THERMAL ANALYSIS OF LUBRICANT FLOW IN A TEXTURED INLET CONTACT

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ABSTRACT

Performance of an inlet textured contact is analysed for a temperature dependent lubricant flow. Thermal effects are analysed for different shear rates. Shearing of lubricant and subsequent heating reduce load carrying capacity and frictional losses compared with isothermal conditions. Load carrying capacity of a parallel surface contact can be improved by using texturing at the inlet when thermal effects are considered. It is also shown that for the different convergence ratios considered, the texture enables the sustaining of a load until a certain critical shear rate is reached. This critical shear rate depends on a number of factors such as the convergence ratio and lubricant parameters including the viscosity-temperature coefficient and the dynamic viscosity at reference temperature.

INTRODUCTION

In hydrodynamic lubrication, convergent gaps help in separating the surfaces in relative motion and generating pressure in the fluid film. As a result, the contact can carry a load while keeping friction as low as possible. With no convergence in the gap, load carrying capacity is zero. It has been shown that surface texturing can provide pressure build-up in parallel surface contacts. Theoretical results reported so far for textured contacts are isothermal and show some improvement of hydrodynamic performance [1–11]. Generation of heat takes place in such a contact as was shown for a step bearing by Dobrica and Fillon [12]. Temperature is in fact increasing through the contact due to lubricant shearing. Since the dynamic viscosity of the lubricant is temperature dependent, variation of the viscosity is expected through the contact. Frictional losses as well as the pressure build-up and the load carrying capacity are then expected to vary. The question is then how much these quantities are affected.

The influence of thermal conditions has not yet been studied for textured geometry. Therefore thermal performance of a textured contact requires investigation. The present work focuses on the study of a two-dimensional inlet textured contact with special emphasis on thermal effects. The results obtained for different values of shear rate and convergence ratio are analysed and compared with isothermal results.

NUMERICAL MODEL

Equations

The code used to model the problem is CFX 10.0. The Navier-Stokes equations, momentum equation Eq.(1) coupled with the continuity equation Eq.(2), are solved over the domain together with the energy equation Eq.(3), using the finite volume method. The flow is considered laminar, steady and two dimensional.

\[
\frac{\partial}{\partial x_i}(\rho u_i u_j) = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right) \quad (1)
\]

\[
\frac{\partial}{\partial x_j}(\rho u_i) = 0 \quad (2)
\]

\[
\frac{\partial}{\partial x_j}(\rho u_i h_{tot}) = \frac{\partial}{\partial x_i} \left( \rho \frac{\partial T}{\partial x_i} \right) + \frac{\partial u_i}{\partial x_j} \left[ \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \delta_{ij} \left( \frac{2}{3} \mu \frac{\partial u_k}{\partial x_k} \right) \right] \quad (3)
\]

where \( \delta_{ij} = 1 \) when \( i = j \), and 0 when \( i \neq j \). The total enthalpy is calculated by the following expression:

\[
h_{tot} = h_{stat} + \frac{1}{2} V^2 \quad (4)
\]

with \( \frac{1}{2} V^2 \) representing the kinetic energy. The static enthalpy is calculated by integrating the following expression:

\[
dh = C_p dT + \frac{1}{\rho} \left[ 1 + \frac{T}{\rho} \frac{\partial}{\partial T} \right] dP \quad (5)
\]

Geometries studied and lubricant properties

The geometry used is that of a textured slider bearing with 3 rectangular dimples located in the inlet region, Fig.1. The following dimensions are used: \( L = 6 \text{ mm} \) (length of the domain) and \( h_0 = 0.03 \text{ mm} \) (height of the domain). The convergence ratio is defined by

\[
k = \frac{h_1 - h_0}{h_0}
\]

The width of each dimple is set to 0.3 mm, whereas the depth \( d \) is set such that \( d/h_0 = 0.75 \). This represents dimples...
smaller than the minimum film thickness. The first dimple is located at 0.2 mm from the inlet and the distance between the dimples is set to 0.4 mm.

![Figure 1: Geometry of the test cases](image)

**Boundary conditions for the Navier-Stokes equation**

At the boundary walls, no slip is assumed between the lubricant and the wall. The upper wall translates with a velocity $U$ along the $x$-axis. The lubricant, in direct contact with the wall, moves with the same velocity. The lubricant has zero velocity at the lower boundary wall. Ambient pressure is imposed at the inlet and outlet of the domain. The velocity gradient perpendicular to the inlet boundary is set to zero in order to impose the flow direction.

**Boundary conditions for the energy equation**

Yu and Sadeghi [13] showed that, for a thrust washer, the temperature of the “runner” is nearly constant. It was also shown according to Tucker and Keogh [14] that variations are not significant in terms of impact on bearing performance. A constant temperature of $50^\circ C$ is set at the upper wall. As the inlet of such contacts is usually cooled, a temperature profile, varying from $40^\circ C$ at the lower wall to $50^\circ C$ at the upper wall, is taken at the inlet boundary.

$$T_{in} = -\frac{T_{up} - T_0}{h_1^2} y^2 + 2 \frac{T_{up} - T_0}{h_1} y + T_0$$

At the lower boundary wall, adiabatic conditions are assumed:

$$\frac{\partial T}{\partial y}_{\text{lower wall}} = 0$$

Thermal effects require careful modelling since many factors such as viscosity, density, specific heat capacity and thermal conductivity of the lubricant are temperature dependent. The expressions in Table 1 are chosen to describe variation of the lubricant properties with temperature. Their influence is estimated in the following with factorial design.

In order to estimate performance of the system under different shear rates, load carrying capacity $W$ and friction $F_r$, given in Eq.(6) and (7) respectively, are integrated at the upper wall.

$$W = \int_0^L p \, dx$$

$$F_r = \int_0^L \mu \frac{\partial u}{\partial y} \, dx$$

Their corresponding non-dimensional values are given by:

$$W^* = \frac{W}{\mu_0 U \left( \frac{h_0}{L} \right)^2}$$

$$F_{r^*} = \frac{F_r}{\mu_0 U \left( \frac{h_0}{L} \right)}$$

The non-dimensional temperature in the contact is calculated as follows:

$$T^* = \frac{T}{T_0}$$

The shear rate is given by:

$$\gamma = \frac{U}{h_0}$$

**Meshing strategy**

The space discretization, i.e. meshing of the domain, induces an error into the numerical simulation. An estimation of this error is possible with a Richardson extrapolation. This method is used to investigate the value of the load for the contact geometry with zero convergence (k = 0) and a velocity set to $U = 5$ m/s. As the real value of the load is not known, this method is expected to give an approximation of the exact value and enables estimation of the grid error [15]. The value of the non-dimensional load is plotted in Fig.2 for grids of 3360, 13440 and 53760 elements. The value of the load error for the grid of 53760 elements is estimated to be 1.5%. The error is small enough to consider the results reliable. This type of grid is used in the following workscope.

![Figure 2: Grid independency study](image)

**RESULTS AND DISCUSSION**

**Factorial design**

The influence of dynamic viscosity $\mu$, density $\rho$, specific heat capacity $C_p$ and thermal conductivity $\lambda$ of the lubricant
are estimated by modelling the dimpled geometry of Fig.1 with \( k = 0 \) by using a factorial design method. Factorial design [16] is a statistical method to evaluate the influence of one factor (main effect) and the interaction of factors (combined effect) on representative quantities.

The dynamic viscosity, density, specific heat capacity and thermal conductivity are either taken constant at 313K (low level: -) or temperature dependent (high level: +). They are combined in all possible ways, which leads to \( 2^4 = 16 \) tests. The high and low level are represented in Table 2. Four different quantities are evaluated: the load (\( W \)), friction (\( F_r \)), friction coefficient (\( f = F_r/W \)) and maximum temperature (\( T_{max} \)).

To evaluate the main effect of one factor on an engineering quantity, the average of the quantities among the 16 runs where the factor is at the low level is substracted from the average of the quantities where the factor is at the high level. A percentage from the isothermal case (run 1) is then given in Table 3. In the same way, to evaluate the combined effect of some factors, the average of the quantities where all the factors chosen are at the low level are substracted from the average of the quantities where all the factors are at the high level.

From Table 3, the viscosity has the largest influence on all the quantities evaluated. When viscosity dependency is considered, the friction and the load significantly decrease since the viscosity decreases under temperature increase.

The other main effect which appears is the density since it has a quite significant effect on the friction coefficient. The specific heat capacity and thermal conductivity have no noticeable main effect. A combined effect of viscosity and density appears to be quite important. However, the most significant combined effect corresponds to the interaction between viscosity, density and specific heat capacity. This combined effect results in a strong influence on the friction coefficient.

The fluid is therefore modelled with temperature dependent viscosity, density and specific heat capacity, whereas the lubricant thermal conductivity is set to the constant value at 313K, i.e. \( \lambda = 0.13 \, W m^{-1} K^{-1} \).

### Table 1: Variation of lubricant properties with temperature

<table>
<thead>
<tr>
<th>Property</th>
<th>Equation</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Viscosity ( \mu )</td>
<td>( \mu = \mu_0 e^{-\beta (T-T_0)} )</td>
<td>( \mu_0 = 0.116 , N , m^{-2} )</td>
</tr>
<tr>
<td>Density ( \rho )</td>
<td>( \rho = -0.6 , T + \rho_0 )</td>
<td>( \rho_0 = 1044 , kg \cdot m^{-3} )</td>
</tr>
<tr>
<td>Specific heat capacity ( C_p )</td>
<td>( C_p = 3.67 , T + C_{p0} )</td>
<td>( C_{p0} = 825.6 , J , kg^{-1} , K^{-1} )</td>
</tr>
</tbody>
</table>
| Conductivity \( \lambda \) | \( \lambda = a \, T^2 + b \, T + \lambda_0 \) | \( a = 2.62 \times 10^{-7} \, W \, m^{-1} \, K^{-3} \)  
|                 |                                               | \( b = -2.4009 \times 10^{-4} \, W \, m^{-1} \, K^{-2} \)  
|                 |                                               | \( \lambda_0 = 0.179481 \, W \, m^{-1} \, K^{-1} \) |

#### Thermal effects

Comparison between a textured and a smooth bearing (without the 3 dimples), is first investigated for different convergence ratios \( k \), see Fig.3. Isothermal flow presents a higher viscosity and thus a higher load. When \( k \) increases, the non dimensional load carrying capacity increases until a certain value of \( k \) is reached and then decreases. For the isothermal smooth case, the maximum value occurs around \( k = 1.2 \) whereas it occurs at larger \( k \) for the thermal smooth case. Inlet backflow observed at large \( k \) values was shown to be responsible for a pressure gradient reduction and thus a loss in load carrying capacity [17]. The appearance of backflow is delayed when thermal effects are considered which explains the shift of the maximum load carrying capacity to larger \( k \). At low convergence ratios, a significant combined effect results in a strong influence on the friction coefficient.

![Figure 3: Non dimensional load as a function of convergence ratio](image)

The fluid is therefore modelled with temperature dependent viscosity, density and specific heat capacity, whereas the lubricant thermal conductivity is set to the constant value at 313K, i.e. \( \lambda = 0.13 \, W m^{-1} K^{-1} \).
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Table 2: Factorial design

<table>
<thead>
<tr>
<th>run</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
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<tbody>
<tr>
<td>$\mu$</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
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<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>$\rho$</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
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<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>$C_p$</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
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<td>+</td>
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<td>-</td>
<td>+</td>
<td></td>
</tr>
<tr>
<td>$\lambda$</td>
<td>-</td>
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<td>-</td>
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<td>-</td>
<td>-</td>
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</tr>
</tbody>
</table>

Table 3: Main and combined effects in percent from the low level of A = $\mu$, B = $\rho$, C = $C_p$ and D = $\lambda$ on the different quantities. The combined effects BC, CD, ACD and BCD are not represented since they are insignificant.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>AB</th>
<th>AC</th>
<th>AD</th>
<th>BD</th>
<th>ABC</th>
<th>ABD</th>
<th>ABCD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W$</td>
<td>-83.7</td>
<td>1.8</td>
<td>0.0</td>
<td>0.0</td>
<td>-0.8</td>
<td>0.1</td>
<td>0.0</td>
<td>-0.3</td>
<td>5.6</td>
<td>-0.5</td>
</tr>
<tr>
<td>$F_r$</td>
<td>-38.2</td>
<td>0.0</td>
<td>0.1</td>
<td>0.0</td>
<td>0.0</td>
<td>0.1</td>
<td>0.0</td>
<td>0.0</td>
<td>2.7</td>
<td>0.0</td>
</tr>
<tr>
<td>$f$</td>
<td>252.7</td>
<td>-11.5</td>
<td>-0.7</td>
<td>0.6</td>
<td>-0.7</td>
<td>0.7</td>
<td>2.1</td>
<td>-25.5</td>
<td>-5.6</td>
<td>-5.2</td>
</tr>
<tr>
<td>$T_{max}$</td>
<td>-0.8</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.1</td>
<td>0.0</td>
</tr>
</tbody>
</table>

values. The transition point between these two behaviours occurs around $k = 0.8$ for the isothermal case and $k = 1$ for the thermal case. Dimples have positive effects on a larger range of $k$ for the thermal case as the recirculation zone is delayed and is slightly less important.

Analysis of textured contact performance is now carried out for various shear rates $\gamma$ and for $k = 0$ and 0.1. Results for load and friction are compared with the isothermal case for two different $k$-values. When thermal conditions are considered, the lubricant is subjected to a temperature increase through the contact due to shear losses. Temperature also increases with shear rate as shown in Fig.4. The maximum temperature is located at the outlet of the contact, as expected. With shear rate increasing, the temperature rise seems to be almost linear, Fig.5. As a consequence of this temperature increase, the viscosity decreases in the sliding direction. The density also decreases whereas the specific heat capacity is increasing. A decrease in viscosity leads to a reduction in the shear stress and consequently in frictional losses.

In Fig.6, the non-dimensional load carrying capacity is compared for the isothermal and thermal cases. In both cases, presence of texture in the inlet part of the contact has a positive effect on load carrying capacity compared to the smooth case. There are however some differences. For the isothermal case, non-dimensional load stays constant whereas a decrease in load is observed for the thermal case. The magnitude of the non-dimensional load is significantly reduced for the thermal case. The ability of the fluid to carry a load is decreased. It can be observed that non-dimensional load becomes zero at a certain shear rate for a convergence ratio $k$ equal to zero. This fact is attributed to the significant difference in dynamic viscosity at the inlet and outlet of the contact. Since dynamic viscosity is much lower in the outlet part of the contact, Fig.4, losses are reduced much more at the outlet than at the inlet of the contact. To build-up pressure, the energy of the moving wall needs to be converted into mechanical energy at the inlet of the contact [17]. Thus, losses need to be reduced at the inlet of the contact while they should be kept at a higher level at the outlet in order to decrease the pressure. In this case, losses decrease at the outlet while pressure gradient increases at the outlet and decreases at the inlet of the contact. Thus, load carrying capacity is decreased when thermal effects are considered. When load carrying capacity becomes zero, this mechanism dominates the effect of pressure generation created by the texture or gap convergence. Lack of load carrying capacity results in a decrease in minimum film thickness. As a consequence of higher
shear rate, temperature will increase and dynamic viscosity will decrease, leading to even thinner films and finally to a collapse of the lubricant film. Disappearance of load carrying capacity can also be expected for \( k \) larger than zero for higher shear rates since the non-dimensional load curve is monotonously decreasing.

Thus, there exists a critical shear rate \( \gamma_c \), \( k \) dependent, at which the fluid film may collapse. The system operates at a safer level when the \( k \)-value is increased. It should be mentioned that, with \( U \) increasing, the dimensional load is constantly increasing for the isothermal case whereas it presents a maximum for the thermal case. This maximum is located near the value of \( U = 5 \) m/s for \( k = 0 \) and \( U = 10 \) m/s for \( k = 0.1 \).

As the viscosity-temperature dependency has the strongest influence on the flow, the right choice of viscosity-temperature coefficient is important when various lubricants are considered in order to increase the critical shear \( \gamma_c \) rate and to extend the safe operating zone. As can be seen in Fig.7, non-dimensional load is reduced for low shear rate but the curve becomes flatter. With the reduced value of the viscosity-temperature coefficient, the viscosity decreases at a slower rate. At high shear rate, the non-dimensional load becomes higher compared to the case with higher \( \beta \). Since the curve is almost linear, higher critical shear rate can be deduced.

Another way to increase margin of safe operation in terms of critical shear rate is to choose a thinner lubricant. Results obtained for a reduced value of \( \mu_0 \) are shown in Fig.7. At low shear rate, the non-dimensional load carrying capacity of the thinner lubricant is similar to the one with higher \( \mu_0 \). At the same time, at higher shear rates, non-dimensional load carrying capacity of the thinner lubricant is much higher. A decrease of \( \beta \) or \( \mu_0 \) makes the viscosity decrease at a slower rate with temperature. Thus, the thermal effects are reduced. Other parameters can also compensate for thermal effects, such as specific heat capacity which should be increased.

Since dynamic viscosity is strongly reduced in the fluid film due to heat generation, the flow is easier to shear. The shear is directly linked to the velocity profile and the derivative of the horizontal velocity with respect to the vertical coordinate. They are both altered in such a way that the integral of the term \( \frac{\partial u}{\partial y} \) at the upper wall is decreased. Friction is more greatly reduced when heat transfer is considered for both \( k \)-values, as seen in Fig.8. It should be mentioned that, with \( \gamma \) increasing, the dimensional friction is constantly increasing for both the isothermal and thermal cases.

The flow recirculation zone inside the dimples is decreased when heat transfer conditions are considered. It is also decreased when the upper wall velocity is increased. When heat transfer is considered, the more the upper wall velocity is increased, the more the heat increases through the contact and the more the dynamic viscosity is decreased. Thus, the lubricant has the ability to be accommodated into the sudden change of surface shape inside the dimples. The height of the recirculation zone in the dimples is thus reduced. In this case, the efficiency of the dimples is improved according to results from a previous study [17].
CONCLUSIONS

A thermal study of an inlet textured contact has been performed for a flow of lubricant with temperature dependent viscosity, density and specific heat capacity submitted to temperature variations. The results obtained were compared with the isothermal case. It has been found that:

• At low convergence ratio $k$, load carrying capacity of an inclined surface contact can be improved by using texturing at the inlet for both isothermal and thermal cases. When backflow occurs at the inlet of the contact, i.e. for large convergence ratios, texture reduces the performance of the bearing.

• There exists a critical shear rate $\gamma_c$ at which load carrying capacity of the textured contact becomes zero. The critical shear rate is dependent on convergence ratio $k$ and lubricant properties such as $\beta$ and $\mu_0$. Some ways to increase it can be realized by increasing $k$ or reducing $\beta$ and $\mu_0$ in order to extend the safe operating zone.

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Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Viscosity-temperature coefficient</td>
<td>$[K^{-1}]$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Shear rate</td>
<td>$[s^{-1}]$</td>
</tr>
<tr>
<td>$\gamma_c$</td>
<td>Critical shear rate</td>
<td>$[s^{-1}]$</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Lubricant thermal conductivity</td>
<td>$[W m^{-1} K^{-1}]$</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Lubricant dynamic viscosity</td>
<td>$[Pa s]$</td>
</tr>
<tr>
<td>$\mu_0$</td>
<td>Lubricant dynamic viscosity at $T = T_0$</td>
<td>$[Pa s]$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Lubricant density</td>
<td>$[kg m^{-3}]$</td>
</tr>
<tr>
<td>$C_p$</td>
<td>Lubricant specific heat capacity</td>
<td>$[J kg^{-1} K^{-1}]$</td>
</tr>
<tr>
<td>$d$</td>
<td>Dimple depth</td>
<td>$[m]$</td>
</tr>
<tr>
<td>$d/h_0$</td>
<td>Depth to minimum film ratio</td>
<td></td>
</tr>
<tr>
<td>$f$</td>
<td>Friction coefficient</td>
<td></td>
</tr>
<tr>
<td>$F_r$</td>
<td>Friction per unit width</td>
<td>$[N/m]$</td>
</tr>
<tr>
<td>$h_0$</td>
<td>Outlet height of the domain</td>
<td>$[m]$</td>
</tr>
<tr>
<td>$h_1$</td>
<td>Inlet height of the domain</td>
<td>$[m]$</td>
</tr>
<tr>
<td>$h_{stat}$</td>
<td>Lubricant static enthalpy</td>
<td>$[J kg^{-1}]$</td>
</tr>
<tr>
<td>$h_{tot}$</td>
<td>Lubricant total enthalpy</td>
<td>$[J kg^{-1}]$</td>
</tr>
<tr>
<td>$k$</td>
<td>Convergence ratio</td>
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<td>Length of the domain</td>
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<td>Lubricant pressure</td>
<td>$[Pa]$</td>
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<td>Lubricant temperature</td>
<td>$[K]$</td>
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<tr>
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<td>$T_{max}$</td>
<td>Maximum temperature</td>
<td>$[K]$</td>
</tr>
<tr>
<td>$U$</td>
<td>Upper wall velocity</td>
<td>$[m/s]$</td>
</tr>
<tr>
<td>$u_i, u_j$</td>
<td>Velocity components in i and j direction</td>
<td>$[m/s]$</td>
</tr>
<tr>
<td>$W$</td>
<td>Load per unit width</td>
<td>$[N/m]$</td>
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<tr>
<td>$x_i, x_j$</td>
<td>Cartesian coordinates in i and j direction</td>
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References


