Making MLSD Decisions Using a Data-Dependent Threshold Device

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Abstract – This paper presents a detector for obtaining MLSD decisions on binary symbols that has been corrupted by intersymbol interference and additive Gaussian noise. The basis of the detector is a bound on a cross-product of the quadratic form in the log-likelihood function for the transmitted sequence. The detector will only make decisions on *some* of the symbols in a transmitted sequence, but those decisions will be the *same* decisions as the MLSD would have made. The number of symbols to be detected is stochastic, varying from sequence to sequence. The detector is simple in structure, consisting of a whitened matched filter and two variable thresholds for each symbol to be detected. Because the detector in general does not detect the complete sequence of symbols, but has a very good performance on those symbols that are detected, it is suitable as a complement to some existing detector that is inferior to the maximum likelihood sequence detector.

1 Introduction

We investigate the detection of blocks of independent antipodally modulated, binary symbols transmitted over a discrete-time, additive Gaussian channel with intersymbol interference (ISI). A simple detector of binary symbols is presented. The output of the detector is ternary, "–1", "+1" and "don't know", and when a decision is made on a symbol, it is the same decision that a maximum-likelihood sequence detector (MLSD) would have made on that symbol. The detector consists of a whitened matched filter and two variable thresholds for each symbol, where the thresholds are dependent on the received signal.

The detector is derived by expanding, with respect to a single symbol, the quadratic form of the log-likelihood function for the entire transmitted sequence. This expansion leads to an expression where only one term depends on the symbol to be detected. This term is a product of that symbol and a difference between the output from a whitened matched filter and a function of the other symbols. This function can be upper and lower bounded by two thresholds that are independent of the undetermined symbols. The symbol is decoded if the output from the matched filter is either above the upper threshold or below the lower threshold. In general, not all of the symbols in each block are decoded, leaving some symbols undetermined, and the number of decoded symbols and their positions are stochastic and vary from one block to another.

The detector offers an efficient way to obtain MLSD decisions on some symbols. Perhaps the detector's most important potential use is as an aid to receivers that are inferior in performance to the MLSD, *e.g.*, decision-feedback equalizers. Two possible principal methods of combining the detector with other receivers are described in [8]. However, in this paper we restrict ourselves to deriving the detector itself and discussing some of its properties.

The presentation proceeds as follows. In Section 2 a model for block transmission systems is described and some auxiliary definitions are given. The proposed detector is derived in Section 3, while a detection example is given in Section 4. Section 5 briefly examines the decision rate of the detector. Finally, in Section 6 a discussion of the results is found.

2 A block transmission system model

Consider the transmission of blocks of binary data, typically interspersed with symbols known to the receiver, through a channel with additive Gaussian noise and *known* intersymbol interference (ISI)[1]. We represent the resulting transmission system with the matrix notation

$$\mathbf{y} = \mathbf{H}\mathbf{b} + \mathbf{n}, \tag{2.1}$$

[1]Throughout the paper we use binary antipodally modulated data symbols in a real-valued symbol-sampled block transmission model. However, the results are generalizable to, *e.g.*, 2^M-level QAM with continuous transmission and complex-valued channel models.

where the transmitted message is coded in $\mathbf{b} \in \{-1,+1\}^N$, $\mathbf{y} \in \mathbb{R}^{N+L-1}$ is a vector of channel observables, $\mathbf{H}$ is a real-valued, deterministic and known $(N+L-1) \times N$ matrix representing the ISI, and $\mathbf{n}$ is a jointly Gaussian, zero-mean random vector with a $\mathrm{N}(\mathbf{0}, \mathbf{R_n})$ distribution [1, 4]. We assume the symbols in $\mathbf{b}$ to be independent, identically distributed with an equal probability of $+1$ and -1 occurring.

We continue with a number of additional definitions that we find useful in what follows. The signal-to-noise ratio (SNR) in this block transmission system we define as

$$\mathrm{SNR} \triangleq \frac{\mathrm{tr}\left\{\mathbf{H}^T \mathbf{R_n}^{-1} \mathbf{H}\right\}}{N}, \tag{2.2}$$

where $\mathrm{tr}\{\mathbf{X}\}$ denotes the trace of the matrix $\mathbf{X}$.

The MLSD is a processor that, given the received signal $\mathbf{y}$, finds the sequence $\mathbf{b}$ for which the *likelihood function* for $\mathbf{b}$ given $\mathbf{y}$ has the largest value. Defining the matrix

$$\mathbf{M} \triangleq \mathbf{H}^T \mathbf{R_n}^{-1} \mathbf{H} \tag{2.3}$$

the MLSD *minimizes*

$$L'(\mathbf{b}, \mathbf{z}) \triangleq \left\|\mathbf{b} - \mathbf{M}^{-1}\mathbf{z}\right\|_{\mathbf{M}}^2, \tag{2.4}$$

where

$$\mathbf{z} \triangleq \mathbf{H}^T \mathbf{R_n}^{-1} \mathbf{y} = \mathbf{M}\mathbf{b} + \mathbf{H}^T \mathbf{R_n}^{-1} \mathbf{n} \tag{2.5}$$

is the output from the *whitened matched filter* [1, 4, 7]. The output of the MLSD then is

$$\widehat{\mathbf{b}}^{\text{MLSD}} \triangleq \left\{\mathbf{b} \mid L'(\mathbf{b}) < L'(\mathbf{b}_0)\ \forall\, \mathbf{b}_0 \neq \mathbf{b},\ \ \mathbf{b}_0 \in \{-1, 1\}^N\right\}. \tag{2.6}$$

In its general form the MLSD performs a complete search over $\{-1,+1\}^N$, and, hence, its computational complexity is proportional to 2^N. With the Viterbi algorithm [3] the computational complexity is proportional to $N2^L$, where, if the channel impulse response is time-invariant, L is the length of the system memory.

3 The proposed detector

We will here develop the detector in a "step-by-step" manner that we hope is intuitively appealing to the reader. Let b_i, $\widehat{b}_i^{\text{MLSD}}$, z_i and $m_{i,j}$ denote the elements in $\mathbf{b}$, $\widehat{\mathbf{b}}^{\text{MLSD}}$, $\mathbf{z}$ and $\mathbf{M}$, respectively, where i is the row index and j is the column index. Focusing on the detection of the kth bit, we rewrite (2.4) and obtain [8]

$$\begin{aligned} L'(\mathbf{b}, \mathbf{z}) = \;& 2b_k\left(\Delta(\mathbf{b}, k) - z_k\right) + \sum_{i\neq k, j\neq k} b_i m_{i,j} b_j \\ & -2\sum_{i\neq k} b_i z_i + m_{k,k} + \mathbf{z}^T\mathbf{M}^{-1}\mathbf{z}, \end{aligned} \tag{3.1}$$

where

$$\Delta(\mathbf{b}, k) \triangleq \sum_{i\neq k} b_i m_{i,k}. \tag{3.2}$$

Observe that all but the first term on the right hand side of (3.1) and $\Delta(\mathbf{b}, k)$ as given by (3.2) are *independent* of b_k. Recall that z_k is the output from the whitened matched filter and note that

$$|\Delta(\mathbf{b}, k)| \leq \sum_{i\neq k} |b_i m_{i,k}| = \sum_{i\neq k} |m_{i,k}|. \tag{3.3}$$

If the output of the whitened matched filter at instant k is above the possible maximum of $\Delta(\mathbf{b}, k)$, *i.e.*, $z_k > \sum_{i\neq k} |m_{i,k}|$, the function $L'(\mathbf{b}, \mathbf{z})$ in (3.1) is minimized by $b_k = +1$, independently of the status of the other bits[2]. Hence, the corresponding output of the MLSD is positive, $\widehat{b}_k^{\text{MLSD}} = +1$. Similarly, if $z_k < -\sum_{i\neq k} |m_{i,k}|$, then $\widehat{b}_k^{\text{MLSD}} = -1$. This is true for all positions k, and the tests can be done independently for all symbols. These observations are sufficient for attempting to decode some symbols in a block using only a matched filter and a threshold device.

[2] If the noise is white, the rightmost part of equation (3.3) becomes the sum of the absolute values of the channel correlation function (with the zero term omitted), which is a measure of the severeness of the ISI.

Assume now that some of the symbols in $\mathbf{b}$ have been decoded as described. Using the decoded symbols as constants, we proceed by minimizing (3.1) with respect to the remaining, undetermined symbols. With the same technique as described above, the function $\Delta(\mathbf{b}, k)$ in (3.2) can be bounded from above by

$$\Delta_k^+ \triangleq \sum_{\{i \mid i \neq k,\ b_i \text{ decoded}\}} m_{i,k} \widehat{b}_i^{\text{MLSD}} + \sum_{\{i \mid i \neq k,\ b_i \text{ not decoded}\}} |m_{i,k}| \tag{3.4}$$

and from below by

$$\Delta_k^- \triangleq \sum_{\{i \mid i \neq k,\ b_i \text{ decoded}\}} m_{i,k} \widehat{b}_i^{\text{MLSD}} - \sum_{\{i \mid i \neq k,\ b_i \text{ not decoded}\}} |m_{i,k}| \tag{3.5}$$

where the difference between (3.4) and (3.5) is the sign of the second sum. We will henceforth refer to Δ_k^+ and Δ_k^- as the upper and lower threshold for bit k, respectively. Notice that the distance between the upper and lower thresholds, at least for some k, decreases when bits are decoded. We can again check whether the output of the whitened matched filter $z_k > \Delta_k^+$, and if it is, assign $\widehat{b}_k^{\text{MLSD}} = +1$. Analogously, if $z_k < \Delta_k^-$ then $\widehat{b}_k^{\text{MLSD}} = -1$. Thus, by comparing the output of the matched filter, z_k, with these new and tighter thresholds, additional symbols could possibly be decoded. By repeating this procedure until no more symbols are decoded, we obtain the final thresholds and the output of the detector. We will view the proposed detector as a matched filter followed by a threshold device, as is illustrated in Figure 1.

The resulting output of the detector is ternary, "−1", "+1" or "*no decision*". The actual identity and the total number of symbols that can be determined depend on the received signal $\mathbf{y}$ (both on the transmitted bits $\mathbf{b}$ and the noise $\mathbf{n}$), and on the channel characteristics through the matrix $\mathbf{M}$. We conclude that the decisions on the symbols that actually are decoded are *identical* to the corresponding decisions of an MLSD.

Figure 1: A block description of the proposed detector.

4 An example of the operation of the detector

We present an example of the operation of the detector where blocks of binary data of length $N = 20$ are transmitted over a time-invariant channel described by (2.1). The impulse response of the system is $[h_0, h_1, h_2] = [1, 0, 1]$ and the noise $\mathbf{n}$ is white, stationary and Gaussian with an SNR of 10 dB. In Figure 2 an example of the operation of the detector is given. The solid lines are the thresholds, $\Delta_k^+(l)$ and $\Delta_k^-(l)$, the stars and the circles are the output of the matched filter $\mathbf{z}$, with the circles denoting decoded symbols and the stars undetermined symbols. The upper and lower thresholds will merge when a sufficient number of symbols in a sequel have been decoded. In the example, the symbols at k equal to 15, 17 and 19 could not be decoded. Note that bit 12 was decoded as positive although the output from the whitened matched filter was negative. In this example the detector made correct decisions on all symbols that were decoded.

In the example displayed in Figure 2, the thresholds shown in the diagram belonging to the fourth iteration are the final thresholds. These define the threshold device in Figure 1.

5 Studying the behaviour of the detector by simulations

The potential of the proposed detector is very dependent on its ability to make decisions. This section presents simulation results where the proposed detector is used to decode symbols in a block transmission system. The probability that the proposed detector makes a decision on symbol k we denote with $P_{\text{D},k}$ and we define the *decision rate* as

$$P_\text{D} \triangleq \frac{1}{N} \sum_{k=1}^{N} P_{\text{D},k}. \tag{5.1}$$

The decision rate is dependent on the ISI and the block length N, as well as on the covariance matrix of the noise.

Consider a time-invariant two-tap channel described by (2.1) with

$$h_k = \delta(k) + \alpha\delta(k - d), \tag{5.2}$$

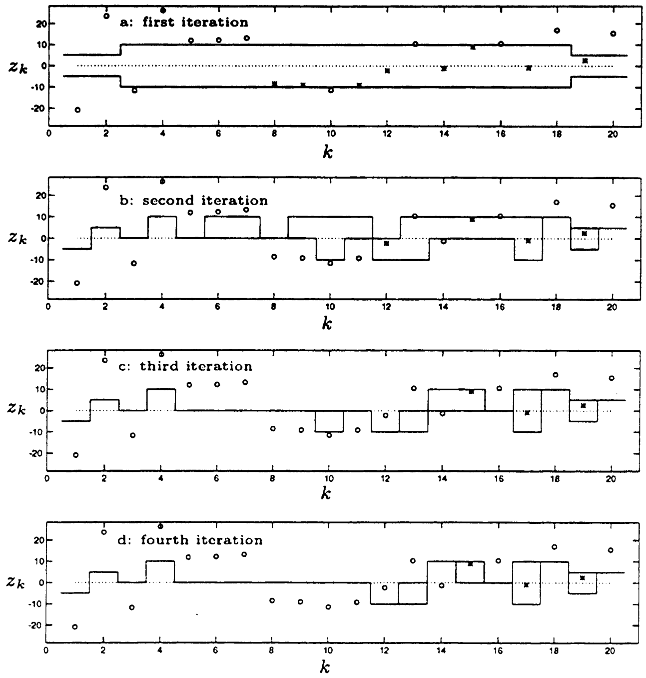

Figure 2: The output of the whitened matched filter plotted together with the thresholds in an example where four iterations are needed to obtain the final thresholds. Symbols that are decoded are marked "o" and undecoded symbols are marked "$*$".

where $\delta(\cdot)$ is the Kronecker delta function, α is the real-valued amplitude of the second tap and d, a positive integer, is the time-spread of the channel. In the left part of Figure 3 the estimated probability of *no* decision, $1 - P_{\mathrm{D}}$, is plotted versus the strength of the second tap, α, for the delay $d \in [1, 2, 5, 10, 15]$, a block length $N = 20$, and an SNR of 10 dB. It can be observed that the probability of no decision, given a certain d, seem to have global maxima at $\alpha = 1$ and $\alpha = -1$, it is low if $|\alpha|$ is small or large enough, and also non-increasing with d.

The right part of Figure 3 shows the estimated probability that bit number k is decoded, $P_{\mathrm{D},k}$, for varying SNR. The plot shows the probability of decision for each individual bit $k \in [1, 2, \ldots, 20]$ separately. The four symbols at the edges of the block, *i.e.*, the symbols corresponding to $k \in [1, 2, 19, 20]$, have the highest probability of being decoded. This is because these symbols are subjected to less ISI than the symbols in the centre part of the block. As these symbols are decoded more frequently, their neighbours, the symbols at $k \in [3, 4, 17, 18]$ are also decoded more frequently. It can be observed that the probability of decision in general decreases with increasing SNR. This may at first seem counter-intuitive, but the quality of the decisions increases with the SNR, while the number of decisions made decreases.

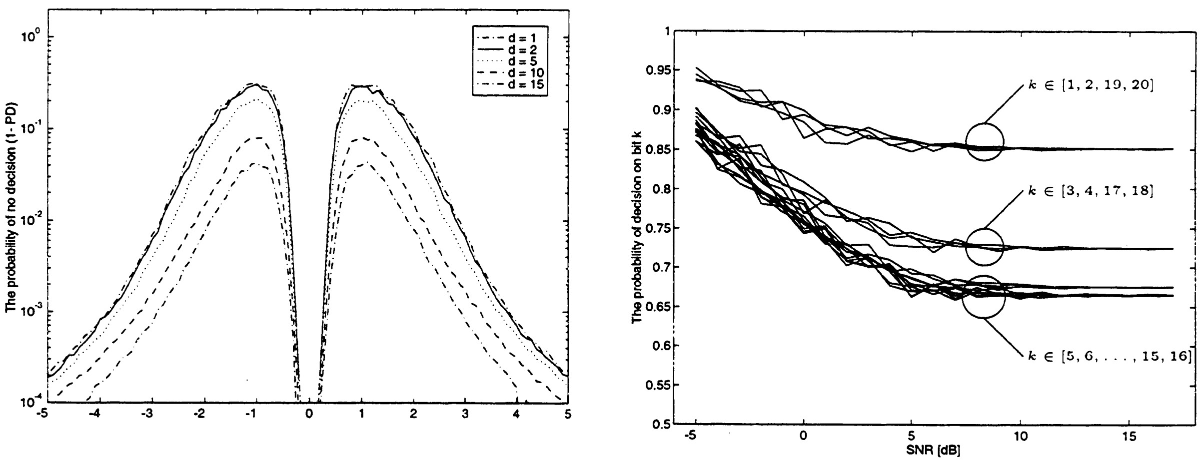

Figure 3: *Left:* The probability of no decision, $1 - P_{\mathrm{D}}$, for the proposed detector plotted versus the strength of the second tap, α, for a delay $d \in [1, 2, 5, 10, 15]$. The SNR is 10 dB. *Right:* The probability of decision for each individual bit k, $P_{\mathrm{D},k}$ versus SNR.

6 Discussion

We have presented a detector of low computational complexity that makes MLSD decisions on a portion of the transmitted symbols. The key components of the detector are a whitened matched filter and a threshold device with two variable thresholds per symbol. The thresholds for a specific bit stem from an upper and lower bounding of a function of the other bits, *cf.* equations (3.2) – (3.5). This function appears in the quadratic term of the log-likelihood function for the transmitted sequence, see equation (3.1).

An important property of the proposed detector, determining its potential, is its decision rate: the average percentage of symbols on which a decision is made. Simulations using a block length $N = 20$ rendered a decision rate better than 65% for all two-tap channel models, while channel models with strong ISI and a high SNR could render much lower decision rates.

We believe that the primary use of the proposed detector is as a pre-processor to sub-optimal receivers inferior in performance to the MLSD. It can aid these receivers by providing MLSD decisions on some symbols, which could be utilized in several ways [8]. Receivers that beneficially could be combined with the proposed detector is, for instance, decision-feedback equalizers and linear equalizers, see, *e.g.*, [4], and combined linear-Viterbi detectors [2, 5, 6].

We have given our presentation in a symbol-sampled block transmission environment. However, the results are generalizable, for instance, to M-level QAM continuous transmission systems with complex-valued channels.

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