirical approach

As a good example of the empirical approach, spread formula for rolling of square bars according to Wusatowski [1] can be used. The formula reads as

\[
\frac{b_{\text{out}}}{b_{\text{in}}} = a c d f \left( \frac{h_{\text{out}}}{h_{\text{in}}} \right)^{-w} \tag{2.1.1}
\]

and

\[
w = 10 \left( \frac{h_{\text{in}}}{D} \right)^{1.269} \left( \frac{b_{\text{in}}}{b_{\text{out}}} \right) \tag{2.1.2}
\]

where \( b_{\text{in}} \) and \( b_{\text{out}} \) are the initial and final width of the bar, \( h_{\text{in}} \) and \( h_{\text{out}} \) are the initial and final height of the bar and \( D \) is the roll radius. The coefficients \( a, c, d \) and \( f \) approximate the influence of the temperature, rolling velocity, material being rolled and the surface quality and material of the rolls, respectively. The values of the coefficients are evaluated using additional equations, tables and graphs.
In general, the profiles of the rolled bars are more complicated than square. Then, the so-called equivalent rectangular method has been applied. Several variants of the method exist, but the basic idea is the same. The arbitrary profiles are transferred into their rectangular equivalents and various spread formulas are applied, using a mean value of the roll radius. Then, the area of the final cross-section of the bar is predicted, which can be fitted into the profile confined by the shape of the roll grooves. An example of the application of such a method can be found in Lee et al. [2].

Similarly, there are several empirical formulas designed for prediction of the roll force and roll torque during rolling of bars. Most of these are based on flat-square rolling with modifications for various pass shapes, contact area, friction and other process parameters. Comparison of a few such models was carried out by Said et al. [3].

The empirical methods are mainly applied for symmetrical rolling passes such as the round-oval, square-oval and square-diamond passes. The obvious drawback of the empirical methods is that they apply correctly only for the conditions for which the models were evaluated. The advantage is that the models are fast and relatively simple to use. The practical significance of the empirical models is obvious. New models are still being developed and validated using either experiments or more advanced numerical models, Lee [4] and Minutolo et al. [5].

2.2 Numerical approach

A numerical approach that aims to model the process of shape rolling has to face a highly non-linear physical problem. The non-linearity has its origins in both kinematic and material effects such as:

1. complex 3D shape
2. large displacements, large rotations and large strains
3. elastic-plastic material behaviour with thermal and viscous effects
4. time-dependent material behaviour due to evolution of microstructure
5. contact and friction.

The finite-element method has been recognized as a numerical approach that is well capable of solving such problems. Two basic formulations are adopted in the non-linear FEM to deal with the large displacements, rotations and strains. These are Lagrangian and Eulerian formulation, respectively, Bathe [6] and Belytschko et al. [7].

In the Lagrangian formulation the mesh is fixed to the material and follows its movements. Using this formulation free surfaces are traced naturally and deformation history can easily be taken into account. The disadvantage of the formulation is that the mesh can become distorted due to the large deformation. The distortion leads to decreased accuracy of the results or even to a premature end of the calculation. To address the problem, a remeshing procedure can be used, in which the old mesh is replaced by a new one. Critical issues in the application of remeshing are the automatic mesh generation and the accuracy of the transfer of information between the old and new meshes.

In the Eulerian formulation the mesh is fixed in the space and the material flows across the
elements. Since the mesh does not move, the distortion is not an issue here. In general, the material boundaries are not equal to the element edges and special procedures are required to trace the free surfaces as well as boundaries between different materials, including contact. Also the transfer of material properties must be taken into account.

Most of the metal-forming processes involve modification of free surfaces and contact. Thus, the majority of the FEM codes for the simulation of metal-forming have applied the Lagrangian formulation or better the updated Lagrangian formulation. The term updated means that all variables are referred to the last calculated (updated) configuration, Bathe [6].

Some of the first applications of 3D FEM for simulation of shape rolling appeared in the late 1980s. The codes used both elastic-plastic and rigid-plastic material formulations and assumed isothermal rolling process, Liu et al. [8], Park et al. [9] and Yanagimoto et al. [10]. In the mid-1990s, the developments in computer technology and FEM reached a level when larger 3D simulations became feasible. At this point Corus launched a project, which resulted in development of a roll-pass design system based on ABAQUS finite element software, Engelmann et al. [11]. The system has some advanced features such as thermo-mechanical coupling and an adaptive meshing algorithm based on the arbitrary Lagrangian Eulerian formulation, Belytschko et al. [7].

The ALE formulation is a combination of the Lagrangian and Eulerian descriptions. In such a formulation the mesh displacement is not necessarily equal to the material displacement nor equal to zero, but can be chosen independently from the material displacement. The ALE formulation can be used to solve problems with free surfaces in an Eulerian formulation or to avoid grid distortion in a Lagrangian formulation.

In general, a rolling-pass simulation is a transient analysis that includes actual threading of the workpiece through the roll gap. The analysis is run until a steady-state solution is reached, which means that a constant shape of the workpiece is observed at the exit of the roll gap. Such an approach emanates from the application of Lagrangian formulation. An alternative approach uses a mesh that is an initial guess of the final configuration. Then the flow of the material through the mesh is calculated using the Eulerian formulation. The calculation is terminated when the configuration of free surfaces has converged. The crucial point for the steady-state approach is the prediction of free surfaces and the tracing of properties for history dependent materials, Kim et al. [12]. Due to the ability to correctly describe the free surfaces, the ALE formulation was applied by Wisselink et al. [13] in a steady-state analysis of shape rolling and slitting rolling.

Shape rolling is a coupled thermo-mechanical problem, where the material properties strongly depend both on temperature and strain rate. The interaction between the mechanical and thermal field can be solved within one time step leading to simultaneous solutions. However, this approach is often associated with high computational cost. Alternatively, staggered solution technique is used, where the mechanical problem is solved at a fixed temperature, followed by the solution of thermal problem at a fixed configuration. The latter approach is commonly applied in thermo-mechanical analysis of shape rolling, Kim et al. [14], but also in general purpose FE codes such as MSC.Marc [15].

Beside the fully 3D FE analysis of shape rolling, simplified methods were developed in effort to minimize the computational costs. Kiuchi et al. [16] proposed the so-called complex
element method. They assumed that the longitudinal displacement is uniform over any cross-section perpendicular to the rolling direction. The method combined the slab method, to impose the force equilibrium in the rolling direction, and 3D rigid-plastic FEM, to analyse the lateral spread and elongation of the slabs. The approach was further developed by Kim et al. [17]. They replaced the 3D FEM with a 2D FEM with generalized plane-strain formulation, reducing the size of models even more. Application of the method was extended for coupled thermo-mechanical analysis, Serajzadeh [18]. The temperature was predicted by 2D FEM neglecting the temperature gradient in the rolling direction. Glowacki [19] utilized the concept of generalized plane-strain, but the slab method was omitted and the force equilibrium in the rolling direction was not evaluated. The method was also coupled with a thermal model using a 2D FEM.

Today, the simplified methods are still being used. They are applied in analyses of multi-pass shape rolling, where they provide approximations of the displacement and temperature field for complex material models. Such models are often physically-based material models that predict the evolution of mechanical properties, such as flow stress, together with microstructural changes, Serajzadeh [18] and Glowacki [20].

In this study, simulations of wire-rod and bar rolling are performed using both 3D FEM and a simplified method combining the slab method and 2D FEM, equivalent to previous work by Kim et al. [17]. The 3D and 2D simulations are performed using the FE program MSC.Marc and an in-house FE code implemented in Matlab, respectively. Both codes are implicit, non-linear FE codes, based on the updated Lagrangian formulation, MSC.Marc [15] and Paper C.

Figure 3. Distribution of equivalent plastic strain in an oval-round rolling pass as predicted by simplified approach presented in Paper B.
3 Contact analysis

Contact analysis is an inherent part of the numerical simulations of forming processes. In this study, the analysis is simplified assuming that the tool is rigid and only the workpiece can experience deformation. Such an assumption is often used in simulations of shape rolling, where the roll forces are not as high as in flat rolling. The concept of the mathematical model of the problem is shown in Figure 4. There, the boundary, $\Gamma^\Omega$, of the deformable domain, $\Omega$, is divided into sub-domains such that

$$\Gamma_u \cup \Gamma_t \cup \Gamma_C = \Gamma^\Omega \quad \text{and} \quad \Gamma_u \cap \Gamma_t = \Gamma_u \cap \Gamma_C = \Gamma_t \cap \Gamma_C = 0$$  \hspace{1cm} (3.1)

where $\Gamma_u$ and $\Gamma_t$ are the boundaries with prescribed displacements and tractions, respectively and $\Gamma_C$ is the contact interface. The latter is defined as

$$\Gamma_C = \Gamma^\Omega \cap \Gamma^{\text{obs}}$$  \hspace{1cm} (3.2)

where $\Gamma^{\text{obs}}$ is the boundary of the rigid obstacle. On the contact interface, neither tractions nor displacements are prescribed and the extent of the boundary itself is unknown beforehand.

What is known, are the conditions that must be fulfilled on the interface. The conditions read as:

1. $p \leq 0$
2. $\tau^n \geq 0$
3. $\tau^t = 0$ assuming no friction or $|\tau^t| - \mu |\tau^n| \leq 0$ assuming Coulomb friction law
4. $\tau^n p = 0$

The first condition restricts interpenetration of the contact bodies by requiring the penetration, $p$, to be zero or negative. The second condition states that the normal contact tractions, $\tau^n$, assumed positive in compression, must be greater or equal zero so sticking does not occur. The third condition treats the tangential contact tractions, $\tau^t$, in the frictionless case and in the case where frictional effect are modelled using the Coulomb friction law. The last condition requires that the compressive $\tau^n$ are generated only in the instance when contact is occurring.

![Figure 4. Contact problem.](image-url)
To impose the contact interface conditions is not a trivial task. In the numerical analysis, a complete mathematical model of contact consists of the following components:
1. contact constraint method, which provides means of calculating the unknown displacements and tractions under contact constraints
2. contact searching algorithm, which searches potential contact nodes and determines the extent of the contact interface
3. friction law, which governs the frictional effects.

3.1 Contact constraint method

The most common methods of imposing the contact constraints are the penalty method, the Lagrange multiplier method and extensions of these methods such as the perturbed Lagrangian and augmented Lagrangian methods. Description of the underlying theories and the various aspects of the implementation of these methods into numerical solution can be found in Belytscko et al. [7], Laursen [21] and Mohammadi [22].

In this study, the contact problem is simplified assuming that one of the contact bodies, representing the tool, is:
1. rigid
2. displacement driven, i.e. spatial configuration is known at any time during the analysis
3. represented by piecewise linear curves, i.e. for each line segment the normal and tangential unit vectors are constant.

Considering these assumptions, a rather straightforward approach of enforcing the contact interface conditions can be applied. The approach is named direction-constraint method and its application is illustrated by an example. We assume small-displacement analysis of a beam bending, where the beam is made of a linear-elastic material, see Figure 5. The beam is deflected due to the frictionless contact with a rigid wall. Applying the boundary conditions we obtain that

\[
\begin{bmatrix}
  k_S & 0 \\
  0 & k_B
\end{bmatrix}
\begin{bmatrix}
  u_{2x} \\
  u_{2y}
\end{bmatrix} =
\begin{bmatrix}
  f_{2x} \\
  f_{2y}
\end{bmatrix}
\]  
\hspace{1cm} (3.1.1)

where the unknown displacements and forces can be transformed into the normal and tangential components as

\[
\begin{align*}
  u_2 &= u_2^n + u_2^t = u_2^n \begin{bmatrix} n_x^n \\ n_y^n \end{bmatrix} + u_2^t \begin{bmatrix} n_x^t \\ n_y^t \end{bmatrix} \quad \text{and} \\
  f_2 &= f_2^n + f_2^t = f_2^n + 0 = f_2^n \begin{bmatrix} n_x^n \\ n_y^n \end{bmatrix}
\end{align*}
\]  
\hspace{1cm} (3.1.2)

where the component of the normal and tangential unit vectors and the displacement representing the penetration, \( u_2^n \), are known, see Figure 5. The transformed displacements and forces are substituted into equation (3.1.1) and we obtain

\[
\begin{bmatrix}
  k_S & 0 \\
  0 & k_B
\end{bmatrix}
\begin{bmatrix}
  u_2^n \\
  u_2^t
\end{bmatrix} =
\begin{bmatrix}
  f_2^n \\
  f_2^t
\end{bmatrix}
\]  
\hspace{1cm} (3.1.3)

Now, we have a system of two equations with two unknowns, and, using algebra, we can solve the unknown displacement \( u_2^t \) as
Then, the contact displacement $u_2$ is obtained from equation (3.1.2) and the contact force $f_2$ is obtained from equation (3.1.1).

The presented approach has been introduced into the system of non-linear finite-element equations and extended to contact with friction, which is presented in Paper C.

$u_2 = (k_S n_1 n_2 + k_B n_1 n_2) - (k_S n_2 n_1 - k_B n_1 n_2)$

(3.1.4)

Figure 5. Illustration problem: a) initial configuration b) contact detection c) final configuration and d) contact displacements and forces.

3.2 Contact searching

Methods that introduce the contact constraints into the system of FE equations were discussed in the previous section. However, before any constraints are applied to a potential contact node, the contact of the node with the obstacle has to be detected. For this purpose a contact searching algorithm, based on that given by Laursen [21], is used.

The algorithm is designed for application in 2D problems. It detects contact of a node, $x$, with a rigid obstacle represented by a polygon line with vertices, $v_i$. The gap, $g$, between the body and the obstacle is calculated using the method of closest-point projection, see Figure 6 a).

The algorithm can be divided into the following steps:
1. find the closest vertex point, $v_k$, for the given contact node, $x$
2. find the line segment that contains the closest projection of the contact node, $x'$
3. calculate the gap, \( g \), between the body and the obstacle.

In step 1, the distance vectors between the node and all vertices are calculated as

\[ d_i = x - v_i. \]  

(3.2.1)

Then, the distance vector of minimum length, \( d_k \), is found, which identifies the closest neighbour vertex. The search is performed in each iteration for all potential contact nodes, i.e. nodes defining the outline of the deformable body. If the model has \( N \) outline nodes and \( V \) vertices, then the global search of the closest vertices requires \((N \cdot V)\) operations. In order to reduce the number of operations in large-scale problems, the global search may be performed only once, at the very beginning of the simulation. Then, in the subsequent iterations, the search for the closest vertex of a particular node is restricted to the vicinity of the previously found closest vertex.

In step 2, the line segment containing the closest projection of the contact node is found by examining the dot products

\[ d_k \cdot n_{k-1}^t \text{ and } d_k \cdot n_k^t \]  

(3.2.2)

where \( n_{k-1}^t \) and \( n_k^t \) are the tangential unit vectors of the line segments sharing the vertex, \( v_k \). Then, one of the following four cases occurs:

a) \( d_k \cdot n_{k-1}^t \leq 0 \) and \( d_k \cdot n_k^t < 0 \) \( \Rightarrow \) line segment between \( v_{k-1} \) and \( v_k \) contains \( x' \)

b) \( d_k \cdot n_{k-1}^t > 0 \) and \( d_k \cdot n_k^t \geq 0 \) \( \Rightarrow \) line segment between \( v_k \) and \( v_{k+1} \) contains \( x' \)

c) \( d_k \cdot n_{k-1}^t \leq 0 \) and \( d_k \cdot n_k^t \geq 0 \) \( \Rightarrow \) either line segment can contain \( x' \)

d) \( d_k \cdot n_{k-1}^t > 0 \) and \( d_k \cdot n_k^t < 0 \) \( \Rightarrow \) vertex point \( v_k \) is \( x' \).

In the case c, the sought segment is not uniquely defined, see Figure 6 b). Then, the calculation of the gap in step 3 is performed for both segments, and the corresponding distances are directly compared. If the gaps are equal, which is very unlikely, either projection is suitable for the subsequent constraint calculation.

In step 3, the gap between the body and the obstacle is calculated as

\[ g = d_k \cdot n_{\text{closest}}^n \]  

(3.2.3)

where \( n_{\text{closest}}^n \) is the normal-unit vector of the closest-projection segment identified in the previous step. Finally, if the calculated gap is negative the node penetrates the obstacle and the contact constraints are imposed. On the other hand, if the gap is positive the node is not in contact and the constraints are not applied.

For the node detected in contact, the depth of penetration is calculated and the normal and tangential unit vectors on the surface of the obstacle in the point of the closest projection are identified. The penetration and the vectors are essential for imposing the contact constraints using the algorithm described in Paper C. Additional information about this and other contact searching algorithms can be found in Laursen [21] and Mohammadi [22].
3.3 Regularization of friction

The two most common friction laws applied in numerical simulations of metal-forming processes are the shear friction law and the Coulomb friction law. The former one is given by

$$\mathbf{t}^r = -m \frac{\bar{\sigma}}{\sqrt{3}} \mathbf{t}$$  \hspace{5mm} (3.3.1)

where $\mathbf{t}^r$ is the tangential contact traction, $m$ is the friction factor, $\bar{\sigma}$ is the yield stress of the material and $\mathbf{t}$ is the unit vector in the direction of the sliding velocity, $\omega$. 

The Coulomb friction law, which is applied in this study, can be written with respect to contact tractions or contact forces as

$$\mathbf{t}^r = -\mu |\mathbf{t}^n| \mathbf{t} \quad \text{or} \quad f^t = -\mu |f^n| \mathbf{t}$$  \hspace{5mm} (3.3.2)

where $\mu$ is the friction coefficient, $\mathbf{t}^n$ is the normal contact traction, $f^t$ and $f^n$ is the tangential and the normal contact force, respectively. The Coulomb friction law, as given in the equation (3.3.2), is a discontinuous function of the sliding velocity. Such discontinuity is difficult to handle by numerical procedures. Therefore, in this study, the friction law is regularized as

$$f^t = -\mu |f^n| \frac{2}{\pi} \tan \left( \frac{\omega}{\omega_r} \right) \mathbf{t}$$  \hspace{5mm} (3.3.3)

where $\omega_r$ is the regularization parameter. Application of equation (3.3.3) leads to continuous representation of friction, as illustrated in Figure 7. It can be seen, that the regularization parameter has a strong influence on the predicted tangential force. The higher the $\omega_r$, the smoother the predicted $f^t$. On the other hand, if the parameter is chosen too high the friction will deteriorate. As an optimum, it is recommended to use values of $\omega_r$ between 1-10% of the sliding velocity, MSC.Marc [15]. During rolling, however, the sliding velocity is not constant across the contact surface. To choose a suitable value of $\omega_r$, the following relation may be used, Hsiang et al. [23]

$$\omega_r = \frac{v_{roll}}{1100}$$  \hspace{5mm} (3.3.4)

where $v_{roll}$ is the circumferential velocity of the roll.
Figure 7. Contact with friction: a) relative load distribution b) effect of regularization.
4 Summary of papers

4.1 Paper A

The first paper is a study on wire-rod rolling. The aim of the study is to predict the evolution of deformation and temperature in an AISI 302 stainless steel during hot rolling in a wire block. The experimental data are obtained by rolling of wire samples with an initial radius of 12.5 mm and a temperature of 1000°C. The rolling is performed in the first four pairs of an industrial wire block with a r-o-r-o-r (r-round, o-oval) pass sequence. By using a special jig, the rolled samples are taken out of the block after passing the 1st, 2nd, 3rd and 4th roll pair, respectively. The samples enter the wire block at a velocity of 12.5 ms⁻¹.

Coupled thermo-mechanical simulations of the rolling trials are performed using the FE program MSC.Marc. The material behaviour of the AISI 302 is described by a simple material model based on dislocation density. The flow stress, \( \sigma \), is related to the dislocation density, \( \rho \), strain rate, \( \dot{\varepsilon} \), and absolute temperature, \( T_K \), by the following equations

\[
\sigma = \sigma_0 + \alpha G b \sqrt{\rho} \quad (4.1.1)
\]

\[
\frac{d \rho}{dt} = \frac{m \dot{\varepsilon}}{bL} \Omega \rho^2 \quad (4.1.2)
\]

\[
\Omega = \Omega_0 \dot{\varepsilon}^n \exp \left( - \frac{Q_\Omega}{RT_K} \right) \quad (4.1.3)
\]

\( G, b \) and \( m \) are the shear modulus, the magnitude of the Burgers vector and the average Taylor factor, respectively. The parameters, \( \sigma_0, \alpha, L, \Omega_0, n \) and \( Q_\Omega \) are adjustable coefficients, evaluated from experimental flow curves of an AISI 304L stainless steel, which is very similar to the studied steel in terms of chemical composition.

![Graph showing experimental and predicted flow curves.](image)

Figure 8. AISI 304L - experimental and predicted flow curves.
The first of the above equations is the classical relation between the dislocation density and flow stress. The second equation describes the time evolution of dislocation density. Combining these two equations, the time evolution of flow stress is also described. The third equation is a pure empirical relation for dynamic recovery, entering temperature dependence into the system of equations. Note that whenever the strain rate, $\dot{\varepsilon}$, is zero, no recovery occurs. This is a simplification of the real material behaviour.

![Graph](image1)

**Figure 9.** Predicted deformation history of a material point in the centre of the wire.

![Graph](image2)

**Figure 10.** Experimental and predicted wire profiles evaluated at room temperature.

Comparison of the experimental and predicted flow curves is shown in Figure 8. The figure also demonstrates the extrapolation capability of the model. This is an important feature because the high rolling velocity generates high strain rates outside the range used in the compression tests.
Deformation history of a material point in the centre of the wire, as predicted by the FE analysis is shown in Figure 9. The predicted temperature and strain rate increase from pass to pass whereas the flow stress decreases. Between the passes, since $\Omega(\dot{e}=0)=0$, no recovery occurs and the model predicts a constant flow stress. This is a drawback of the model because in reality, static recovery will take place relaxing the flow stress.

Finally, profiles of the wire predicted by the FE analysis and the profiles obtained in the wire-block trials are compared in Figure 10. As can be seen, the predicted profiles are very close to the measured ones.

4.2 Paper B

In this study an alternative approach for the prediction of roll force and roll torque in shape rolling is evaluated. The approach is a combination of the slab method and a 2D FEM with a generalized plane-strain formulation, an approach originally presented by Kim et al. [17]. The basic assumption of the approach is that the longitudinal displacement of the material is uniform over any cross-section perpendicular to the rolling direction. The idea is illustrated in Figure 11. The 2D FEM is applied to calculate the lateral spread and the uniform thickness change of a material slice passing through the roll gap. The slab method is applied to impose the force balance in the rolling direction and to calculate the roll force and roll torque by integrating the contact forces acting on the slice. In general, the incoming velocity of the material, that provides the force balance in the rolling direction, is unknown. Thus, an iterative procedure has to be applied, which is described in this paper. The whole approach, including a non-linear 2D FE code, is implemented in Matlab. Details of the non-linear 2D FE code are described in Paper C.

![Figure 11. Illustration of the generalized plane-strain formulation in shape rolling.](image)

The accuracy of the approach, in terms of roll force and roll torque, is evaluated. Seven various shape rolling passes are simulated using the 2D FEM and 3D FEM. The FE program MSC.Marc is used for the 3D simulations. The results of the analysis show that the 2D approach overestimates the roll forces and roll torques, compared to the 3D FEM. Therefore an ad-hoc correction factor is proposed. The factor scales the force in the rolling direction,
which is obtained by the slab method and applied to the slice. The factor, \( s \), is given by

\[
s = \left( \frac{l_{out}^c}{l_{out}} \right)^2
\]

(4.2.1)

where \( l_{out} \) and \( l_{out}^c \) is the total outline and contact outline of the deformed slice, respectively. By applying the ad-hoc factor, the prediction of roll force is improved, which is illustrated in Figure 12.

![Figure 12. Predicted roll force with and without correction](image)

4.3 Paper C

In this paper, details of a 2D FE code for solution of non-linear mechanical problems with contact and friction are presented. The code is applied for simulation of shape rolling using a combination of the slab method and a 2D FEM with a generalized plane-strain formulation, as presented in Paper B.

The code is implemented in Matlab and it is based on the following formulations and methods:

1. updated Lagrangian formulation
2. implicit time integration with full Newton-Raphson iterative method
3. 2-dimensional with generalized plane-strain formulation
4. hypoelastic-plastic material model with additive decomposition of the rate-of-deformation tensor into elastic and plastic parts
5. J2 flow-theory based on von Mises yield surface with isotropic hardening and associative flow rule for the evaluation of plastic response
6. radial return algorithm for the integration of rate constitutive equations
7. objective time-stepping algorithm for stress update, based on the Jaumann rate of Cauchy stress and the notion of rotated configuration
8. 4-node quadrilateral elements with selective reduced integration
9. contact model based on a so-called “direction constraint method” and a regularized Coulomb friction model, both presented in this paper.

In order to verify the implementation of the in-house FE code and the proposed contact algorithm, benchmark contact simulations are performed. Compression of a rectangular workpiece made of an elastic-plastic material is analysed using the in-house FE code and the FE program MSC.Marc. Plane-strain conditions are assumed and the friction is modelled using the Coulomb model with a friction coefficient of 0.1 and 0.3, respectively. The results provided by the two codes are identical.

Then, the in-house FE code is applied for the same type of simulations of shape rolling as described in Paper B. Round-to-oval and oval-to-round passes are analysed, with the aim of comparing the solutions provided by the 2D and 3D approach, respectively. Since the roll radius has a strong influence on the free spread (area reduction), roll force and torque, each of the passes is analysed using three different roll radii. Thermal and viscous effects are neglected and a simple elastic-plastic material model is used. In terms of roll force and roll torque, the results provided by the 2D and 3D approach show good agreement. The predicted area reduction, however, is underestimated by the 2D approach. In other words, the material has more lateral spread in the 2D than in the 3D simulations. The smaller the roll radius the greater the error, see Figure 13. This type of analysis has not been found in the literature, but it is important for the practical application of the 2D approach.

![Figure 13. Profiles predicted for round-to-oval and oval-to-round passes using a roll radius of 110 mm (left-hand side) and 200 mm (right-hand side).](image-url)
5 Discussion and future work

The finite-element method is applied for process simulation of wire-rod and bar rolling, starting with an analysis of rolling in a wire block. Typical for such a process is the high rolling velocity, which results in an adiabatic-like deformation process. If the rolling proceeds too fast, the temperature and strain rate can locally reach the limit where the load-carrying capacity of the workpiece is exceeded and defects evolve. The heat generated by the plastic work depends on the strain rate and flow stress of the material. Thus, in order to predict the temperatures correctly, a good prediction of the flow stress is needed. In this study a simple model, based on dislocation theory, is used. The model can correctly reproduce the experimental flow curves obtained by hot compression tests. The model, however, is applied in simulations of a process where the strain rates are almost two orders of magnitude higher than the highest strain rate applied in the compression tests. The confidence to flow stress predicted by the model may be questioned, but, in general, there are no experimental data available for the extreme conditions, experienced during wire-rod rolling. The study includes a sequence of four roll passes. In reality, the wire block consists of eight roll pairs and, before arriving to the wire block, the material could have experienced more than twenty roll passes. In order to model such a multi-step hot-forming process a material model incorporating processes of micro-structural changes, such as recrystallization and grain growth is needed. Since these processes are deformation and temperature driven a good thermo-mechanical model of the rolling process is needed as well.

Even today, simulations of that many forming steps, using the 3D FEM, results in a very long computational time. Therefore a simplified approach to mechanical simulations of shape rolling is evaluated. The approach is a combination of the slab method and a 2D FEM with a generalized plane-strain formulation. The combination of the two methods strongly reduces the size of a shape-rolling model, making the approach very computationally efficient. When applied, the approach has a tendency to overestimate the roll force and roll torque. Considering the prediction of final profile, the 2D method underestimates the area reduction predicting more lateral spread than a 3D FEM model. The applied generalized plane-strain formulation simplifies the displacement field in the rolling direction, leaving more freedom for the lateral flow of the material. Also in a 2D model, material outside the deformation zone is not included, which otherwise poses constraints on the material deformed between the rolls. The error in the prediction of roll force and roll torque can be reduced by using an ad-hoc factor, proposed in the study. The error in the predicted spread is not reduced by the factor, and the deviation increases when roll radius decreases. This is a disadvantage, having consequences for the practical application of the method because, in practice, smaller roll radii are used. These provide more effective reduction of the stock in a single rolling pass.

Despite the observed discrepancies, the 2D approach gives a close approximation of the 3D deformation in a shape-rolling pass. Such an approximation may be utilized in models of microstructure evolution during multi-pass rolling. In order to do that, the mechanical FE code built in the study, has to be coupled with a thermal model. Another option is to implement the approach into an existing thermo-mechanical FE code. Work will continue on the existing dislocation-density-based material model to include the effect of static recovery and possibly recrystallization. The complexity of such a model shall be kept low so that it is applicable in the context of FEM. The intention is to combine the gained computational resources of the 2D approach with more sophisticated material models in the analysis of shape rolling.
References


