# Spin measurements in $l p \rightarrow h X$ deep inelastic scattering ${ }^{\text {}}$ 

M. Anselmino ${ }^{1}$, M. Boglione ${ }^{1}$, J. Hansson ${ }^{2}$ and F. Murgia ${ }^{3}$<br>(1) Dipartimento di Fisica Teorica, Università di Torino and INFN, Sezione di Torino, Via P. Giuria 1, I-10125 Torino, Italy<br>(2) Department of Physics, Luleå University of Technology, S-97187 Luleå, Sweden<br>(3) INFN, Sezione di Cagliari, Via A. Negri 18, I-09127 Cagliari, Italy


#### Abstract

The production of hadrons in polarized lepton-nucleon deep inelastic scattering is discussed. The helicity density matrix of the hadron is computed within the QCD hard scattering formalism and its elements are shown to yield information on the spin structure of the nucleon and the spin dependence of the quark fragmentation process. The case of $\rho$ vector mesons is considered in more detail and estimates are given.


According to the QCD hard scattering scheme and the factorization theorem [1]-[5] the helicity density matrix of the hadron $h$ inclusively produced in the DIS process $\ell^{\uparrow} N^{\uparrow} \rightarrow h^{\uparrow} X$ is given by

$$
\begin{align*}
& \rho_{\lambda_{h}, \lambda_{h}^{\prime}}^{(s, S)}(h) \frac{E_{h} d^{3} \sigma^{\ell, s+N, S \rightarrow h+X}}{d^{3} \mathbf{p}_{h}}=\sum_{q ; \lambda_{\ell}, \lambda_{q}, \lambda_{q}^{\prime}} \int \frac{d x}{\pi z} \frac{1}{16 \pi x^{2} s^{2}} \times  \tag{1}\\
& \rho_{\lambda_{\ell}, \lambda_{\ell}}^{\ell, s} \rho_{\lambda_{q}, \lambda_{q}^{\prime}}^{q / N, S} f_{q / N}(x) \hat{M}_{\lambda_{\ell}, \lambda_{q} ; \lambda_{\ell}, \lambda_{q}}^{q} \hat{M}_{\lambda_{\ell}, \lambda_{q} ; \lambda_{\ell}, \lambda_{q}^{\prime}}^{q *} D_{\lambda_{h}, \lambda_{h}^{\prime}}^{\lambda_{q}, \lambda_{q}^{\prime}}(z),
\end{align*}
$$

where $\rho^{\ell, s}$ is the helicity density matrix of the initial lepton with spin $s, \rho^{q / N, S}$ is the helicity density matrix of quark $q$ inside the polarized nucleon $N$ with spin $S$ and $f_{q / N}(x)$ is the number density of unpolarized quarks $q$ with momentum fraction $x$ inside an unpolarized nucleon. The $\hat{M}_{\lambda_{\ell}, \lambda_{q} ; \lambda_{\ell}, \lambda_{q}}^{q}$ are the helicity amplitudes for the elementary process $\ell q \rightarrow \ell q$. The final lepton spin is not observed and helicity conservation of perturbative QCD and QED has already been taken into account in the above equation. As a consequence only the diagonal elements of $\rho^{\ell, s}$ contribute to $\rho(h)$, and non-diagonal elements, present in case of transversely polarized leptons, do not contribute. $D_{\lambda_{h}, \lambda_{h}^{\prime}}^{\lambda_{q}, \lambda_{q}^{\prime}}(z)$ is the product of fragmentation amplitudes

$$
\begin{equation*}
D_{\lambda_{h}, \lambda_{h}^{\prime}}^{\lambda_{q}, \lambda_{q}^{\prime}}(z)=\mathscr{\&}_{X, \lambda_{X}} \mathcal{D}_{\lambda_{X}, \lambda_{h} ; \lambda_{q}} \mathcal{D}_{\lambda_{X}, \lambda_{h}^{\prime} ; \lambda_{q}^{\prime}}^{*} \tag{2}
\end{equation*}
$$

[^0]where $\not_{X, \lambda_{X}}$ stands for a spin sum and phase space integration of the undetected particles, considered as a system $X$. The usual unpolarized fragmentation function $D_{h / q}(z)$, i.e. the density number of hadrons $h$ resulting from the fragmentation of an unpolarized quark $q$ and carrying a fraction $z$ of its momentum, is given by
\[

$$
\begin{equation*}
D_{h / q}(z)=\frac{1}{2} \sum_{\lambda_{q}, \lambda_{h}} D_{\lambda_{h}, \lambda_{h}}^{\lambda_{q}, \lambda_{q}}(z)=\frac{1}{2} \sum_{\lambda_{q}, \lambda_{h}} D_{h_{\lambda_{h}} / q_{\lambda_{q}}}(z) \tag{3}
\end{equation*}
$$

\]

where $D_{\lambda_{h} h}^{\lambda_{q}, \lambda_{h}}(z) \equiv D_{h_{\lambda_{h}} / q_{\lambda_{q}}}$ is a polarized fragmentation function, i.e. the density number of hadrons $h$ with helicity $\lambda_{h}$ resulting from the fragmentation of a quark $q$ with helicity $\lambda_{q}$.

Collinear configuration (intrinsic $\mathbf{k}_{\perp}=0$ ) together with angular momentum conservation in the forward fragmentation process imply

$$
\begin{equation*}
D_{\lambda_{h}, \lambda_{h}^{\prime}}^{\lambda_{q}, \lambda_{q}^{\prime}}=0 \quad \text { when } \quad \lambda_{q}-\lambda_{q}^{\prime} \neq \lambda_{h}-\lambda_{h}^{\prime} \tag{4}
\end{equation*}
$$

Eq. (1) holds at leading twist, leading order in the coupling constants and large $Q^{2}$ values. The intrinsic $\mathbf{k}_{\perp}$ of the partons has been integrated over and collinear configurations dominate both the distribution functions and the fragmentation processes. For simplicity of notations we have not indicated the $Q^{2}$ scale dependences in $f$ and $D$. The variable $z$ is related to $x$ by the usual imposition of energy momentum conservation in the elementary $2 \rightarrow 2$ process. More technical details can be found in Ref. [5].

The quark helicity density matrix $\rho^{q / N, S}$ can be decomposed as

$$
\begin{equation*}
\rho^{q / N, S}=P_{P}^{q / N, S} \rho^{N, S}+P_{A}^{q / N, S} \rho^{N,-S}, \tag{5}
\end{equation*}
$$

where $P_{P(A)}^{q / N, S}$ (which, in general, depends on $x$ ) is the probability that the spin of the quark inside the polarized nucleon $N$ is parallel (antiparallel) to the nucleon $\operatorname{spin} S$ and $\rho^{N, S(-S)}$ is the helicity density matrix of the nucleon with spin $S(-S)$. Notice that

$$
\begin{equation*}
P^{q / N, S}=P_{P}^{q / N, S}-P_{A}^{q / N, S} \tag{6}
\end{equation*}
$$

is the component of the quark polarization vector along the parent nucleon spin direction.

We choose $x z$ as the hadron production plane with the lepton moving along the $z$-axis and the nucleon in the opposite direction in the lepton-nucleon centre of mass frame. As usual we indicate by an index $L$ the (longitudinal) nucleon spin orientation along the $z$-axis, by an index $S$ the (sideway) orientation along the $x$-axis and by an index $N$ the (normal) orientation along the $y$-axis.

Some elements of the helicity density matrix of the produced hadrons can be measured via the angular distribution of the final hadron $h$ decay. Typical examples are the $\rho \rightarrow \pi \pi$ and $\Lambda \rightarrow p \pi$ decays.

For spin- 1 hadrons $(V)$ one can measure $\rho_{0,0}$ and $\rho_{1,0}$.
The general formulae for polarized protons and unpolarized leptons are $(T=S, N)$ :

$$
\begin{equation*}
\rho_{0,0}^{\left(S_{T}\right)}(V) d^{3} \sigma=\sum_{q} \int \frac{d x}{\pi z} f_{q / N} d \hat{\sigma}^{q} D_{V_{0} / q_{+}} \tag{7}
\end{equation*}
$$

$$
\begin{align*}
\rho_{0,0}^{\left(S_{L}\right)}(V) & =\rho_{0,0}^{\left(S_{T}\right)}(V)  \tag{8}\\
\rho_{1,0}^{\left(S_{S}\right)}(V) d^{3} \sigma & =\sum_{q} \int \frac{d x}{\pi z} f_{q / N} \frac{P^{q / N, S_{S}}}{2}\left[\operatorname{Re} \hat{M}_{+}^{q} \hat{M}_{-}^{q *}\right] D_{1,0}^{+,-}  \tag{9}\\
\rho_{-1,0}^{\left(S_{S}\right)}(V) & =\rho_{1,0}^{\left(S_{S}\right)}(V)  \tag{10}\\
\rho_{1,0}^{\left(S_{N}\right)}(V) & =-\rho_{-1,0}^{\left(S_{N}\right)}(V)=i \rho_{1,0}^{\left(S_{S}\right)}(V) \tag{11}
\end{align*}
$$

These formulae involve the non-diagonal fragmentation functions (2). $\hat{M}_{ \pm}^{q}$ is a short notation for $\hat{M}_{+, \pm ;+, \pm}^{q} / 4 \sqrt{\hat{s}}$. Such measurements supply information on the polarized quark fragmentation process and the polarized distribution functions.

With $S U(6)$ wavefunctions and simple assumptions for $D_{1,0}^{+,-}$[5] we get

$$
\begin{align*}
\rho_{0,0}(\rho) & =\frac{1}{3}  \tag{12}\\
\operatorname{Re} \rho_{1,0}^{\left(S_{S}\right)}\left(\rho^{+, 0}\right) & \simeq 0.10-0.15 \tag{13}
\end{align*}
$$

both for $\sqrt{s}=23 \mathrm{GeV}$ and 314 GeV , almost independently of $p_{T}$ and $\left|x_{F}\right|$, although they have to be high enough to justify the use of Eq. (1) and the valence quark approximation.

As we noticed after Eq. (1) only the diagonal elements of the lepton helicity density matrix $\rho^{\ell, s}$ contribute to $\rho(h)$, so that only longitudinal polarizations affect the results. For longitudinally polarized leptons one obtains the same results as in the unpolarized lepton case for the non-diagonal matrix elements and slightly different ones for the diagonal elements. Thus, two different measurements might yield more information. Further discussion can be found in Ref. [5].

We also remind that according to the $S U(6)$ wavefunction the entire $\Lambda$ polarization, which we did not discuss in detail here [5], is due to the strange quark. Any non-zero value would offer valuable information on the much debated issue of strange quark polarization, $\Delta s$, inside a polarized nucleon.

This work has been supported by the European Community under contract CHRX-CT94-0450.
[1] J.C. Collins, Nucl. Phys. B 394 (1993) 169.
[2] J.C. Collins, Nucl. Phys. B 396 (1993) 161.
[3] J.C. Collins, S.H. Heppelmann and G.A. Ladinsky, Nucl. Phys. B 420 (1994) 565.
[4] For earlier work on spin asymmetries and helicity density matrices in hard scattering see also J. Babcock, E. Monsay and D. Sivers, Phys. Rev. D 19 (1978) 1483;
M. Anselmino and P. Kroll, Phys. Rev. D 30 (1984) 36;
N.S. Craigie, K. Hidaka, M. Jacob and F.M. Renard, Phys. Rep. 99 (1983) 69.
[5] M. Anselmino, M. Boglione, J. Hansson and F. Murgia, Phys. Rev. D 54 (1996) 828.


[^0]:    * Talk delivered by J. Hansson at the XII International Symposium on High Energy Spin Physics, Amsterdam, Sept. 10-14, 1996.

